A Structural Overview: Entry, Exit, Firm Dynamics, and Aggregate Fluctuations

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Outline

Introduction

Objective

Show how firm entry and exit affect aggregate dynamics

Model

Set-up:

- Time is discrete with infinite horizon
- Perfectly competitive output market
- Normalize price of output and capital to 1
- Labor Supply Function $L_s(w) = w^\gamma$
- Two categories of agents:
  - Incumbents
  - Potential Entrants

Production

Production Function:

$$y_t = z_t s_t (k_t^{\alpha} l_t^{1-\alpha})^{\theta}$$

- $\alpha, \theta \in (0, 1)$
- k_t =physical capital
- l_t =labor
- Productivity:
  - z_t = aggregate $\rightarrow log(z_{t+1}) = \rho_z log(z_t) + \sigma_z \varepsilon_{z,t+1}$
  - s_t = idiosyncratic $\rightarrow log(s_{t+1}) = \rho_s log(s_t) + \sigma_s \varepsilon_{s,t+1}$

Model: Incumbent

Each Period:

- Distribution of operating firms denoted $\Gamma_t(k, s)$
- Observe λ, k, s and pay cost c_f drawn from distribution G
- Choose labor solving: max $\pi(\lambda, k, s) = sz[k^{\alpha} l^{1-\alpha}]^{\theta} - wl$

Policy Function (Choice Rule)

Incumbent chooses:
$x(\lambda, k, s)$ and $l(\lambda, k, s)$

Law of Motion

$k' = k(1 - \delta) + x$

Model: Incumbent

Note: $g(x,k)$ = the cost of adjusting capital stock by x

Value of Exit:
$V_x(k) = k(1-\delta) - g[-k(1-\delta), k]$

Value of Staying:
$\max \tilde{V}(\lambda,k,s) = -x - g(x,k) + \frac{1}{R} \int\int V(\lambda',k',s')dH(s'|s)dJ(\lambda'|\lambda)$

Value Function

$V(\lambda,k,s) = \pi(\lambda,k,s) + \int \max \{V_x(k), \tilde{V}(\lambda,k,s) - c_f\}dG(c_f)$

Model: Potential Entrant

Each Period:

- Observes aggregate state (λ) and receives idiosyncratic signal (q)
- Pays cost c_e and invests k' if they choose to enter
- Does not choose labor or produce output

Policy Function (Choice Rule)

Entrant chooses:
$k'(\lambda, q)$

Law of Motion

$k' = k(1-\delta) + x$

Model: Potential Entrant

Value of Entry

$$\max V_e(\lambda, q) = -k' + \frac{1}{R} \int\int V(\lambda', k', s') dH(s'|q) dJ(\lambda'|\lambda)$$

Enters if $V_e(\lambda, q) \geq c_e$

The Stationary Case

Set Up:

- Will converge to equilibrium where all aggregate variables are constant
- No aggregate shocks, $\sigma_z = 0$

