PhD Macro Core Part I:
Lecture 17 – Monetary Policy in NK Models

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Today

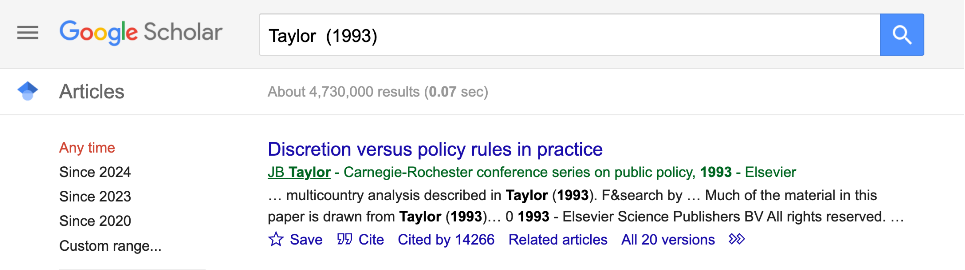
- Sticky prices create room for monetary policy
- How do central banks conduct monetary policy?
- What is the optimal monetary policy design?
- We will find it out today!

Interest Rate Rules

- Characterize monetary policy with simple interest rate rules instead of exogenous money supply rules
- Why? Prices (interest rates) are more informative than quantities!
- What to include in the interest rate rules?
- Must be "determinate" or "unique" endogenous variables
- The "grandfather" of interest rate rules: "Taylor Rule" by John B. Taylor

Taylor Rule (1993)

- The second most influencing paper in the Carnegie-Rochester Conference Series on Public Policy



Taylor Rule

- A policy rule of the following form (log linearized):

$$i_t = \phi_\pi \pi_t + \phi_x x_t$$

  where π_t is inflation, x_t is the gap between actual and potential output, and i_t is the interest rate

- Taylor argued that values of $\phi_\pi = 1.5$ and $\phi_x = 0.5$ fit the data well. He argued that the coefficient of inflation needed to be greater than 1. This came to be known as the "Taylor principle."

- Taylor's Logic: Stabilizing!

- Need a sufficient reaction to inflation to generate a stable root to keep the system from exploding

Determinacy in a Model with Flexible Prices

- A very simple model: No capital, prices are flexible, money in the utility function

- The demand side of the economy is summarized by the Euler/IS equation:

$$y_t = \mathbb{E}_t y_{t+1} - (i_t - \mathbb{E}_t \pi_{t+1})$$

- Suppose the policy rule just reacts to inflation with a random, mean zero shock, u_t:

$$i_t = \phi_\pi \pi_t + u_t$$

- Suppose that u_t follows a stationary (e.g. $0 < \rho < 1$ AR(1) process):

$$u_t = \rho u_{t-1} + e_t$$

Determinacy in a Model with Flexible Prices

- Suppose that real output is both exogenous and constant. This means that $y_t = \mathbb{E}_t y_{t+1} = 0$.

- The Euler equation then becomes:
$$i_t = \mathbb{E}_t \pi_{t+1}$$

- If we combine this expression with the policy rule, we get:
$$\mathbb{E}_t \pi_{t+1} = \phi_\pi \pi_t + u_t$$

- If $\phi_\pi < 1$, then there is no unique solution - any value of π_t will have expected inflation goes to zero in the limit for any u_t. To see this as cleanly as possible, suppose $u_t = 0$. Then, solving forward:
$$\mathbb{E}_t \pi_{t+T} = \phi_\pi^T \pi_t$$

Determinacy in a Model with Flexible Prices

- If $\phi_\pi < 1$ (ruling out negative values), then $\phi_\pi^T \to 0$ for T big. This means any value of π_t is consistent with inflation, not exploding.

- In contrast, if $\phi_\pi > 1$, then $\phi_\pi^T \to \infty$. The only for inflation to not explode is then if $\pi_t = 0$; that would be the unique solution.

- For the more general situation in which we allow $u_t \neq 0$, we can solve for the unique solution by guess that $\pi_t = au_t$. Doing so, we get:

$$a\mathbb{E}_t u_{t+1} = \phi_\pi a u_t + u_t$$

Since $\mathbb{E}_t u_{t+1} = \rho u_t$, we have:

$$a\left(\rho - \phi_\pi\right) = 1$$

So we get $\pi_t = \frac{1}{\rho - \phi_\pi} u_t$ as the solution (which is unique, provided $\phi_\pi > 1$).

