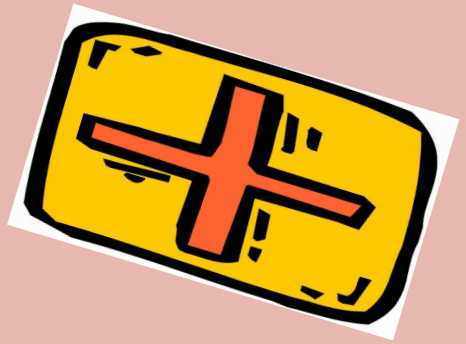
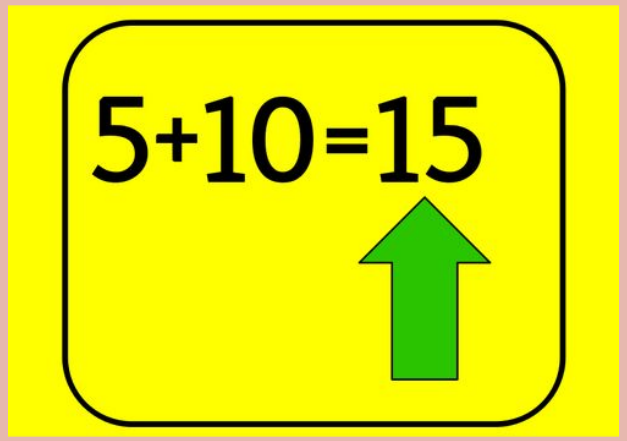
Foundations of Addition & Subtraction





EDEC 303

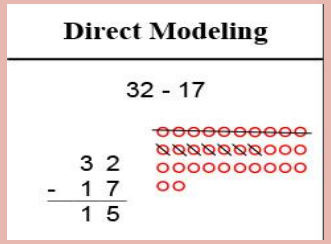
$$\begin{array}{r} {\scriptstyle 10} \\ 2\not{3}6 \\ -\ 62 \\ \hline 174 \end{array}$$

- *Describe the general sequence of activities used to help children develop meaning for the operations.*
- *Identify and give examples of the three distinct types of situations that lead to addition/subtraction*
- *Describe the three-phase process for helping children learn basic facts*
- *Describe the key thinking strategies for learning basic facts for addition and subtraction.*

Vocabulary

Types of Problems : Change, Part-Part-Whole & Compare Problems

Direct Modeling



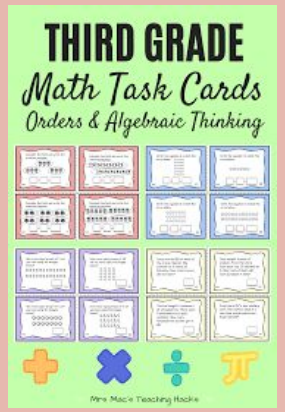
Number Model, Equation, Number Sentence

Story or Word Problems and Graphic Organizers for Representing Problems

Basic Facts and Extended Facts; Derived Facts

Properties: Commutative (+ & *) , Associative (+ & *), Identity (+ & *)

Distributive Property of Multiplication over Addition



DAP Considerations (Materials, Language, Instruction)