Recall

$log(z_{t+1}) = \rho_z log(z_t) + \sigma_z \varepsilon_{z,t+1}$

The Stationary Case

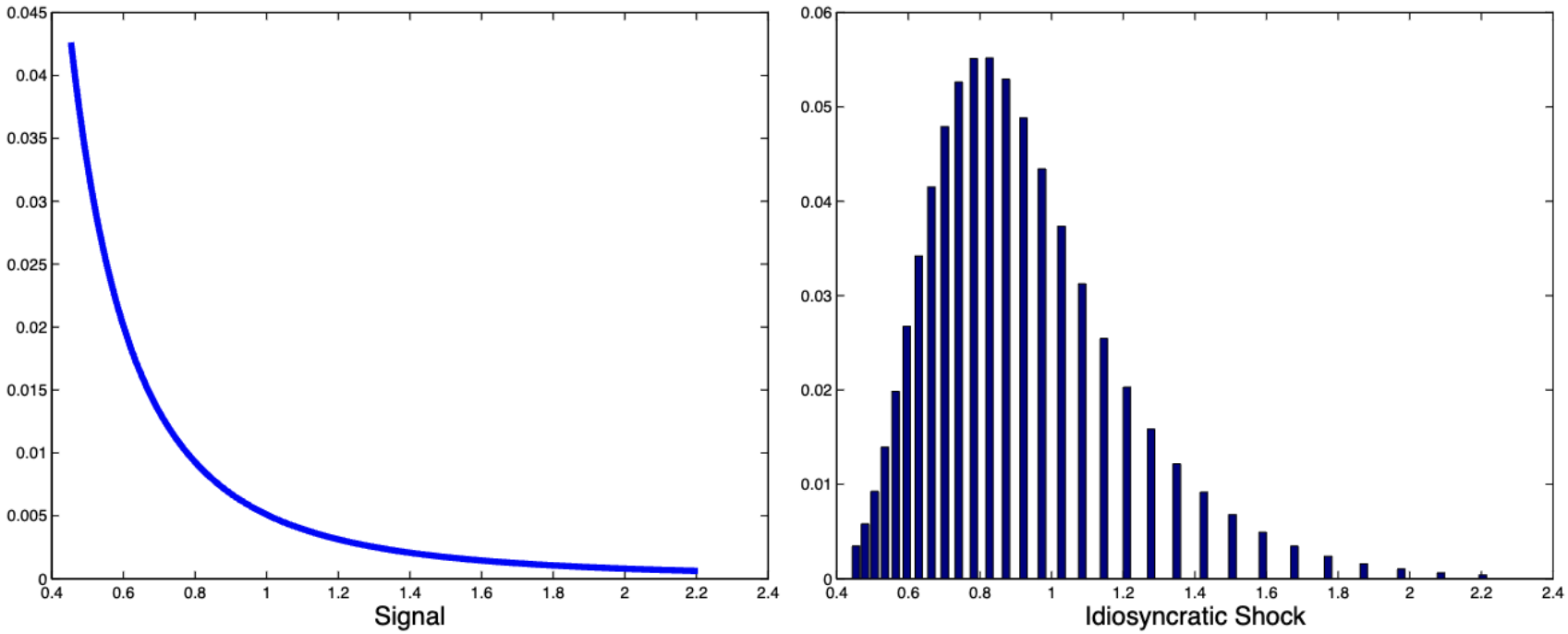

Figure 2: Signal's distribution (left) and productivity distribution of actual entrants.

The Stationary Case

Description	Symbol	Value
Capital share	α	0.3
Span of control	θ	0.8
Depreciation rate	δ	0.1
Interest rate	R	1.04
Labor supply elasticity	γ	2.0
Mass of potential entrants	M	1,766.29
Persistence idiosync. shock	ρ_s	0.55
Variance idiosync. shock	σ_s	0.22
Operating cost – mean parameter	μ_{c_f}	-5.63872
Operating cost – var parameter	σ_{c_f}	0.90277
Fixed cost of investment	c_0	0.00011
Variable cost of investment	c_1	0.03141
Pareto exponent	ξ	2.69
Entry cost	c_e	0.005347

Table 1: Parameter Values.

The Stationary Case

Statistic	Model	Data
Mean investment rate	0.153	0.122
Std. Dev. investment rate	0.325	0.337
Investment autocorrelation	0.059	0.058
Inaction rate	0.067	0.081
Entry rate	0.062	0.062
Entrants' relative size	0.58	0.60
Exiters' relative size	0.47	0.49

Table 2: Calibration Targets.

Aggregate Fluctuations - Mechanics

- Integrates labor market dynamics, aggregate productivity shocks, and entry/exit mechanisms.
- Explains economic behavior based on these interactions.

Forecasting Equation:

$$\log w_{t+1} = \beta_0 + \beta_1 \log w_t + \beta_2 \log z_{t+1} + \beta_3 \log z_t + \epsilon_{t+1}$$

- w_t: Current wage.
- z_t, z_{t+1}: Aggregate productivity (current and next-period).
- ϵ_{t+1}: Zero-mean error term.

Calibration Summary

Purpose:

- Match empirical data and extend the model to include aggregate productivity shocks and labor elasticity.

Key Points:

- **Labor Supply Elasticity (γ):** Determines labor supply responsiveness to wage changes.
  - Higher γ: More elastic adjustment, amplifies output responses to shocks.
- **Productivity Dynamics (P_z and σ_z):**
  - P_z: Persistence of productivity shocks.
  - σ_z: Magnitude of productivity shocks.
  - Positive shocks increase wages, employment, and output, but effects diminish over time due to mean reversion.

Parameter Values

Calibrated Parameters:

Parameter	**Value**
Labor supply elasticity (γ)	2.0
Persistence of aggregate shocks (ρ_z)	0.685
Std. Dev. of aggregate shocks (σ_z)	0.0163

The Forecasting Rule

Purpose:

- Models how firms predict future wages to optimize decisions on:
  - Labor demand.
  - Investment.
  - Entry and exit.

Forecasting Equation:

$$\log(w_{t+1}) = 0.38385 + 0.65105 \log(w_t) + 0.53075 \log(z_{t+1}) - 0.21508 \log(z_t) + \epsilon_{t+1}$$

- w_t: Current wage.
- z_t, z_{t+1}: Current and next-period aggregate productivity.
- ϵ_{t+1}: Zero-mean error term.

