TH2 - Übung 2

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1 CSP Trace Semantik

1.1 Traces für konkrete Prozesse berechnen

Gegeben seien die folgenden Prozesse:

$$P = (a \rightarrow b \rightarrow Skip) \,\square\, (b \rightarrow d \rightarrow Stop)$$

$$Q = (x \rightarrow Skip) \,\triangle\, (y \rightarrow Stop)$$

$$R = (P;\ Q) \setminus \{x, y\}$$

$$S = P \,|[\,\{a, b\}\,|\,\{x, y\}\,]|\ Q$$

Berechnen Sie die Trace-Semantik der Prozesse auf der Basis der Trace-Definitionen im CSP-Guide (Kap. 6). Geben Sie die für die Berechnung verwendeten Regeln an. Referenzversion ist Version 43 des CSP-Guide.

$$P = (a \rightarrow b \rightarrow Skip) \;\square\; (b \rightarrow d \rightarrow Stop)$$

$$
\begin{aligned}
traces(P) &= traces((a \rightarrow b \rightarrow Skip) \;\square\; (b \rightarrow d \rightarrow Stop) \\
&= traces(a \rightarrow b \rightarrow Skip) \cup traces(b \rightarrow d \rightarrow Stop) \qquad \text{Choice, 6.14}
\end{aligned}
$$

$$traces(a \rightarrow b \rightarrow Skip) = \{\langle\rangle\} \cup \{\langle a \rangle \frown tr \mid tr \in traces(b \rightarrow Skip)\} \quad \text{Prefixing, 6.10}$$

$$
\begin{aligned}
traces(b \rightarrow Skip) &= \{\langle\rangle\} \cup \{\langle b \rangle \frown tr \mid tr \in traces(Skip)\} \qquad \text{Prefixing, 6.10} \\
&= \{\langle\rangle\} \cup \{\langle b \rangle \frown tr \mid tr \in \{\langle\rangle, \langle\checkmark\rangle\}\} \qquad\qquad \text{Skip, 6.4} \\
&= \{\langle\rangle, \langle b \rangle, \langle b, \checkmark \rangle\}
\end{aligned}
$$

$$
\begin{aligned}
traces(a \rightarrow b \rightarrow Skip) &= \{\langle\rangle\} \cup \{\langle a \rangle \frown tr \mid tr \in \{\langle\rangle, \langle b \rangle, \langle b, \checkmark \rangle\}\} \\
&= \{\langle\rangle, \langle a \rangle, \langle a, b \rangle, \langle a, b, \checkmark \rangle\}
\end{aligned}
$$

$$traces(b \rightarrow d \rightarrow Stop) = \{\langle\rangle\} \cup \{\langle b \rangle \frown tr \mid tr \in traces(d \rightarrow Stop)\} \quad \text{Prefixing, 6.10}$$

$$
\begin{aligned}
traces(d \rightarrow Stop) &= \{\langle\rangle\} \cup \{\langle d \rangle \frown tr \mid tr \in traces(Stop)\} \qquad \text{Prefixing, 6.10} \\
&= \{\langle\rangle\} \cup \{\langle d \rangle \frown tr \mid tr \in \{\langle\rangle\}\} \qquad\qquad\quad \text{Stop, 6.3} \\
&= \{\langle\rangle, \langle d \rangle\}
\end{aligned}
$$

$$
\begin{aligned}
traces(b \rightarrow d \rightarrow Stop) &= \{\langle\rangle\} \cup \{\langle b \rangle \frown tr \mid tr \in \{\langle\rangle, \langle d \rangle\}\} \\
&= \{\langle\rangle, \langle b \rangle, \langle b, d \rangle\}
\end{aligned}
$$

$$
\begin{aligned}
traces(P) &= \{\langle\rangle, \langle a \rangle, \langle a, b \rangle, \langle a, b, \checkmark \rangle\} \cup \{\langle\rangle, \langle b \rangle, \langle b, d \rangle\} \\
&= \{\langle\rangle, \langle a \rangle, \langle a, b \rangle, \langle a, b, \checkmark \rangle, \langle b \rangle, \langle b, d \rangle\}
\end{aligned}
$$

