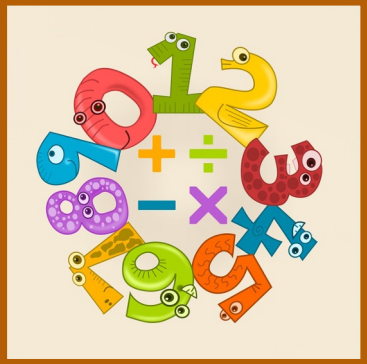
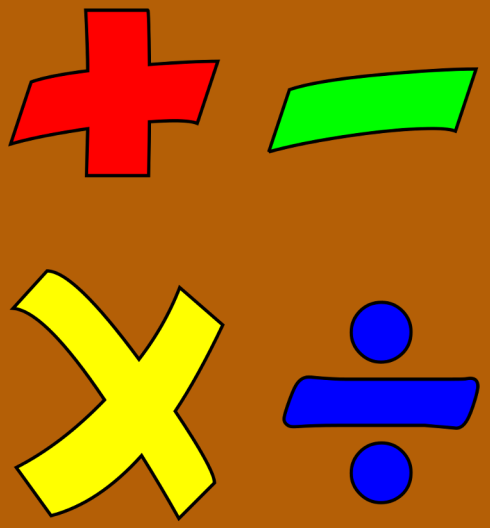


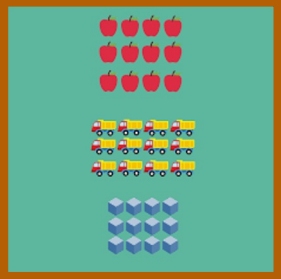
Understanding the Operations

Multiplication and Division



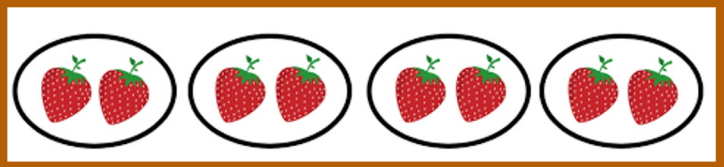
Learning Outcomes

- *Describe the general sequence of activities used to help children develop meaning for the operations.*
- *Identify and give examples of the four distinct types of situations that lead to multiplication/division*
- *Describe the three-phase process for helping children learn basic facts*
- *Describe the key thinking strategies for learning basic facts for multiplication and division.*



Vocabulary

Equal Groups



Arrays (Rows & Columns)

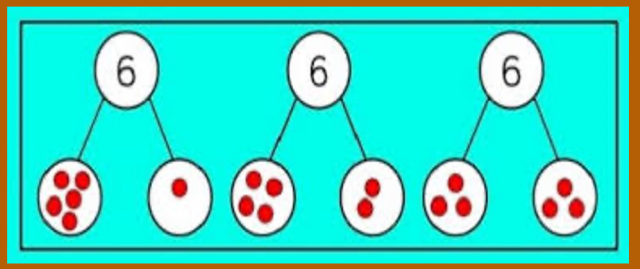
Factors, Product

Combination Problems

Divisor, Quotient, Dividend

Measurement Division

Partition (Partitive) Division



Related PDESAS Standards

2nd Grade CC.2.2.2.A.3: Work with equal groups of objects to gain foundations for multiplication.

3rd Grade: CC.2.1.3.B.1: Apply place value understanding and properties of operations to perform multi-digit arithmetic.

CC.2.2.3.A.1: Represent and solve problems involving multiplication and division.

CC.2.2.3.A.2: Understand properties of multiplication and the relationship between multiplication and division.

CC.2.2.3.A.3: Demonstrate multiplication and division fluency.

CC.2.2.3.A.4: Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Related PDESAS Standards

4th Grade:

CC.2.1.4.B.1: Apply place value concepts to show an understanding of multi-digit whole numbers.

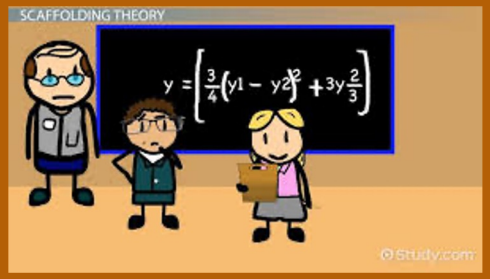
CC.2.1.4.B.2: Use place value understanding and properties of operations to perform multi-digit arithmetic.

