Entry, Exit, Firm Dynamics, and Aggregate Fluctuations

Clementi and Palazzo (2016)

Eric Bodnar, Ella Walling, Milton Bronstein

Introduction

- What is the role of entry and exit of firms on aggregate productivity shocks?
- Both propagate the effects of such shocks
- Assumptions:
    - Demand for firms’ product and supply for their input are infinitely elastic
    - Labor supply has finite elasticity
- Findings:
    - Exit risk is decreases with age
    - Employment growth decreases with size and age
    - Entry rate is procyclical and output rate is counter-cyclical (with respect to output)

The Model

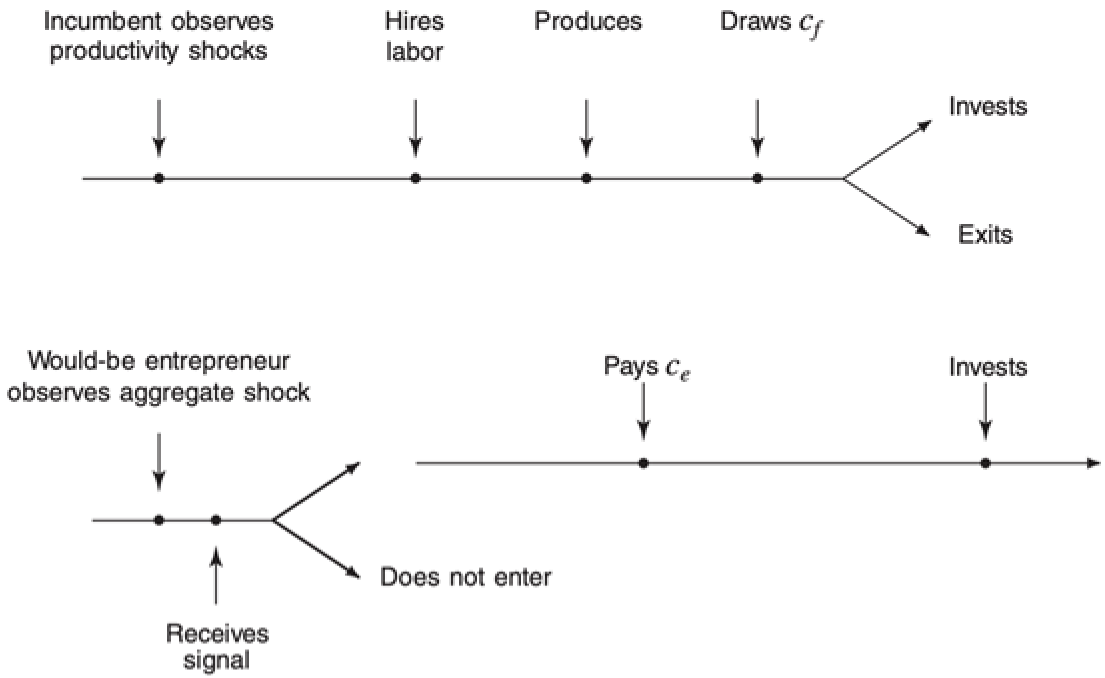

FIGURE 1. TIMING IN PERIOD t

The Model: Existing Firms

- At time t, a positive number of firms produce a homogeneous good
  - With production function $y_t = z_t s_t(k_t^{\alpha}\ l_t^{1-\alpha})^{\theta}$, with $\alpha, \theta \in (0, 1)$
- k_t is capital
  - Adjusting k_t by x incurs a cost $g(x, k)$
  - Capital depreciates at rate $\delta \in (0, 1)$
- l_t is labor
  - Firms hire labor at wage rate w_t
  - Labor supply is given by function $L_s(w) = w^{\gamma}$, $\gamma > 0$

The Model: Existing Firms

- z_t represents aggregate random disturbances
  - Driven by stochastic process $\log(z_{t+1}) = \rho_z \log(z_t) + \sigma_z \varepsilon_{z,\, t+1}$ where $\varepsilon_{z,\, t+1} \sim N(0,1)$

- s_t represents idiosyncratic random disturbances
  - Driven by dynamic process $\log(s_{t+1}) = \rho_s \log(s_t) + \sigma_s \varepsilon_{s,\, t+1}$ where $\varepsilon_{s,\, t+1} \sim N(0,1)$
  - Conditional distribution of s_{t+1}: $H(s_{t+1} | s_t)$

The Model: Existing Firms

- For all $t \geq 0$, the distribution of firms over the two dimensions of heterogeneity is denoted by $\Gamma_t(k,s)$