Determinacy in a Basic New Keynesian Model

- A standard New Keynesian model: Euler/IS equation, Phillips curve, exogenous output process
- The equations of the model are:

$$\begin{aligned} \pi_t &= \gamma\left(y_t - y_t^f\right) + \beta \mathbb{E}_t \pi_{t+1} \\ y_t &= \mathbb{E}_t y_{t+1} - (i_t - \mathbb{E}_t \pi_{t+1}) \\ y_t^f &= \rho y_{t-1}^f + s_y \varepsilon_t \end{aligned}$$

  where the slope coefficient is $\gamma = \frac{(1-\phi)(1-\phi\beta)}{\phi}(1+\chi)$.

- Suppose that the nominal interest rate obeys a simple Taylor rule of the form:

$$i_t = \phi_\pi \pi_t + \phi_x \left(y_t - y_t^f\right)$$

Determinacy in a Basic New Keynesian Model

- What restrictions on ϕ_π and ϕ_x to ensure a determinate rational expectations equilibrium?

- To see this, eliminate i_t and form a three-variable system. The Euler/IS equation becomes:

$$y_t = \mathbb{E}_t y_{t+1} - \phi_\pi \pi_t - \phi_x y_t + \phi_x y_t^f + \mathbb{E}_t \pi_{t+1}$$

- This is a difference equation in y_t and π_t. The Phillips Curve is already that (since i_t doesn't enter directly into the Phillips Curve, we don't need to do any substitution into it). In essence, i_t is a static variable, and we want to substitute it in writing down a system of equations to solve. Given these two remaining equations, plus the exogenous process for y_t^f, we can form a vector system as follows:

$$\mathbb{E}_t \begin{bmatrix} \pi_{t+1} \\ y_{t+1} \\ y_{t+1}^f \end{bmatrix} = \begin{bmatrix} \frac{1}{\beta} & -\frac{\gamma}{\beta} & \frac{\gamma}{\beta} \\ \phi_\pi - \frac{1}{\beta} & 1 + \phi_x + \frac{\gamma}{\beta} & -\frac{\gamma}{\beta} - \phi_x \\ 0 & 0 & \rho \end{bmatrix} \begin{bmatrix} \pi_t \\ y_t \\ y_t^f \end{bmatrix}$$

Determinacy in a Basic New Keynesian Model

- Our goal now is to find the eigenvalues of this system of equations.

- One of them is clearly ρ, which is than one, and hence stable. This is the eigenvalue associated with the (exogenous) state variable, y_t^f.

- For equilibrium determinacy, we need two unstable eigenvalues associated with the remaining jump variables, y_t and π_t. To find the other two eigenvalues, we just need to find the eigenvalues of the upper 2×2 block of the coefficient matrix.

- That is, we need to find the λ which makes:

$$\det \begin{bmatrix} \frac{1}{\beta} - \lambda & -\frac{\gamma}{\beta} \\ \phi_\pi - \frac{1}{\beta} & 1 + \phi_x + \frac{\gamma}{\beta} - \lambda \end{bmatrix} = 0$$

Determinacy in a Basic New Keynesian Model

- The determinant of a 2×2 matrix is just the difference of the product of the diagonals:

$$\left(\frac{1}{\beta} - \lambda\right)\left(1 + \phi_x + \frac{\gamma}{\beta} - \lambda\right) + \frac{\gamma}{\beta}\left(\phi_\pi - \frac{1}{\beta}\right) = 0$$

- Two useful facts about eigenvalues and determinants.

- The determinant and trace of the upper 2×2 matrix are:

$$\lambda_1 \lambda_2 = \det \begin{bmatrix} \frac{1}{\beta} & -\frac{\gamma}{\beta} \\ \phi_\pi - \frac{1}{\beta} & 1 + \phi_x + \frac{\gamma}{\beta} \end{bmatrix} = \frac{1}{\beta} + \frac{\phi_x}{\beta} + \frac{\gamma\phi_\pi}{\beta}$$

$$\lambda_1 + \lambda_2 = \operatorname{trace} \begin{bmatrix} \frac{1}{\beta} & -\frac{\gamma}{\beta} \\ \phi_\pi - \frac{1}{\beta} & 1 + \phi_x + \frac{\gamma}{\beta} \end{bmatrix} = \frac{1}{\beta} + 1 + \phi_x + \frac{\gamma}{\beta}$$

