PhD Macro Core Part I: Lecture 16 – Fiscal Policy

Min Fang
University of Florida

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Today

- Fiscal policy in the RBC model:
- "Lump-sum" Taxes
- "Distortionary" Taxes

Lump-Sum Finance

- All government revenue is raised through lump-sum taxes (unrealistic)
- The amount of spending is exogenously given
- Analyze the effects of changes in government spending
- Ricardian Equivalence

Household

- A representative household that solves a standard problem

$$\max_{C_t, N_t, K_{t+1}, B_{t+1}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\ln C_t - \theta \frac{N_t^{1+\chi}}{1+\chi} + h\left(G_t\right) \right)$$

$$\text{s.t.}$$

$$C_t + K_{t+1} - (1-\delta) K_t + B_{t+1} - B_t \leqslant w_t N_t + R_t^k K_t + \Pi_t - T_t + r_{t-1} B_t$$

- The first-order conditions for the household problem are standard:

$$\frac{1}{C_t} = \beta \mathbb{E}_t \left[\frac{1}{C_{t+1}} \left(R_{t+1}^k + (1-\delta) \right) \right] \tag{1}$$

$$\frac{1}{C_t} = \beta \mathbb{E}_t \left[\frac{1}{C_{t+1}} \left(1 + r_t \right) \right] \tag{2}$$

$$\theta N_t^{\chi} = \frac{1}{C_t} w_t \tag{3}$$

Firm

- A representation firm that solves the standard static firm problem:

$$\max_{N_t, K_t} \quad \Pi_t = A_t K_t^{\alpha} N_t^{1-\alpha} - w_t N_t - R_t^k K_t$$

- FOCs to equate factor prices with marginal products, again entirely standard:

$$w_t = (1-\alpha) A_t K_t^{\alpha} N_t^{-\alpha} \tag{4}$$

$$R_t^k = \alpha A_t K_t^{\alpha-1} N_t^{1-\alpha} \tag{5}$$

Government

- The government chooses spending, G_t, exogenously
- Finances spending with the aforementioned lump-sum taxes, T_t, and by issuing new debt, D_{t+1}
- Inherits an existing stock of debt, D_t, from history
- Debt is held by all households
- The government budget constraint binding with equality:

$$T_t = G_t + (1 + r_{t-1}) D_t - D_{t+1}$$

Equilibrium

- **Def:** A competitive equilibrium is a set of prices $\left(r_t, R_t^k, w_t\right)$ and allocations $(C_t, K_{t+1}, N_t, B_{t+1}, D_{t+1})$ such that (i) household and firm optimality conditions all hold, (ii) the firm hires all the labor and capital supplied by the household, (iii) the household and firm budget constraints hold with equality, and (iv) household bond-holdings equal government debt issuance in all periods (i.e. $B_{t+1} = D_{t+1}$, and we require that $B_t = D_t$ initially), given values and stochastic processes of G_t and A_t, as well as initial values of government debt and household bond-holdings, which must be equal (e.g. $B_t = D_t$).

Derive the Equation System

- The government budget constraint binding with equality means that:

$$T_t = G_t + (1 + r_{t-1})\, D_t - D_{t+1}$$

- Plug this, along with the definition of profit, into the household budget constraint at equality:

$$C_t + K_{t+1} - (1-\delta)K_t = A_t K_t^{\alpha} N_t^{1-\alpha} - G_t - (1 + r_{t-1})\, D_t + D_{t+1} + (1 + r_{t-1})\, B_t - B_{t+1}$$

- Using the fact that $D_t = B_t$ and $D_{t+1} = B_{t+1}$, we have:

$$C_t + K_{t+1} - (1-\delta)K_t + G_t = A_t K_t^{\alpha} N_t^{1-\alpha} \tag{7}$$

Endogenous Equation System

- Defining $I_t = K_{t+1} - (1-\delta)K_t$ and $Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}$, we can summarize the equilibrium as:

$$\frac{1}{C_t} = \beta E_t \left[\frac{1}{C_{t+1}} \left(R_{t+1}^k + (1-\delta) \right) \right] \tag{8}$$