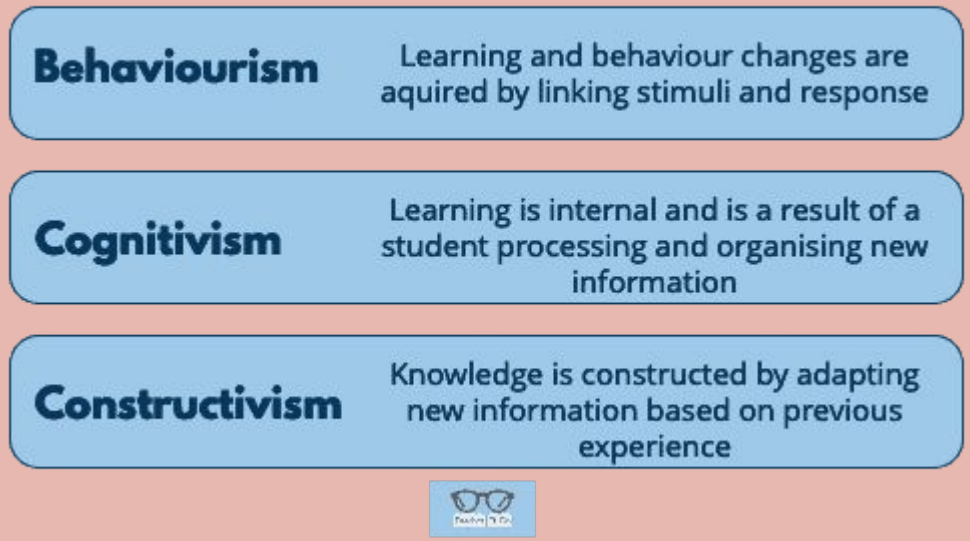
Bruner: Concrete, Representational, Abstract

Piaget: Prior Knowledge & Experience; Stages (Sensori-Motor; Preoperational; Concrete Operational; Formal Operational); Constructivism

Vygosky: Zone of Proximal Development

More Knowledgeable Other (MKO)

Scaffolding; Social Constructivism



Learning Trajectory (Pre-requisites & the Path)

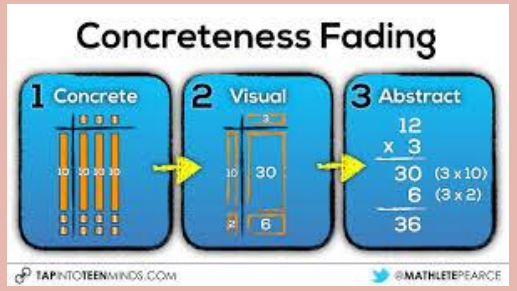
Counting

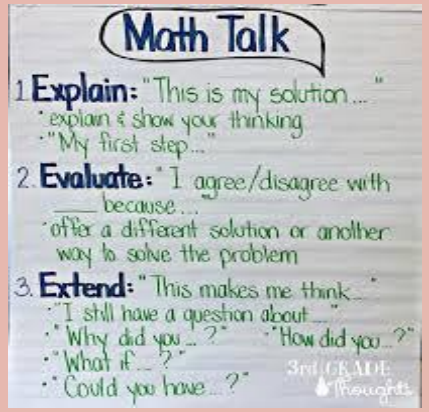
Concrete Situations

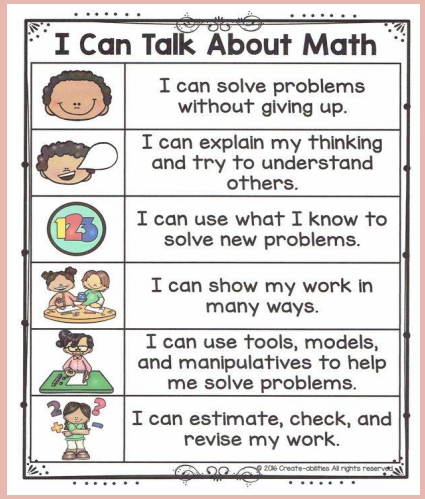
Familiarity with Many Problem Contexts

Talking & Writing about Mathematical Ideas

(Pictures, Words & Symbols)







Essential Understanding #1

Addition and subtraction are used to represent and solve many different kinds of problems.

– Sequential counting

– Inverse relationship of addition and subtraction

– Part, part, whole relationship

– Number sentence representation

– Context for problems and different representations

Essential Understanding #2

The mathematical foundation for understanding computational procedures for addition and subtraction of whole numbers are the properties of addition and place value.