$$Q = (x \to Skip) \triangle (y \to Stop)$$

$$
\begin{aligned}
traces(Q) &= traces((x \to Skip) \triangle (y \to Stop)) \\
&= traces(x \to Skip) \cup \\
&\quad \{tr_1 \frown tr_2 \mid tr_1 \in traces(x \to Skip) \wedge \checkmark \notin \sigma(tr_1) \wedge tr_2 \in traces(y \to Stop)\} \quad \text{Interrupt, 6.53}
\end{aligned}
$$

$$
\begin{aligned}
traces(x \to Skip) &= \{\langle\rangle\} \cup \{\langle x \rangle \frown tr \mid tr \in traces(Skip)\} \quad &\text{Prefixing, 6.10} \\
&= \{\langle\rangle\} \cup \{\langle x \rangle \frown tr \mid tr \in \{\langle\rangle, \langle\checkmark\rangle\}\} \quad &\text{Skip, 6.4} \\
&= \{\langle\rangle, \langle x \rangle, \langle x, \checkmark \rangle\}
\end{aligned}
$$

$$
\begin{aligned}
traces(y \to Stop) &= \{\langle\rangle\} \cup \{\langle y \rangle \frown tr \mid tr \in traces(Stop)\} \quad &\text{Prefixing, 6.10} \\
&= \{\langle\rangle\} \cup \{\langle y \rangle \frown tr \mid tr \in \{\langle\rangle\}\} \quad &\text{Stop, 6.3} \\
&= \{\langle\rangle, \langle y \rangle\}
\end{aligned}
$$

$$
\begin{aligned}
traces(Q) &= \{\langle\rangle, \langle x \rangle, \langle x, \checkmark \rangle\} \cup \\
&\quad \{tr_1 \frown tr_2 \mid tr_1 \in \{\langle\rangle, \langle x \rangle, \langle x, \checkmark \rangle\} \wedge \checkmark \notin \sigma(tr_1) \wedge tr_2 \in \{\langle\rangle, \langle y \rangle\}\} \\
&= \{\langle\rangle, \langle x \rangle, \langle x, \checkmark \rangle\} \cup \{\langle\rangle, \langle y \rangle, \langle x \rangle, \langle x, y \rangle\} \\
&= \{\langle\rangle, \langle x \rangle, \langle x, \checkmark \rangle, \langle y \rangle, \langle x, y \rangle\}
\end{aligned}
$$

$$R = (P;\ Q) \setminus \{x, y\}$$

$$
\begin{aligned}
traces(R) &= traces((P;\ Q) \setminus \{x, y\}) \\
&= \{tr \setminus \{x, y\} \mid tr \in traces(P;\ Q) \qquad \text{Hiding, 6.48}
\end{aligned}
$$

$$
\begin{aligned}
traces(P;\ Q) &= \{tr \mid tr \in \{\langle\rangle, \langle a\rangle, \langle a, b\rangle, \langle a, b, \checkmark\rangle, \langle b\rangle, \langle b, d\rangle\} \wedge \checkmark \notin \sigma(tr)\} \\
&\quad \cup \{tr_1 \frown tr_2 \mid tr_1 \frown \langle\checkmark\rangle \in \{\langle\rangle, \langle a\rangle, \langle a, b\rangle, \langle a, b, \checkmark\rangle, \langle b\rangle, \langle b, d\rangle\} \wedge \\
&\qquad tr_2 \in \{\langle\rangle, \langle x\rangle, \langle x, \checkmark\rangle, \langle y\rangle, \langle x, y\rangle\}\} \qquad \text{Seq. Comp., 6.51} \\
&= \{\langle\rangle, \langle a\rangle, \langle a, b\rangle, \langle b\rangle, \langle b, d\rangle\} \cup \\
&\quad \{tr_1 \frown tr_2 \mid tr_1 \in \{\langle a, b\rangle\} \wedge tr_2 \in \{\langle\rangle, \langle x\rangle, \langle x, \checkmark\rangle, \langle y\rangle, \langle x, y\rangle\} \\
&= \{\langle\rangle, \langle a\rangle, \langle a, b\rangle, \langle b\rangle, \langle b, d\rangle\} \cup \\
&\quad \{\langle a, b\rangle, \langle a, b, x\rangle, \langle a, b, x, \checkmark\rangle, \langle a, b, y\rangle, \langle a, b, x, y\rangle\} \\
&= \{\langle\rangle, \langle a\rangle, \langle a, b\rangle, \langle b\rangle, \langle b, d\rangle, \langle a, b, x\rangle, \langle a, b, x, \checkmark\rangle, \langle a, b, y\rangle, \langle a, b, x, y\rangle\}
\end{aligned}
$$

