

# Large Language Models in Economics

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## **The revolution of LLMs**

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## What is a LLM?

- ChatGPT, a chatbot built on top of the GPT LLM released on November 28, 2022, has popularized deep learning models trained with a text corpus.
- Language models learn a probability distribution over language:

$$P(w_1, \dots, w_m)$$

For example, what is the most likely word after “European Central” in an article at the FT?

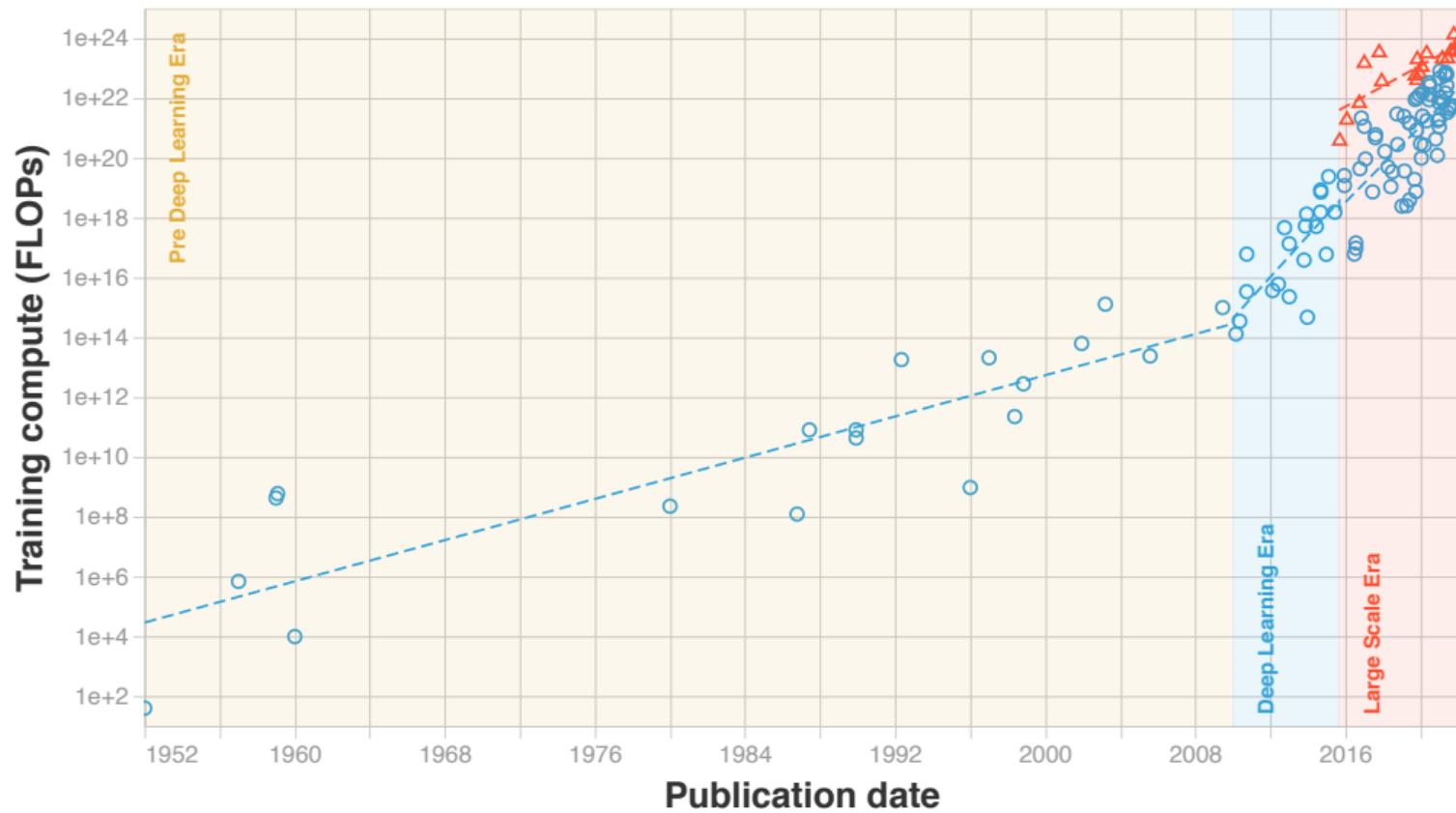
- A language model can use many different probability structures and not necessarily a deep neural networks (even if the latter have gained much popularity).
- Large in terms of training data (e.g., Common Crawl, Wikipedia, GitHub, ...) and parameters (e.g., PaLM has 540 billion parameters; GPT-4 rumored to have 1 trillion).

Dataset	Sampling prop.	Epochs	Disk size
CommonCrawl	67.0%	1.10	3.3 TB
C4	15.0%	1.06	783 GB
Github	4.5%	0.64	328 GB
Wikipedia	4.5%	2.45	83 GB
Books	4.5%	2.23	85 GB
ArXiv	2.5%	1.06	92 GB
StackExchange	2.0%	1.03	78 GB

**Table 1: Pre-training data.** Data mixtures used for pre-training, for each subset we list the sampling proportion, number of epochs performed on the subset when training on 1.4T tokens, and disk size. The pre-training runs on 1T tokens have the same sampling proportion.

# Training compute (FLOPs) of milestone Machine Learning systems over time

n = 118



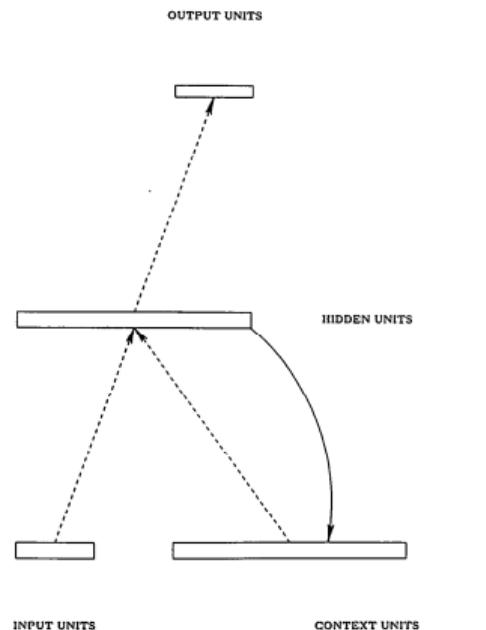
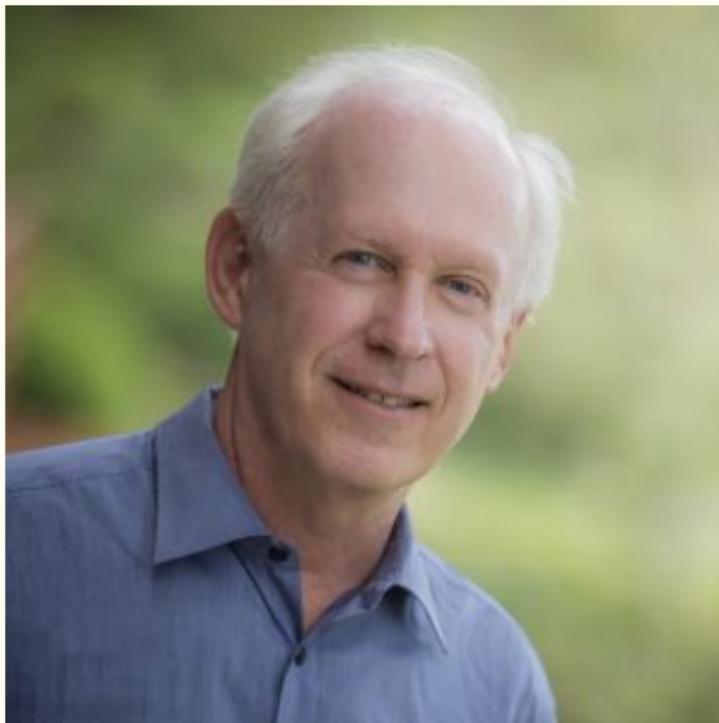
# Training compute (FLOPs) of milestone Machine Learning systems over time

n = 99



# Location, location, location

- Original contribution by **Elman (1990)**: **Finding Structure in Time**.
- **Key idea**: exploit the location of words within a text.



**Figure 2.** A simple recurrent network in which activations are copied from hidden layer to context layer on a one-for-one basis, with fixed weight of 1.0. Dotted lines represent trainable connections.

# The transformers

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- Why now?
- Conjunction of:
  1. A pathbreaking algorithmic revolution: transformer models based on self-attention (December 2017).
  2. GPUs: attention multiheads can run on separate GPUs openings.
  3. We have learned that we want to train LLMs according to power laws linking complexity and data.  
**Hoffman et al., 2022:** for every doubling of model size the number of training tokens should also be doubled.
- This is the reason behind the “T” in GPT (generative pre-trained transformer).

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# Attention Is All You Need

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## 454.69 USD

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+391.97 (624.95%) ↑ past 5 years

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After hours 456.54 +1.85 (0.41%)

1D | 5D | 1M | 6M | YTD | 1Y | 5Y | Max



Open

465.83

Mkt cap

1.12T

CDP score

B

High

480.88

P/E ratio

236.30

52-wk high

480.88

Low

450.60

Div yield

0.035%

52-wk low

108.13

# Training Compute-Optimal Large Language Models

Jordan Hoffmann\*, Sebastian Borgeaud\*, Arthur Mensch\*, Elena Buchatskaya, Trevor Cai, Eliza Rutherford,  
Diego de Las Casas, Lisa Anne Hendricks, Johannes Welbl, Aidan Clark, Tom Hennigan, Eric Noland,  
Katie Millican, George van den Driessche, Bogdan Damoc, Aurelia Guy, Simon Osindero, Karen Simonyan,  
Erich Elsen, Jack W. Rae, Oriol Vinyals and Laurent Sifre\*

\*Equal contributions

We investigate the optimal model size and number of tokens for training a transformer language model under a given compute budget. We find that current large language models are significantly under-trained, a consequence of the recent focus on scaling language models whilst keeping the amount of training data constant. By training over 400 language models ranging from 70 million to over 16 billion parameters on 5 to 500 billion tokens, we find that for compute-optimal training, the model size and the number of training tokens should be scaled equally: for every doubling of model size the number of training tokens should also be doubled. We test this hypothesis by training a predicted compute-optimal model, *Chinchilla*, that uses the same compute budget as *Gopher* but with 70B parameters and 4 $\times$  more data. *Chinchilla* uniformly and significantly outperforms *Gopher* (280B), GPT-3 (175B), Jurassic-1 (178B), and Megatron-Turing NLG (530B) on a large range of downstream evaluation tasks. This also means that *Chinchilla* uses substantially less compute for fine-tuning and inference, greatly facilitating downstream usage. As a highlight, *Chinchilla* reaches a state-of-the-art average accuracy of 67.5% on the MMLU benchmark, greater than a 7% improvement over *Gopher*.