CC.2.2.4.A.1: Represent and solve problems involving the four operations.

CC.2.2.4.A.2: Develop and/or apply number theory concepts to find factors and multiples.

CC.2.2.4.A.4: Generate and analyze patterns using one rule

DAP Considerations (Materials, Language, Instruction)

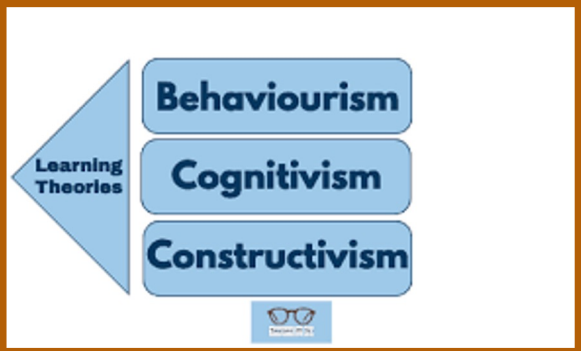


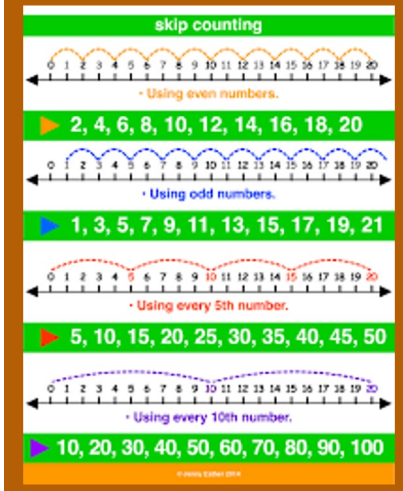
Bruner: Concrete, Representational, Abstract

Piaget: Prior Knowledge & Experience; Stages (Sensory-Motor; Preoperational; Concrete Operational; Formal Operational); Constructivism

Vygotsky: Zone of Proximal Development; More Knowledgeable Other (MKO)

Scaffolding; Social Constructivism



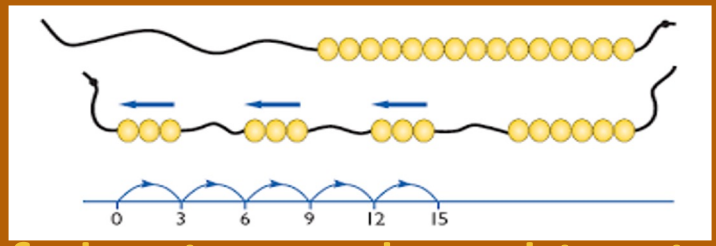


Big Idea #1: Inverse Operations

Multiplication & Division are connected.
(like tying & untying a shoe)

Multiplication has many applications:

- Quick counting
- Scaling up or down
- Area

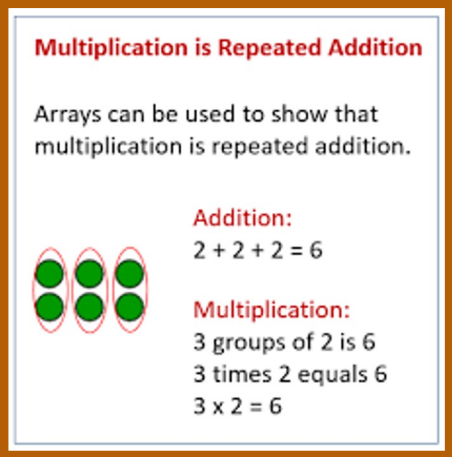


Division reminds us of sharing or breaking into smaller parts.

Multiplication is the opposite of division.

