Computational Economics Lecture 11: SMM Estimation of Heterogeneous Firm Models

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Outline

Motivation

- We have learned how to solve the heterogeneous firm models
- How about the parameters that we choose?
  - The parameters should reflect the reality that we live in
  - A good way to capture that mapping is to use the simulated method of moments
- We would like to start with a canonical model: (Cooper and Haltiwanger, 2006)
- "On the Nature of Capital Adjustment Costs," Review of Economic Studies (2008)
- This model is often taken as a reference for heterogeneous-firm model estimation
- I follow largely Russell Cooper's own notation

Data Facts

- Data: Longitudinal Research Database (U.S. Census)
- Balanced panel of about 7,000 large continuing plants from LRD: 1972-88
- Focus on investment (equipment) rates: purchases and retirements

TABLE 1

Summary statistics

Variable	LRD
Average investment rate	12·2% (0·10)
Inaction rate: investment	8·1% (0·08)
Fraction of observations with negative investment	10·4% (0·09)
Spike rate: positive investment	18·6% (0·12)
Spike rate: negative investment	1·8% (0·04)
Serial correlation of investment rates	0·058 (0·003)
Correlation of profit shocks and investment	0·143 (0·003)

LRD, Longitudinal Research Database.

Data Facts: Investment Rate Distribution

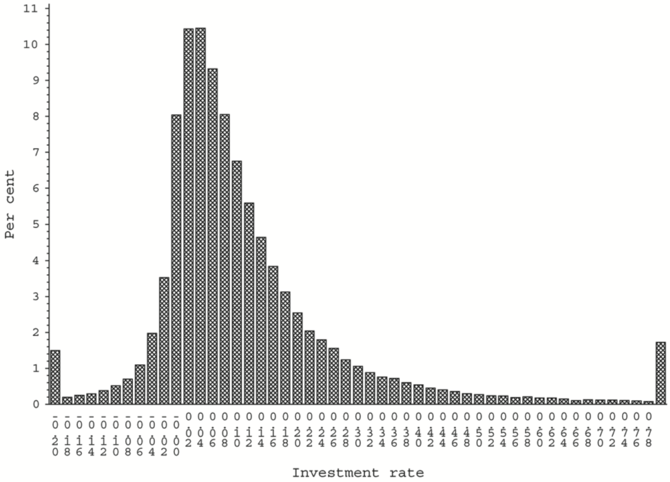

FIGURE 1

Investment rate distribution

Data Facts: Upward Slopping Hazard Function

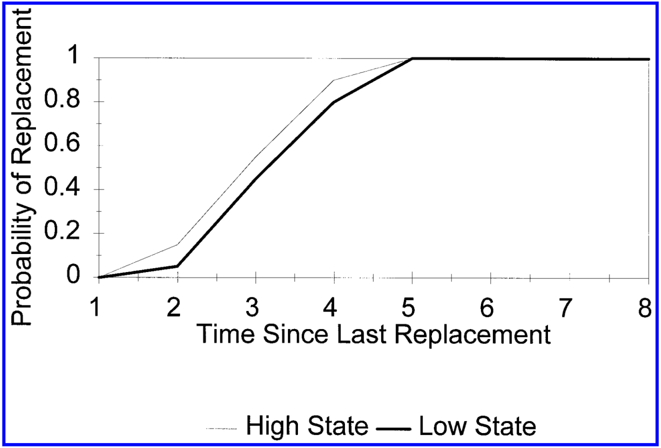

FIGURE 1. THEORETICAL HAZARD FOR MACHINE REPLACEMENT

Model Overview

- Firm optimization with convex and nonconvex costs of adjustment and irreversibility
- Dynamic optimization of choosing capital investment

$$V(A, K) = \max_{K'} \Pi(A, K) - C(I, A, K) + \beta E_{A'|A} V\left(A', K'\right)$$

- Investment: $I = K' - (1 - \delta)K$; Investment Rate: I/K
- Four models of the adjustment process:
    1. no adjustment cost
    2. convex adjustment cost
    3. nonconvex adjustment cost
    4. irreversible investment