Accuracy of the Forecasting Rule

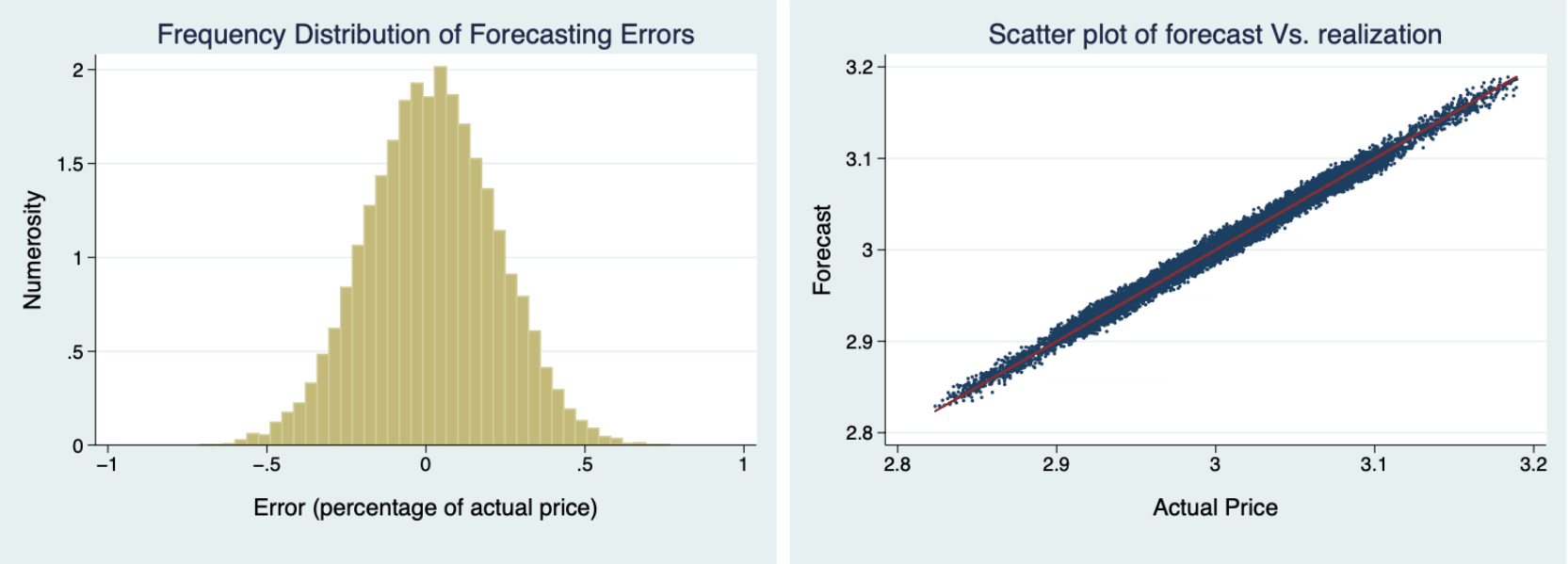

Figure 7: Accuracy of the Forecasting Rule.

Entry and Exit

Entry Dynamics:

- **Drivers:**
  - Expected profitability, influenced by future wages and productivity.
  - Firms enter if the expected value exceeds the fixed entry cost.

Exit Dynamics:

- Continuation value relative to exit value.
- Less productive firms or those facing adverse conditions are more likely to exit.

Aggregate Conditions:

- **Positive Shocks:** Increase continuation value, reducing exits.
- **Recessions:** Lower continuation value, increasing exits.

Aggregate Impacts:

- **Amplification:** Booms see more firms entering, raising output and employment.
- **Propagation:** New entrants sustain economic activity over time.

Cyclical Behavior of Entry and Exit

Key Variables and Their Behavior:

Variable	Correlation	Explanation
Entry Rate	0.402	Entry rate is pro-cyclical, meaning more firms enter the market when industry output is high.
Exit Rate	-0.779	Exit rate is counter-cyclical, indicating firms are more likely to exit when industry output is low.
Entrants' Size	-0.725	Entrants are smaller during periods of high output, as larger firms are less likely to enter when competition is strong.
Exiters' Size	-0.892	Exiters tend to be larger during periods of low output, likely due to decreased profitability for larger firms.

Impulse Response Analysis

- The analysis illustrates how the economy reacts to a positive aggregate productivity shock.
- Tracks key variables such as productivity, wages, entry rate, output, employment, and exit rate.