- $\lambda_t \in \Lambda$ is the vector of aggregate state variables with transition operator $J(\lambda_{t+1} | \lambda_t)$

The Model: Existing Firms Optimization Problem

- Aggregate state λ, capital k, and idiosyncratic productivity s are given
- The firm maximizes profit through the following static problem:

$$\pi(\lambda, k, s) = \max_l sz[k^{\alpha} l^{1-\alpha}] - wl$$

- Upon exiting the market, a firm receives

$$V_x(k) = k(1-\boldsymbol{\delta}) - g[-k(1-\boldsymbol{\delta}), k]$$

- [undepreciated portion of capital] - [net adjustment cost of dismantling]

The Model: Existing Firms Optimization Problem

Using the previous two equations, the start of period value for an existing firm is given by $V(\lambda, k, s)$, which solves the following functional equation:

$$V(\lambda, k, s) = \pi(\lambda, k, s) + \int \max \{V_x(k), \tilde{V}(\lambda, k,s) - c_f\}\, dG(c_f)$$

where

$$\tilde{V}(\lambda, k,s) = \max_x -x - g(x, k) + 1/R \int_{\Lambda} \int_{\Re} V(\lambda', k', s')\, dH(s'|s)\, dJ(\lambda'|\lambda)$$

such that $k' = k(1-\delta) + x$

Finally, note that after leaving the market firms cannot re-enter at a later stage and repossess their undepreciated capital stock

The Model: Prospective Firms

- Each period there is a constant number of prospective firms $M > 0$
  - Receives a signal q about their prospective productivity, where $q \sim Q(q)$

- Conditional on entry, the distribution of idiosyncratic shock in the first period, s', is $H(s'|q)$
  - $H(s'|q)$ is strictly decreasing in q

- Potential firms that decide to enter the market will incur entry cost $c_e > 0$

The Model: Prospective Firm Value Function

Given aggregate state λ, the value of a prospective firm with a signal q is:

$$V_e(\lambda, q) = \max_{k'} -k' + 1/R \int_{\Lambda} \int_{\Re} V(\lambda', k', s')\, dH(s' | s)\, dJ(\lambda' | \lambda)$$

Note that a prospective firm will invest and start operating if and only if $V_e(\lambda, q) \geq c_e$

The Model: Recursive Competitive Equilibrium

Given Γ_0, a recursive competitive equilibrium is defined by:

(i) Value functions $V(\lambda, k, s)$, $\tilde{V}(\lambda, k, s)$, and $V_e(\lambda, q)$

(ii) Policy functions $x(\lambda, k, s)$, $l(\lambda, k, s)$, and $k'(\lambda, q)$

(iii) Bounded sequences of wages $\{w_t\}^{\infty}_{t=0}$

Incumbents' measures $\{\Gamma_t\}^{\infty}_{t=1}$

Entrants' measures $\{\varepsilon_t\}^{\infty}_{t=1}$

Basically everyone solves their optimization problems and all markets clear

The Stationary Case: functional forms

- No aggregate shocks (σ_z=0). Only the idiosyncratic ones

- First period shock (t=1): $\log(s) = \rho_s \log(q) + \sigma_s \eta$,
    - with $\eta \sim N(0,1)$

- All aggregate variables converge to constants

The Stationary Case: functional forms

- Investment adjustment costs: $g(x,k) = \chi(x)c_0k + c_1(x/k)^2k$
  - With c0, c1 ≥ 0 and $\chi(0) = 0$ and $\chi(x) = 1$ for $x \neq 0$
- Productivity signal (q) follows Pareto distribution such that: $Q(q) = (\underline{q}/q)^{\zeta}$
  - With $q \geq \underline{q} \geq 0$ and $\zeta > 1$
- Operating costs (c_f) distribution G is log-normal with μ_{cf} and σ_{cf}

The Stationary Case: entry, investment, and exit

- $V(\lambda, k, s)$ is weakly increasing in s
  - The incumbent's value may increase with the idiosyncratic shock
- The distribution $H(s'|q)$ is decreasing in q.
  - A greater signal today leads to a distribution around higher shocks tomorrow
- $V_e(\lambda, q)$ is strictly increasing in q
  - The entrant's value increases with the signal
- Thus, there is a cut-off q^*
  - Every firm outside the market observing a signal $q \geq q^*$ will enter because they are optimist about their productivity

