# Topic

Event

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**Abstract**

Write a short abstract. Do not give to much details here, but arouse the readers interest. A nice opportunity to comment the text is given by the following: Especially if you write in german, you sometimes need to specify the hyphanation, as e.g. for the word thisisaverylongwordwhosehyphanationmustbedefined.

# 1 Introduction

Begin your paper with an introduction into your topic.

## 1.1 hallo

tschuess.

# 2 Preliminaries

## 2.1 Virtual Substitution

In 1993, the concept of Virtual Substitution (VS) was first introduced. Initially it was a procedure to eliminate quantifier/variable elimination for linear real arithmetic formulas. Further, VS became a procedure of quantifier elimination for non-linear arithmetic formulas. But one of the most significant limitation of VS is that it cannot eliminate quantified variables whose degree is higher than 2.

VS is a procedure to eliminate a quantified variable. Let $\varphi^{\mathbb{R}}$ is a quantifier-free real-arithmetic formula where $x \in p(x)$ and $p(x) \sim 0, \sim \in \{=, <, >, \leq, \geq, \neq\}$ is a constraint of $\varphi^{\mathbb{R}}$. Degree of x in $p(x)$ must be $\leq 2$. Then, after quantifier elimination by VS we get the following equivalence,

$$\exists x. \varphi^{\mathbb{R}} \iff \bigvee_{t \in T(x, \varphi^{\mathbb{R}})} (\varphi^{\mathbb{R}}[t \backslash\backslash x] \wedge C_t)$$

where T is a finite set of all possible test candidates for x and $C_t$ is a side condition of $t \in T$.

## 2.2 Test Candidates and Side Condition

To solve non-linear equalities with VS first we have to choose a variable, $x \in p(x)$ to eliminate and then compute all possible test candidates(TCs). $\varphi^{\mathbb{R}}$ is satisfied if there is a test candidate (TC) $t \in T$ such that $\varphi^{\mathbb{R}}[t \backslash\backslash x] = p_1[t \backslash\backslash x] \wedge \cdots p_n[t \backslash\backslash x] \wedge C_t$ is satisfiable.

So, the indices of the substitutions are the side conditions of the TC it considers and the labels on the edges to a substitutions are the constraints which provide TC. A detailed explanation of how to construct TCs with side condition is provided in the section 3.1.

## 2.3 Square Root Expression

A square root expression(SRE) has the form,

$$\frac{p + q\sqrt{r}}{s} \qquad \text{, where } p, q, r, s \in P$$

and the set of all square root can be expressed by,

$$SqrtEx := \{\frac{p + q\sqrt{r}}{s} |\ p, q, r, s \in P\}$$

**Definition 2.1 (Polynomial)** *A polynomial is a mathematical expression consisting of a sum of terms, each term including a variable or variables raised to a power and multiplied by a coefficient. If a polynomial has only one variable, it is called univariate. An univariate of degree d has the following form where $a_d \neq 0$,*

$$p(x) = a_d x^d + a_{d-1} x^{d-1} + \ldots + a_0 x^0$$

*If a polynomial has two or more variables, it is called multivariate. A multivariate (two variables) of degree d has the following form where $a_{dd} \neq 0$,*

$$p(x, y) = a_{dd} x^d y^d + a_{d(d-1)} x^d y^{d-1} + a_{(d-1)d} x^{d-1} y^d + \ldots + a_{10} x^1 y^0 + a_{10} x^0 y^1 + a_{00} x^0 y^0$$

*The following expression is a quantifier-free real-arithmetic formula where a, b, c are the polynomials and the set of all polynomials in $\varphi^{\mathbb{R}}$ is $P = \{a, b, c\}$,*

$$\varphi^{\mathbb{R}} = (a \leq 0 \vee b = 0) \wedge (b < 0 \vee c \neq 0)$$

Let, $p(x) = ax^2 + bx + c = 0$ is a quadratic equation of variable $x$ where $a, b, c \in P$ and $x \notin a \cup b \cup c$. Now, the solution formula for $x$ in $p(x) = a_d x^d + a_{d-1} x^{d-1} + \ldots + a_0 x^0$ considers the following four cases,

$$x_0 = -\frac{c}{b} \qquad \text{, if } a = 0 \wedge b \neq 0 \tag{2.1}$$

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \qquad \text{, if } a \neq 0 \wedge b^2 - 4ac \geq 0 \tag{2.2}$$

$$x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \qquad \text{, if } a \neq 0 \wedge b^2 - 4ac \geq 0 \tag{2.3}$$

$$x_3 = -\infty \qquad \text{, if } a = 0 \wedge b = 0 \wedge c = 0 \tag{2.4}$$

Note that, $x_0$ is a real zero of $p(x)$ for linear equation, for quadratic equation $x_1$ and $x_2$ are two real zeros of $p(x)$. $x_4$ is any real number which is also a solution for $x$.

Now, we can express the symbolic zero of x in a polynomial, which is quadratic in x by a SRE $\frac{p+q\sqrt{r}}{s}$ as given in table 2.1.

**Remark** We can construct TCs by the comparison with SRE (table 2.1) and also considering that TCs can be supplemented by an infinitesimal $\varepsilon$.

**Table 2.1** Comparison with SRE $\frac{p+q\sqrt{r}}{s}$

| Equation No. | p | q | r | s |
|---|---|---|---|---|
| 2.1 | $-c$ | 0 | 1 | $b$ |
| 2.2 | $-b$ | 1 | $b^2-4ac$ | $2a$ |
| 2.3 | $-b$ | $-1$ | $b^2-4ac$ | $2a$ |
| 2.4 | 0 | 1 | 0 | 0 |

# 3 Solving Non-linear Equalities with Virtual Substitution

# 4 Conclusion

Give a conclusion on your topic. Give a few sentences to summarize the topic. If possible, point out the quality of the result and give $\exists$ a small prospect of subsequent works.