## Three kinds of LLM

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- Generic language models: predicting the next token. I will center on this one type.
- Instruction tuned.
- Dialog tuned: ChatGPT (the base model is hard to interact with).

## The uses of LLM

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- All three types share that they are trained to tackle text-based tasks:
  1. Text classification.
  2. Text summarization (including sentiment analysis).
  3. Text generation (including translation and coding).
  4. Questions/Answers.
  5. Common sense reasoning.
- Because of these capabilities, we can consider LLMs as a part of generative AI: models capable of generating new content.
- This is the reason behind the “G” in GPT (generative pre-trained transformer).

- Some authors are even talking about foundation models: instead of multiple pipelines for each task, we have a common one.
- Key reason: embedding.
- Adapted models and pluggings.
- Emerging properties we do not fully understand:
  1. For example, LLMs seem to have a theory of the mind.
  2. Related to old ideas in F.A. Hayek's **The Sensory Order**.

## Data

Text



Images



Speech



Structured Data



3D Signals



Dialog



Video



Control



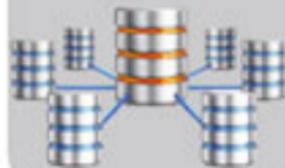
## Training



## Foundation Model



## Search Engine



few-shot  
prompting

## Tasks

Question  
Answering



Sentiment



Information  
Extraction



Image  
Captioning



Object  
Recognition



Instruction  
Following



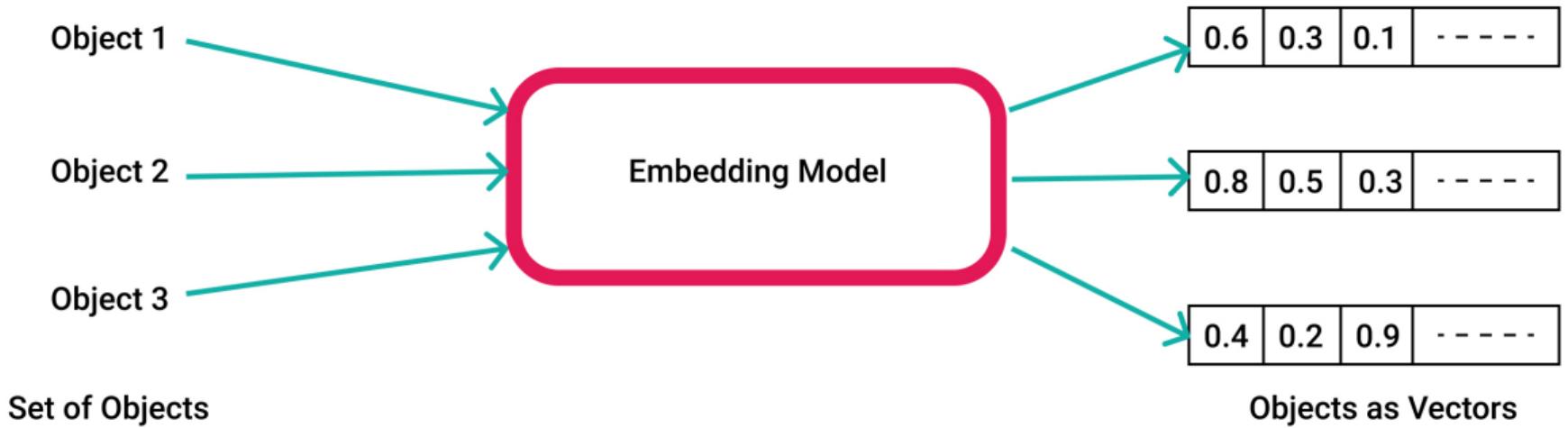
Image  
Generation



Video  
Creation



fine-tuning





THE COLLECTED WORKS OF  
**F·A·HAYEK**

VOLUME

14

*THE SENSORY  
ORDER*

*and Other Writings on  
the Foundations of  
Theoretical Psychology*

Edited by

**Viktor J. Vanberg**

## Foundation models II

- How far away are we from human-level artificial general intelligence (AGI)? (what is intelligence anyway?).
- Questions:
  1. Hallucinations?
  2. Safety vs. accuracy?
  3. Existential risk from AI?
- <https://munkdebates.com/debates/artificial-intelligence>

# On the Opportunities and Risks of Foundation Models

Rishi Bommasani\* Drew A. Hudson Ehsan Adeli Russ Altman Simran Arora  
Sydney von Arx Michael S. Bernstein Jeannette Bohg Antoine Bosselut Emma Brunskill  
Erik Brynjolfsson Shyamal Buch Dallas Card Rodrigo Castellon Niladri Chatterji  
Annie Chen Kathleen Creel Jared Quincy Davis Dorottya Demszky Chris Donahue  
Moussa Doumbouya Esin Durmus Stefano Ermon John Etchemendy Kawin Ethayarajh  
Li Fei-Fei Chelsea Finn Trevor Gale Lauren Gillespie Karan Goel Noah Goodman  
Shelby Grossman Neel Guha Tatsunori Hashimoto Peter Henderson John Hewitt  
Daniel E. Ho Jenny Hong Kyle Hsu Jing Huang Thomas Icard Saahil Jain  
Dan Jurafsky Pratyusha Kalluri Siddharth Karamcheti Geoff Keeling Fereshte Khani  
Omar Khattab Pang Wei Koh Mark Krass Ranjay Krishna Rohith Kuditipudi  
Ananya Kumar Faisal Ladhak Mina Lee Tony Lee Jure Leskovec Isabelle Levent  
Xiang Lisa Li Xuechen Li Tengyu Ma Ali Malik Christopher D. Manning  
Suvir Mirchandani Eric Mitchell Zanele Munyikwa Suraj Nair Avanika Narayan  
Deepak Narayanan Ben Newman Allen Nie Juan Carlos Niebles Hamed Nilforoshan  
Julian Nyarko Giray Ogut Laurel Orr Isabel Papadimitriou Joon Sung Park Chris Piech  
Eva Portelance Christopher Potts Aditi Raghunathan Rob Reich Hongyu Ren  
Frieda Rong Yusuf Roohani Camilo Ruiz Jack Ryan Christopher Ré Dorsa Sadigh  
Shiori Sagawa Keshav Santhanam Andy Shih Krishnan Srinivasan Alex Tamkin  
Rohan Taori Armin W. Thomas Florian Tramèr Rose E. Wang William Wang Bohan Wu  
Jiajun Wu Yuhuai Wu Sang Michael Xie Michihiro Yasunaga Jiaxuan You Matei Zaharia  
Michael Zhang Tianyi Zhang Xikun Zhang Yuhui Zhang Lucia Zheng Kaitlyn Zhou  
Percy Liang<sup>\*1</sup>

Center for Research on Foundation Models (CRFM)  
Stanford Institute for Human-Centered Artificial Intelligence (HAI)  
Stanford University

## General vs. specialized LLM

- Either for general purpose or specialized corpora of documents.
- You can pre-train the LLM in a large dataset and adapt it to a smaller corpus (even zero-shot learning).
  - For example, all documents within the Fed, all the NBER working papers, all articles at the FT.
- This is the reason behind the “P” in GPT (generative pre-trained transformer).
- Parameter-efficient fine-tuning methods, prompt training, and supervised learning.
- **Key idea:** Transduction (particular→particular) vs. induction (particular→general→particular).
- Related to the failure of the project of building a universal formal grammar in the 1970s (we will return to this point later on).

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# Language Models are Few-Shot Learners

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**Tom B. Brown\***

**Benjamin Mann\***

**Nick Ryder\***

**Melanie Subbiah\***

**Jared Kaplan<sup>†</sup>**

**Prafulla Dhariwal**

**Arvind Neelakantan**

**Pranav Shyam**

**Girish Sastry**

**Amanda Askell**

**Sandhini Agarwal**

**Ariel Herbert-Voss**

**Gretchen Krueger**

**Tom Henighan**

**Rewon Child**

**Aditya Ramesh**

**Daniel M. Ziegler**

**Jeffrey Wu**

**Clemens Winter**

**Christopher Hesse**

**Mark Chen**

**Eric Sigler**

**Mateusz Litwin**

**Scott Gray**

**Benjamin Chess**

**Jack Clark**

**Christopher Berner**

**Sam McCandlish**

**Alec Radford**

**Ilya Sutskever**

**Dario Amodei**

OpenAI



Statistics for  
Engineering and  
Information Science

Vladimir N. Vapnik

The Nature  
of Statistical  
Learning Theory

Second Edition



## Chatbots vs. APIs

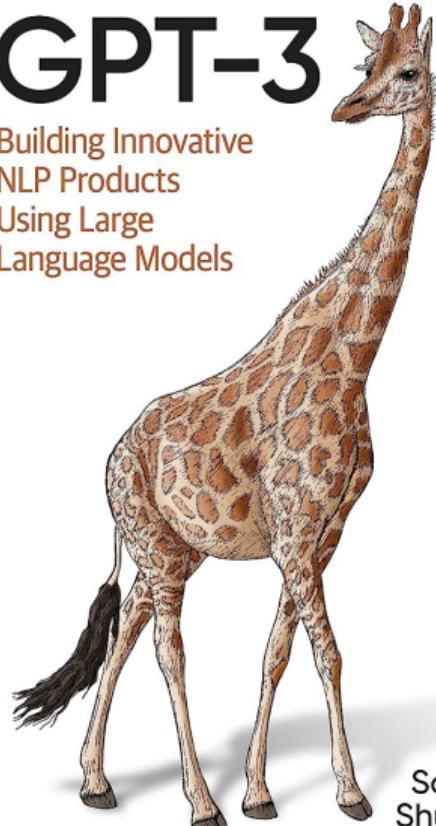
- You might have used the chatbot for ChatGPT. This is the reason behind the “Chat” in ChatGPT (chatbot for generative pre-trained transformer).
- However, for more systematic research, one can use APIs (application programming interfaces) and focus on prompt design:

```
1 import openai
2
3 openai.ChatCompletion.create(
4     model="gpt-3.5-turbo",
5     messages=[
6         {"role": "system", "content": "You are a helpful assistant."},
7         {"role": "user", "content": "Who won the world series in 2020?"},
8         {"role": "assistant", "content": "The Los Angeles Dodgers won the World Series in 2020."}
9         {"role": "user", "content": "Where was it played?"}
10    ]
11 )
```

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# GPT-3

Building Innovative  
NLP Products  
Using Large  
Language Models



Sandra Kublik &  
Shubham Saboo

1. ChatGPT: designed for chatbots and conversational AI.
2. Llama 2: best open source model, trained on 1-1.4T tokens.
3. Bard: Google.
4. LangChain: designed for translation.
5. Cohere: designed for text classification, summarization, and sentiment analysis.
6. Many others.