The Stationary Case: entry, investment, and exit

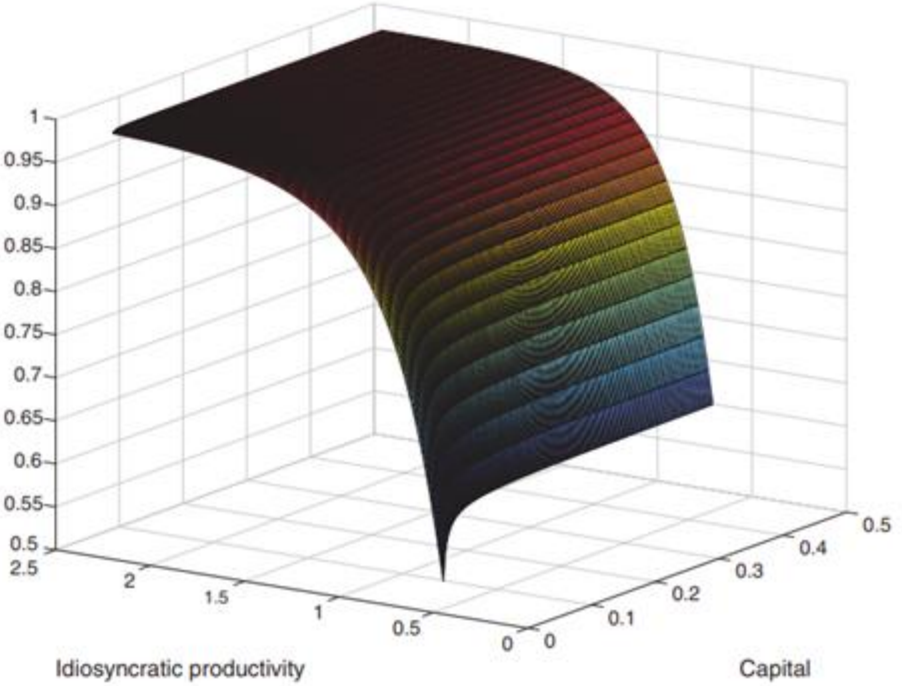

FIGURE 3. CONDITIONAL PROBABILITY OF SURVIVAL

The Stationary Case: entry, investment, and exit

- Entrants with greater signals make greater investments and start with greater capital (k)
- Incumbents draw their cost (c_f) and exit if $V_x(k) > \tilde{V}(\lambda, k,s) - c_f$
  - Findings (figure 3):
    - Exit probability is decreasing in idiosyncratic shock as the value of staying is increasing in such shock while the exit value is not
    - Increasing the capital stock has a greater impact on the value of staying than in the value of exit
    - Survival (staying) probability is increasing in k and s

The Stationary Case: calibration

TABLE 1—PARAMETER VALUES

Description	Symbol	Value
Capital share	α	0.3
Span of control	θ	0.8
Depreciation rate	δ	0.1
Interest rate	R	1.04
Labor supply elasticity	γ	2.0
Mass of potential entrants	M	1,766.29
Persistence idiosyncratic shock	ρ_s	0.55
Variance idiosyncratic shock	σ_s	0.22
Operating cost – mean parameter	μ_{c_f}	–5.63872
Operating cost – var parameter	σ_{c_f}	0.90277
Fixed cost of investment	c_0	0.00011
Variable cost of investment	c_1	0.03141
Pareto exponent	ξ	2.69
Entry cost	c_e	0.005347

The Stationary Case: calibration

TABLE 2—CALIBRATION TARGETS

Statistic	Model	Data
Mean investment rate	0.153	0.122
SD investment rate	0.325	0.337
Investment autocorrelation	0.059	0.058
Inaction rate	0.067	0.081
Entry rate	0.062	0.062
Entrants' relative size	0.58	0.60
Exiters' relative size	0.47	0.49

- Parameters are chosen based on literature and in order to match these statistics with the observed ones
  - Arbitrary M. But a greater value increases equilibrium wage and, hence, reduce the firms size. Then, higher exit and entry
  - Set c_e equal to the average operating cost c_f

Aggregate Fluctuations: mechanics

- Aggegate productivity shocks z affect every firm
- Equilibrium wage (w) at time t satisfies:

$$(1) \qquad \log w_t = \frac{\log[(1-\alpha)\theta z_t]}{1+\gamma[1-(1-\alpha)\theta]} + \frac{1-(1-\alpha)\theta}{1+\gamma[1-(1-\alpha)\theta]}\Omega_t,$$

with $\quad \Omega_t = \log\left[\int \left(s k^{\alpha\theta}\right)^{\frac{1}{1-(1-\alpha)\theta}} d\Gamma_t(k,s)\right]$

- It depends on aggregate shocks as well as a function of idiosyncratic shocks and capital in the distribution of incumbent firms