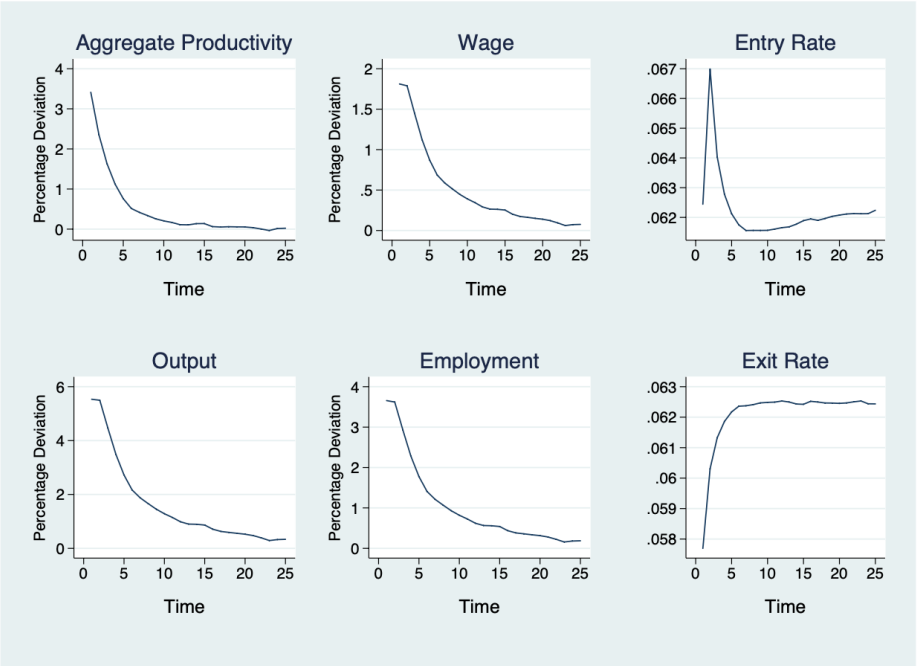

Figure 9: Response to a positive productivity shock.

Impulse Response Analysis

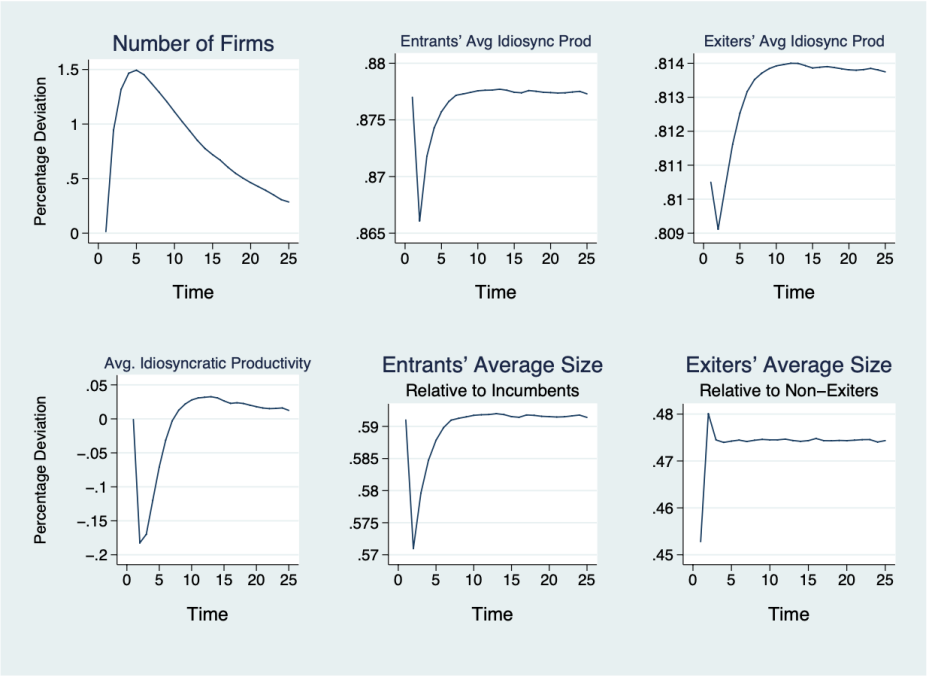

Figure 10: Response to a positive productivity shock.

Alternative Assumptions on the Discount Factor

Overview:

- The discount factor determines how future cash flows are valued.
- It plays a critical role in shaping firm entry decisions and aggregate economic dynamics.
- Its impact depends on its relationship with aggregate productivity (z_t).

Key Scenarios:

- **Pro-Cyclical Scenario:**
  - When z_t is high (booms), the discount factor is high.
  - Entry becomes more attractive, amplifying economic fluctuations.
- **Counter-Cyclical Scenario:**
  - When z_t is high, the discount factor is low.
  - Entry becomes less responsive to productivity shocks, dampening fluctuations.