Large language model

# Llama 2: open source, free for research and commercial use

We're unlocking the power of these large language models. Our latest version of Llama – Llama 2 – is now accessible to individuals, creators, researchers, and businesses so they can experiment, innovate, and scale their ideas responsibly.

[Download the Model](#)



huggingface.co/spaces/HuggingFaceH4/open\_llm\_leaderboard

**Spaces** HuggingFaceH4 / **open\_llm\_leaderboard** like 4.07k Running App Files Community 159

## Open LLM Leaderboard

The Open LLM Leaderboard aims to track, rank and evaluate LLMs and chatbots as they are released.

任何人都可以从社区提交模型进行自动评估，只要它是带有权重的 🤗 Transformers 模型。我们还支持对非商业授权模型的 delta-weights 评估，例如原始的 LLaMa 发布。

其他酷炫的 LLM 测量指标在 HuggingFace 上开发，去看看吧：👉👉 [human and GPT4 evals](#), 🚦 [performance benchmarks](#)

Base pretrained model – Finetuned model – Model using RL (read more details in "About" tab)

LLM Benchmark About Submit here!

Select columns to show

<input checked="" type="checkbox"/> Average	<input checked="" type="checkbox"/> ARC	<input checked="" type="checkbox"/> HellaSwag	<input checked="" type="checkbox"/> MMLU	<input checked="" type="checkbox"/> TruthfulQA
<input type="checkbox"/> Type	<input type="checkbox"/> Hub License	<input type="checkbox"/> #Params (B)	<input type="checkbox"/> Hub	<input type="checkbox"/> Model sha

Search for your model and press ENTER...

Filter model types

<input checked="" type="radio"/> all	<input type="radio"/>  base	<input type="radio"/>  finetuned	<input type="radio"/>  RL-tuned
--------------------------------------	---	---	--

T	Model	Average	ARC	HellaSwag	MMLU	TruthfulQA
	<a href="#">upstage/Llama-2-70b-instruct-v2</a>	73	71.1	87.9	70.6	62.2
◆	<a href="#">upstage/Llama-2-70b-instruct</a>	72.3	70.9	87.5	69.8	61
◆	<a href="#">stabilityai/StableBeluga2</a>	71.4	71.1	86.4	68.8	59.4
◆	<a href="#">automa/qCamel-70-x</a>	71	68.3	87.9	70.2	57.5
◆	<a href="#">jondurbin/airoboros-12-70b-gpt4-1.4.1</a>	70.9	70.4	87.8	70.3	55.2
◆	<a href="#">TheBloke/llama-2-70b-Guanaco-QLoRA-fp16</a>	70.6	68.3	88.3	70.2	55.7
◆	<a href="#">upstage/llama-65b-instruct</a>	70	68.9	86.4	64.8	59.7
◆	<a href="#">stabilityai/StableBeluga1-Delta</a>	68.7	68.2	85.9	64.8	55.8

## Leaderboard

[Vote](#) | [Blog](#) | [GitHub](#) | [Paper](#) | [Dataset](#) | [Twitter](#) | [Discord](#)

 This leaderboard is based on the following three benchmarks.

- [Chatbot Arena](#) - a crowdsourced, randomized battle platform. We use 50K+ user votes to compute Elo ratings.
  - [MT-Bench](#) - a set of challenging multi-turn questions. We use GPT-4 to grade the model responses.
  - [MMLU](#) (5-shot) - a test to measure a model's multitask accuracy on 57 tasks.

Code: The Arena Elo ratings are computed by this [notebook](#). The MT-bench scores (single-answer grading on a scale of 10) are computed by [fastchat.llm\\_judge](#). The MMLU scores are computed by [InstructEval](#) and [Chain-of-Thought Hub](#). Higher values are better for all benchmarks. Empty cells mean not available.

Model	⭐ Arena Elo rating	📈 MT-bench (score)	MMLU	License
GPT-4	1206	8.99	86.4	Proprietary
Claude-1	1166	7.9	77	Proprietary
Claude-instant-1	1138	7.85	73.4	Proprietary
Claude-2	1135	8.06	78.5	Proprietary
GPT-3.5-turbo	1122	7.94	70	Proprietary
Vicuna-33B	1096	7.12	59.2	Non-commercial
Vicuna-13B	1051	6.57	55.8	Llama 2 Community
MPT-30B-chat	1046	6.39	50.4	CC-BY-NC-SA-4.0
WizardLM-13B-v1.1	1040	6.76	50	Non-commercial
Guanaco-33B	1038	6.53	57.6	Non-commercial
PaLM-Chat-Bison-001	1015	6.4		Proprietary
Vicuna-7B	1006	6.17	49.8	Llama 2 Community

## **On the role of LLMs in economics**

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## How can LLMs change my workflow?

- **Text as data** by M. Gentzkow, B.T. Kelly, and M. Taddy: general introductory survey.
- **Text algorithms in economics** by E. Ash and S. Hansen: general introductory survey.
- **A User's Guide to GPT and LLMs for Economic Research** by K. Bryan: examples of how to use LLM in your daily research.
- Second half of <https://youtu.be/bZQun8Y4L2A> by A. Karpathy: nice tricks for good prompting.
- **Language Models and Cognitive Automation for Economic Research** by A. Korinek: application of LLM for ideation, writing, background research, data analysis, coding, and mathematical derivations.

## How can I apply LLMs to learn about the economy?

- Hedonic prices and quality-adjusted price indices powered by AI by P. Bajari *et al.*: use product description text to predict product prices.
- Bloated Disclosures: Can ChatGPT Help Investors Process Financial Information? by A. Kim, M. Muhn, and V. Nikolaev: probe the economic usefulness of LLMs in summarizing complex corporate disclosures using the stock market as a laboratory.
- Asset Embeddings by X. Gabaix, R.S.J. Koijen, and M. Yogo: learn asset embeddings from investors' holdings data.
- Work2vec: Using language models to understand wage premia by S.H. Bana: uncover the premia associated with eight in-demand certifications.
- Out of One, Many: Using Language Models to Simulate Human Samples by L.P. Argyle *et al.*: using LLMs to synthesize data from undersample populations.

## What are the effects of LLMs on the economy? (positive and normative)

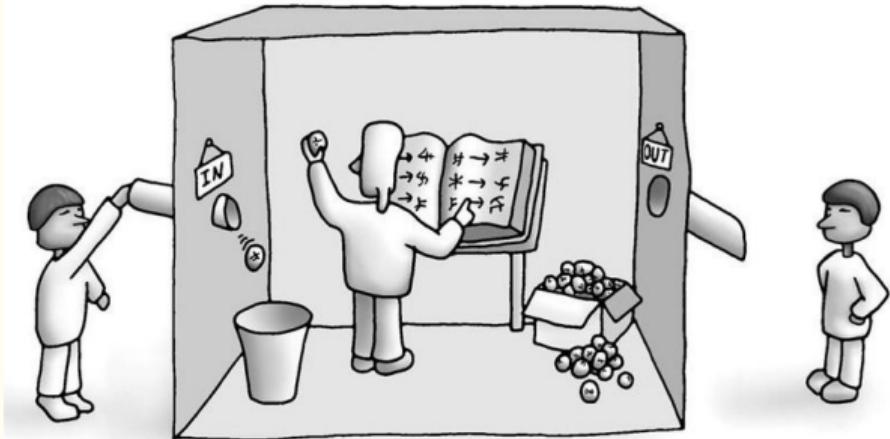
- Large Language Models as Simulated Economic Agents: What Can We Learn from Homo Silicus? by J.J. Horton: LLM as an approximation of bounded-rational agents.
- Economics, Hayek, and Large Language Models by T. Cowen: a podcast about how LLM might change our conception of how economies work.
- Generative AI at Work by E. Brynjolfsson, D. Li, and L.R. Raymond.
- Preparing for the (Non-Existent?) Future of Work by A. Korinek and M. Juelfs.
- GPTs are GPTs: An Early Look at the Labor Market Impact Potential of Large Language Models by T. Eloundou, S. Manning, P. Mishkin, and D. Rock.
- Regulating Transformative Technologies by D. Acemoglu and T. Lensman.
- Power and Progress by D. Acemoglu and S. Johnson.

## Natural language processing

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# Natural language processing

- Natural language processing (NLP): field specialized in how computers can deal with language as it appears in “natural” contexts (speech, text, ...).
- One of the very first applications of computers: Georgetown-IBM experiment in automatic translation in 1954.
- Classical NLP was based on symbolic rules (John Searle's Chinese room experiment and ELIZA) and Chomskyan theories of linguistics.
  - After some early success, the field stagnated.
- In comparison, modern NLP is built around statistical models.
  - Base of its recent success.



Welcome to

EEEEEE	LL	IIII	ZZZZZZ	AAAAAA
EE	LL	II	ZZ	AA AA
EEEEEE	LL	II	ZZZ	AAAAAAA
EE	LL	II	ZZ	AA AA
EEEEEE	LLLLL	IIII	ZZZZZZ	AA AA

Eliza is a mock Rogerian psychotherapist.

The original program was described by Joseph Weizenbaum in 1966.  
This implementation by Norbert Landsteiner 2005.

ELIZA: Is something troubling you ?

YOU: Men are all alike.

ELIZA: What is the connection, do you suppose ?

YOU: They're always bugging us about something or other.

ELIZA: Can you think of a specific example ?

YOU: Well, my boyfriend made me come here.

ELIZA: Is it important to you that your boyfriend made you come here ?

YOU: He says I'm depressed much of the time.

ELIZA: I am sorry to hear that you are depressed.

YOU: It's true. I am unhappy.

ELIZA: Can you explain what made you unhappy ?