Aggregate Fluctuations: mechanics

- Based on Krussell & Smith (1998), the author affirm that $\mathbf{\Omega}_{t+1}$ depends on $\mathbf{\Omega}_t$ and z_{t+1} and boil equation (1) down to

$$(2) \qquad \log w_{t+1} = \beta_0 + \beta_1 \log w_t + \beta_2 \log z_{t+1} + \beta_3 \log z_t + \varepsilon_{t+1},$$

where $E(\varepsilon_{t+1})=0$

- That is, firms forecast the equilibrium wage depending on its current value and the aggregate productivity shocks

Aggregate Fluctuations: calibration

TABLE 3—PARAMETER VALUES

Description	Symbol	Value
Labor supply elasticity	γ	2.0
Persistent aggregate shock	ρ_z	0.685
Standard deviation aggregate shock	σ_z	0.0163

TABLE 4—ADDITIONAL CALIBRATION TARGETS

Statistic	Model	Data
Standard deviation output growth	0.032	0.032
Autocorrelation output growth	0.069	0.063
Standard deviation employment growth (relative to output growth)	0.656	0.667

- Parameters targeted to these statistics observed in the US non-farm private sector (1947-2008)

Aggregate Fluctuations: calibration

- As a result, firms forecast the equilibrium wage as follows:

$$\log(w_{t+1}) = 0.38385 + 0.65105 \log(w_t) + 0.53075 \log(z_{t+1}) - 0.21508 \log(z_t) + \varepsilon_{t+1}$$

- Wage is persistent but it reverts to its mean
- A positive aggregate shock increases the demand for labor, hence the equilibrium wage
- A past positive shock makes firm expect a smaller one for the current time period. Thus, the demand for labor and the wage decrease

Aggregate Fluctuations: entry and exit

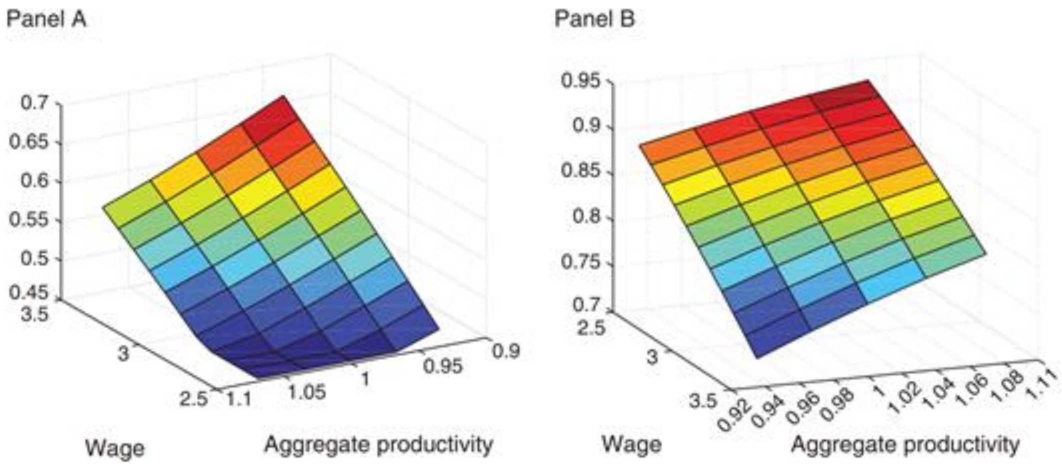

FIGURE 8

Notes: Left: Entry threshold on the signal space. Right: Survival probability.

- Each firm solves their problem and makes their choice
- Potential entrants enter if they observe a signal q is greater than the threshold (increasing in w and decreasing in z)
- For incumbents, their probability of staying is decreasing in w and increasing in z (conditional on their capital and idiosyncratic shock)

Aggregate dynamics (counterfactual)

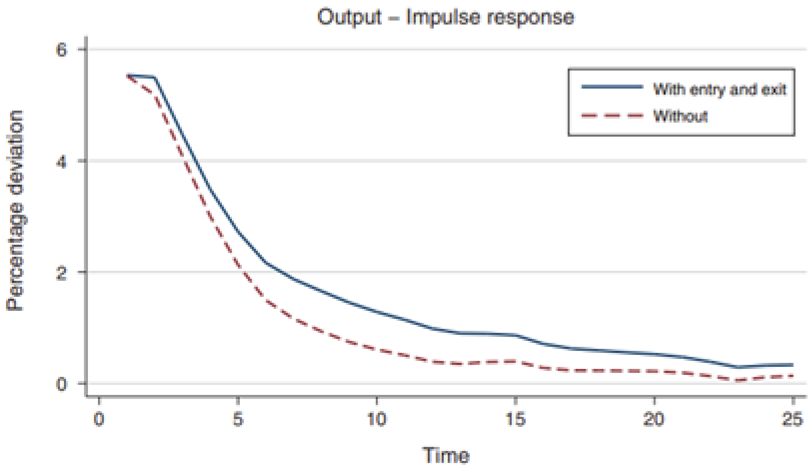

FIGURE 11. THE EFFECT OF ENTRY AND EXIT ON OUTPUT DYNAMICS

- If there was no entry and exit, aggregate shocks would be less persistent.
- Output still reverts to its mean, but more slowly than in this counterfactual
- As entrants become more productive, the reallocation of output favors them