Example: 2 x 3 = 6 6 ÷ 3 = 2

Big Idea #2:
The Four Arithmetic Operations are Related

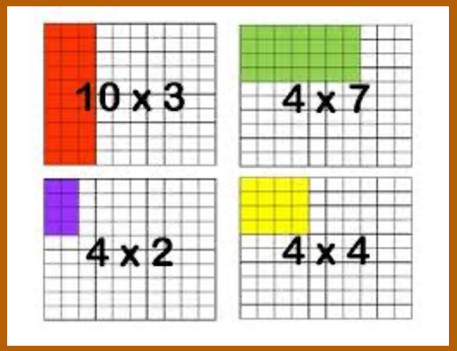


Multiplication has some similarities to Addition.

Division has some similarities to Subtraction.

Multiplication & Division build on solid Number Sense.

Multiplication & Division are Inverse Operations.

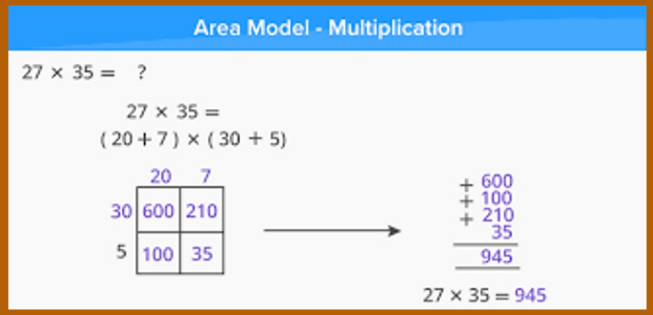


Big Idea #3: Models:

Graphic Organizers and Manipulatives

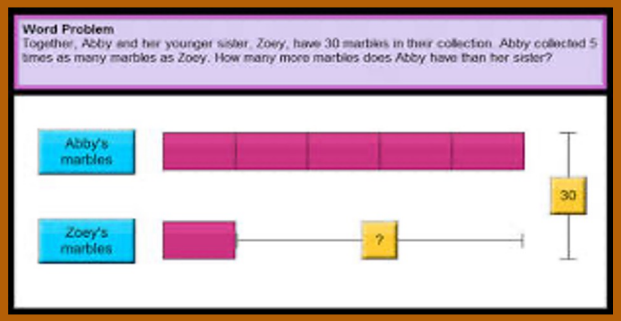
Models can be used to represent and solve contextual (story/word) problems.

- Models can be used to understand the problem.
- Models can be used regardless of the size of the numbers.
- Some models : bar models, arrays and area models



Big Idea #4:

Multiplication (*) and Division (/) Problem Types

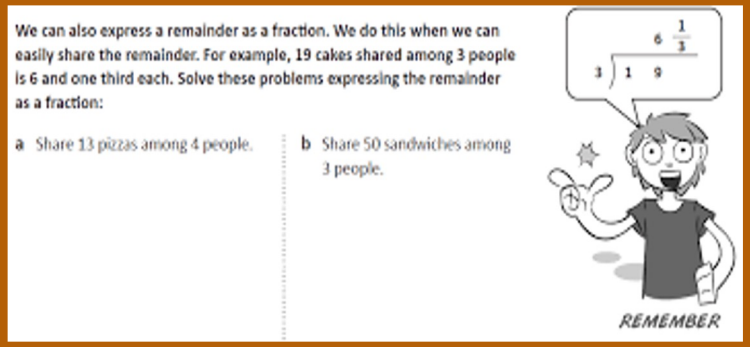


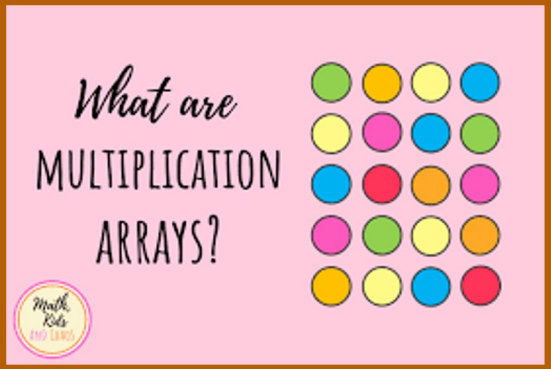
Equal Groups

Multiplicative Comparison

Combination

Area or Arrays





A Familiar kind of Story Problem

Andrew had three boxes of trading cards. Each box holds 8 cards. How many cards does Andrew have altogether?

3 boxes * 8 cards/box = 24 cards in all

The first factor is 3.