YOU:



# Modern language models refute Chomsky's approach to language

Steven T. Piantadosi<sup>a,b</sup>

<sup>a</sup>UC Berkeley, Psychology <sup>b</sup>Helen Wills Neuroscience Institute

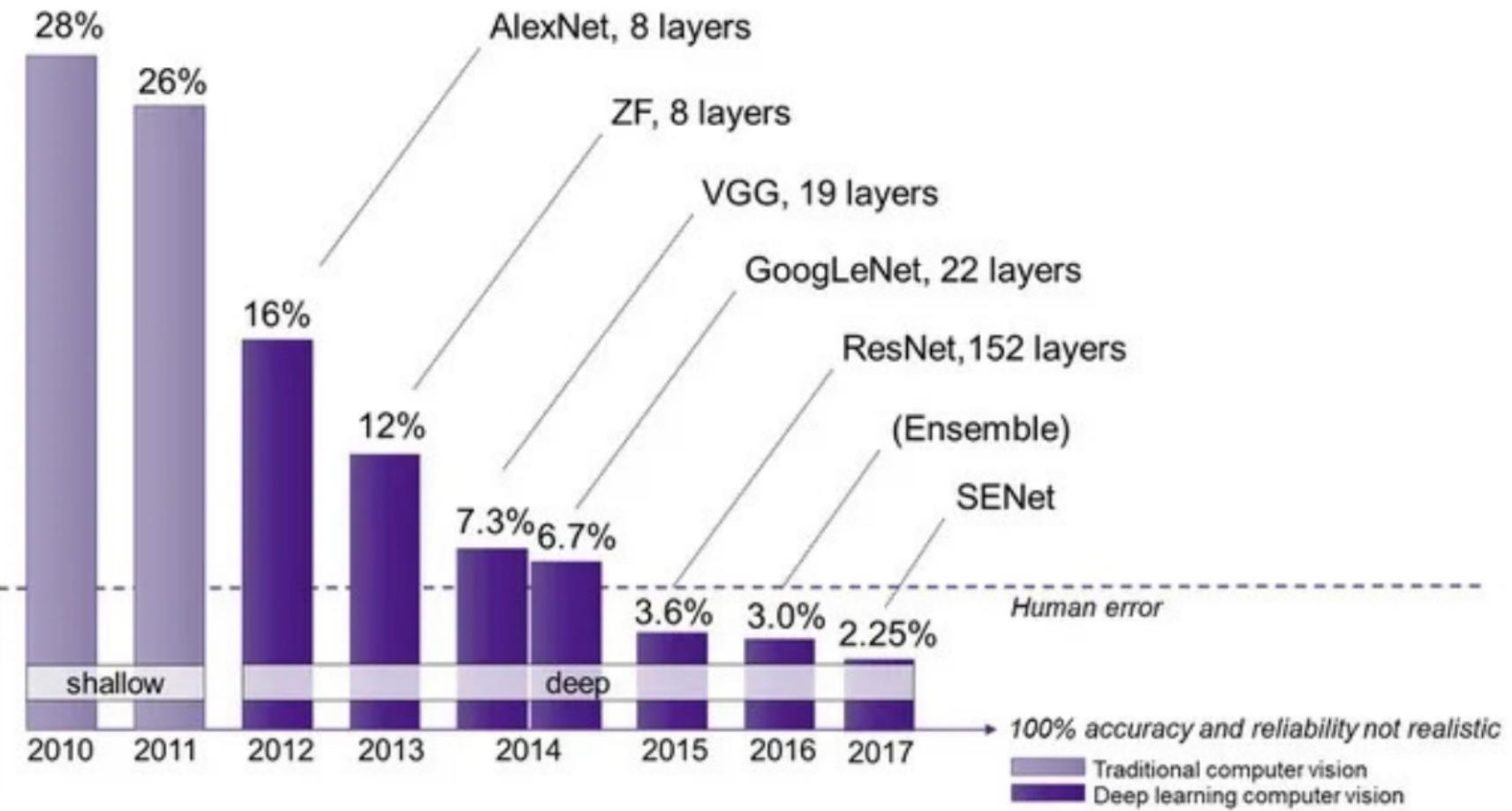
The rise and success of large language models undermines virtually every strong claim for the innateness of language that has been proposed by generative linguistics. Modern machine learning has subverted and bypassed the entire theoretical framework of Chomsky's approach, including its core claims to particular insights, principles, structures, and processes. I describe the sense in which modern language models implement genuine *theories* of language, including representations of syntactic and semantic structure. I highlight the relationship between contemporary models and prior approaches in linguistics, namely those based on gradient computations and memorized constructions. I also respond to several critiques of large language models, including claims that they can't answer "why" questions, and skepticism that they are informative about real life acquisition. Most notably, large language models have attained remarkable success at discovering grammar without using any of the methods that some in linguistics insisted were necessary for a science of language to progress.

## The transformer model

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## The transformer model

- Deep convolutional neural networks, introduced in 2012, greatly impacted computer vision.
- But NLP (at the time, built around RNN and CNN) had lagged.
- **Vaswani et al. (2017)**: [Attention Is All You Need](#). Group of researchers affiliated with Google.
- 83,844 Google Scholar citations as of August 2, 2023.
- Transformers applied to other fields outside natural language processing (Visual transformers, DALL-E). In fact, anything that is set-to-set.
- Built around two ideas:
  1. (Self-)Attention.
  2. Encoder/decoder structure.

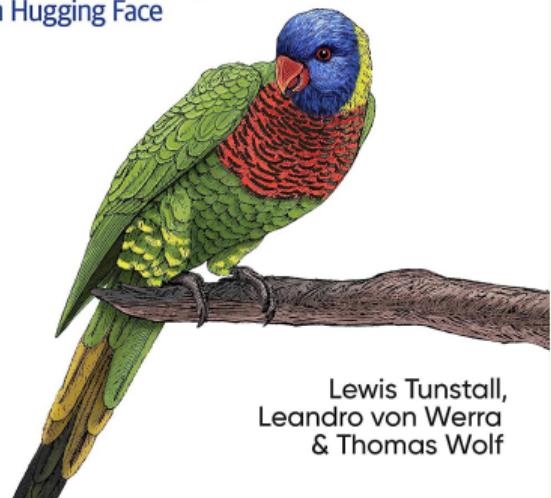


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Revised  
Edition

# Natural Language Processing with Transformers

Building Language Applications  
with Hugging Face



Lewis Tunstall,  
Leandro von Werra  
& Thomas Wolf



## TRANSFORMERS FOR MACHINE LEARNING A Deep Dive

Uday Kamath  
Kenneth L. Graham  
Wael Emara

 CRC Press  
Taylor & Francis Group  
A CHAPMAN & HALL BOOK

Artificial Intelligence: Foundations, Theory, and Algorithms

Gerhard Paß  
Sven Giesselbach

## Foundation Models for Natural Language Processing

Pre-trained Language Models  
Integrating Media

OPEN ACCESS

 Springer

## Steps to build a transformer model

1. Formalizing text (see block 11).
2. Text wrangling (see block 11).
3. Tokenization.
4. Embedding.
5. Attention.
6. Output.
7. Training.
8. Extensions.

## **Step III: Tokenization**

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## Tokenization

- Tokenization is splitting a raw character string into useful semantic pieces for processing called tokens.
- For example, we chop the string of characters:

“The European Central Bank is in Frankfurt”

into

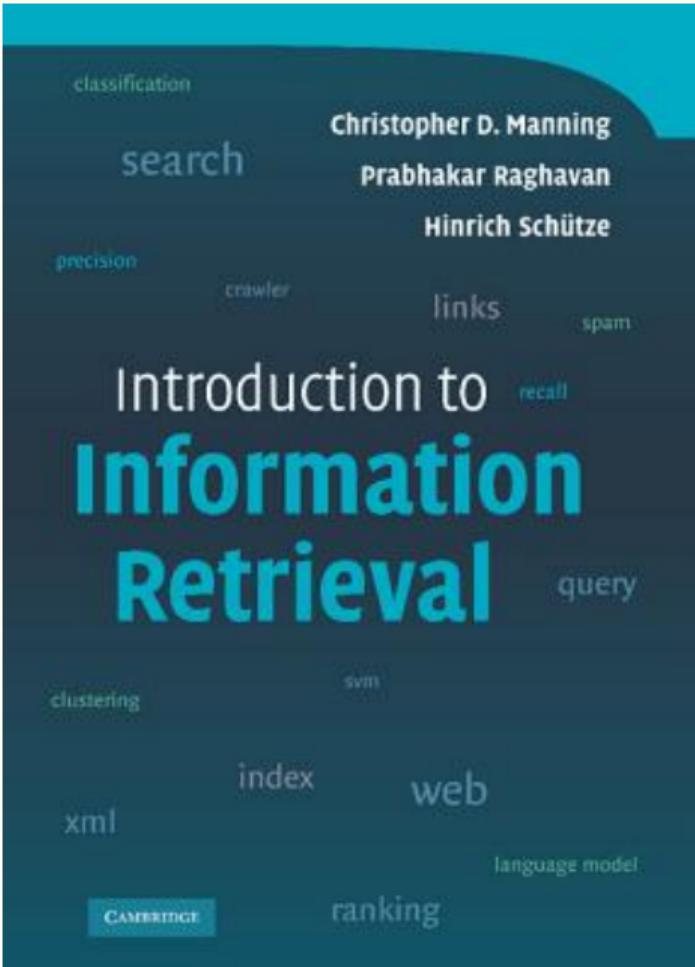
“The”, “European”, “Central”, “Bank”, “is”, “in”, “Frankfurt”.

- Often, tokens are words, but there may be characters, numbers, punctuation, and white spaces.
- Simple rules work well, but not perfectly. For example, splitting on white space and punctuation will separate hyphenated phrases, as in “risk-averse agent” and contractions, as in “aren’t”.
- While, in practice, one uses a specialized library for tokenization, it is important to understand tokenization in some more detail.

## Vocabularies

---

- Tokenization relies on a vocabulary: a list of all allowed tokens.
- Oxford English dictionary  $\approx$  170k words in current use (vs. more than one mil ever used).
- We take advantage of that, in practice, we only use around 40k words (with a clear Zipf's law distribution). Other words are mapped into the 40k or masked as unknown.
- For specialized LLM, we might want to have specific vocabularies.
- How?
  1. Domain knowledge.
  2. Stop-words removal.
  3. Linguistic roots.
  4. Multi-word phrases.



## Tokenization in GPT-3

- GPT-3 tokenizer here: <https://platform.openai.com/tokenizer>.
- GPT-3 uses byte pair encoding (<https://github.com/openai/tiktoken>):
  1. Common words are a single token, less frequent words are represented by multiple tokens:  
“Enconding” is tokenized as “Enc” and “oding”.
  2. Odd words are dropped.
- We assign every token an ID from a vocabulary with a total of 50257 tokens. For memory reasons, one may want to cap the vocabulary at  $2^{16} = 65536$  tokens.
- Example: “European Central Bank” → “European”, “Central”, “Bank” → [22030, 5694, 5018].
- More precisely, we represent each integer as a one-hot vector  $w_{1 \times 50227}$  with a 1 in the corresponding entry.