Relevance of Idiosyncratic Shocks

- Idiosyncratic shocks include:
  - Technical efficiency.
  - Demand-side factors (e.g., consumer preferences or market conditions).
- Even with demand shocks, firm dynamics (entry/exit) behave similarly, aligning with the model.

Empirical Evidence:

- Data from U.S. manufacturing (Census of Manufacturing, 1977–1997) shows:
  - Demand-side factors (e.g., price competition) significantly influence firm dynamics.
  - Entrants have productivity levels similar to incumbents but face more volatile demand.
  - Total Factor Productivity (TFP) dynamics reflect both supply- and demand-side shocks, validating the model's generalizability.

Amplification and Propagation

- Entry and exit amplify aggregate shocks through changes in firm dynamics.
- Key mechanism:
  - A rise in aggregate productivity increases entry, leading to more small firms with high growth potential.
  - Over time, surviving entrants grow larger, propagating the effects of the initial shock.
- Aggregate output Y_t depends on z_t, the firm distribution Γ_t, the number of firms N_t, and aggregate employment L_t.

$$Y_t = z_t \left[\int \Gamma_t(s) s^{\frac{1}{1-\alpha}} ds \right]^{1-\alpha} N_t^{1-\alpha} L_t^{\alpha} \tag{1}$$

- Entry increases the number of firms N_t, while firm growth affects the distribution Γ_t.

Comparison with Benchmark

- In the benchmark model without entry or exit, output dynamics rely only on aggregate productivity z_t and capital accumulation.

$$Y_t^{\text{benchmark}} = z_t K_t^{\alpha} L_t^{1-\alpha} \tag{2}$$

- The difference between the models is:

$$\Delta Y_t = Y_t - Y_t^{\text{benchmark}} \approx c_N \Delta N_t + c_\Gamma \Delta \Gamma_t(s) \tag{3}$$

  - c_N: changes in the number of firms N_t on output.
  - c_Γ: shifts in the productivity distribution $\Gamma_t(s)$.

Comparison with Benchmark

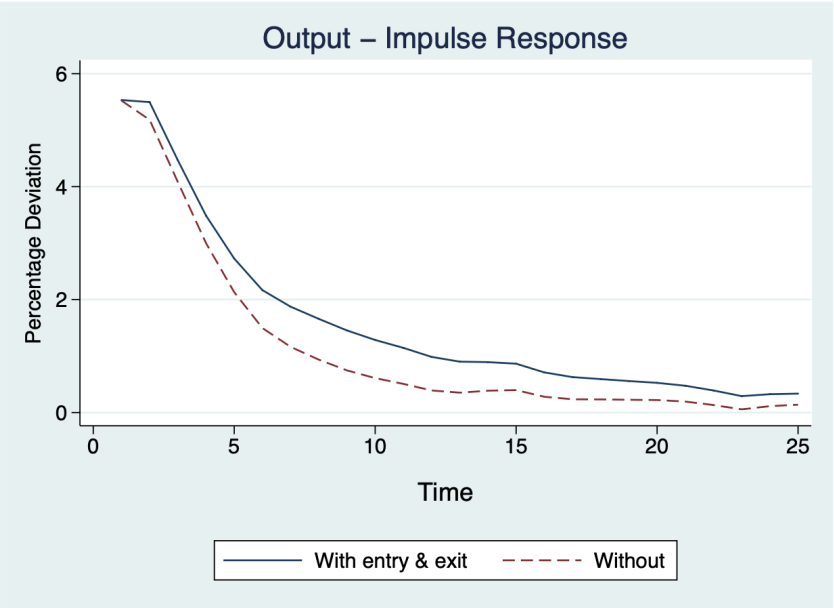

Figure 11: The effect of entry & exit on output dynamics.