– Commutative /Associative Properties

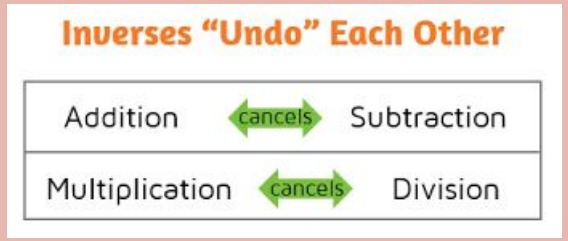
– Subtraction is NOT Commutative

– Place value allows for decomposition and composition

– Properties of addition are essential to justifying the correctness of computational algorithms.

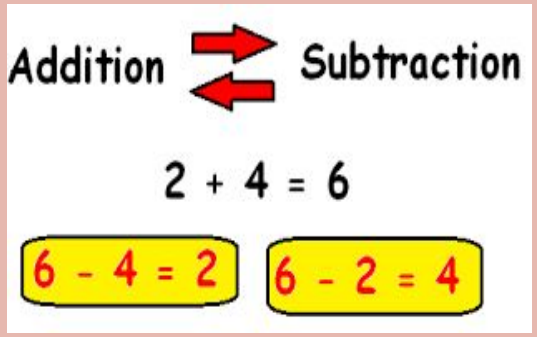
Big Idea #1: Inverse Operations

- Addition & Subtraction are connected.
  (like inhaling & exhaling)



- Addition names the whole in terms of the parts.
  ____ + ____ = Whole

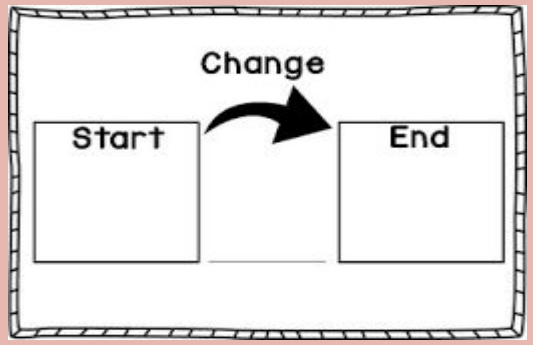
- Subtraction names a missing part.
  ____ - ____ = Missing Part



Big Idea #2: Models:

Graphic Organizers and Manipulatives

- Models can be used to represent and solve contextual (story/word) problems.



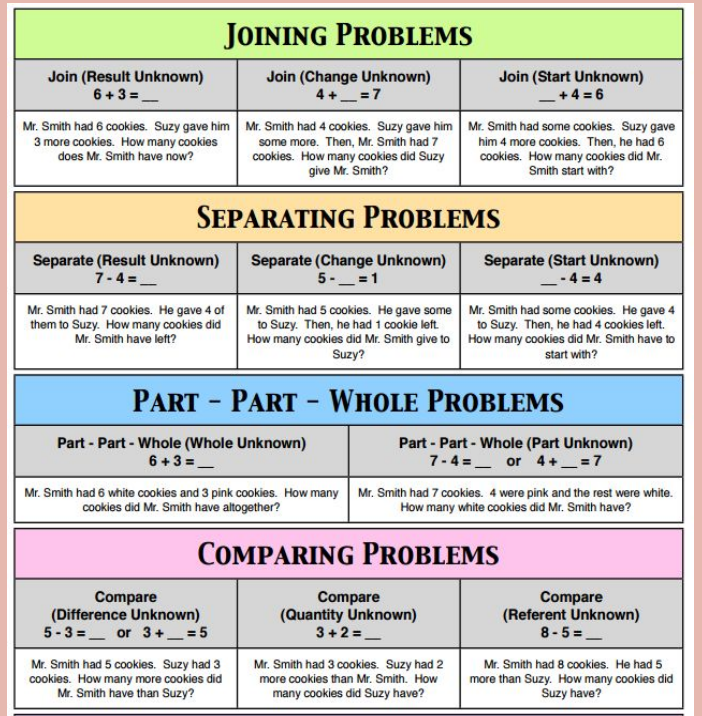
- Models can be used to understand the problem.
- Models can be used regardless of the size of the numbers.