## **Step IV: Embedding**

---

# Embedding

- In natural language, words bundle in predictable patterns:

$$P(\text{Bank}|\text{European} + \text{Central}) \gg 0$$

but

$$P(\text{Giraffe}|\text{European} + \text{Central}) \approx 0$$

- This means we can use probabilities to generate predictions.
- We can capture this idea with an embedding: a representation of a token as a vector.
- We can estimate static embeddings with a simple logistic classifier (Word2vec).
- Useful for tasks such as document classification or sentiment analysis.
- However, static embeddings are not powerful enough for many interesting problems.
- We want more complex models that can incorporate contextual information.

## Contextual embedding into vectors

- We take each token and embed it into a dense  $n$ -dim vector, to which we will add some context information.
- Why do we do this?
  1. Dimensionality reduction.
  2. More importantly: projection into a more informative space (interpretability?).
- Also, we usually do this in blocks of tokens: it will train the transformer to make predictions within the block.

## Embedding in GPT-3

- GPT-3 uses input blocks of  $m = 2048$  tokens (even if it needs to leave space empty):  $B_{2048 \times 50227}$  where each row is one token and a  $12288$ -dim embedding.
- More concretely, we get a sequence-embeddings matrix:

$$E_{2048 \times 12228} = B_{2048 \times 50227} * W_{50227 \times 12228}^E$$

where  $W_{50227 \times 12228}^E$  is an embedding weight matrix (we will see later how we pick it).

- For example:

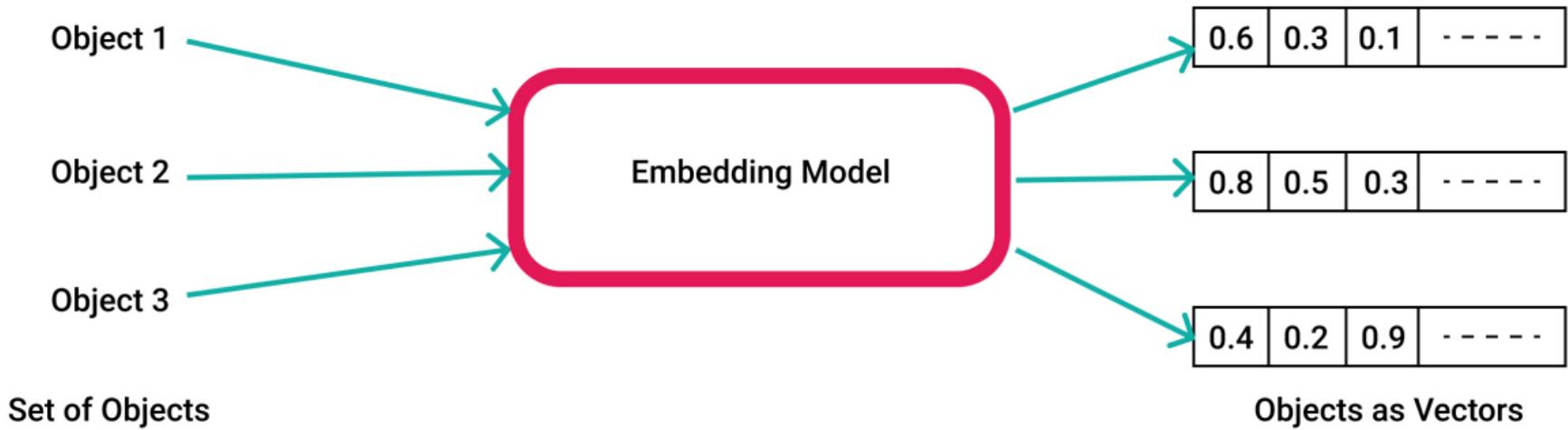
$$\text{"European"} = [0.01, -0.99, \dots, 0.34, 0.12]$$

## Why is embedding key?

- Since we have a vector representation for each token, we can define standard vector operations by looking at the closest embedding:
  - Sum:  $\text{Bank} = \text{European} + \text{Central}$

$$[0.03, -0.9, \dots, 0.42, 0.36] \approx [0.01, -0.99, \dots, 0.34, 0.12] + [0.02, 0.09, \dots, 0.08, 0.24]$$

- Subtraction:  $\text{Frankfurt} = \text{European Commission} + \text{Brussels} - \text{European Central Bank.}$
- We can map any piece of information into an  $n$ -dimension vector. Whether there are tokens from text, pixels from photographs, Fourier weights from a recording, etc, is irrelevant → foundation models.



## Encoding

- We mentioned before we want to incorporate context into our embedding.
- Distributional semantics: “A word is characterized by the company it keeps” ([Firth, 1957](#)).
- Think about the sentence: “I seat in the bank inside the bank office by the river bank where you bank.”
- We capture these relations by looking at the position of a token within a block: encoding.
- Quite different ways to do it, language-dependent (i.e., compare English, an analytic language, with Latin, a synthetic one!)



JOHN RUPERT FIRTH

## Positional encoding in GPT-3

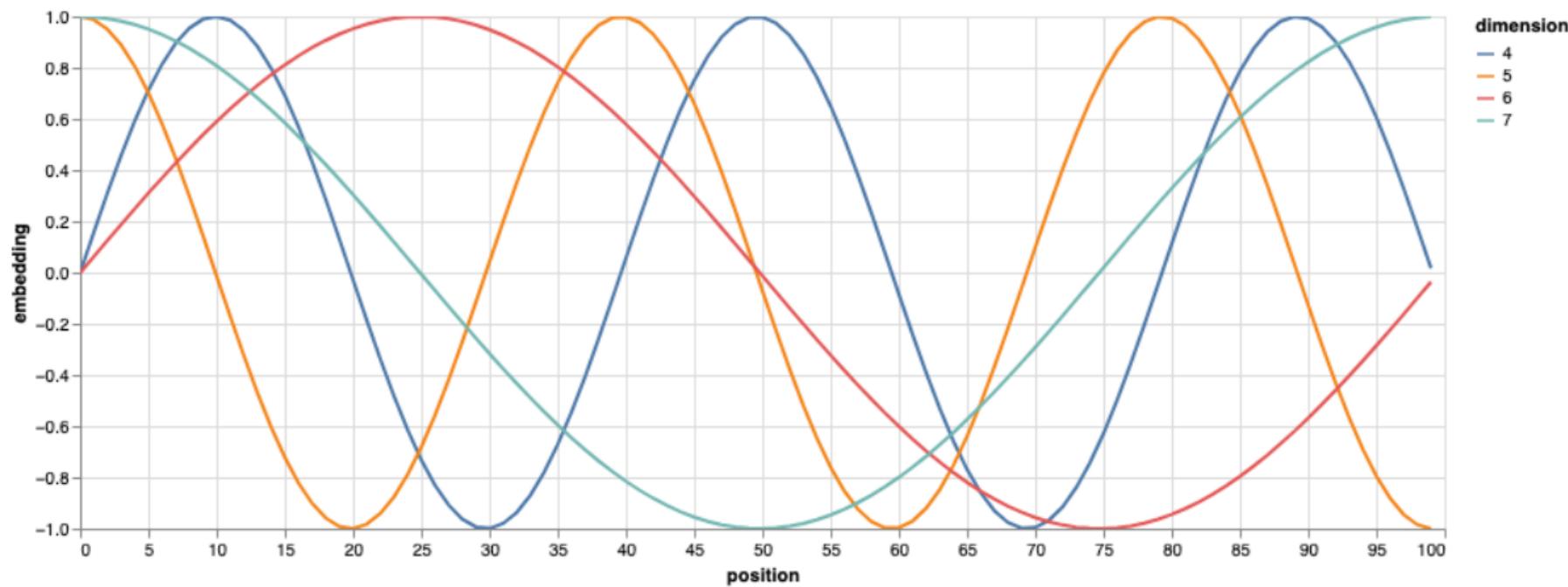
- We take the position of each token within the block  $[0 - 2047]$  through 12288 sinusoidal functions, each with a different frequency.
- Thus, we get a sequence-positional-encodings matrix:

$$S_{2048 \times 12228} = \sin_{12288}(B_{2048 \times 50227})$$

Extrapolate easily.

- We sum the sequence-embeddings matrix and sequence-positional-encodings matrix:

$$SE_{2048 \times 12228} = E_{2048 \times 12228} + S_{2048 \times 12228}$$



## **Step V: Attention**

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# Attention

- Train the neural network to focus on some input data (e.g., some tokens) and lower the weights of other inputs by sharing communication among tokens.
- Mimics human cognition.
- Generalization of ideas floating since the 1990s (multiplicative modules, sigma pi units, and hyper-networks).
- Permutation invariant (unless we introduce positional encoding).
- Particularly easy to parallelize with GPUs because it avoids the previous approach of using recurrence.
- A more detailed introduction:

[https://www.youtube.com/watch?v=AIiwuClvH6k&ab\\_channel=GoogleDeepMind](https://www.youtube.com/watch?v=AIiwuClvH6k&ab_channel=GoogleDeepMind).

Table 1: Maximum path lengths, per-layer complexity and minimum number of sequential operations for different layer types.  $n$  is the sequence length,  $d$  is the representation dimension,  $k$  is the kernel size of convolutions and  $r$  the size of the neighborhood in restricted self-attention.

Layer Type	Complexity per Layer	Sequential Operations	Maximum Path Length
Self-Attention	$O(n^2 \cdot d)$	$O(1)$	$O(1)$
Recurrent	$O(n \cdot d^2)$	$O(n)$	$O(n)$
Convolutional	$O(k \cdot n \cdot d^2)$	$O(1)$	$O(\log_k(n))$
Self-Attention (restricted)	$O(r \cdot n \cdot d)$	$O(1)$	$O(n/r)$

## Query, key, and value I

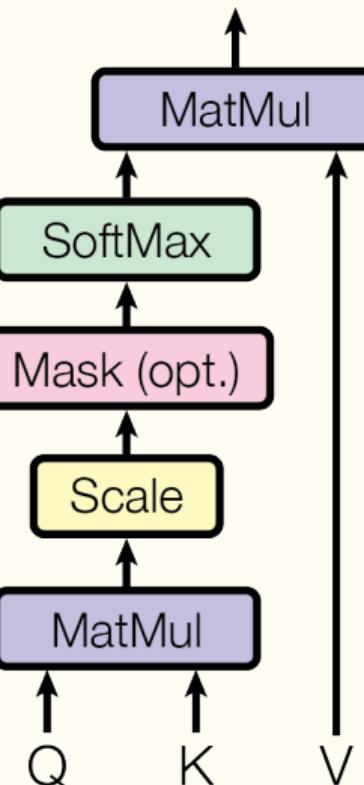
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- We take a sequence  $\mathbf{x} = (x_1, \dots, x_m)$  of  $n$ -dim input vectors and produce a sequence  $\mathbf{y} = (y_1, \dots, y_m)$  of  $p$ -dim output vectors.
- $p$  is the head size.
- With our previous example of GPT-3,  $m = 2048$  and  $n = 12288$ .
- In a database, you have a query and obtain a value.
- Often, you want to have a key for each value.