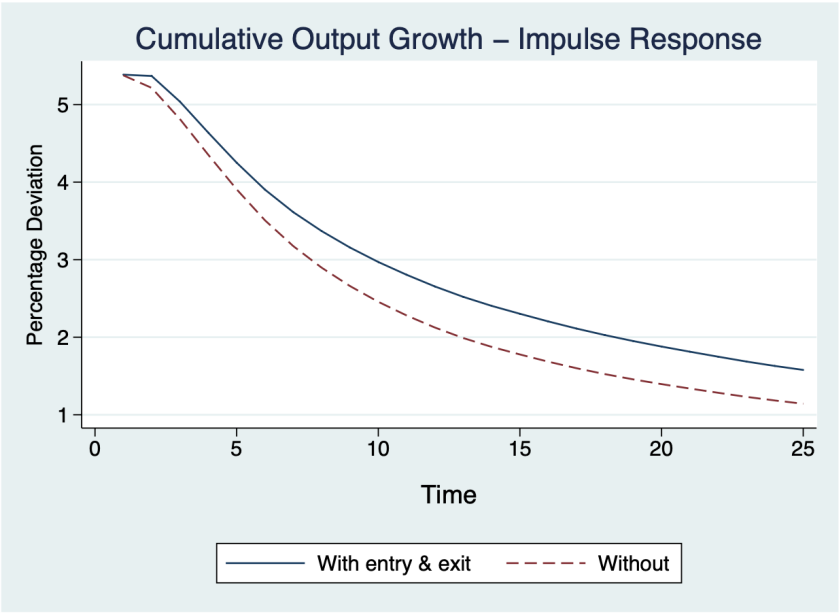

Figure 12: Cumulative Output Growth.

- The Entry-Exit model shows higher output over time due to amplification effects (Figure 11).
- It also demonstrates greater cumulative output growth, highlighting propagation effects (Figure 12).

Productivity Decomposition

- According to Haltiwanger (1997), aggregate productivity growth can be decomposed as:

$$\Delta \log(TFP_t) = \underbrace{\sum_{i \in \mathcal{C}_t} \phi_{i,t-k} \Delta \log(TFP_{it})}_{\text{Within}} + \underbrace{\sum_{i \in \mathcal{C}_t} [\log(TFP_{i,t-k}) - \log(TFP_{t-k}] \Delta \phi_{it}}_{\text{Between}} +$$

$$\underbrace{\sum_{i \in \mathcal{C}_t} \Delta \log(TFP_{it}) \Delta \phi_{it}}_{\text{Covariance}} + \underbrace{\sum_{i \in \mathcal{E}_t} [\log(TFP_{it}) - \log(TFP_{t-k}] \phi_{it}}_{\text{Entry}} -$$

$$\underbrace{\sum_{i \in \mathcal{X}_t} [\log(TFP_{i,t-k}) - \log(TFP_{t-k}] \phi_{i,t-k}}_{\text{Exit}} \tag{4}$$

Productivity Decomposition

k	Within	Between	Covariance	Entry	Exit	Net Entry
1	-8.7778	-4.1623	12.4366	-0.2964	-0.8011	0.5047
5	-15.1437	-13.3884	27.5833	-0.7556	-1.7104	0.9548

Table 6: Productivity Decomposition (percentages).

- In the model, net entry (Entry - Exit) positively contributes to aggregate productivity growth.
- Entrants are typically more productive than exiters, and their role grows over time as they expand.

Cyclical Variation of Cross-sectional Moments

- The first moment (mean) of the productivity distribution is counter-cyclical, consistent with empirical evidence:
  - Foster et al. (2008)
- The third moment (skewness) is pro-cyclical, aligning with observed patterns:
  - Cabral and Mata (2003)
- The second moment (variance) shows mixed results and is not robust across parameter values:
  - Bachmann and Bayer (2013); Bloom et al. (2014)

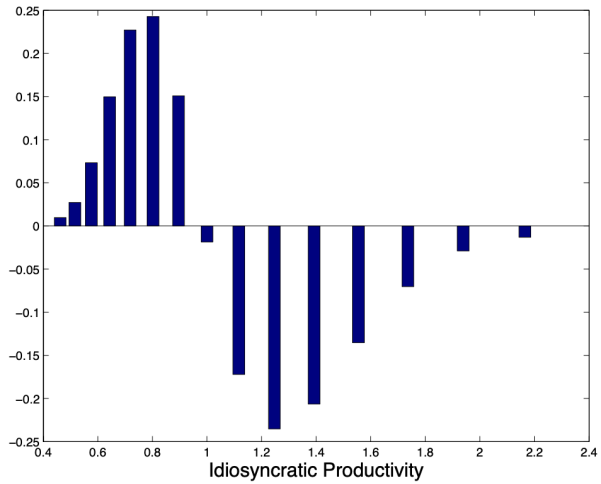

Figure 13: Change in the Cross-Sectional Distribution.

Characteristics of the 2007-2009 Recession

- Significant decline in total employment and new firm entry.
- Analysis of cohorts reveals (2006 cohort - benchmark):
  - Extensive margin[1]: lower survival rates than the benchmark and persistent.
  - Plant demographics may have had a large and persistent effect on outputs.

