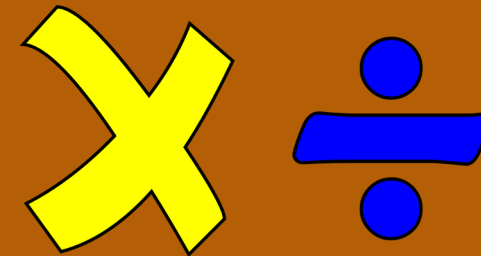
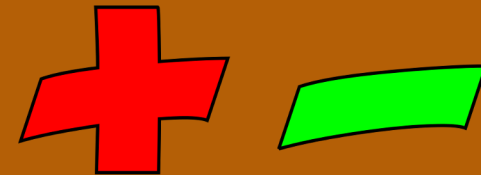


Multiplication Chart

×	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

Understanding the Operations

Multiplication and Division

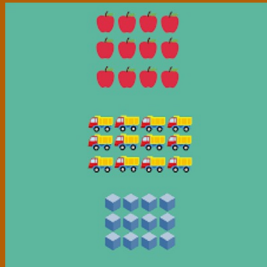




Learning Outcomes

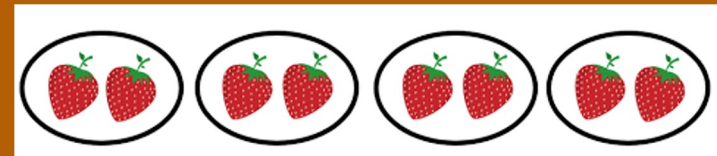
- *Describe the general sequence of activities used to help children develop meaning for the operations.*
- *Identify and give examples of the four distinct types of situations that lead to multiplication/division*
- *Describe the three-phase process for helping children learn basic facts*
- *Describe the key thinking strategies for learning basic facts for multiplication and division.*





Vocabulary

Equal Groups



Arrays (Rows & Columns)

Factors, Product

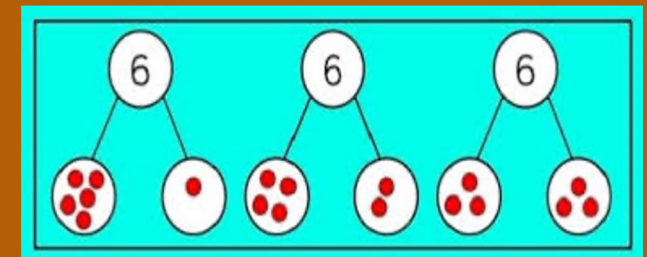
Combination Problems



Divisor, Quotient, Dividend

Measurement Division

Partition (Partitive) Division



Related PDESAS Standards

2nd Grade CC.2.2.2.A.3: Work with equal groups of objects to gain foundations for multiplication.

3rd Grade: CC.2.1.3.B.1: Apply place value understanding and properties of operations to perform multi-digit arithmetic.

CC.2.2.3.A.1: Represent and solve problems involving multiplication and division.

CC.2.2.3.A.2: Understand properties of multiplication and the relationship between multiplication and division.

CC.2.2.3.A.3: Demonstrate multiplication and division fluency.

CC.2.2.3.A.4: Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Related PDESAS Standards

4th Grade:

CC.2.1.4.B.1: Apply place value concepts to show an understanding of multi-digit whole numbers.

CC.2.1.4.B.2: Use place value understanding and properties of operations to perform multi-digit arithmetic.

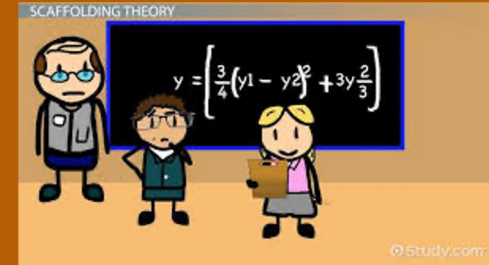
CC.2.2.4.A.1: Represent and solve problems involving the four operations.

CC.2.2.4.A.2: Develop and/or apply number theory concepts to find factors and multiples.