## Query, key, and value II

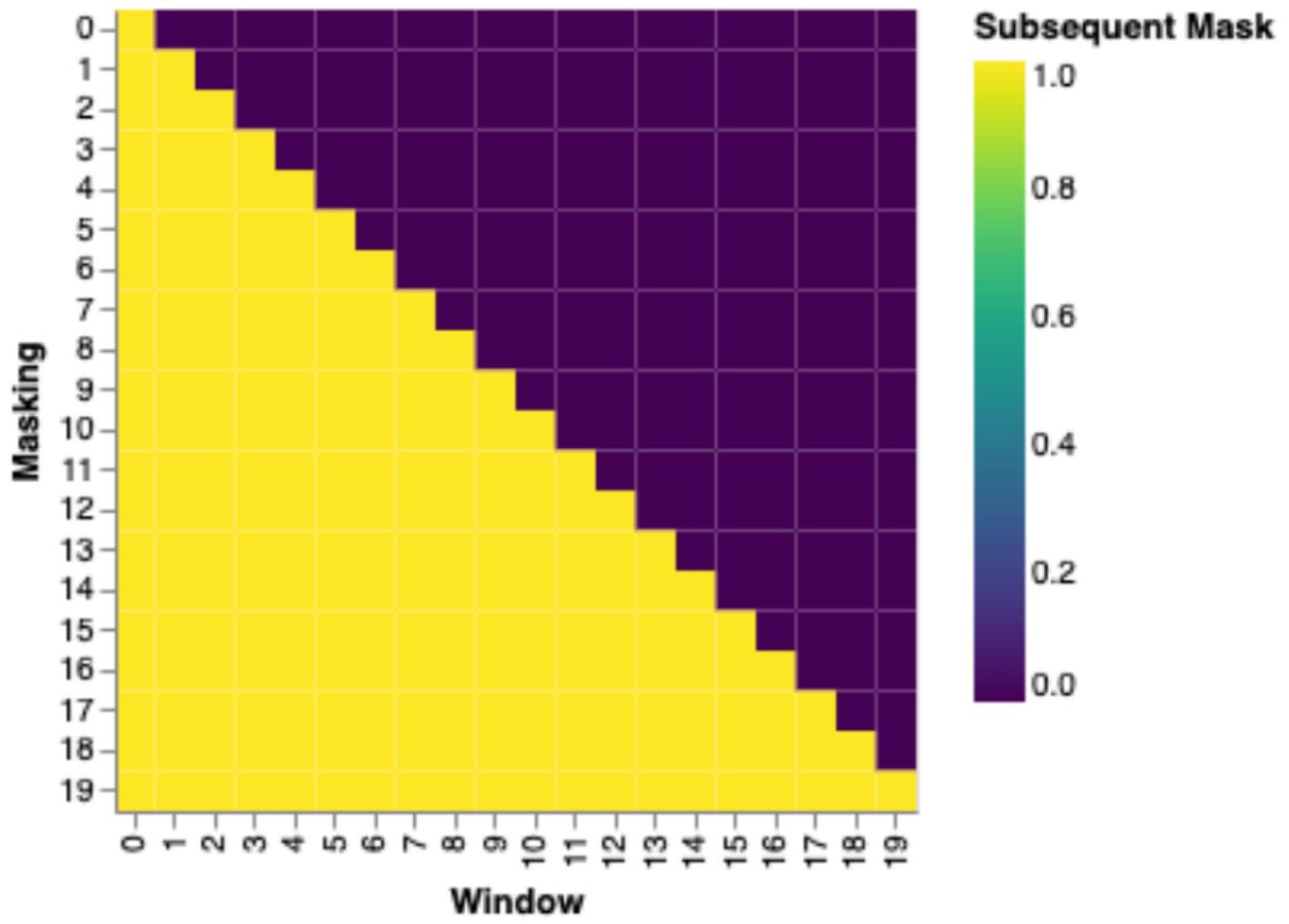
- Every token emits a query (“what am I looking for?”) and a key (“what do I contain?”) vector.
- Three components:
  1. Q: query  $Q_{m \times p} = \text{softmax}(SE_{m \times n} W_{n \times p}^Q)$ .
  2. K: key  $K_{m \times p} = \text{softmax}(SE_{m \times n} W_{n \times p}^K)$ .
  3. V: value  $V_{m \times p} = \text{softmax}(SE_{m \times n} W_{n \times p}^V)$ .
- Importance matrix  $\text{softmax}(QK^T)$  represents the relative importance of each token with respect to all others (“affinities”).
- Then:
$$\text{Attention}(Q, K, V) = \text{softmax}(QK^T)V$$
- You can think about  $\text{Attention}(Q, K, V)$  as a refined embedding.

# Scaled Dot-Product Attention

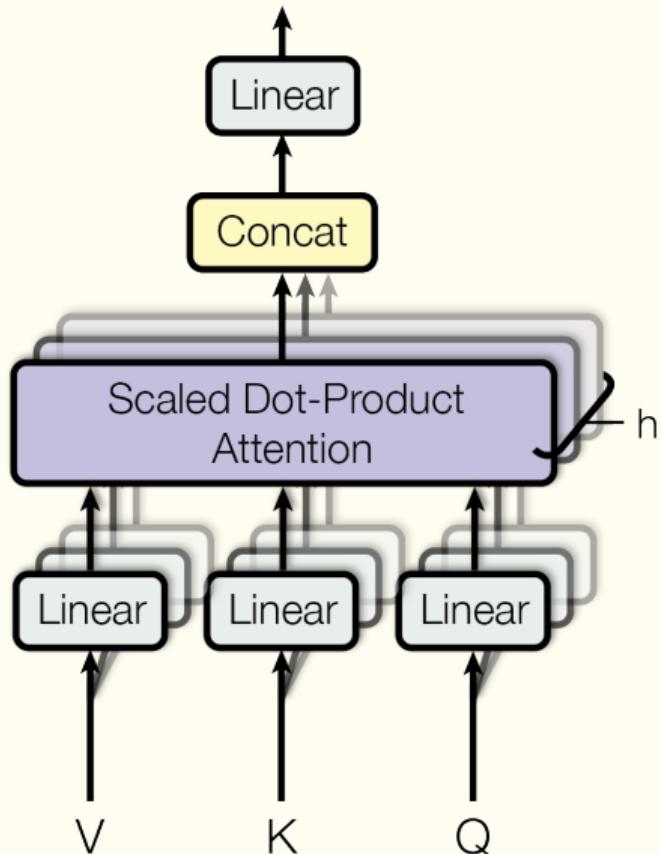


## Scaling, masking, and multiheads

- Scaling: we can scale  $(QK^T)$  by  $\sqrt{n}$  before applying softmax.
- Masking: some words in the input sequence are masked. Many possible reasons: GPT-3 to avoid having an encoder.
- Multiheads: We build multiple attention weights  $W^{Q,r}, W^{K,r}, W^{V,r}$ , where  $r$  is the index of the self-attention path.
- There is a simpler implementation of query, key, value: dot product.



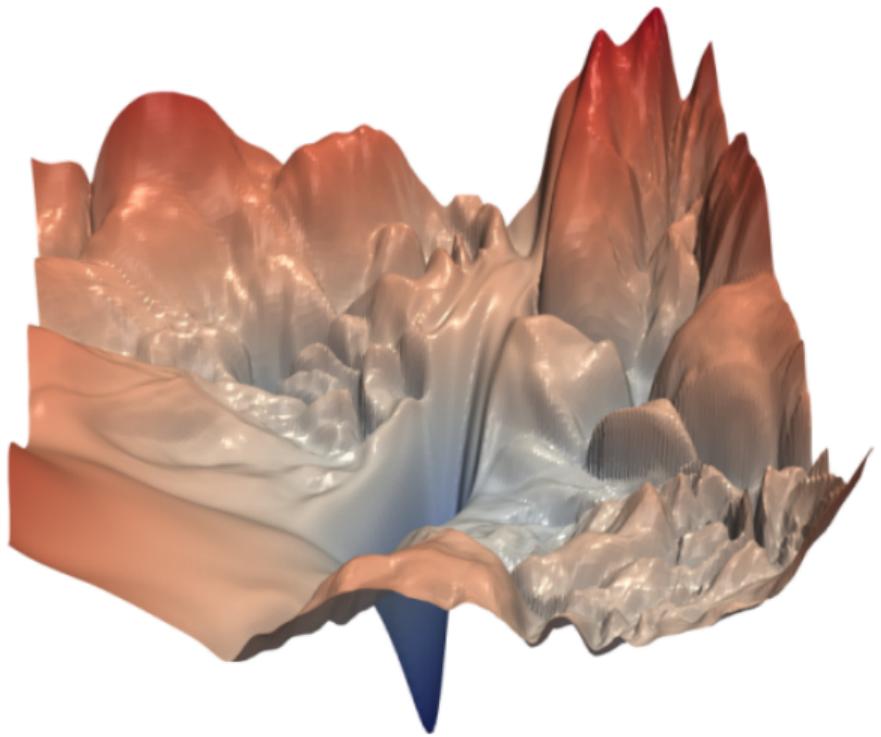
## Multi-Head Attention



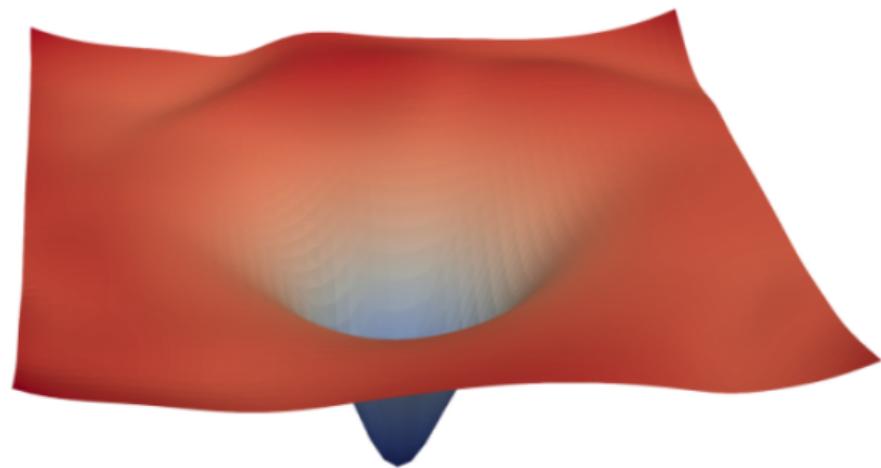
## Scaling, masking, and multiheads in GPT-3

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- In GPT-3, we have  $p = 128$  and 96 attention weights for a total of 12228 (same as  $n$ ).
- Also, we multiply by a new weight matrix  $W_O$ , add original  $SE$ , and normalize to get an output  $\text{Attention}_{\text{norm}}(Q, K, V)_{2048 \times 12228}$ .
  1. Why sum? Skip connection (also known as a residual or shortcut connection).
  2. Why normalization?