Stationary Replication

-Use Tauchen 1986 method and np.leggauss to approximate the 2 AR(1) processes

-Take in parameters calibrated in other papers (μ_z,μ_m, ρ_z, ρ_m,σ_z,σ_m)

-Generate capital grid as done by McGrattan 1999

Stationary Replication part 2

- Perform VFI on incumbent (kgrid, zgrid, adj, ztrans, vgi, labor) to get desired values(vf, kpol,eprob,etastar)

-Create signal grid and transition matrix from signal to shock from idiosyncratic grid

-Use this grid to calculate conditional distribution of idiosyncratic shock

Stationary Replication part 3

-Compute entry threshold using interpolation, compute mass on threshold

-Compute distribution of entrants using conditional expectation

-Simulate until equilibrium wage is reached to get stationary distribution, compute summary statistics

Aggregate Fluctuations Replication

-Guess values of wage forecast

-Modify bellman optimization code to also include wage grid

-Revise according to

Process on the right

$$V_1(w,z,k,s) = \pi(w,z,k,s) + Pr[c_f > c_f^*(w,z,k,s)]V_x(k)$$
$$+ Pr[c_f \leq c_f^*(w,z,k,s)]\left[\widetilde{V}(w,z,k,s) - E[c_f | c_f \leq c_f^*(w,z,k,s)]\right]$$

subject to

$$\pi(w,z,k,s) = \frac{1-(1-\alpha)\theta}{(1-\alpha)\theta} w^{-\frac{\theta(1-\alpha)}{1-\theta(1-\alpha)}} [(1-\alpha)\theta s z k^{\alpha\theta}]^{\frac{1}{1-\theta(1-\alpha)}},$$

$$V_x(k) = k(1-\delta) - g[-k(1-\delta), k],$$

$$\widetilde{V}(w,z,k,s) = \max_{k' \in \Psi_k} \left\{ -x - c_0 k \chi - c_1 \left(\frac{x}{k}\right)^2 k + \frac{1}{R} \sum_j \sum_i \sum_n V_0(w_i, z_j, k', s_n) H(s_n|s) J(w_i|w,z,z_j) G(z_j|z) \right\},$$

$$x = k' - k(1-\delta),$$

$$\chi = 1 \text{ if } k' \neq k \text{ and } \chi = 0 \text{ otherwise},$$

Aggregate Fluctuations Replication part 2

-Modify the ‘entry problem’ over triplets instead

-Equate labor demand and supply

-Simulate and run the Krusell Smith 1998 regression

Impulse Response and Decomposition Replication

-Simulate simple model without entry or exit

-Use code from Haltiwanger 1997

$$Y_t = z_t \left[\int \widehat{\Gamma}_t(s) s^{\frac{1}{1-\alpha}} ds \right]^{1-\alpha} N_t^{1-\alpha} L_t^{\alpha}.$$

$$\Delta \log(TFP_t) = \sum_{i \in \mathcal{C}_t} \phi_{i,t-k} \Delta \log(TFP_{it}) + \sum_{i \in \mathcal{C}_t} [\log(TFP_{i,t-k}) - \log(TFP_{t-k})] \Delta \phi_{it} +$$

$$\sum_{i \in \mathcal{C}_t} \Delta \log(TFP_{it}) \Delta \phi_{it} + \sum_{i \in \mathcal{E}_t} [\log(TFP_{it}) - \log(TFP_{t-k})] \phi_{it} -$$

$$\sum_{i \in \mathcal{X}_{t-k}} [\log(TFP_{i,t-k}) - \log(TFP_{t-k})] \phi_{i,t-k}$$

Extensions

- In the model, the authors normalize the output price into 1. What if the demand is not infinitely elastic? What if the output price was endogenous?

- Here, entrants only choose their initial investment. What if we let entrants hire labor besides the capital stock (as incumbents do)?

- What if firms faced any capacity constraint? They could play a role as a limit investments for entrants and capital adjustment for incumbents (per period)