

Topic

Event

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### Abstract

Write a short abstract. Do not give too much details here, but arouse the readers interest. A nice opportunity to comment the text is given by the following: Especially if you write in german, you sometimes need to specify the hyphanation, as e.g. for the word `thisisaverylongwordwhosehyphanationmustbedefined`.

## 1 Introduction

Begin your paper with an introduction into your topic.

### 1.1 hallo

tschuess.

## 2 Preliminaries

### 2.1 Virtual Substitution

In 1993, the concept of Virtual Substitution (VS) was first introduced. Initially it was a procedure to eliminate quantifier/variable elimination for linear real arithmetic formulas. Further, VS became a procedure of quantifier elimination for non-linear arithmetic formulas. But one of the most significant limitation of VS is that it cannot eliminate quantified variables whose degree is higher than 2.

VS is a procedure to eliminate a quantified variable. Let  $\varphi^{\mathbb{R}}$  is a quantifier-free real-arithmetic formula where  $x \in p(x)$  and  $p(x) \sim 0, \sim \in \{=, <, >, \leq, \geq, \neq\}$  is a constraint of  $\varphi^{\mathbb{R}}$ . Degree of  $x$  in  $p(x)$  must be  $\leq 2$ . Then, after quantifier elimination by VS we get the following equivalence,

$$\exists x. \varphi^{\mathbb{R}} \iff \bigvee_{t \in T(x, \varphi^{\mathbb{R}})} (\varphi^{\mathbb{R}}[t \setminus x] \wedge C_t)$$

where  $T$  is a finite set of all possible test candidates for  $x$  and  $C_t$  is a side condition of  $t \in T$ .

### 2.2 Test Candidates and Side Condition

To solve non-linear equalities with VS first we have to choose a variable,  $x \in p(x)$  to eliminate and then compute all possible test candidates (TCs).  $\varphi^{\mathbb{R}}$  is satisfied if there is a test candidate (TC)  $t \in T$  such that  $\varphi^{\mathbb{R}}[t \setminus x] = p_1[t \setminus x] \wedge \dots \wedge p_n[t \setminus x] \wedge C_t$  is satisfiable.

So, the indices of the substitutions are the side conditions of the TC it considers and the labels on the edges to a substitutions are the constraints which provide TC. A detailed explanation of how to construct TCs with side condition is provided in the section 3.1.

### 2.3 Square Root Expression

A square root expression(SRE) has the form,

$$\frac{p + q\sqrt{r}}{s}, \text{ where } p, q, r, s \in P$$

and the set of all square root can be expressed by,

$$SqrtEx := \left\{ \frac{p + q\sqrt{r}}{s} \mid p, q, r, s \in P \right\}$$

**Definition 2.1 (Polynomial)** *A polynomial is a mathematical expression consisting of a sum of terms, each term including a variable or variables raised to a power and multiplied by a coefficient. If a polynomial has only one variable, it is called univariate. An univariate of degree  $d$  has the following form where  $a_d \neq 0$ ,*

$$p(x) = a_dx^d + a_{d-1}x^{d-1} + \dots + a_0x^0$$

*If a polynomial has two or more variables, it is called multivariate. A multivariate (two variables) of degree  $d$  has the following form where  $a_{dd} \neq 0$ ,*

$$p(x, y) = a_{dd}x^d y^d + a_{d(d-1)}x^d y^{d-1} + a_{(d-1)d}x^{d-1} y^d + \dots + a_{10}x^1 y^0 + a_{01}x^0 y^1 + a_{00}x^0 y^0$$

*The following expression is a quantifier-free real-arithmetic formula where  $a, b, c$  are the polynomials and the set of all polynomials in  $\varphi^{\mathbb{R}}$  is  $P = \{a, b, c\}$ ,*

$$\varphi^{\mathbb{R}} = (a \leq 0 \vee b = 0) \wedge (b < 0 \vee c \neq 0)$$

Let,  $p(x) = ax^2 + bx + c = 0$  is a quadratic equation of variable  $x$  where  $a, b, c \in P$  and  $x \notin a \cup b \cup c$ . Now, the solution formula for  $x$  in  $p(x) = a_dx^d + a_{d-1}x^{d-1} + \dots + a_0x^0$  considers the following four cases,

$$x_0 = -\frac{c}{b}, \text{ if } a = 0 \wedge b \neq 0 \quad (2.1)$$

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \text{ if } a \neq 0 \wedge b^2 - 4ac \geq 0 \quad (2.2)$$

$$x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}, \text{ if } a \neq 0 \wedge b^2 - 4ac \geq 0 \quad (2.3)$$

$$x_3 = -\infty, \text{ if } a = 0 \wedge b = 0 \wedge c = 0 \quad (2.4)$$

Note that,  $x_0$  is a real zero of  $p(x)$  for linear equation, for quadratic equation  $x_1$  and  $x_2$  are two real zeros of  $p(x)$ .  $x_4$  is any real number which is also a solution for  $x$ .

Now, we can express the symbolic zero of  $x$  in a polynomial, which is quadratic in  $x$  by a SRE  $\frac{p+q\sqrt{r}}{s}$  as given in table 2.1.

**Remark** We can construct TCs by the comparison with SRE (table 2.1) and also considering that TCs can be supplemented by an infinitesimal  $\varepsilon$ .

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**Table 2.1** Comparison with SRE  $\frac{p+q\sqrt{r}}{s}$ 

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Equation No.	p	q	r	s
2.1	$-c$	0	1	$b$
2.2	$-b$	1	$b^2 - 4ac$	$2a$
2.3	$-b$	$-1$	$b^2 - 4ac$	$2a$
2.4	0	1	0	0

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### 3 Topic

This section concerns the main topic. In the following you can see a small illustration of how to use itemizings and enumerations.

- Point 1.
- Point 2.

1. Point 1.
2. Point 2.

- I) Point 1.
- II) Point 2.

1. Point 1.
2. Point 2.

**Term one:** Description of term one.

**Term two:** Description of term two.

In Algorithm 1 you can see how we define an algorithm.

#### 3.1 Example

Give an example to illustrate the idea of your topic. Import images in the following way. Store the images in a separate folder as precasted in our template.

### 4 Conclusion

Give a conclusion on your topic. Give a few sentences to summarize the topic. If possible, point out the quality of the result and give  $\exists$  a small prospect of subsequent works.

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**Algorithm 1** Describe the purpose of the algorithm. For more information see the newalg-Manual.

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```
VOID METHOD ( typeA argumentA, typeB argumentB )
1  write the algorithm in pseudocode
2  it should not go into detail, but display main idea
3  however, keep being consistent
4   $x \leftarrow 1$  (this is how to assign a value to a variable)
5  while a condition being True or False
6  do do something
7      and something else
8
9  if a condition being True or False
10     then point 1
11
12     else if another condition
13         then point 2
14
15     else point 3
16         return True
```

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Figure 1: Proseminar supervisor's pet.