$$
\begin{aligned}
traces(R) &= \{\langle\rangle, \langle a\rangle, \langle a, b\rangle, \langle b\rangle, \langle b, d\rangle, \langle a, b, x\rangle, \langle a, b, x, \checkmark\rangle, \langle a, b, y\rangle, \langle a, b, x, y\rangle\} \setminus \{x, y\} \\
&= \{\langle\rangle, \langle a\rangle, \langle a, b\rangle, \langle b\rangle, \langle b, d\rangle, \langle a, b, \checkmark\rangle\}
\end{aligned}
$$

$$S = P \ |[\ \{a, b\}\ |\ \{x, y\}\ ]|\ Q$$

$$
\begin{aligned}
traces(S) = \{ & tr \in TRACES \mid tr \upharpoonright \{a, b, \checkmark\} \in traces(P) \wedge \\
& tr \upharpoonright \{x, y, \checkmark\} \in traces(Q) \wedge \sigma(tr) \subseteq \{a, b, x, y, \checkmark\}\} \qquad \text{Alph. Parallel, 6.18-20} \\
= \{ & tr \in TRACES \mid tr \upharpoonright \{a, b, \checkmark\} \in \{\langle\rangle, \langle a\rangle, \langle a, b\rangle, \langle a, b, \checkmark\rangle, \langle b\rangle, \langle b, d\rangle\} \wedge \\
& tr \upharpoonright \{x, y, \checkmark\} \in \{\langle\rangle, \langle x\rangle, \langle x, \checkmark\rangle, \langle y\rangle, \langle x, y\rangle\} \wedge \\
& \sigma(tr) \subseteq \{a, b, x, y, \checkmark\}\} \\
= \{ & \langle\rangle, \\
& \langle a\rangle, \langle b\rangle, \langle a, b\rangle, \langle b, d\rangle, \\
& \langle x\rangle, \langle y\rangle, \langle x, y\rangle, \\
& \langle a, x\rangle, \langle x, a\rangle, \langle b, x, \rangle, \langle x, b\rangle, \\
& \langle a, b, x\rangle, \langle a, x, b\rangle, \langle x, a, b\rangle, \\
& \langle b, d, x\rangle, \langle b, x, d\rangle, \langle x, b, d\rangle, \\
& \langle a, y\rangle, \langle y, a\rangle, \langle b, y\rangle, \langle y, b\rangle, \\
& \langle a, b, y\rangle, \langle a, y, b\rangle, \langle y, a, b\rangle, \\
& \langle b, d, y\rangle, \langle b, y, d\rangle, \langle y, b, d\rangle, \\
& \langle a, x, y\rangle, \langle x, a, y\rangle, \langle x, y, a\rangle, \\
& \langle b, x, y\rangle, \langle x, b, y\rangle, \langle x, y, b\rangle, \\
& \langle a, b, x, y\rangle, \langle a, x, b, y\rangle, \langle a, x, y, b\rangle, \langle x, a, y, b\rangle, \langle x, y, a, b\rangle, \langle x, a, b, y\rangle, \\
& \langle b, d, x, y\rangle, \langle b, x, d, y\rangle, \langle x, b, d, y\rangle, \langle x, b, y, d\rangle, \langle x, y, b, d\rangle, \langle b, x, y, d\rangle, \\
& \langle a, b, x, \checkmark\rangle, \langle a, x, b, \checkmark\rangle, \langle x, a, b, \checkmark\rangle\}
\end{aligned}
$$