The second factor is 8.

The product is 24.

Equal Groups Problems

Andrew has some boxes of trading cards. Each box holds 8 cards. Altogether, Andrew has 24 cards. How many boxes of cards does Andrew have? □ * 8 = 24

Andrew has three boxes of trading cards. Each box holds several cards. Altogether, Andrew has 24 cards. How many cards are in each box??

*3 * □ = 24*

The Multiplication & Addition Connection

- In equal groups problems we are counting groups of the same size.

- This is why multiplication is sometimes called “repeated addition.”
- 3 X 8 = 8 + 8 + 8 = 24
- This is not the only definition of multiplication.

Check for Understanding

- Write an equal groups problem. What key information are you giving? solving for? Formulate an equation related to your problem.
- Swap problems with a classmate. Identify the key information. Solve each other's problem by using manipulatives/pictures.
- Identify the Concrete, the Representation and the Abstract Models for your problem.

Multiplicative Comparison Problems

Hilary spent $5 on gifts for her family. Jeff spent three times as much as Hillary did for his family's gifts. How much money did Jeff spend?

3 (multiplicative factor) * 5 (set to be copied) = 15 (product)

Solving Multiplicative Comparison Problems

Hilary spent $5 on gifts for her family. Jeff spent three times as much as Hilary did for his family's gifts. How much money did Jeff spend?

Hilary's spending ($5)

Jeff's spending (3 times what Hilary spent) shown in a bar model.

Area and Array Story Problems

Important:

Arrays do not show up in story problems often, however, arrays are used frequently as a hands-on or visual representation of multiplication. 2 X 4 can be shown as an array:

* * * *

* * * *

Choose your favorite multiplication fact. Use manipulatives to make an array.

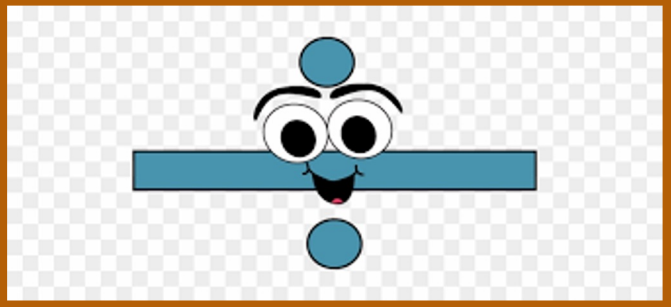
An Array Story Problem

A farmer wants to plant twenty-four ears of corn in three rows. How many ears of corn should be planted in each row?

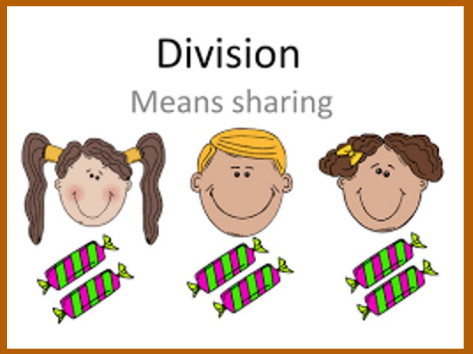
Number in each row * Number in each column = Total

Check for Understanding

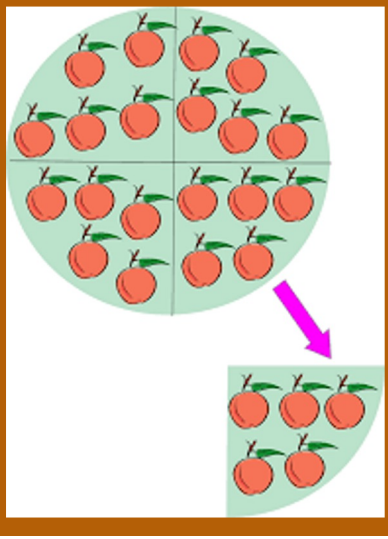
- Look at the examples of Multiplicative Comparison and Area/Array problems.
- What similarities do they have in their wording?
- Write a multiplicative comparison problem. Draw a bar model for your problem.
- What are the advantages & disadvantages of using manipulatives to solve multiplication/division problems?