- Some models : Change diagram, bar model, Number bonds

Big Idea #3: + and - Problem Structures

Addition & Subtraction problems fall into three Problem Structures:

- Change Problems (Join or Separate)
- Part-Part-Whole Problems
- Compare Problems



JOINING PROBLEMS

Join (Result Unknown) 6 + 3 = __	Join (Change Unknown) 4 + __ = 7	Join (Start Unknown) __ + 4 = 6
Mr. Smith had 6 cookies. Suzy gave him 3 more cookies. How many cookies does Mr. Smith have now?	Mr. Smith had 4 cookies. Suzy gave him some more. Then, Mr. Smith had 7 cookies. How many cookies did Suzy give Mr. Smith?	Mr. Smith had some cookies. Suzy gave him 4 more cookies. Then, he had 6 cookies. How many cookies did Mr. Smith start with?

SEPARATING PROBLEMS

Separate (Result Unknown) 7 - 4 = __	Separate (Change Unknown) 5 - __ = 1	Separate (Start Unknown) __ - 4 = 4
Mr. Smith had 7 cookies. He gave 4 of them to Suzy. How many cookies did Mr. Smith have left?	Mr. Smith had 5 cookies. He gave some to Suzy. Then, he had 1 cookie left. How many cookies did Mr. Smith give to Suzy?	Mr. Smith had some cookies. He gave 4 to Suzy. Then, he had 4 cookies left. How many cookies did Mr. Smith have to start with?

PART – PART – WHOLE PROBLEMS

Part - Part - Whole (Whole Unknown) 6 + 3 = __	Part - Part - Whole (Part Unknown) 7 - 4 = __ or 4 + __ = 7
Mr. Smith had 6 white cookies and 3 pink cookies. How many cookies did Mr. Smith have altogether?	Mr. Smith had 7 cookies. 4 were pink and the rest were white. How many white cookies did Mr. Smith have?

COMPARING PROBLEMS

Compare (Difference Unknown) 5 - 3 = __ or 3 + __ = 5	Compare (Quantity Unknown) 3 + 2 = __	Compare (Referent Unknown) 8 - 5 = __
Mr. Smith had 5 cookies. Suzy had 3 cookies. How many more cookies did Mr. Smith have than Suzy?	Mr. Smith had 3 cookies. Suzy had 2 more cookies than Mr. Smith. How many cookies did Suzy have?	Mr. Smith had 8 cookies. He had 5 more than Suzy. How many cookies did Suzy have?

Problems

Take a look at the two problems below. Discuss with those at your table the similarities and differences between them.

1. Carl has 5 blocks. Rita gives him 8 more blocks. How many blocks does Carl have now?

2. Alicia counted 5 squirrels in the park on Tuesday. On Wednesday, she counted 8 squirrels. How many squirrels did she count on Tuesday and Wednesday? What did you discuss?

Change Problems

(either Join or Separate)

can be directly modeled/acted out

- Start
- Change
- End (Result)

Change Problems

There were 28 students at recess. A group of 7 students came outside to join them. How many students are there now?

What type of change is this? (Join or Separate)

What information do you have?

- Start
- Change
- End (Result)

What information are you missing?

- Start
- Change
- End (Result)

Change Problems

Shane has 12 pencils. He gives some pencils to his friends. Now, he has 7 left. How many pencils did he give away?

What type of change is this? (Join or Separate)

What information do you have?

- Start
- Change
- End (Result)

What information are you missing?

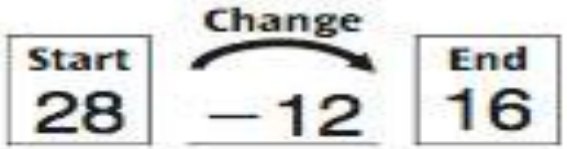
- Start
- Change
- End (Result)