Common Elements in the Specification

- Assume $A_{it} = a_t \varepsilon_{it}$ with aggregate and plant-specific shocks
- Profit function is
$$\Pi(A, K) = AK^{\theta}$$
- Estimate θ and shock process using observed capital and calculated profits
- Discount rate $\beta = .95$; depreciation rate $= .069$
- Obviously, there is no need to explain the no adjustment cost model (**No AC**)

Convex Cost of Adjustment Model (CON)

- Adjustment cost:
$$C(I, A, K) = pI + \frac{\gamma}{2}[I/K]^2 K$$
- Optimality
$$i = I/K = (1/\gamma)\left[EV_k\left(A', K'\right) - p\right]$$
- Properties of solution as below:
- No inaction; Bursts from idiosyncratic shocks
- Hazard is downward sloping in contrast to the data
- Not a Q model ($\theta < 1$): Cooper-Ejarque findings apply

Non Convex Cost of Adjustment Model (NC-F) or (NC-λ)

- Similar to what you just learned

$$V(A,K) = \max\left\{V^i(A,K), V^a(A,K)\right\}$$

- Inactive

$$V^i(A,K) = \Pi(A,K) + \beta E_{A'|A} V\left(A', K(1-\delta)\right)$$

- Active

$$V^a(A,K) = \max_I \Pi(A,K)\lambda - FK - I + \beta E_{A'|A} V\left(A', K'\right)$$

- **NC-F**: costs in profit
- **NC-λ**: costs in capital

Non Convex Cost of Adjustment Model (NC-F) or (NC-λ)

- Non-convex costs of adjustment
- Lump sum: $F > 0$
- Opportunity cost: : $\lambda < 1$
- Are bursts more likely in good times or bad?
- Inaction and bursts: stochastic replacement cycle
- Hazard functions: bursts more likely as capital ages

Transactions Costs of Adjustment Model (TRAN)

- Value function:

$$V(A,K) = \max\left\{V^b(A,K), V^s(A,K), V^i(A,K)\right\}$$

- Buying capital

$$V^b(A,K) = \max_{I} \Pi(A,K) - I + \beta E_{A'|A} V\left(A', K(1-\delta) + I\right)$$

- Selling capital

$$V^s(A,K) = \max_{R} \Pi(A,K) + p_s R + \beta E_{A'|A} V\left(A', K(1-\delta) - R\right)$$

- Inaction

$$V^i(A,K) = \Pi(A,K) + \beta E_{A'|A} V\left(A', K(1-\delta)\right)$$

- Gets bursts and inaction

Example Models

TABLE 2

Parameterization of illustrative models

Model	γ	F	λ	p_s	p_b
No AC	0	0	1	1	1
CON	2	0	1	1	1
NC-F	0	0·01	1	1	1
NC-λ	0	0	0·95	1	1
TRAN	0	0	1	0·75	1

TABLE 3

Moments from illustrative models

Moment	LRD	No AC	CON	NC-F	NC-λ	TRAN
Fraction of inaction	0·081	0·0	0·038	0·616	0·588	0·69
Fraction with positive investment bursts	0·18	0·298	0·075	0·212	0·213	0·120
Fraction with negative investment bursts	0·018	0·203	0·0	0·172	0·198	0·024
Corr (i_{it},i_{it-1})	0·058	−0·053	0·732	−0·057	−0·06	0·110
Corr (i_{it}, a_{it})	0·143	0·202	0·692	0·184	0·196	0·346

LRD, Longitudinal Research Database.