(a) without skip connections



(b) with skip connections

---

# Layer Normalization

---

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**Jamie Ryan Kiros**  
University of Toronto  
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## Abstract

Training state-of-the-art, deep neural networks is computationally expensive. One way to reduce the training time is to normalize the activities of the neurons. A recently introduced technique called batch normalization uses the distribution of the summed input to a neuron over a mini-batch of training cases to compute a mean and variance which are then used to normalize the summed input to that neuron on each training case. This significantly reduces the training time in feed-forward neural networks. However, the effect of batch normalization is dependent on the mini-batch size and it is not obvious how to apply it to recurrent neural networks. In this paper, we transpose batch normalization into layer normalization by computing the mean and variance used for normalization from all of the summed inputs to the neurons in a layer on a *single* training case. Like batch normalization, we also give each neuron its own adaptive bias and gain which are applied after the normalization but before the non-linearity. Unlike batch normalization, layer normalization performs exactly the same computation at training and test times. It is also straightforward to apply to recurrent neural networks by computing the normalization statistics separately at each time step. Layer normalization is very effective at stabilizing the hidden state dynamics in recurrent networks. Empirically, we show that layer normalization can substantially reduce the training time compared with previously published techniques.

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## **Step VI: Output**

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## Decoding

- Next, we pass the result  $\text{Attention}_{\text{norm}}(Q, K, V)$  through a feed forward neural network with ReLUs to get  $Y^F_{2048 \times 12228}$ .
  - Why? Forecasting.
- We sum  $Y^E_{2048 \times 12228} = \text{Attention}_{\text{norm}}(Q, K, V) + Y^F$  and normalize.
- Finally, we get  $Y_E$  with the inverse of our embedding weight matrix:

$$Y^E(W^E)^{(-1)}$$

- We apply softmax and select a word among the top-k probabilities.
- Also, we can use human alignment.

## **Step VII: Training**

---

## Practical implementation I

- GPT-3 uses 499 billion tokens in the full training data. The Common Crawl data set contains 410 of those.
- Loss function to select all the relevant weights: the average negative log-likelihood per token.
- Dropout.
- Powerful optimizer.
- Length of training vs. size of model and data.

Table 2: The Transformer achieves better BLEU scores than previous state-of-the-art models on the English-to-German and English-to-French newstest2014 tests at a fraction of the training cost.

Model	BLEU		Training Cost (FLOPs)	
	EN-DE	EN-FR	EN-DE	EN-FR
ByteNet [18]	23.75			
Deep-Att + PosUnk [39]		39.2		$1.0 \cdot 10^{20}$
GNMT + RL [38]	24.6	39.92	$2.3 \cdot 10^{19}$	$1.4 \cdot 10^{20}$
ConvS2S [9]	25.16	40.46	$9.6 \cdot 10^{18}$	$1.5 \cdot 10^{20}$
MoE [32]	26.03	40.56	$2.0 \cdot 10^{19}$	$1.2 \cdot 10^{20}$
Deep-Att + PosUnk Ensemble [39]		40.4		$8.0 \cdot 10^{20}$
GNMT + RL Ensemble [38]	26.30	41.16	$1.8 \cdot 10^{20}$	$1.1 \cdot 10^{21}$
ConvS2S Ensemble [9]	26.36	<b>41.29</b>	$7.7 \cdot 10^{19}$	$1.2 \cdot 10^{21}$
Transformer (base model)	27.3	38.1		<b><math>3.3 \cdot 10^{18}</math></b>
Transformer (big)	<b>28.4</b>	<b>41.8</b>		$2.3 \cdot 10^{19}$

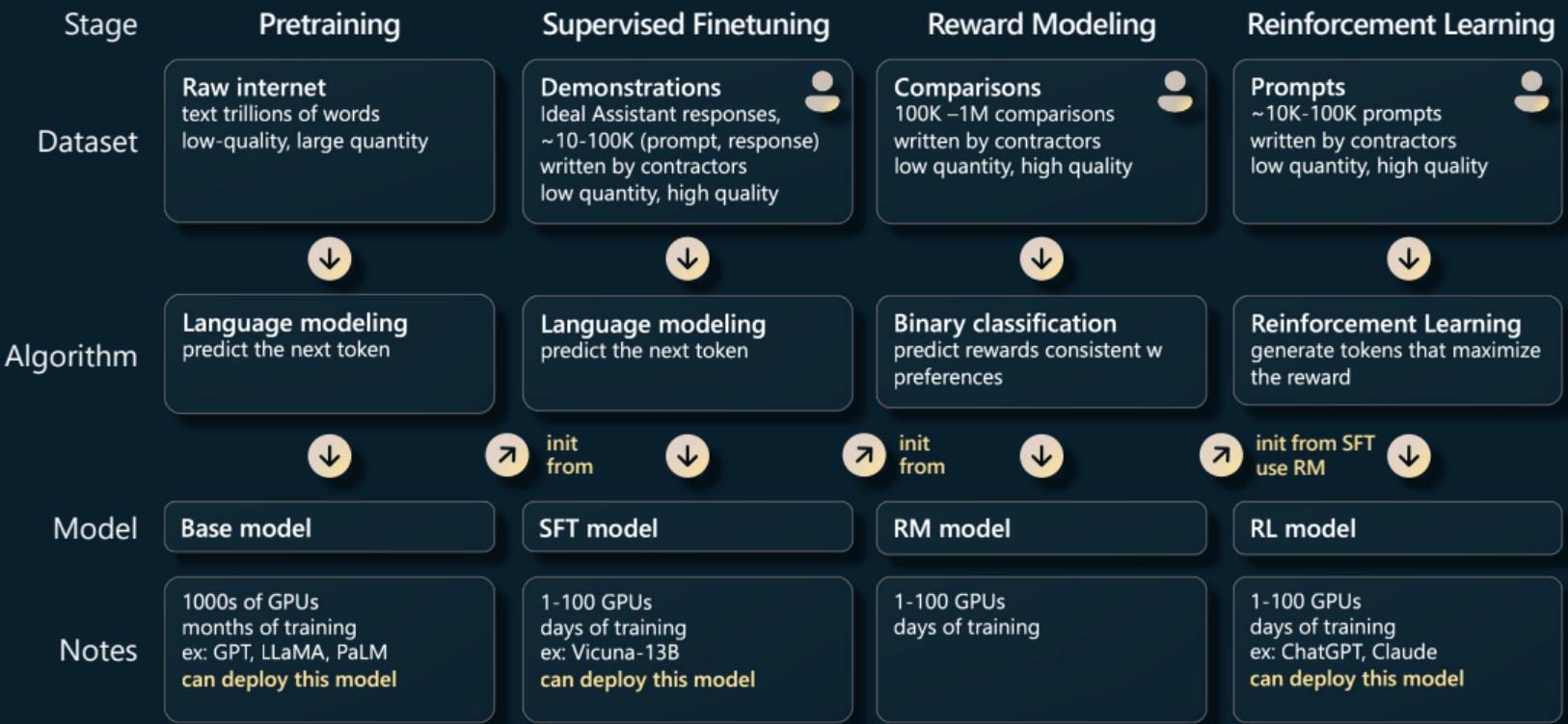
Table 3: Variations on the Transformer architecture. Unlisted values are identical to those of the base model. All metrics are on the English-to-German translation development set, newstest2013. Listed perplexities are per-wordpiece, according to our byte-pair encoding, and should not be compared to per-word perplexities.

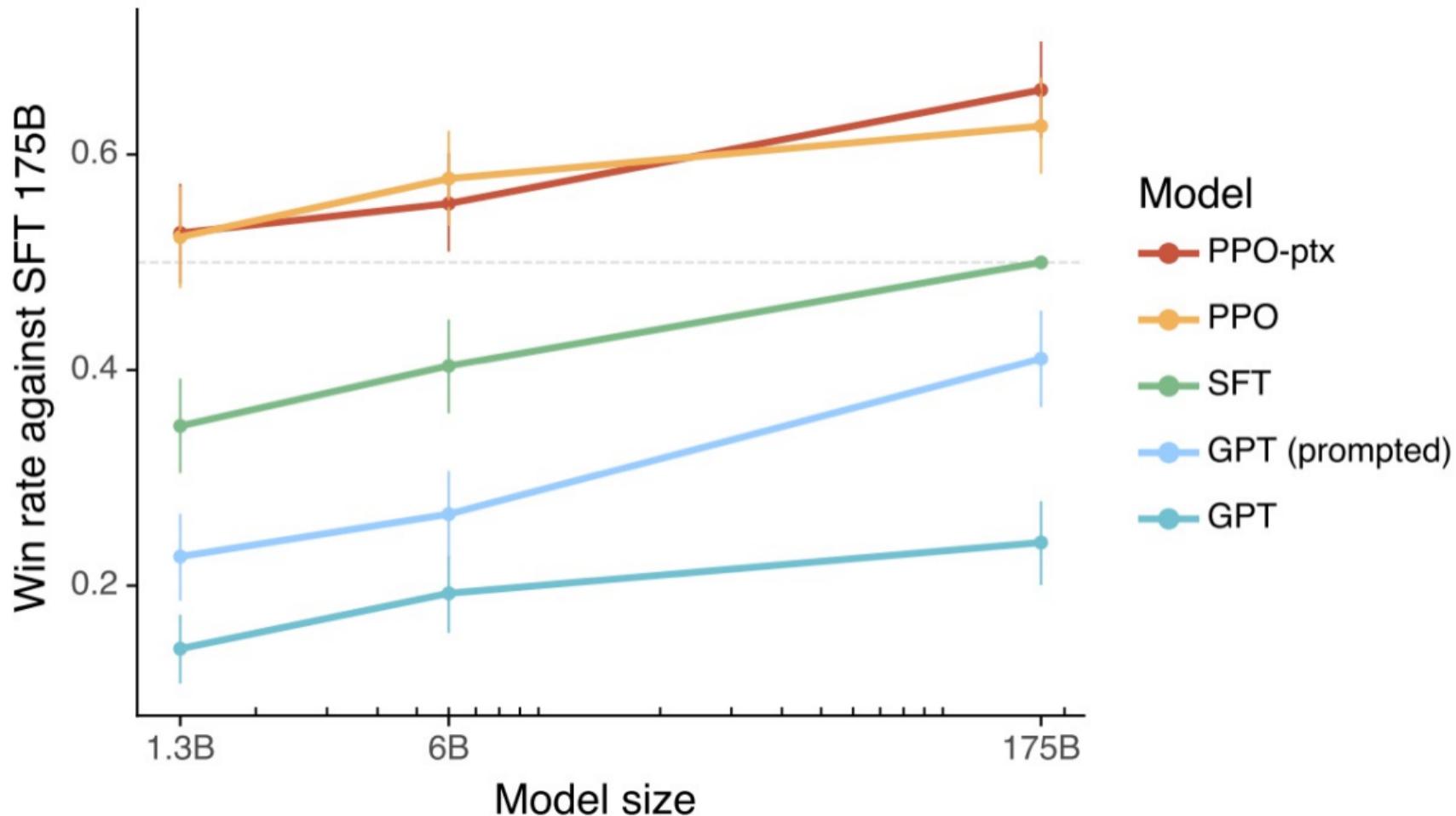
	$N$	$d_{\text{model}}$	$d_{\text{ff}}$	$h$	$d_k$	$d_v$	$P_{\text{drop}}$	$\epsilon_{ls}$	train steps	PPL (dev)	BLEU (dev)	params $\times 10^6$
base	6	512	2048	8	64	64	0.1	0.1	100K	4.92	25.8	65
(A)				1	512	512				5.29	24.9	
				4	128	128				5.00	25.5	
				16	32	32				4.91	25.8	
				32	16	16				5.01	25.4	
(B)					16					5.16	25.1	58
										5.01	25.4	60
(C)				2						6.11	23.7	36
				4						5.19	25.3	50
				8						4.88	25.5	80
				256		32	32			5.75	24.5	28
				1024		128	128			4.66	26.0	168
				1024		5.12	25.4			53		
				4096		4.75	26.2			90		
(D)							0.0			5.77	24.6	
							0.2			4.95	25.5	
							0.0			4.67	25.3	
										5.47	25.7	
(E)	positional embedding instead of sinusoids									4.92	25.7	
big	6	1024	4096	16			0.3	300K		<b>4.33</b>	<b>26.4</b>	213