$$\frac{1}{C_t} = \beta E_t \left[\frac{1}{C_{t+1}} (1 + r_t) \right] \tag{9}$$

$$w_t = (1-\alpha) A_t K_t^{\alpha} N_t^{-\alpha} \tag{10}$$

$$R_t = \alpha A_t K_t^{\alpha-1} N_t^{1-\alpha} \tag{11}$$

$$\theta N_t^{\chi} = \frac{1}{C_t} w_t \tag{12}$$

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha} \tag{13}$$

$$Y_t = C_t + I_t + G_t \tag{14}$$

$$K_{t+1} = I_t + (1-\delta) K_t \tag{15}$$

Exogenous Stochastic Processes

- Close the model with exogenous stochastic processes for government spending and productivity:

$$\ln G_t = (1-\rho_G)\ln(\omega Y) + \rho_G \ln G_{t-1} + s_G \varepsilon_{g,t} \tag{16}$$
$$\ln A_t = \rho_A \ln A_{t-1} + s_A \varepsilon_{a,t} \tag{17}$$

Where $\varepsilon_{a,t}$ and $\varepsilon_{g,t}$ are drawn from standard normal distributions.

Ricardian Equivalence

- **Ricardian Equivalence:** The method of government budget finance (the mix between taxes and debt) is irrelevant conditional on a time path of government spending, G_t.

- Conditions: an infinitely-lived representative agent exists, and all taxes are lump sum.

- How to think about it? Add an equation to (8)-(17): the government's flow budget constraint:

$$G_t + r_{t-1}D_t = T_t + D_{t+1} - D_t \tag{18}$$

- The model is indeterminate: we would have 11 equations but 12 variables (adding D_t and T_t)

- Since D_t and T_t don't appear in (8)-(17), how exactly D_t and T_t are set doesn't matter for the equilibrium allocations and prices.

Ricardian Equivalence: Simplified Version

- First, let's assume that the real interest rate is constant, $r_{t+j} = r$ for any j (this is, of course, not consistent with general equilibrium, but it makes the exposition follow a bit cleaner and is not necessary).

- Second, let's assume that household income is given by Y_t and there is no investment, so we don't have to keep writing factor prices. From the household's perspective, this is like an endowment economy model.

- Third, let us assume that the initial values of household savings and government debt are both zero, $D_t = B_t = 0$. None of these assumptions are necessary; they make things cleaner.

Ricardian Equivalence: Simplified Version

- The household's flow budget constraint is:

$$C_t + B_{t+1} = Y_t - T_t$$

- In period $t+1$, the budget constraint will be:

$$C_{t+1} + B_{t+2} = Y_{t+1} - T_{t+1} + (1+r)B_{t+1}$$

- So:

$$B_{t+1} = \frac{1}{1+r}C_{t+1} - \frac{1}{1+r}\left[Y_{t+1} - T_{t+1}\right] + \frac{1}{1+r}B_{t+2}$$

- But B_{t+2} will look similarly:

$$B_{t+2} = \frac{1}{1+r}C_{t+2} - \frac{1}{1+r}\left[Y_{t+2} - T_{t+2}\right] + \frac{1}{1+r}B_{t+3}$$

Ricardian Equivalence: Simplified Version

- But then plugging this in for the B_{t+1} expression, we have:

$$B_{t+1} = \frac{1}{1+r}C_{t+1} + \left(\frac{1}{1+r}\right)^2 C_{t+2} - \frac{1}{1+r}(Y_{t+1} - T_{t+1}) - \left(\frac{1}{1+r}\right)^2 (Y_{t+2} - T_{t+2}) + \left(\frac{1}{1+r}\right)^2 B_{t+3}$$

- If we keep doing this, we are going to end up with the following:

$$B_{t+1} = \sum_{j=1}^{\infty} \frac{C_{t+j}}{(1+r)^j} - \sum_{j=1}^{\infty} \frac{Y_{t+j} - T_{t+j}}{(1+r)^j} + \lim_{T\to\infty} \left(\frac{1}{1+r}\right)^T B_{t+T+1}$$

- The transversality condition eliminates the last condition. So we are left with:

$$B_{t+1} = \sum_{j=1}^{\infty} \frac{C_{t+j}}{(1+r)^j} - \sum_{j=1}^{\infty} \frac{Y_{t+j} - T_{t+j}}{(1+r)^j}$$

Ricardian Equivalence: Simplified Version

- Plugging this into the period- t budget constraint, we get the intertemporal budget constraint for the household:

$$\sum_{j=0}^{\infty} \frac{C_{t+j}}{(1+r)^j} = \sum_{j=0}^{\infty} \frac{Y_{t+j}}{(1+r)^j} - \sum_{j=0}^{\infty} \frac{T_{t+j}}{(1+r)^j} \tag{19}$$