Determinacy in a Basic New Keynesian Model

- The necessary condition for stability is that:

$$(\lambda_1 - 1)(\lambda_2 - 1) > 0, \qquad \text{or } \lambda_1\lambda_2 - (\lambda_1 + \lambda_2) > -1$$

- Plug in our expressions from above and simplify:

$$\begin{aligned}
\frac{1}{\beta} + \frac{\phi_x}{\beta} + \frac{\gamma\phi_\pi}{\beta} - \left(\frac{1}{\beta} + 1 + \phi_x + \frac{\gamma}{\beta}\right) &> -1 \\
\phi_x\left(\frac{1}{\beta} - 1\right) + \frac{\gamma\phi_\pi}{\beta} - \frac{\gamma}{\beta} &> 0 \\
\phi_x(1 - \beta) + \gamma\phi_\pi - \gamma &> 0
\end{aligned}$$

- The last line follows from multiplying both sides by β. Now divide both sides by γ and simplify:

$$\phi_x\frac{1-\beta}{\gamma} + \phi_\pi > 1$$

- If $\beta \approx 1$, then unless γ is very small the determinacy condition is still roughly $\phi_\pi > 1$.

Summary on Interest Rate Rules

- Taylor Rule with $\phi_\pi > 1$ is a good guideline of central bank monetary policy
- Why? Because the purpose of monetary policy is to stabilize the business cycle
- The Taylor Rule serves this purpose very well!
- More details are in the handouts
- Does the central bank follow the Taylor Rule?
- Is the Taylor Rule the optimal formula for monetary policy?
- Let's find it out!

Optimal Monetary Policy in the New Keynesian Model

- The basic model: the IS curve and the Phillips curve:

$$\pi_t = \gamma x_t + \beta \mathbb{E}_t \pi_{t+1} \tag{1}$$

$$x_t = \mathbb{E}_t x_{t+1} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t \pi_{t+1} - r_t^f\right) \tag{2}$$

- The Federal Reserve in the US has a "dual mandate" in that it wants to promote price stability and full employment. We can define a loss function for the central bank as follows:

$$\mathcal{L} = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \frac{1}{2}\left[\pi_{t+j}^2 + \omega x_{t+j}^2\right] \tag{3}$$

- In particular, in the most basic version of the NK model, the value of ω ought to be:

$$\omega = \frac{\gamma}{(1+\chi\epsilon)\epsilon}$$

Optimal Policy Under Discretion vs. Commitment

- Two versions of an optimal policy problem: discretion and commitment
- Under discretion, the policymaker will choose its instrument to minimize the period loss function, taking all future values in the loss function as given.
  - Basically, the policymaker is unable to commit to anything about the future, and so it just minimizes the loss function in the present, taking the future as given.
- Under commitment, in contrast, the policymaker picks an entire sequence of its policy instrument to minimize the entire loss function.
  - What's the purpose of commitment? Is commitment trustable?

Discretion

- The instrument is the short-term nominal interest rate, i_t.

- Derive an optimal targeting rule to pick i_t to minimize the loss function:

$$\min_{i_t} \mathcal{L} = \frac{1}{2}\pi_t^2 + \frac{\omega}{2}x_t^2$$

s.t.

$$\pi_t = \gamma x_t + \beta \mathbb{E}_t \pi_{t+1}$$

$$x_t = \mathbb{E}_t x_{t+1} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t \pi_{t+1} - r_t^f\right)$$

- The policymaker takes r_t^f, $\mathbb{E}_t \pi_{t+1}$, and $\mathbb{E}_t x_{t+1}$ as given

- Plugging x_t into the Phillips Curve to eliminate x_t gives:

$$\pi_t = \gamma \mathbb{E}_t x_{t+1} - \frac{\gamma}{\sigma}\left(i_t - \mathbb{E}_t \pi_{t+1} - r_t^f\right) + \beta \mathbb{E}_t \pi_{t+1}$$

Discretion

- We can write the policy problem as an unconstrained problem of choosing i_t :

$$\min_{i_t} \mathcal{L} = \frac{1}{2}\left[\gamma \mathbb{E}_t x_{t+1} - \frac{\gamma}{\sigma}\left(i_t - \mathbb{E}_t \pi_{t+1} - r_t^f\right) + \beta \mathbb{E}_t \pi_{t+1}\right]^2 + \frac{\omega}{2}\left[\mathbb{E}_t x_{t+1} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t \pi_{t+1} - r_t^f\right)\right]^2$$