## Practical implementation II

- Train and validation data.
- Different approaches:
  1. Supervised fine-tuning (SFT): The raw model is pre-trained on a large dataset and then trained on smaller but higher-quality datasets.
  2. Reinforcement Learning from Human Feedback (RLHF).
  3. Generating vs. ranking answers.
- Check <https://thegradient.pub/ai-is-domestication/>.
- Also, on prompt engineering:  
<https://lilianweng.github.io/posts/2023-03-15-prompt-engineering/>.

# GPT Assistant training pipeline





## Code

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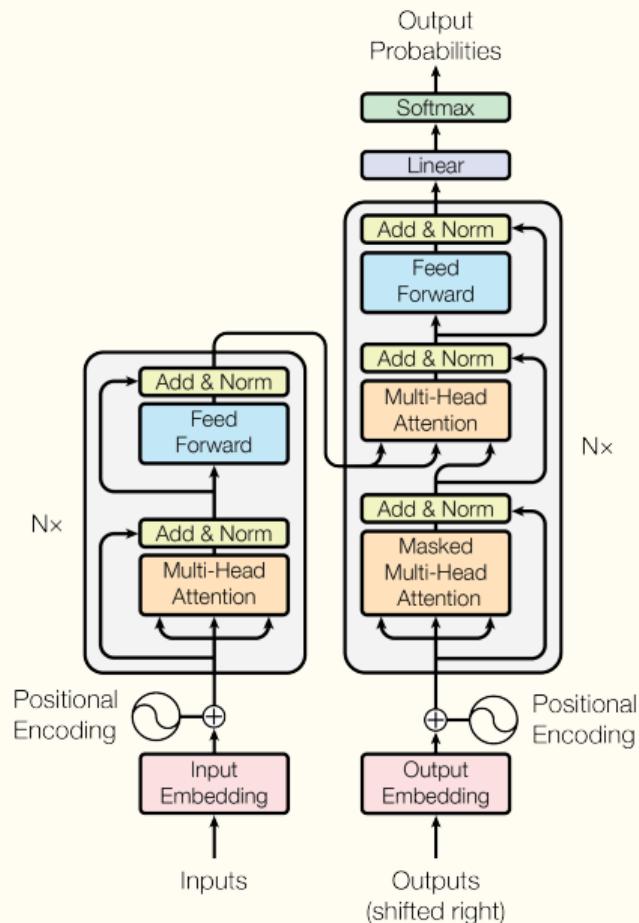
- Python + PyTorch allow for an easy implementation of this architecture.
- Code online:
  1. <http://nlp.seas.harvard.edu/annotated-transformer/>.
  2. <https://www.youtube.com/watch?v=kCc8FmEb1nY> and <https://github.com/karpathy/nanoGPT>.
- You want to run the code on GPUs.

## **Step VIII: Extensions**

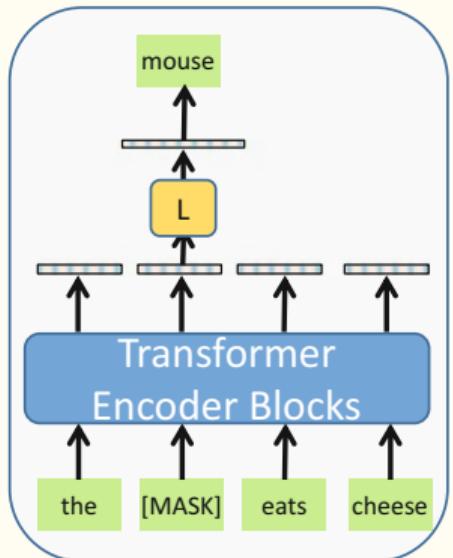
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## Simplifying

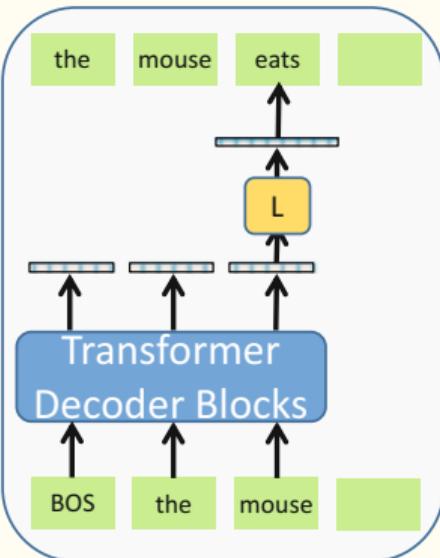
- The original transformer architecture also has a decoder component. Why?
- It turns out we do not need an encoder or a decoder.
- We can dispense with one of the two.
  1. Autoencoders: BERT.
  2. Autoregressive language models: GPT.
- Cross-attention: Q's and K's come from outside sources of information.



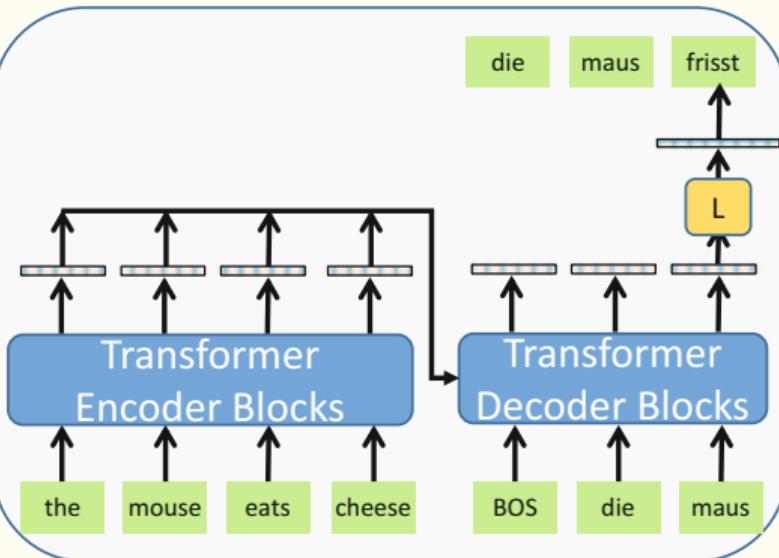
BERT Autoencoder

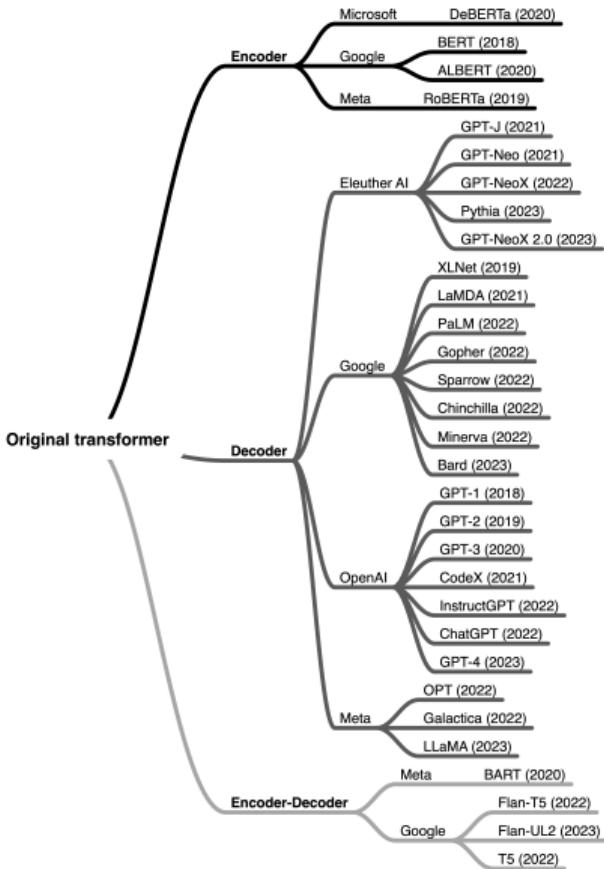


GPT Language Model



Transformer Encoder-Decoder





## Frontier work: QAs and Assistants

- QAs: either quote from a text or created from scratch.
- Hope to avoid domain knowledge.
- Design of assistants through prompt design and pre-train.
- Unfortunately, some of the details of frontier models are not public.
- But check: <https://youtu.be/bZQun8Y4L2A>.