- In other words, the presented discounted value of consumption must equal the present discounted value of net income. Now go to the government's budget constraint assuming no initial debt:

$$G_t + D_{t+1} = T_t$$

- In $t+1$, this will be:

$$G_{t+1} + D_{t+2} = T_{t+1} + (1+r)D_{t+1}$$

- Re-arranging terms:

$$D_{t+1} = \frac{T_{t+1}}{1+r} - \frac{G_{t+1}}{1+r} + \frac{D_{t+2}}{1+r}$$

Ricardian Equivalence: Simplified Version

- Solving forward one period, we would have:

$$D_{t+1} = \frac{T_{t+1}}{1+r} + \frac{T_{t+2}}{(1+r)^2} - \frac{G_{t+1}}{1+r} - \frac{G_{t+2}}{(1+r)^2} + \frac{D_{t+3}}{(1+r)^2}$$

- If we keep going, we are left with:

$$D_{t+1} = \sum_{j=1}^{\infty} \frac{T_{t+j} - G_{t+j}}{(1+r)^j} + \lim_{T\to\infty} \frac{D_{t+1+T}}{(1+r)^T}$$

Ricardian Equivalence: Simplified Version

- This says that the government's new debt issued today, in period t (which is D_{t+1}), equals the present discounted value of primary surpluses (primary surpluses are defined as tax revenue minus spending, not counting any interest payments).

- Plugging this into the period t government budget constraint to eliminate debt, we have:

$$\sum_{j=0}^{\infty} \frac{G_{t+j}}{(1+r)^j} = \sum_{j=0}^{\infty} \frac{T_{t+j}}{(1+r)^j} \tag{20}$$

- In other words, the present discounted value of government spending must equal the present discounted value of taxes. This is the government's intertemporal budget constraint.

- We can combine (20) with (19):

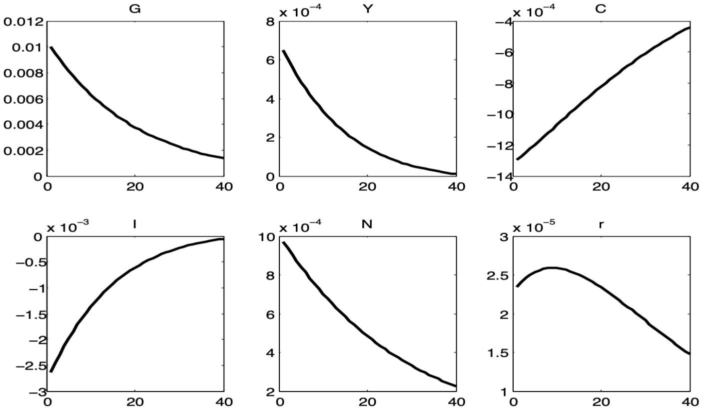
$$\sum_{j=0}^{\infty} \frac{C_{t+j}}{(1+r)^j} = \sum_{j=0}^{\infty} \frac{Y_{t+j}}{(1+r)^j} - \sum_{j=0}^{\infty} \frac{G_{t+j}}{(1+r)^j} \tag{21}$$

Equilibrium Effects of Government Spending Shocks

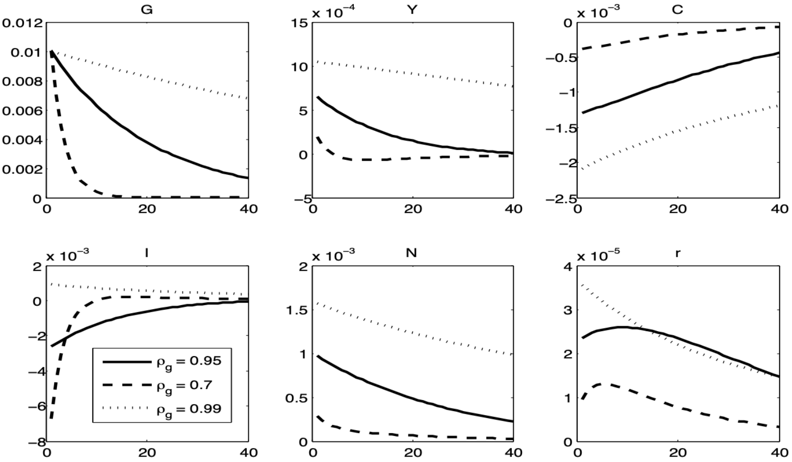
- Solve the model via log-linearizing about the non-stochastic steady state
- Use the following parameter values:
  $\alpha = 1/3, \beta = 0.99, \chi = 1, \delta = 0.025, \theta = 4, \rho_A = 0.97, \rho_A = 0.95, \omega = 0.2$
- Set the standard deviations of both the productivity and government spending shocks to 0.01