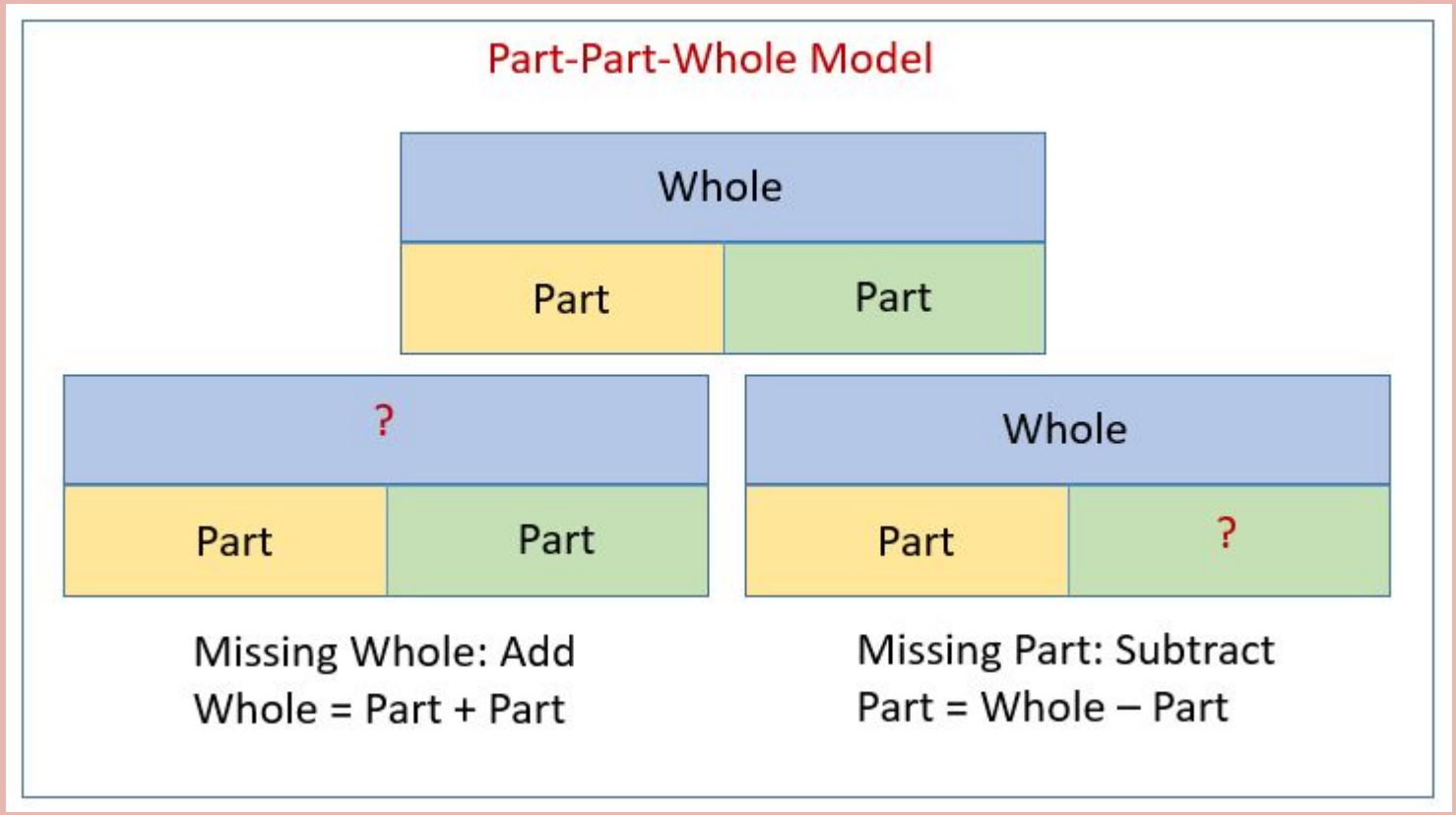
Change Diagram

- The following diagram could be used for Join or Separate problems.

change diagram A diagram used to represent addition or subtraction problems in which a given quantity is increased or decreased. The diagram includes the starting quantity, the ending quantity, and the amount of the change.