...AND NOW SOME



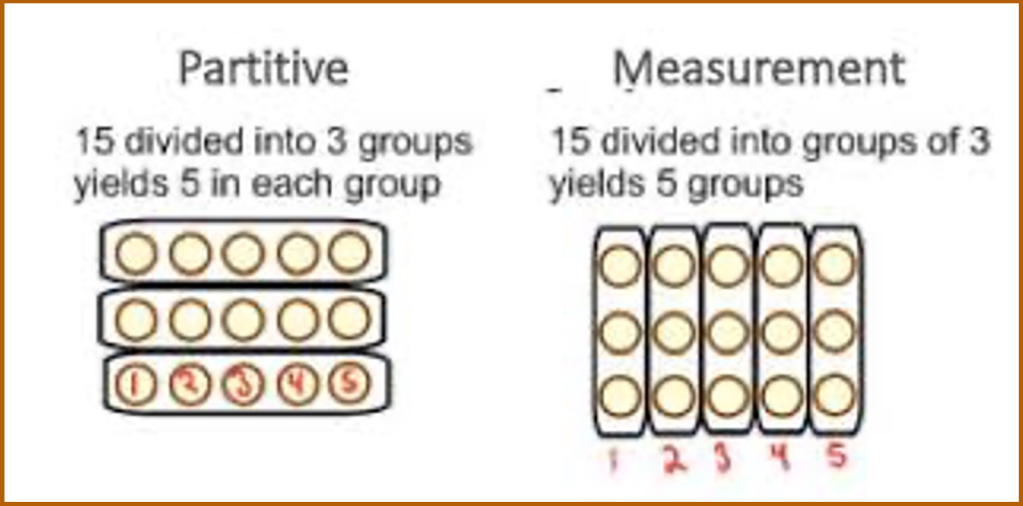
DIVISION

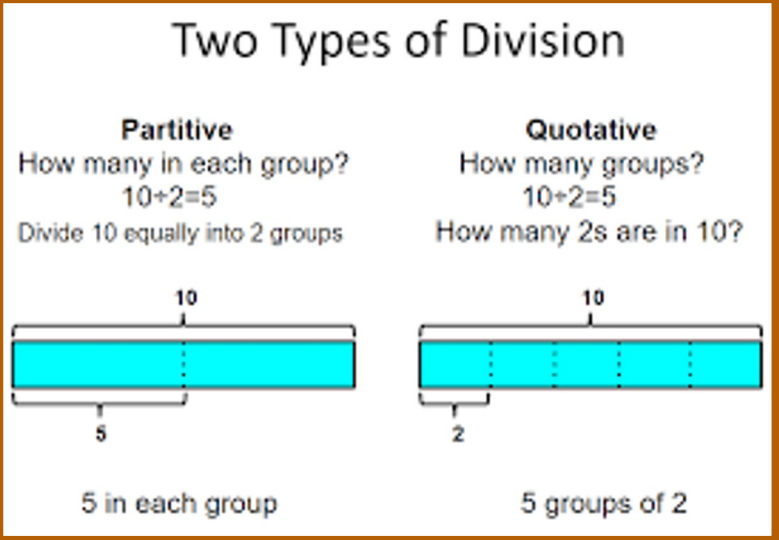


The Various Types of Problems

In order to help children develop a deep understanding of multiplication and division, they need to experience a variety of problem types that lead to these operations.

Teachers need to be aware of these problem types in order to make sure students see the variety.





2 Division Situations

I have 32 Snickers . If I put 2 Snickers in each Halloween treat bag, how many treat bags can I fill?

To solve this, a student could pull out 2 Snickers and keep doing this until there were none left.

This is called “Measurement Division.”

(repeated subtraction)

We knew the size of the group and the total.

Division Situations

Consider this problem:

Ted has 32 Snickers . If he wants to share the Snickers equally among four people (Ana, Bill, Carly & Dan), how many Snickers will each person get?

To solve this, a student could deal out the Snickers (1 for Ana, 1 for Bill, 1 for Carly, 1 for Dan) and keep doing this until there were none left.

This is called “Partitive Division” or fair sharing.

We knew the total and the number of groups.

