Computational Economics Lecture 5: Intro to Iteration Methods II: Finite Periods

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Outline

1. **Motivation**
2. **Finite Periods**
3. **Backward Iteration**
4. **Forward Simulation**
5. **The Idea of MIT Shocks**

Motivation

- We have learned various iteration methods to solve an infinite periods problem
- How about the iteration in finite periods?
  - Some problems are naturally in finite periods (i.e., economic transition)
  - Sometimes, we can hardly solve the problem using forward iteration (i.e., long-term bonds)
- Obviously, we can still use the same methods, i.e., VFI. But what is different?
  - 1. Deal with the starting (same as infinite periods)
  - 2. Deal with the finishing (same as infinite periods)
  - 3. Deal with the transition (exciting new stuff)
- The next question is: What triggers the transition? (The idea of MIT shocks)
- Finite periods are quite useful, although much less taught!
- Please read the handout by Prof. Jesus Fernandez-Villaverde: Lecture on DP

Bellman Equation in the Finite Periods Problem

- Let s_t be the state and $\alpha_t\left(s_t\right)$ be the policy function
- If $T < \infty$, DP is equivalent to backward induction. In the terminal period α_T is:

$$\alpha_T\left(s_T\right) = \arg\max_{a_T \in A(s_T)} u\left(s_T, a_T\right)$$

- And $V_T\left(s_T\right) = u\left(s_T, \alpha_T\left(s_T\right)\right)$.
- For periods $t = 1, \ldots, T-1$, we can find V_t and α_t by recursion:

$$\alpha_t\left(s_t\right) = \arg\max_{a_t \in A(s_t)} \left[u\left(s_t, a_t\right) + \beta \int V_{t+1}\left(s_{t+1}\right) p\left(ds_{t+1} \mid s_t, a_t\right)\right]$$

$$V_t\left(s_t\right) = u\left(s_t, \alpha_t\left(s_t\right)\right) + \beta \int V_{t+1}\left(s_{t+1}\right) p\left(ds_{t+1} \mid s_t, \alpha_t\left(s_t\right)\right)$$

- It could be the case that $a_t = \alpha_t\left(s_t, a_{t-1}, s_{t-1}, \ldots\right)$ depend on the whole history
- But with separability and the Markovian property of p imply that $a_t = \alpha_t\left(s_t\right)$
- It could also be that after long periods, ending conditions matter less and less

Finite Periods Problems

- Problems where there is a terminal condition:
    1. Life cycle models (i.e., OLG)
    2. Investment decisions with an expiration date
    3. Finite games models
- Problems that an initial condition of complex objects is hard to get:
    1. Long-term bond pricing models with forward-looking prices
    2. Portfolio investment models with long-term future returns
- Problems where there is a determined transition:
    1. Emerging economy transition periods to developed economy
- Why are finite time problems nicer in these situations? Backward induction!
- You can think about them as a particular case of multivariate optimization

Initial Value in Finite Periods Problems

- Problems where there is a terminal condition:
    - Usually, the economics of the problem provides natural choices.
    - Example: the final value of an optimal expenditure problem is zero.
    - However, sometimes there are subtle issues.
    - Example: what is the value of dying? And of bequests? OLG.
- Problems that an initial condition of complex objects is hard to get:
    - We still need to guess wisely so we can solve faster
- Problems that from a steady state to a (new) steady state:
    - The initial value is just the steady state we solved in the infinite periods

Backward Iteration (with VFI)

- We begin with the Bellman operator:

$$\Gamma\left(V^{t}\right)(s)=\max_{a \in A(s)}\left[u(s, a)+\beta \int V^{t'}\left(s'\right) p\left(d s' \mid s, a\right)\right]$$

- Specify V^T and apply Bellman operator:

$$V^{T-1}(s)=\max_{a \in A(s)}\left[u(s, a)+\beta \int V^{T}\left(s'\right) p\left(d s' \mid s, a\right)\right]$$

- Iterate until the first period:

$$V^{1}(s)=\max_{a \in A(s)}\left[u(s, a)+\beta \int V^{2}\left(s'\right) p\left(d s' \mid s, a\right)\right]$$

- Or iterate until the value function convergence (if mimicking infinite periods):

$$V^{t}(s)-V^{t+1}(s) \leq \epsilon$$

where t could be negative, as small as you need!

Forward Simulation

- After backward iterations, we solve for the value and policy functions $\{V^t(s), \alpha_t\}_{t=t_0}^T$
- But we have not solved the economic allocation, i.e., $\{s_t, c_t\}_{t=t_0}^T$
- Therefore, we need to do a forward simulation starting from some initial states
- In the end, we would have solved all the policy functions and allocations together
- This is particularly useful when we have heterogeneous firms
- You need to simulate the transition of distribution only once!

The Idea of MIT Shocks

- An MIT shock is a "surprise shock" introduced in a deterministic model that happens a single time, and no agent in that model doubts that the model remains deterministic after the shock. Rumor guesses that it is named based on Tom Sargent's comments.
- And Kurt MIT"-shock-"man and his coauthors write an important paper about it on JEDC: Exploiting MIT shocks in heterogeneous-agent economies, JEDC, 2018
- With an MIT shock, we could simulate transition dynamics of the economy using a finite period method: Starting from a steady state and ending with a steady state
- This helps to reduce the dimension of the aggregate state
- It is widely used in macro and certainly could be used in IO/Public for any kind of aggregate regulation changes or public policy changes
- We will discuss it in detail in our practice next Monday!
- We will play with "Project1-Fang2023Nonconvex" on GitHub
- The paper is a modified version of Khan and Thomas, ECTA, 2008
- But much easier to solve because of the MIT shock setup!