CC.2.2.4.A.4: Generate and analyze patterns using one rule

DAP Considerations (Materials, Language, Instruction)

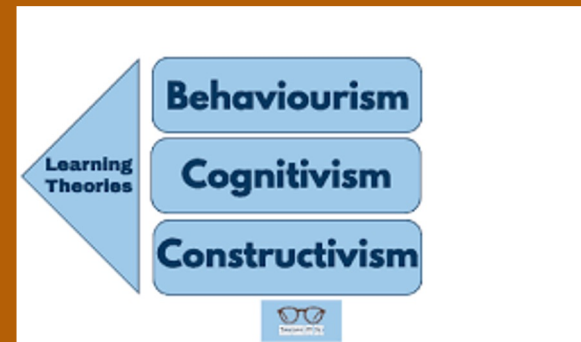
Bruner: Concrete, Representational, Abstract

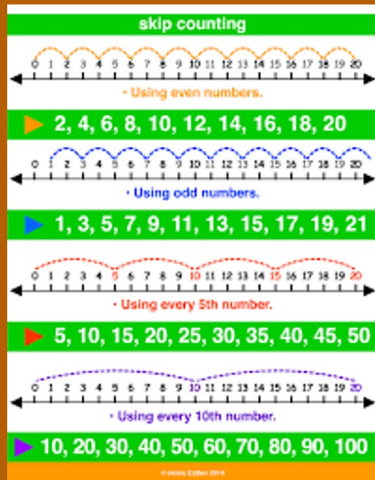


Piaget: Prior Knowledge & Experience; Stages (Sensory-Motor; Preoperational; Concrete Operational; Formal Operational); Constructivism

Vygotsky: Zone of Proximal Development; More Knowledgeable Other (MKO)

Scaffolding; Social Constructivism



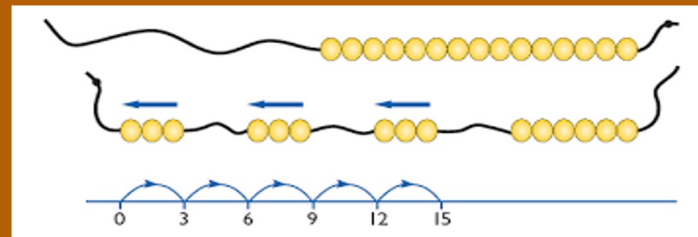


Big Idea #1: Inverse Operations

Multiplication & Division are connected.
(like tying & untying a shoe)

Multiplication has many applications:

- Quick counting
- Scaling up or down
- Area



Division reminds us of sharing or breaking into smaller parts.

Multiplication is the opposite of division.

Example: $2 \times 3 = 6$ $6 \div 3 = 2$



Big Idea #2: The Four Arithmetic Operations are Related

Multiplication has some similarities to
Addition.

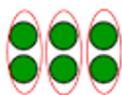
Division has some similarities to
Subtraction.

Multiplication & Division build on solid
Number Sense.

Multiplication & Division are Inverse
Operations.

Multiplication is Repeated Addition

Arrays can be used to show that
multiplication is repeated addition.



Addition:

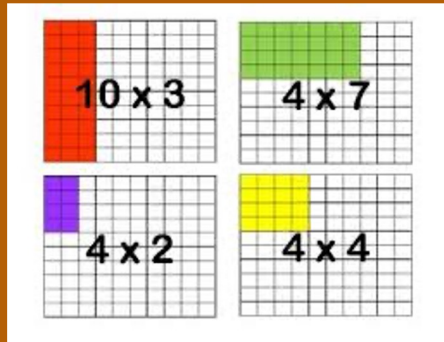
$$2 + 2 + 2 = 6$$

Multiplication:

3 groups of 2 is 6

3 times 2 equals 6

$$3 \times 2 = 6$$

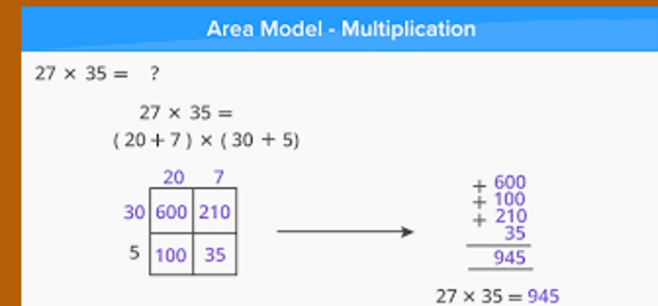


Big Idea #3: Models:

Graphic Organizers and Manipulatives

Models can be used to represent and solve contextual (story/word) problems.

- Models can be used to understand the problem.
- Models can be used regardless of the size of the numbers.
- Some models : bar models, arrays and area models



Big Idea #4:

Multiplication (*) and Division (/) Problem Types

Equal Groups
Multiplicative Comparison

Combination
Area or Arrays

Word Problem
Together, Abby and her younger sister, Zoey, have 30 marbles in their collection. Abby collected 5 times as many marbles as Zoey. How many more marbles does Abby have than her sister?