- The derivative of the loss function with respect to i_t is:

$$\frac{\partial \mathcal{L}}{\partial i_t} = -\frac{\gamma}{\sigma}\pi_t - \frac{\omega}{\sigma}x_t$$

- Setting the derivative equal to zero and simplifying yields: "lean against the wind"

$$x_t = -\frac{\gamma}{\omega}\pi_t \tag{4}$$

- Eq.(4) is an implicit inflation target: when inflation is high, it allows the output gap to go negative

Discretion

- To solve for the required path of the interest rate to implement this lean against the wind condition, plug it into the Phillips curve to eliminate x_t :

$$\pi_t = -\frac{\gamma^2}{\omega}\pi_t + \beta\mathbb{E}_t\pi_{t+1}, \quad \text{or} \quad \mathbb{E}_t\pi_{t+1} = \frac{\left(1+\frac{\gamma^2}{\omega}\right)}{\beta}\pi_t$$

- This is an explosive difference equation in inflation. The coefficient $\frac{\left(1+\frac{\gamma^2}{\omega}\right)}{\beta} > 1$.

- The only non-explosive solution for inflation is $\pi_t = 0$. But if $\pi_t = 0$, then $x_t = 0$.

- Under discretion in this model, we can reach the global minimum of the loss function at 0!

- If $\pi_t = x_t = 0$ in period t, then agents will expect the same for $t+1$: $\mathbb{E}_t\pi_{t+1} = \mathbb{E}_t x_{t+1} = 0$.

- Optimal policy: $i_t = r_t^f$, moving the interest rate one-for-one with the natural rate of interest

Commitment: Brief Introduction

- The central bank doesn't just choose the current instrument, i_t; it chooses all future instruments
- CB doesn't need to take future inflation and the output gap as given when choosing the current i_t.
- For any $j \geqslant 1$, the first order conditions boil down to: "future lean against the wind condition"

$$x_{t+j} = -\frac{\gamma}{\omega} \sum_{s=0}^{j} \pi_{t+s} = -\frac{\gamma}{\omega} \left(\ln P_{t+j} - \ln P_{t-1} \right) \tag{11}$$

- **Time Inconsistency**: The basic idea of time inconsistency is that a policymaker wants to plan to do something in the future, but when the future comes around, if the policymaker can re-optimize, she will want to deviate from the plan.

Commitment vs. Discretion

- The optimal targeting rule under discretion is a "lean against the wind" condition in terms of the output gap and the inflation rate:

$$x_t = -\frac{\gamma}{\omega}\pi_t \tag{12}$$

- The optimal targeting rule under commitment has a similar feel, but it is an implicit price level target: the policymaker is targeting a relationship between the output gap and the price level:

$$x_t = -\frac{\gamma}{\omega}\ln P_t \tag{13}$$

- How do these targeting rules differ, and what are the gains from commitment over discretion?

Divine Coincidence

- In the basic NK model, it turns out that there are no gains from commitment over discretion
- Since discretion achieves $\mathcal{L} = 0$, there can be no gain from commitment
- A term deemed the "Divine Coincidence" (Blanchard and Gali, 2007)
- Does the "Divine Coincidence" hold in more complex reality?

Cost-Push Shocks and a Monetary Policy Trade-off

- Introduce a "cost-push" shock into the Phillips Curve:

$$\pi_t = \gamma x_t + \beta \mathbb{E}_t \pi_{t+1} + u_t$$

- If $u_t \neq 0$, then it is not going to be possible for $\pi_t = x_t = 0$ at all times

- Plug in the FOC into the Phillips Curve to eliminate x_t :

$$\pi_t = -\frac{\gamma^2}{\omega}\pi_t + \beta \mathbb{E}_t \pi_{t+1} + u_t$$

  or

$$\mathbb{E}_t \pi_{t+1} = \frac{\omega + \gamma^2}{\omega\beta}\pi_t - \frac{1}{\beta}u_t, \quad \text{def} \quad a \equiv \frac{\omega + \gamma^2}{\omega\beta}$$

  Let's assume that u_t follows an exogenous $\mathrm{AR}(1)$ process:

$$u_t = \rho_u u_{t-1} + s_u \varepsilon_{u,t}$$

Cost-Push Shocks and a Monetary Policy Trade-off

- Let's solve the difference equation forward. Going forward to $t + 2$, we have:

$$\mathbb{E}_t \pi_{t+2} = a \mathbb{E}_t \pi_{t+1} - \frac{1}{\beta} \mathbb{E}_t u_{t+1}$$

  which is

$$\mathbb{E}_t \pi_{t+2} = a^2 \pi_t - a \frac{1}{\beta} u_t - \frac{1}{\beta} \rho_u u_t$$