For example, the change diagram here represents this subtraction problem: *Rita had $28 in her wallet. She spent $12 at the store. How much money is in Rita's wallet now?*

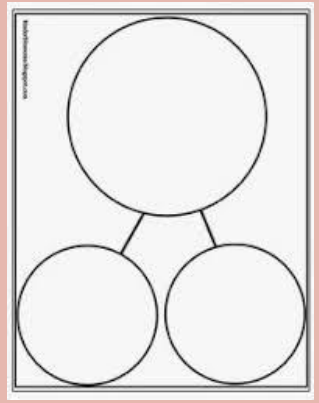




Connie has 15 red marbles and 28 blue marbles. How many marbles does she have?

Part-Part-Whole Problems

There are 9 bakeries in our neighborhood. 2 bakeries are brand new stores. The rest of the bakeries are older. How many older bakeries are there?

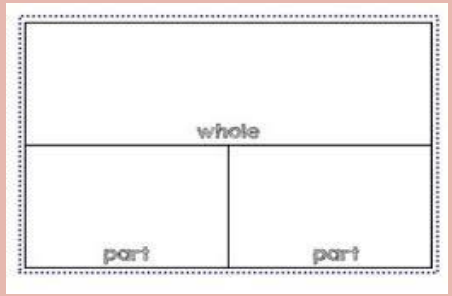


What information does the problem give you?

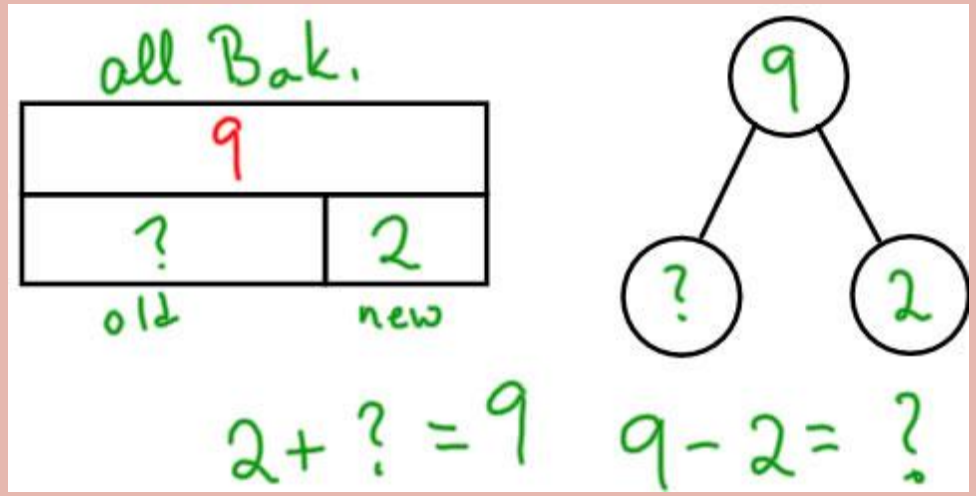
- Part
- Part
- Whole

What information are you looking for ?

- Part
- Part
- Whole



Part-Part-Whole Problems



Bar Model

Number Bonds

Compare Problems

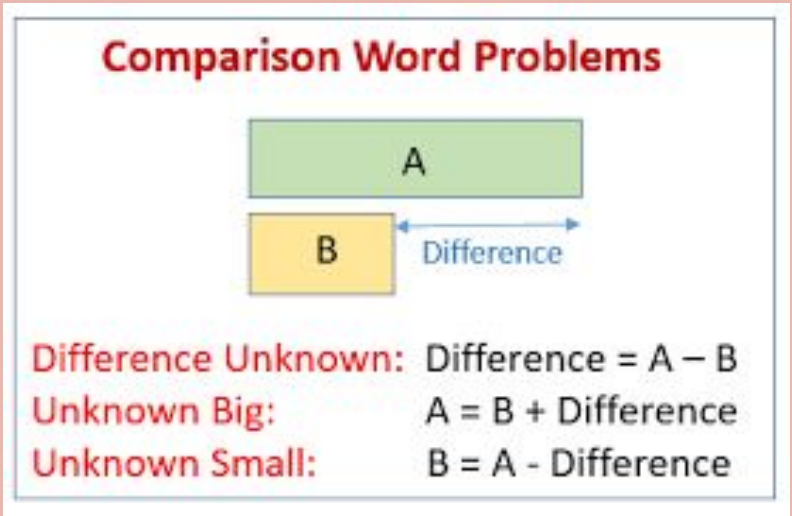
Jessa collected 78 shells on the beach.
Susan collected 6 more shells than Jessa.
How many shells did Susan collect?