Abby's marbles: 5 segments
Zoey's marbles: 1 segment

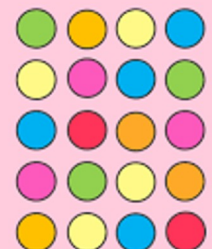
Total: 30

We can also express a remainder as a fraction. We do this when we can easily share the remainder. For example, 19 cakes shared among 3 people is 6 and one third each. Solve these problems expressing the remainder as a fraction:

- a Share 13 pizzas among 4 people.
b Share 50 sandwiches among 3 people.



What are
MULTIPLICATION
ARRAYS?



A Familiar kind of Story Problem

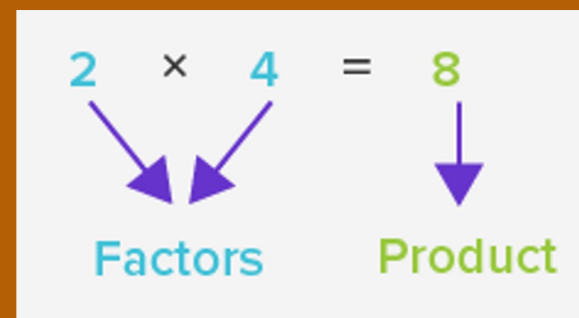
Andrew had three boxes of trading cards. Each box holds 8 cards. How many cards does Andrew have altogether?

3 boxes * 8 cards/box = 24 cards in all

The first factor is 3.

The second factor is 8.

The product is 24.



Equal Groups Problems

Andrew has some boxes of trading cards. Each box holds 8 cards. Altogether, Andrew has 24 cards. How many boxes of cards does Andrew have? $\square * 8 = 24$

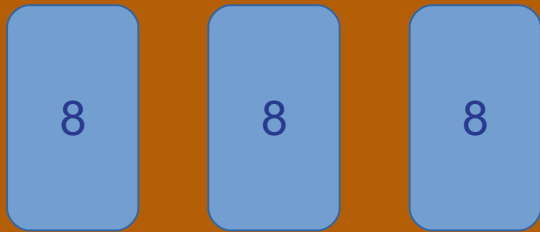
Andrew has three boxes of trading cards. Each box holds several cards. Altogether, Andrew has 24 cards. How many cards are in each box??

$$3 * \square = 24$$



The Multiplication & Addition Connection

- In equal groups problems we are counting groups of the same size.



- This is why multiplication is sometimes called “repeated addition.”
- $3 \times 8 = 8 + 8 + 8 = 24$
- This is not the only definition of multiplication.



Check for Understanding



- Write an equal groups problem. What key information are you giving? solving for? Formulate an equation related to your problem.
- Swap problems with a classmate. Identify the key information. Solve each other's problem by using manipulatives/pictures.
- Identify the Concrete, the Representation and the Abstract Models for your problem.



Multiplicative Comparison Problems

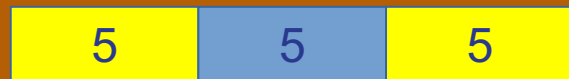
Hilary spent \$5 on gifts for her family. Jeff spent three times as much as Hillary did for his family's gifts. How much money did Jeff spend?

3 (multiplicative factor) * 5 (set to be copied) = 15 (product)

Solving Multiplicative Comparison Problems

Hilary spent \$5 on gifts for her family. Jeff spent three times as much as Hilary did for his family's gifts. How much money did Jeff spend?

Hilary's spending (\$5)



Jeff's spending (3 times what Hilary spent) shown in a bar model.

Area and Array Story Problems

Important:

Arrays do not show up in story problems often, however, arrays are used frequently as a hands-on or visual representation of multiplication. 2×4 can be shown as an array:

* * * *

* * * *

Choose your favorite multiplication fact. Use manipulatives to make an array.

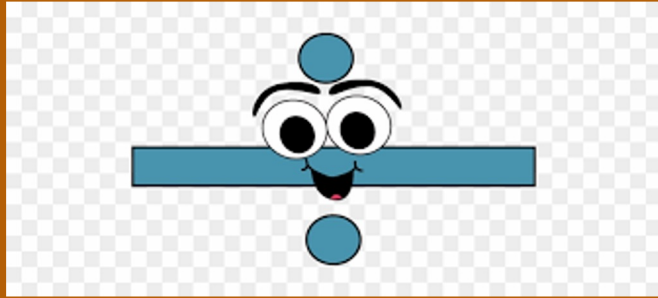
An Array Story Problem

A farmer wants to plant twenty-four ears of corn in three rows. How many ears of corn should be planted in each row?