Equilibrium Effects of Government Spending Shocks

Below are the impulse responses to a shock to government spending:



Equilibrium Effects of Government Spending Shocks



Distortionary Taxes

- Allow for distortionary tax rates on capital and labor income, τ_t^k and τ_t^n
- Assume that there is a stochastic component to the tax rates so that we can analyze the equilibrium effects of changes in tax rates
- Two setups: one in which the government can still use lump-sum taxes, and other in which there is no lump-sum taxes.

Tax Shocks with Lump Sum Finance

- The household's budget constraint is now:

$$C_t + K_{t+1} - (1-\delta)K_t + B_{t+1} - B_t \leqslant (1-\tau_t^n)\, w_t N_t + \left(1-\tau_t^k\right) R_t^k K_t + \Pi_t - T_t + r_{t-1}B_t$$

where τ_t^n and τ_t^k are potentially time-varying tax rates on labor and capital income, respectively.

- FOCs for the household problem can now be written:

$$\frac{1}{C_t} = \beta E_t \left[\frac{1}{C_{t+1}} \left(\left(1-\tau_{t+1}^k\right) R_{t+1}^k + (1-\delta)\right)\right] \tag{26}$$

$$\frac{1}{C_t} = \beta E_t \left[\frac{1}{C_{t+1}} (1+r_t)\right] \tag{27}$$

$$\theta N_t^{\chi} = \frac{1}{C_t} (1-\tau_t^n)\, w_t \tag{28}$$

Tax Shocks with Lump Sum Finance

- The government's budget constraint is now given by:

$$G_t + r_{t-1}D_t \leqslant \tau_t^k R_t^k K_t + \tau_t^n w_t N_t + T_t + D_{t+1} - D_t \tag{29}$$

  where the government earns revenue from taxing both capital and labor income. It can still levy lump-sum taxes and issue debt.

- In equilibrium, we require that $D_t = B_t$ and $D_{t+1} = B_{t+1}$. Since $\Pi_t = Y_t - w_t N_t - R_t K_t$, the aggregate resource constraint still boils down to:

$$Y_t = C_t + I_t + G_t \tag{30}$$

Endogenous Equation System

- The equilibrium conditions are then:

$$\frac{1}{C_t} = \beta E_t \left[\frac{1}{C_{t+1}} \left(\left(1 - \tau^k_{t+1}\right) R^k_{t+1} + (1 - \delta) \right) \right] \tag{31}$$

$$\frac{1}{C_t} = \beta E_t \left[\frac{1}{C_{t+1}} (1 + r_t) \right] \tag{32}$$

$$w_t = (1 - \alpha) A_t K_t^{\alpha} N_t^{-\alpha} \tag{33}$$

$$R^k_t = \alpha A_t K_t^{\alpha - 1} N_t^{1-\alpha} \tag{34}$$

$$\theta N_t^{\chi} = \frac{1}{C_t} (1 - \tau^n_t) w_t \tag{35}$$

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha} \tag{36}$$

$$Y_t = C_t + I_t + G_t \tag{37}$$

$$K_{t+1} = I_t + (1 - \delta) K_t \tag{38}$$

- Same as we had before, except that the capital/labor tax rates show up.

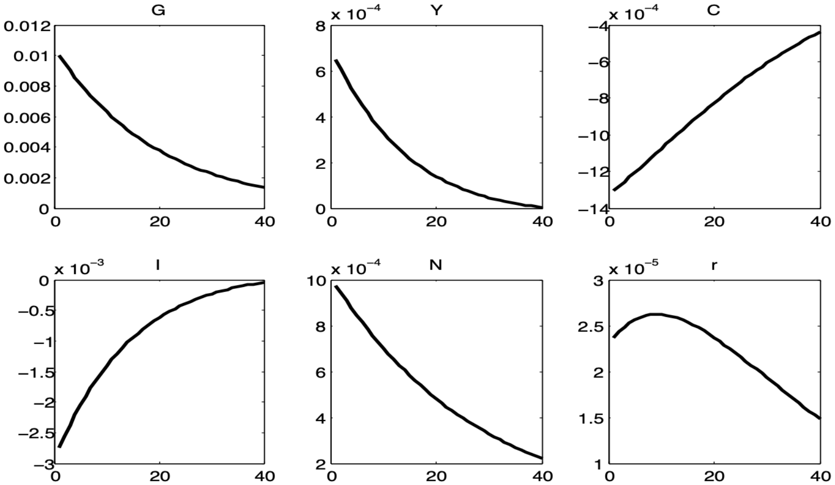
Exogenous Stochastic Processes

- Assumes that the tax rates obey stationary $\mathrm{AR}(1)$ processes with shocks, with tax rates without a time subscript denoting exogenous steady-state values:

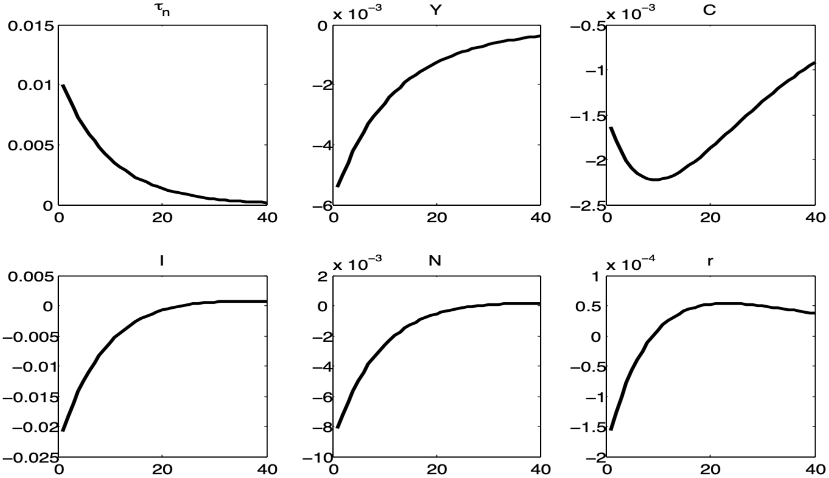
$$\tau_t^k = (1-\rho_k)\,\tau^k + \rho_k \tau_{t-1}^k + \epsilon_{k,t} \tag{39}$$

$$\tau_t^n = (1-\rho_n)\,\tau^n + \rho_n \tau_{t-1}^n + \epsilon_{n,t} \tag{40}$$

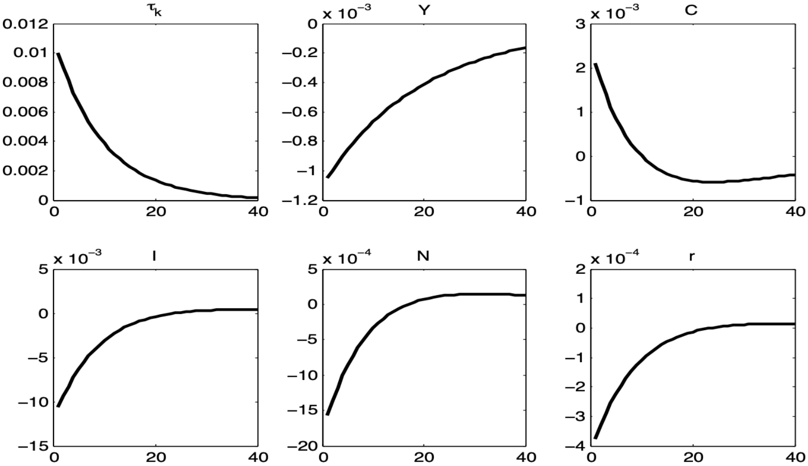
Gov't Expense Shocks with Lump Sum Taxes