What information does the problem give you?

- Larger Set
- Smaller Set
- Difference

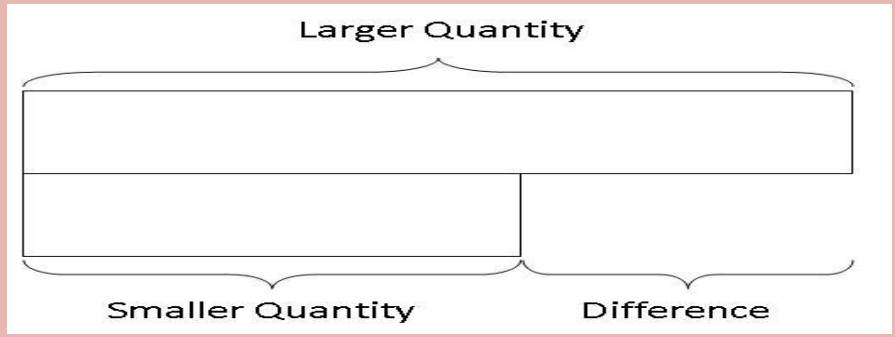
What information are you looking for ?

- Larger Set
- Smaller Set
- Difference

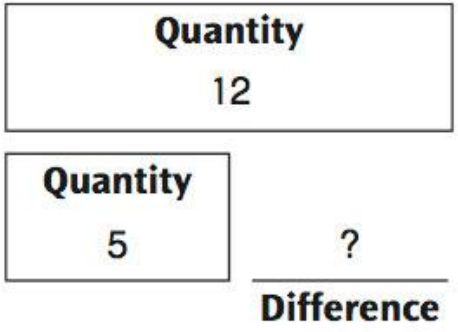


Compare Problems

- Larger Set
- Smaller Set
- Difference

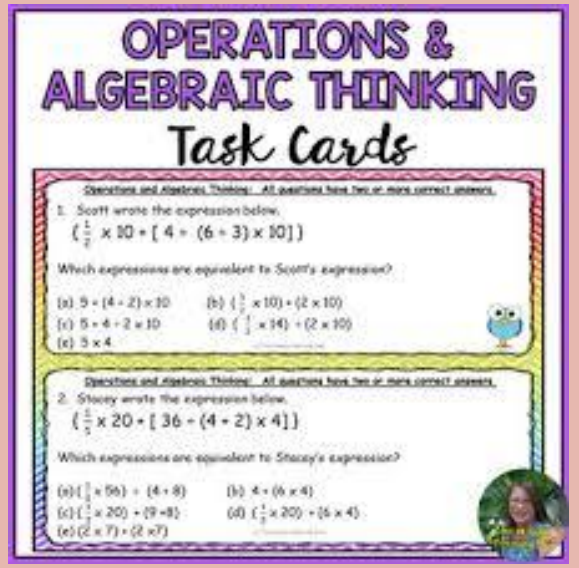


comparison diagram A diagram used to organize the information from a comparison number story. For example, the diagram below organizes the information from Anthony's cookie story above.