Number in each row * Number in each column = Total

Check for Understanding

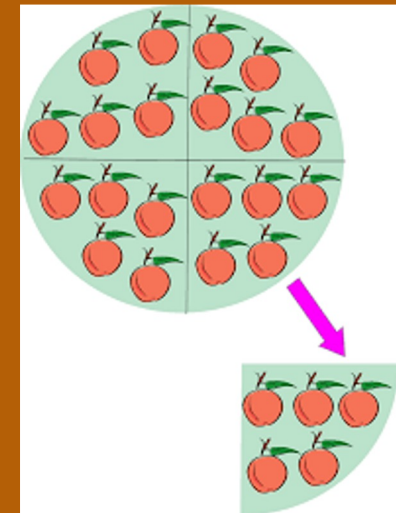
- Look at the examples of Multiplicative Comparison and Area/Array problems.
- What similarities do they have in their wording?
- Write a multiplicative comparison problem. Draw a bar model for your problem.
- What are the advantages & disadvantages of using manipulatives to solve multiplication/division problems?



...AND NOW SOME




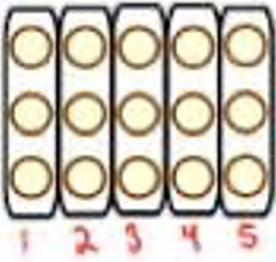
DIVISION



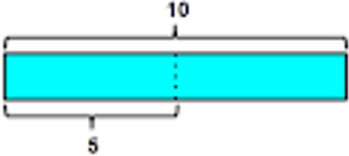
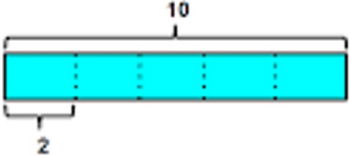
The Various Types of Problems

In order to help children develop a deep understanding of multiplication and division, they need to experience a variety of problem types that lead to these operations.

Teachers need to be aware of these problem types in order to make sure students see the variety.

Partitive	Measurement
15 divided into 3 groups yields 5 in each group	15 divided into groups of 3 yields 5 groups
	

Two Types of Division

Partitive	Quotative
How many in each group? $10 \div 2 = 5$ Divide 10 equally into 2 groups	How many groups? $10 \div 2 = 5$ How many 2s are in 10?
	
5 in each group	5 groups of 2



2 Division Situations



I have 32 Snickers . If I put 2 Snickers in each Halloween treat bag, how many treat bags can I fill?

To solve this, a student could pull out 2 Snickers and keep doing this until there were none left.

This is called “Measurement Division.”

(repeated subtraction)

We knew the size of the group and the total.





Division Situations



Consider this problem:

Ted has 32 Snickers . If he wants to share the Snickers equally among four people (Ana, Bill, Carly & Dan), how many Snickers will each person get?

To solve this, a student could deal out the Snickers (1 for Ana, 1 for Bill, 1 for Carly, 1 for Dan) and keep doing this until there were none left.

This is called “Partitive Division” or fair sharing.

We knew the total and the number of groups.

Instructional Sequence

- Concrete –Pictures- Abstract
- Story Problems and Questions
- Representing Story Problems using manipulative
- and later pictures
- Representing Story Problems using symbols
 - ~ Teaching the signs ($*$ & x) and ($/$) and ($=$)
 - ~ ($=$) means “is the same as” ; a balancing point
- Linking the manipulatives, the symbols, the problem and the number model/equation representation (Reys,



Three circular portraits of educational theorists are arranged vertically on the left. To the right of each portrait is their name and key concepts.

- Lev Vygotsky**
 - *Zone of Proximal Development (ZPD) – The range of skills that are a little bit more difficult than what a student can do independently.
 - *Scaffolding – Changing the level of support to meet the ability of the child.
- Jean Piaget**
 - *Schema – The things a child already knows. Students connect new knowledge to an existing schema.
 - *Constructivism – Students learn by doing, rather than by being told.
- B.F. Skinner**
 - *Behaviorism – Positive reinforcement (praise, rewards, etc.) strengthens behavior or increases the likelihood that the behavior will be repeated. Negative reinforcement (punishment) pushes a child away from a behavior.