Instructional Sequence

- Concrete –Pictures- Abstract
- Story Problems and Questions

- Representing Story Problems using manipulative
- and later pictures

- Representing Story Problems using symbols
  - ~ Teaching the signs (* & x) and (/) and (=)
  - ~ (=) means “is the same as” ; a balancing point

- Linking the manipulatives, the symbols, the problem

and the number model/equation representation (Reys,

5 Representations of Mathematics

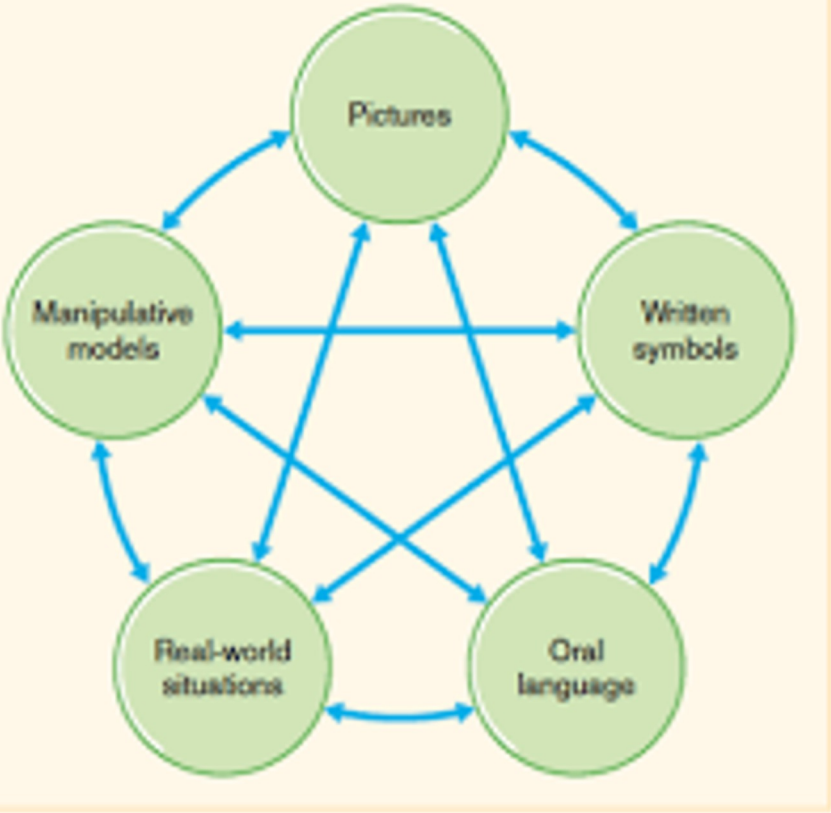


FIGURE 2.11 Five different representations of mathematical ideas. Translations between and within each can help develop new concepts.

Student Strategies

- Direct Modeling using objects or pictures

- Counting Strategies
  - Skip Counting
  - Supports: number line; 100 chart
- Derived Facts (Strategies)
  - I know 7x4=28 because 7x2=14 and double that is 28. (The 4 x's Table is double the 2x's Table.)
- Mental Math - consolidated facts (automaticity)

Basic Fact Instruction

Start Where Children Are: Counting , Adding & Subtracting

Counting On, Counting Back, Skip Counting; Basic +/- Facts

Build Understanding of the Basic Facts & Thinking Strategies (Fact Families)

- Commutative Property (3*6 and 6*3) Mult./Divide 0 or 1 (8*0 or 7/1)
- Perfect Squares (4*4)
- Add another Equal Group 5x7=35 so 6x7 would be 35 + another group of 7 or 35+7=42
- The 9 x's Table

Focus on How to Remember the Facts

- 5-10 minute daily practice (Similar to skill building in sports or music)
- Targeted Practice & Distributed Practice
- Make the practice interesting & a way to build confidence & skill

Combination Problems (least frequent type)

How many different ice cream cones can you make from three choices of ice cream (vanilla, chocolate, strawberry) and two choices for a cone (cake cone or sugar cone)?

Vanilla – cake cone

Chocolate – cake cone

Strawberry – cake cone

Vanilla – sugar cone

Chocolate – sugar cone

Strawberry – sugar cone

3 (flavors) x 2 (cone types) = 6 possible cones