- Now, similarly, going forward another period, we have:

$$\mathbb{E}_t \pi_{t+3} = a \mathbb{E}_t \pi_{t+2} - \frac{1}{\beta} \mathbb{E}_t u_{t+2}$$

  which is

$$\mathbb{E}_t \pi_{t+3} = a^3 \pi_t - \frac{1}{\beta} \left(\rho_u^2 + a \rho_u + a^2 \right) u_t$$

- Go forward another period. We have: $\mathbb{E}_t \pi_{t+4} = a \mathbb{E}_t \pi_{t+3} - \frac{1}{\beta} \rho_u^3 u_t$

Discretion

- Skipping about 30 steps here...... (read the handouts)

- Under discretion, the solution for π_t is:

$$\pi_t = \frac{\omega}{\omega\left(1 - \rho_u \beta\right) + \gamma^2} u_t \tag{16}$$

- Therefore, the solution for x_t is:

$$x_t = -\frac{\gamma}{\omega\left(1 - \rho_u \beta\right) + \gamma^2} u_t \tag{17}$$

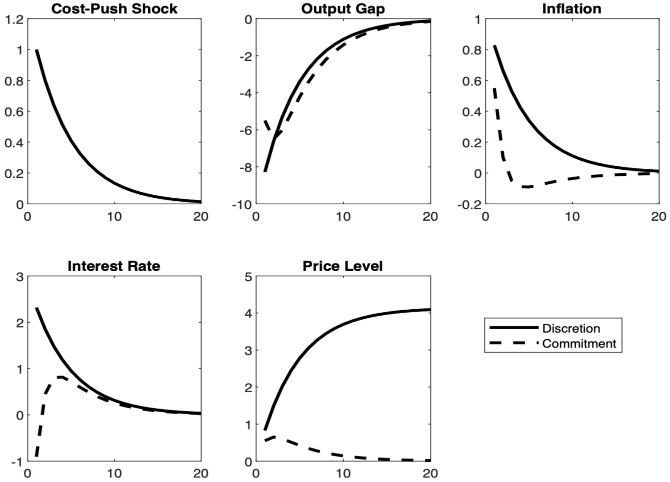
- (16) and (17) are actually pretty intuitive expressions. Suppose that $\omega \to \infty$. This means that the policymaker really wants to stabilize the output gap. Then, the coefficient for x_t will go to 0 , so that $x_t = 0$. Alternatively, suppose that $\gamma \to \infty$ (so that prices are flexible). Then both coefficients go to zero, so neither inflation nor the output gap react to the cost-push shock.

Commitment

- Analytical solutions? No way! Dynare could help!
- Just solve the model in a program like Dynare with $x_t = -\frac{\gamma}{\omega} \ln P_t$
- Under commitment, the price level will be stationary, whereas under discretion, it will not

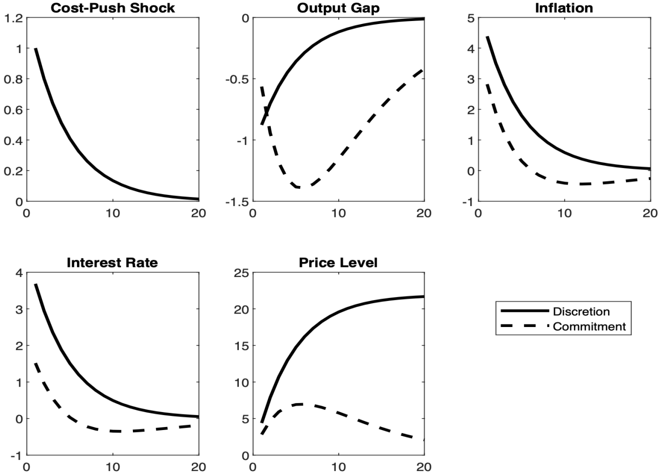
Simulation

Figure 1: Cost-Push Shock: Commitment vs. Discretion ($\omega = 0.01$)



Simulation

Figure 2: Cost-Push Shock: Commitment vs. Discretion ($\omega = 0.5$)



Simulation

Figure 3: Cost-Push Shock: Commitment vs. Discretion ($\omega = 1$)