Operation Understanding & Mathematics Connections

- Number Development & Relationships
- Counting
- Basic Facts
- Whole-Number Place Value & Computation
- Algebraic Thinking (representing story problems as equations/number models)



Instructional Sequence

- Concrete –Pictures- Abstract
- Story Problems and Questions

- Representing Story Problems using manipulatives and later pictures

- Representing Story Problems using symbols
  - ~ Teaching the signs (+) and (-) and (=)
  - ~ (=) means “is the same as” ; a balancing point

- Linking the manipulatives, the symbols, the problem and the number model/equation representation (Reys, p. 94)

5 Representations of Mathematics

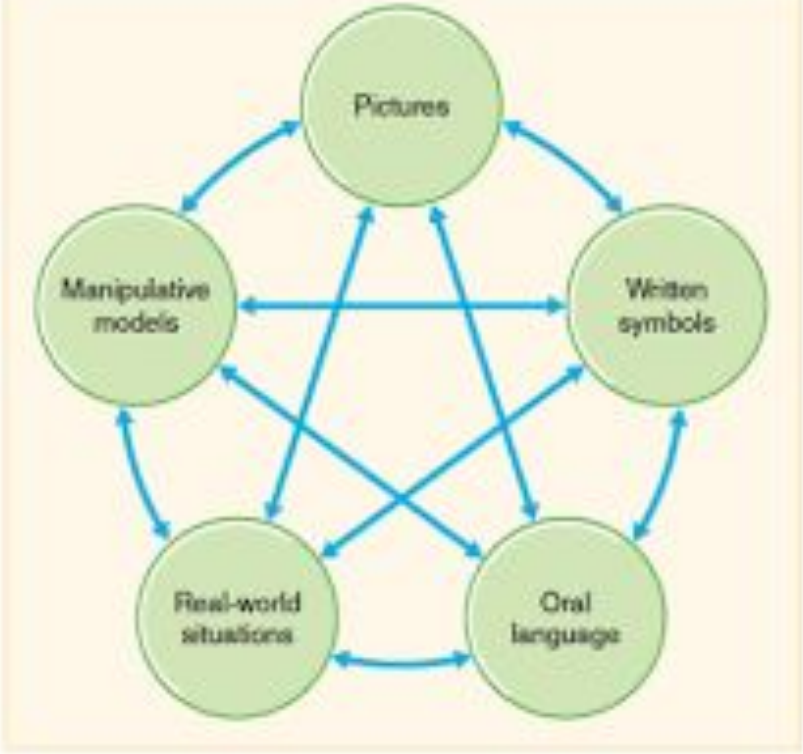


FIGURE 2.11 Five different representations of mathematical ideas. Translations between and within each can help develop new concepts.

Student Strategies

- Direct Modeling using objects or pictures

- Counting Strategies
  - Counting on
  - Counting back
  - Supports: number line; 100 chart; fingers

- Derived Facts (Strategies)
  - I know 9+4=13 because 9+1=10 and 3 more is 13.
- Mental Math - consolidated facts (automaticity)

Basic Fact Instruction

Start Where Children Are: Counting
Counting On, Counting Back, Comparing Numbers, Numbers on a number line

Build Understanding of the Basic Facts & Thinking Strategies (Fact Families)

- Commutative Property (3+6 and 6+3)
- Add/Subtract 1 or 0 (8+1 or 7-0)
- Doubles (5+5)
- Near Doubles (5+6 or 4+5)
- Combinations to 10 (1+9, 2+8, 3+7, 4+6, 5+5)
- Adding to 10 and Beyond: 7+4 (Think 7+3 =10 and 1 more is 11.)

Focus on How to Remember the Facts

- 5-10 minute daily practice (Similar to skill building in sports or music)
- Targeted Practice & Distributed Practice
- Make the practice interesting & a way to build confidence & skill

Building +/- Understanding and Facts (Algebraic Concepts 2.2) in PA

CC.2.2.PREK.A.1: Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

CC.2.2.K.A.1: Extend concepts of putting together and taking apart to add and subtract within 10.

CC.2.2.1.A.1: Represent and solve problems involving addition and subtraction within 20.

CC.2.2.1.A.2: Understand and apply properties of operations and the relationship between addition and subtraction.

CC.2.2.2.A.2: Use mental strategies to add and subtract within 20.