Tax Shocks with Lump Sum Taxes



Tax Shocks with Lump Sum Taxes



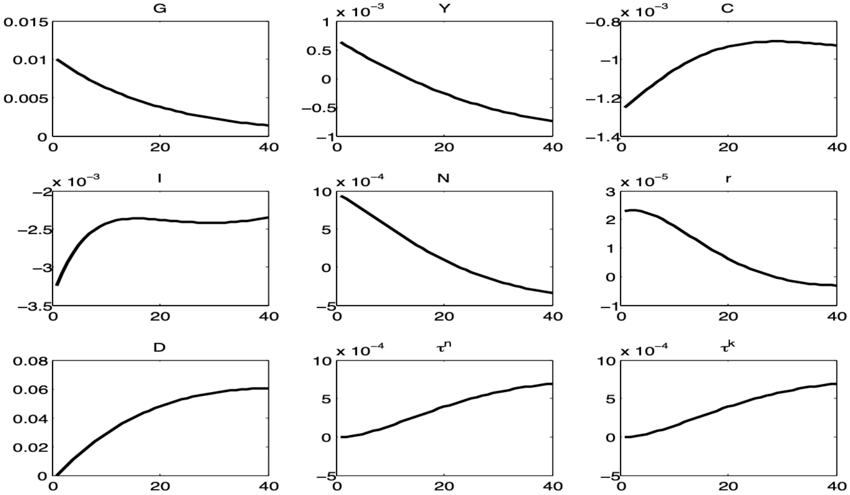
Tax Shocks with Lump Sum Taxes

- An endogenous reaction of government revenue to a change in government spending
- The impulse responses to a government spending shock will not necessarily be the same here as in the model with lump-sum taxes only

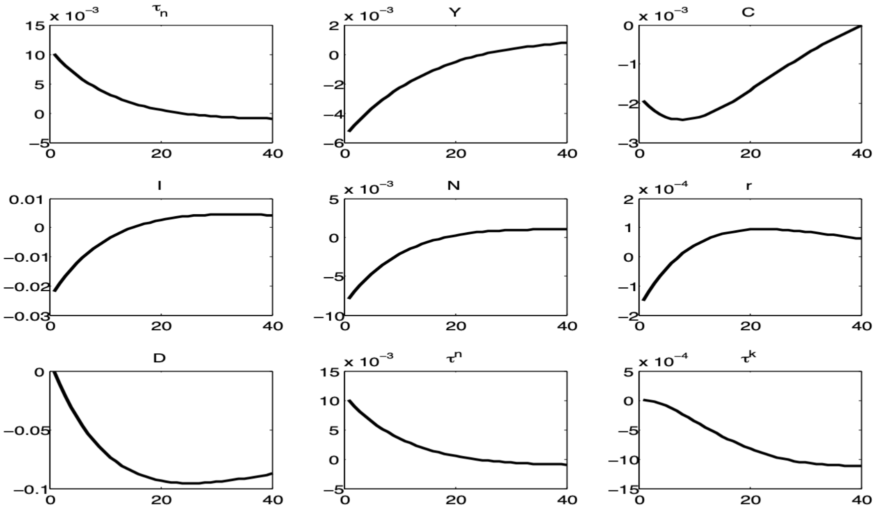
Tax Shocks with No Lump Sum Taxes

- Assume that there are distortionary tax rates on labor and capital income
- Not allowing the government to use lump-sum taxes
- As a result, government debt will not drop out of the equilibrium conditions
- Debt is going to matter
- See the handouts for details of equations

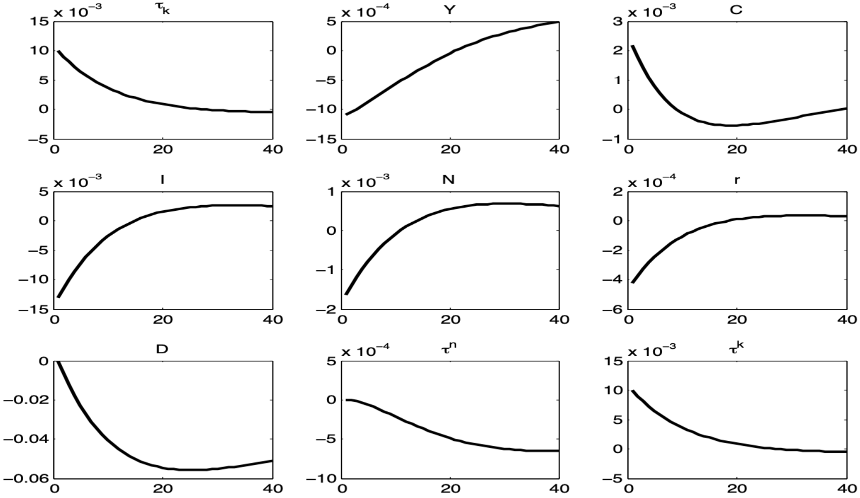
Tax Shocks with No Lump Sum Taxes



Tax Shocks with No Lump Sum Taxes



Tax Shocks with No Lump Sum Taxes



Tax Shocks with No Lump Sum Taxes

- The immediate impact effects are pretty similar to what we had before
- The responses are different in important ways out at further forecast horizons
- The increase in either tax rate causes debt to fall
- Falling debt automatically works to lower tax rates at some horizon
- Output eventually higher than where it started in both cases after a sufficiently long forecast horizon