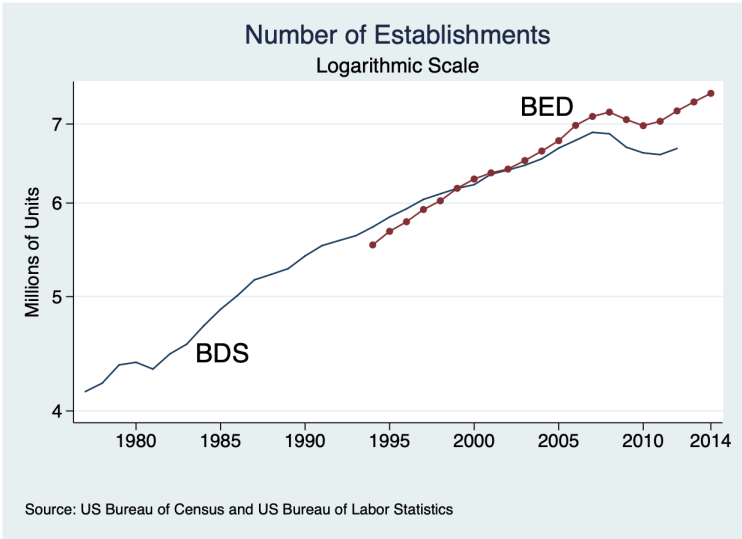

Figure 14: Number of private–sector establishments.

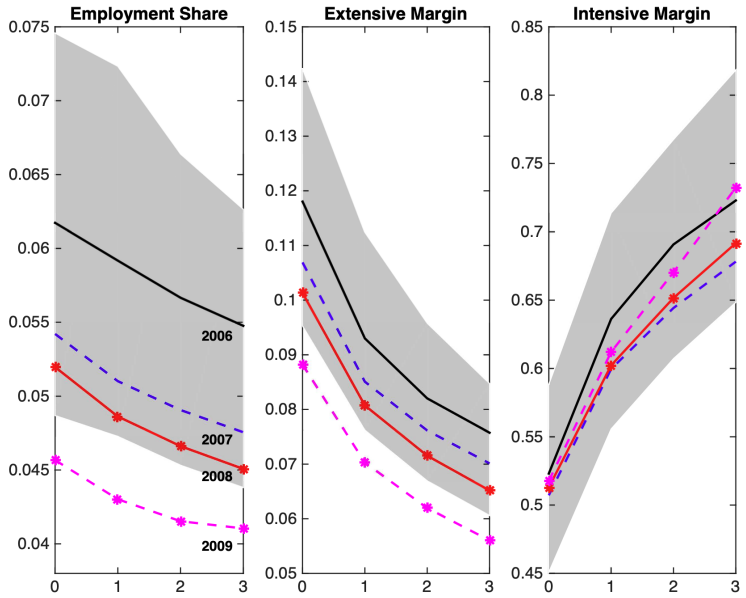

Figure 15: Young Firms during the Great Recession.

[1] Proportion of establishments born in a given year that survive at a later time ($n_{t,s}/N_{t+s}$).

Explaining Observed Patterns with the Model

- Comparison between TFP-only and Entry-specific shocks:

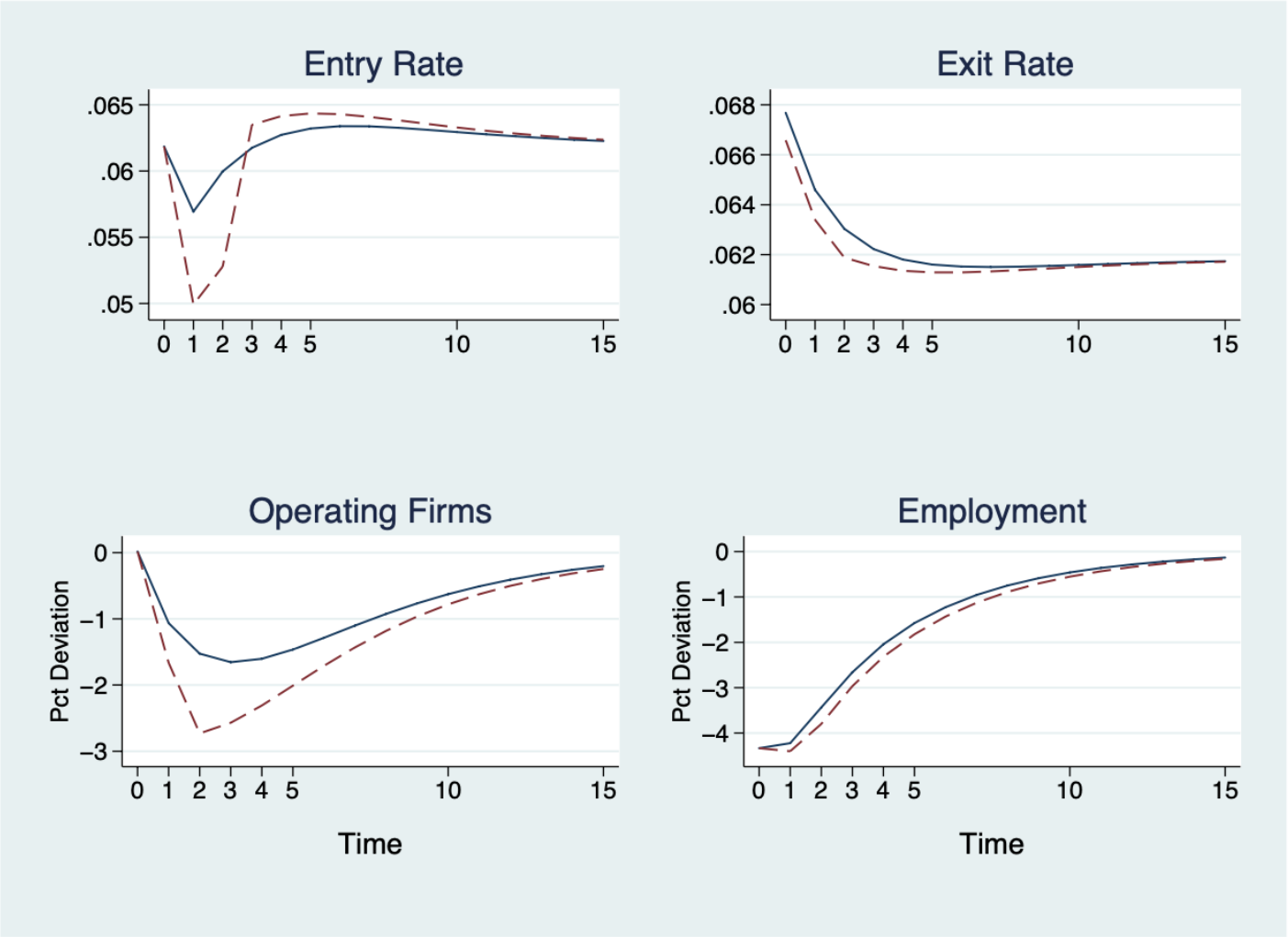

Figure 16: Solid line: TFP shock only. Dashed line: TFP plus entry-specific shock.

Explaining Observed Patterns with the Model

- Comparing data (Figure 15, left) and model predictions (Figure 17, right):
  - Similar patterns except for initial employment.
  - The model explains the drop in employment share and extensive margin.

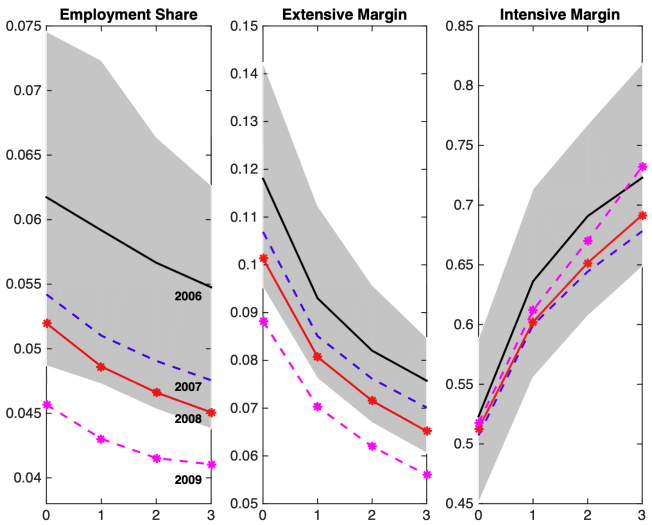

Figure 15: Young Firms during the Great Recession.

Figure 17: Solid: TFP shock only. Dashed: TFP plus entry-specific shock. Dot-dashed: Average cohort.

Conclusion

- Entry and exit amplify and propagate aggregate shocks.
- Firm dynamics are crucial for persistence and variation in aggregate outcomes.
- Plan to replicate the paper:
  - Programming Language: Python or Julia.
  - Steps:
    1. Replicate the structure of the core model.
    2. Match the paper's results from the stationary case
    3. Continue calibrate parameters to match empirical moments.
    4. Simulate scenarios with and without entry/exit dynamics.

Conclusion

- Potential Extensions:
  - H: Explore alterations to the labor market clearing condition to allow for dominant buyers in the labor market.
  - N: Policy Shocks like Subsidies and Tax incentive for firms Entry and Exit
  - Y: Patent Rights into a DynamicModel of Firm Entry and Exit.

Thank you :)