TH2 - Übung 4

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1 CSP Laws

1.1 SCP Laws beweisen

a) $a \to (P_1 \sqcap P_2) =_T (a \to P_1) \sqcap (a \to P_2)$

Gleichheit von Prozessen im Trace-Modell:
$a \to (P_1 \sqcap P_2) =_T (a \to P_1) \sqcap (a \to P_2)$
$\equiv traces(a \to (P_1 \sqcap P_2)) = traces((a \to P_1) \sqcap (a \to P_2))$

$\to$ zu zeigen: $traces(a \to (P_1 \sqcap P_2)) = traces((a \to P_1) \sqcap (a \to P_2))$

$$
\begin{aligned}
traces(a \to (P_1 \sqcap P_2)) &= \{\langle\rangle\} \cup \{\langle a\rangle \frown tr \mid tr \in traces(P_1 \sqcap P_2)\} && \text{Prefixing, 6.10}\\
&= \{\langle\rangle\} \cup \{\langle a\rangle \frown tr \mid tr \in traces(P_1) \cup traces(P_2)\} && \text{Choice, 6.16}\\
&= \{\langle\rangle\} \cup \{\langle a\rangle \frown tr \mid tr \in traces(P_1)\} && \text{Vereinigung von Mengen}\\
&\quad \cup \{\langle a\rangle \frown tr \mid tr \in traces(P_2)\}\\
&= \{\langle\rangle\} \cup \{\langle a\rangle \frown tr \mid tr \in traces(P_1)\} && \text{Idempotenz von Mengen}\\
&\quad \cup \{\langle\rangle\} \cup \{\langle a\rangle \frown tr \mid tr \in traces(P_2)\}\\
&= traces(a \to P_1) \cup traces(a \to P_2) && \text{Prefixing, 6.10}\\
&= traces((a \to P_1) \sqcap (a \to P_2)) && \text{(Internal-)Choice, 6.16}
\end{aligned}
$$

$\square$

b) $Skip \triangle P =_T Skip \sqcap P$

Gleichheit von Prozessen im Trace-Modell:
$Skip \triangle P =_T Skip \sqcap P \quad \equiv \quad traces(Skip \triangle P) = traces(Skip \sqcap P)$

$\to$ zu zeigen: $traces(Skip \triangle P) = traces(Skip \sqcap P)$

$$traces(Skip \, \triangle \, P) = traces(Skip) \cup \{tr_1 \frown tr_2 \mid tr_1 \in traces(Skip) \wedge \checkmark \notin \sigma(tr_1) \quad \text{Interrupt, 6.53}$$
$$\wedge \; tr_2 \in traces(P)\}$$
$$= \{\langle\rangle, \langle\checkmark\rangle\} \cup \{tr_1 \frown tr_2 \mid tr_1 \in \{\langle\rangle, \langle\checkmark\rangle\} \wedge \checkmark \notin \sigma(tr_1) \qquad \text{Skip, 6.4}$$
$$\wedge \; tr_2 \in traces(P)\}$$
$$\mid NB_1 : tr_1 \in \{\langle\rangle, \langle\checkmark\rangle\} \wedge \checkmark \notin \sigma(tr_1) \quad \Leftrightarrow \quad tr_1 = \{\langle\rangle\}$$
$$\mid NB_2 : \{a \frown b\} = \{b\} \Leftrightarrow a = \{\langle\rangle\}$$
$$= \{\langle\rangle, \langle\checkmark\rangle\} \cup \{\langle\rangle \frown tr_2 \mid tr_2 \in traces(P)\} \qquad\qquad NB_1, NB_2$$
$$= \{\langle\rangle, \langle\checkmark\rangle\} \cup traces(P) \qquad\qquad\qquad\qquad NB_2$$
$$= traces(Skip) \cup traces(P) \qquad\qquad\qquad\qquad \text{Skip, 6.4}$$
$$= traces(Skip \sqcap P) \qquad\qquad\qquad\qquad \text{Choice, 6.14}$$

$\square$

1.2 CSP Laws anwenden

$P1 = ((a \rightarrow c \rightarrow Skip) \,\square\, (b \rightarrow c \rightarrow Skip))$
$P2 = (d \rightarrow Skip)$
$P3 = c \rightarrow d \rightarrow Skip$

$$\text{zu zeigen: } (P1; \; P2) \setminus \{a, b\} =_T P3$$

$$(P1; \; P2) \setminus \{a, b\} = (((a \rightarrow c \rightarrow Skip) \,\square\, (b \rightarrow c \rightarrow Skip)); \; (d \rightarrow Skip)) \setminus \{a, b\}$$
$$=_T ((a \rightarrow c \rightarrow Skip) \sqcap (b \rightarrow c \rightarrow Skip); \; (d \rightarrow Skip)) \setminus \{a, b\} \qquad \text{choice-equiv}_T, 8.15$$
$$= (a \rightarrow c \rightarrow Skip) \setminus \{a, b\} \sqcap (b \rightarrow c \rightarrow Skip) \setminus \{a, b\}; \qquad \sqcap\text{-hide-dist}_T, 8.81, NB$$
$$(d \rightarrow Skip) \setminus \{a, b\}$$
$$= (c \rightarrow Skip) \setminus \{a, b\} \sqcap (c \rightarrow Skip) \setminus \{a, b\}; \qquad\qquad 3 \text{ x hide-step}_1, 8.79$$
$$d \rightarrow Skip \setminus \{a, b\}$$
$$= c \rightarrow Skip \setminus \{a, b\} \sqcap c \rightarrow Skip \setminus \{a, b\}; \qquad\qquad 2 \text{ x hide-step}_1, 8.79$$
$$d \rightarrow Skip \setminus \{a, b\}$$
$$= (c \rightarrow Skip) \sqcap (c \rightarrow Skip); \; d \rightarrow Skip \qquad\qquad 3 \text{ x hide-term, 8.85}$$
$$= c \rightarrow Skip; \; d \rightarrow Skip \qquad\qquad\qquad\qquad \square\text{-idem, 8.16}$$
$$= c \rightarrow d \rightarrow Skip \qquad\qquad\qquad\qquad\qquad \text{;-unit-l, 8.94}$$
$$= P3$$

$\square$

2 Failures Semantik

2.1 Stable Failures in Transitionsgraphen