SMM Estimation

- Minimization problem

$$f(\Theta) = \min_{\Theta} \left[\Psi^d - \Psi^s(\Theta)\right]' W \left[\Psi^d - \Psi^s(\Theta)\right]$$

- Θ is the vector of parameters
- W is the inverse of the moments variance-covariance matrix
- W can be obtained by bootstrap or direct calculation
- Heterogeneity is taken care of through idiosyncratic shocks
- LRD moments from the first table

Case 1: Estimation with fixed costs: $F > 0, \lambda = 1$

- Fix $\beta = 0.95, \delta = 0.069$
- Estimate $\Pi(A, K) = AK^{\theta}$ at plant-level (logs, quasi-differencing)
- Find $\theta = 0.592$, also est. ρ, σ for agg. and idio. shocks
- Estimate F, γ, p_s using SMM

Case 1: Estimation with fixed costs: $F > 0, \lambda = 1$

TABLE 4

Parameter estimates: $\lambda = 1$

	Structural parameter estimates (S.E.)			Moments				
Spec.	γ	F	p_s	Corr (i, i_{-1})	Corr (i, a)	Spike$^+$	Spike$^-$	£$(\hat{\Theta})$
LRD				0·058	0·143	0·186	0·018	
all	0·049	0·039	0·975	0·086	0·31	0·127	0·030	6399·9
	(0·002)	(0·001)	(0·004)					
γ only	0·455	0	1	0·605	0·540	0·23	0·028	53,182·6
	(0·002)							
p_s only	0	0	0·795	0·113	0·338	0·132	0·033	7673·68
			(0·002)					
F only	0	0·0695	1	−0·004	0·213	0·105	0·0325	7390·84
		(0·00046)						

LRD, Longitudinal Research Database.

Case 2: Estimation with opportunity costs: $F = 0, \lambda < 1$

- Re-estimate $\Pi(\cdot)$ and shocks through indirect inference
- Curvature and parameters of shocks do not change much
- Estimation λ, γ, p_s using SMM
- Dominates $F > 0$ case
- Allowing both λ and F does not improve fit
- Average adjustment cost paid: $= 0.0091K, = 0.031 * \Pi$

Case 2: Estimation with opportunity costs: $F = 0, \lambda < 1$

TABLE 5

Parameter estimates: $F = 0$

	Structural parameter estimates (S.E.)			Moments				
Specification	γ	λ	p_s	Corr (i, i_{-1})	Corr (i, a)	Spike$^+$	Spike$^-$	£$(\hat{\Theta})$
LRD				0·058	0·143	0·186	0·018	
λ only	0	0·796	1·0	−0·009	0·06	0·107	0·042	9384·06
		(0·0040)						
All	0·153	0·796	0·981	0·148	0·156	0·132	0·023	2730·97
	(0·0056)	(0·0090)	(0·0090)					

LRD, Longitudinal Research Database.

Sectoral Results (All Included)

TABLE 6

Sectoral parameter estimates

	Parameter estimates (S.E.)				Moments				
Sector	γ	F	λ	p_s	$\mathrm{corr}(i, i_{-1})$	$\mathrm{corr}(i, a)$	spike^+	spike^-	$£(\hat{\Theta})$
331-LRD					0·086	0·133	0·064	0·02	n/a
331-est.	0·0	0·07	1	0·946	−0·03	0·249	0·076	0·022	62·19
	(0·0003)	(0·0017)		(0·0046)					
	0·015	0	0·70	0·76	0·041	0·146	0·073	0·018	8·13
	(0·0037)		(0·0344)	(0·0167)					
371-LRD					0·082	0·132	0·204	0·013	n/a
371-est.	0·012	0·069	1	0·962	0·069	0·338	0·112	0·037	452·96
	(0·0026)	(0·0057)		(0·0106)					
371-est.	0·051	0	0·679	0·8082	0·251	0·132	0·096	0·027	318·04
	(0·0068)		(0·0398)	(0·0218)					

LRD, Longitudinal Research Database; est., estimates.

Aggregate Implications

- Moments

Data	$c\left(i, i_{-1}\right)$	$c(i, a)$
Plant	0.058	0.143
Agg.	0.46	0.51
best fit	0.63	0.54

- Quad model with $\gamma = 0.195$ matched aggregate simulated data best: $R^2 = 0.859$

Conclusions

- $\gamma > 0, \lambda < 1, p_s < 1$!
- Non-convexity exists

Appendix

References

Cooper, Russell W and John C Haltiwanger (2006). “On the nature of capital adjustment costs”. In: *The Review of Economic Studies* 73.3, pp. 611–633.