TH2 - Übung 4

Andreas Krohn, Benjamin Jochheim, Theodor Nolte, Benjamin Vetter

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1 CSP Laws

1.1 SCP Laws beweisen

a) $a \rightarrow (P_1 \sqcap P_2) =_T (a \rightarrow P_1) \sqcap (a \rightarrow P_2)$

Gleichheit von Prozessen im Trace-Modell:
$a \rightarrow (P_1 \sqcap P_2) =_T (a \rightarrow P_1) \sqcap (a \rightarrow P_2)$
$\equiv \mathit{traces}(a \rightarrow (P_1 \sqcap P_2)) = \mathit{traces}((a \rightarrow P_1) \sqcap (a \rightarrow P_2))$

$\rightarrow$ zu zeigen: $\mathit{traces}(\mathit{Skip} \triangle P) = \mathit{traces}(\mathit{Skip} \sqcap P)$

$$\begin{aligned}
\mathit{traces}(a \rightarrow (P_1 \sqcap P_2)) &= \{\langle\rangle\} \cup \{\langle a\rangle \frown tr \mid tr \in \mathit{traces}(P_1 \sqcap P_2)\} && \text{Prefixing, 6.10} \\
&= \{\langle\rangle\} \cup \{\langle a\rangle \frown tr \mid tr \in \mathit{traces}(P_1) \cup \mathit{traces}(P_2)\} && \text{Choice, 6.16} \\
&= \{\langle\rangle\} \cup \{\langle a\rangle \frown tr \mid tr \in \mathit{traces}(P_1)\} && \text{Vereinigung von Mengen} \\
&\quad \cup \{\langle a\rangle \frown tr \mid tr \in \mathit{traces}(P_2)\} \\
&= \{\langle\rangle\} \cup \{\langle a\rangle \frown tr \mid tr \in \mathit{traces}(P_1)\} && \text{Idempotenz von Mengen} \\
&\quad \cup \{\langle\rangle\} \cup \{\langle a\rangle \frown tr \mid tr \in \mathit{traces}(P_2)\} \\
&= \mathit{traces}(a \rightarrow P_1) \cup \mathit{traces}(a \rightarrow P_2) && \text{Prefixing, 6.10} \\
&= \mathit{traces}((a \rightarrow P_1) \sqcap (a \rightarrow P_2)) && \text{(Internal-)Choice, 6.16}
\end{aligned}$$

$\square$

b) $\mathit{Skip} \triangle P =_T \mathit{Skip} \sqcap P$

Gleichheit von Prozessen im Trace-Modell:
$\mathit{Skip} \triangle P =_T \mathit{Skip} \sqcap P \quad \equiv \quad \mathit{traces}(\mathit{Skip} \triangle P) = \mathit{traces}(\mathit{Skip} \sqcap P)$

$\rightarrow$ zu zeigen: $\mathit{traces}(\mathit{Skip} \triangle P) = \mathit{traces}(\mathit{Skip} \sqcap P)$

$$
\begin{aligned}
traces(Skip \mathbin{\triangle} P) &= traces(Skip) \cup \{tr_1 \frown tr_2 \mid tr_1 \in traces(Skip) \land \checkmark \notin \sigma(tr_1) && \text{Interrupt, 6.53}\\
&\qquad\qquad \land\, tr_2 \in traces(P)\}\\
&= \{\langle\rangle, \langle\checkmark\rangle\} \cup \{tr_1 \frown tr_2 \mid tr_1 \in \{\langle\rangle, \langle\checkmark\rangle\} \land \checkmark \notin \sigma(tr_1) && \text{Skip, 6.4}\\
&\qquad\qquad \land\, tr_2 \in traces(P)\}\\
&\mid NB_1 : tr_1 \in \{\langle\rangle, \langle\checkmark\rangle\} \land \checkmark \notin \sigma(tr_1) \quad \Leftrightarrow \quad tr_1 = \{\langle\rangle\}\\
&\mid NB_2 : \{a \frown b\} = \{b\} \Leftrightarrow a = \{\langle\rangle\}\\
&= \{\langle\rangle, \langle\checkmark\rangle\} \cup \{\langle\rangle \frown tr_2 \mid tr_2 \in traces(P)\} && NB_1, NB_2\\
&= \{\langle\rangle, \langle\checkmark\rangle\} \cup traces(P) && NB_2\\
&= traces(Skip) \cup traces(P) && \text{Skip, 6.4}\\
&= traces(Skip \sqcap P) && \text{Choice, 6.14}
\end{aligned}
$$

□

1.2 CSP Laws anwenden

$P1 = ((a \to c \to Skip) \,\Box\, (b \to c \to Skip))$
$P2 = (d \to Skip)$
$P3 = c \to d \to Skip$

zu zeigen: $(P1;\ P2) \setminus \{a, b\} =_T P3$

$$
\begin{aligned}
(P1;\ P2) \setminus \{a,b\} &= (((a \to c \to Skip) \,\Box\, (b \to c \to Skip));\ (d \to Skip)) \setminus \{a,b\}\\
&=_T ((a \to c \to Skip) \sqcap (b \to c \to Skip);\ (d \to Skip)) \setminus \{a,b\} && \text{choice-equiv}_T\text{, 8.15}\\
&= (a \to c \to Skip) \setminus \{a,b\} \sqcap (b \to c \to Skip) \setminus \{a,b\}; && \sqcap\text{-hide-dist}_T\text{, 8.81, NB}\\
&\quad (d \to Skip) \setminus \{a,b\}\\
&= (c \to Skip) \setminus \{a,b\} \sqcap (c \to Skip) \setminus \{a,b\}; && \text{3 x hide-step}_1\text{, 8.79}\\
&\quad d \to Skip \setminus \{a,b\}\\
&= c \to Skip \setminus \{a,b\} \sqcap c \to Skip \setminus \{a,b\}; && \text{2 x hide-step}_1\text{, 8.79}\\
&\quad d \to Skip \setminus \{a,b\}\\
&= (c \to Skip) \sqcap (c \to Skip);\ d \to Skip && \text{3 x hide-term, 8.85}\\
&= c \to Skip;\ d \to Skip && \Box\text{-idem, 8.16}\\
&= c \to d \to Skip && \text{;-unit-l, 8.94}
\end{aligned}
$$

□

2 Failures Semantik

2.1 Stable Failures in Transitionsgraphen