5 Representations of Mathematics

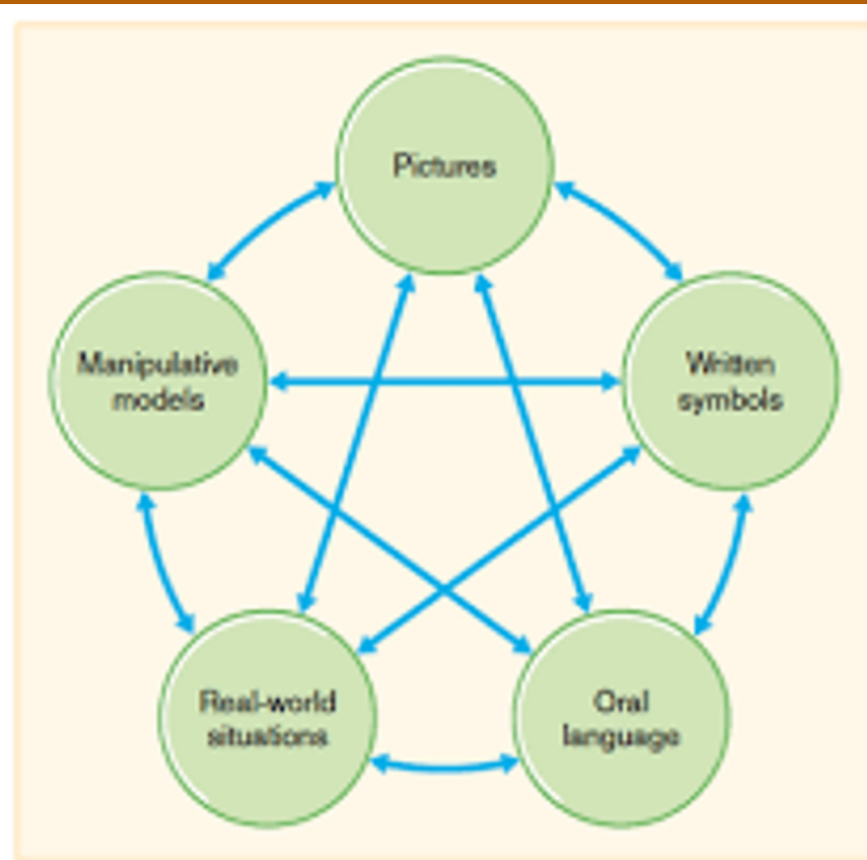


FIGURE 2.11 Five different representations of mathematical ideas. Translations between and within each can help develop new concepts.

Student Strategies

- Direct Modeling using objects or pictures

- Counting Strategies

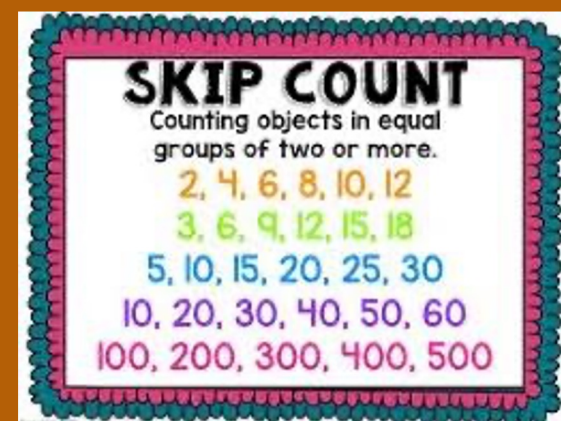
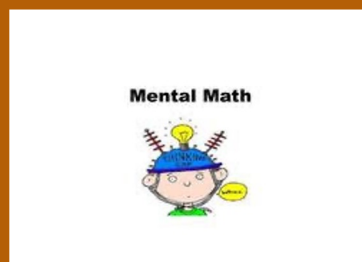
Skip Counting

Supports: number line; 100 chart

- Derived Facts (Strategies)

I know $7 \times 4 = 28$ because $7 \times 2 = 14$ and double that is 28. (The 4 x's Table is double the 2x's Table.)

- Mental Math - consolidated facts (automaticity)



Basic Fact Instruction

Start Where Children Are: Counting , Adding & Subtracting

Counting On, Counting Back, Skip Counting; Basic +/- Facts

Build Understanding of the Basic Facts & Thinking Strategies (Fact Families)

- Commutative Property (3×6 and 6×3) Mult./Divide 0 or 1 (8×0 or $7/1$)
- Perfect Squares (4×4)
- Add another Equal Group $5 \times 7 = 35$ so 6×7 would be $35 +$ another group of 7 or $35 + 7 = 42$
- The 9 x's Table

Focus on How to Remember the Facts

- 5-10 minute daily practice (Similar to skill building in sports or music)
- Targeted Practice & Distributed Practice
- Make the practice interesting & a way to build confidence & skill

Combination Problems (least frequent type)

How many different ice cream cones can you make from three choices of ice cream (vanilla, chocolate, strawberry) and two choices for a cone (cake cone or sugar cone)?

Vanilla – cake cone

Chocolate – cake cone

Strawberry – cake cone

Vanilla – sugar cone

Chocolate – sugar cone

Strawberry – sugar cone



$3 \text{ (flavors)} \times 2 \text{ (cone types)} = 6 \text{ possible cones}$

