PhD Macro Core Part I:
Lecture 14 – Sticky Prices and Menu Costs Part I

Min Fang
University of Florida

Fall 2024

Why Sticky Prices Matter?

- Evidence that demand shocks affect output
- Monetary shocks, fiscal shocks, etc, all have effects on output
- Major challenge: How to explain this empirical finding?
- In RBC models, demand shocks have small or NO effects on output
- Leading explanation: Prices adjust sluggishly to shocks

Sticky Prices and the Business Cycle

- Monetary shock: Increase in money supply
  - Flexible prices: Prices increase while output and real rates are unchanged
  - Sticky prices: Reduction in nominal interest rate reduces real rates
- Fiscal shock: Increase in government spending
  - Flexible prices: Real rates rise, which crowds out private spending
  - Sticky prices: Real rate sluggish unless nominal rate moves, output increases more
- We will start with the macro-type models to include stick prices

Today

- The Calvo model
- The Rotemberg model
- A comparison between the two

The Calvo Model

The Calvo Model

- Reduced-form way of capturing price rigidities.
- Each period, a given firm has a probability of θ that they will have the same price as the last period.
- Complementary probability $1 - \theta$ that they can update their price.
- Reference: Guillermo Calvo (1983), "Staggered Prices in a Utility-Maximising Framework", Journal of Monetary Economics 12 (3), 383-398.
- Reference: Jordi Gali (2007), "Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework", Princeton University Press

Setup

- A baseline model with nominal rigidities
- Monopolistic competition firms (The KEY)
- Sticky prices (staggered price setting)
- Competitive labor markets, Closed economy, No capital accumulation

Households

- Utility

$$\max U_t = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{C_s^{1-\sigma}}{1-\sigma} - \frac{N_s^{1+\varphi}}{1+\varphi} \right],$$

where:

$$C_t = \left[\int_0^1 C_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

- Firms produce differentiated products $C_t(j)$ in a monopolistic competitive markets

- Budget constraint:

$$\int_0^1 C_t(j) P_t(j) dj + Q_t B_t \leqslant B_{t-1} + W_t N_t - T_t$$

Households Optimization

- Optimal allocation of expenditures

$$\max C_t = \left[\int_0^1 C_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

  s.t.

$$\int_0^1 P_t(j) C_t(j) = Z_t$$

- The first order condition is

$$C_t(j) = (\lambda_t P_t(j))^{-\varepsilon} C_t$$

  where λ_t is the Lagrange multiplier associated with the constraint.

- Substituting in the definition of the consumption index

$$\lambda_t = \left[\int_0^1 P_t(j)^{1-\varepsilon} dj\right]^{\frac{-1}{1-\varepsilon}}$$

Households Optimization

- So, if we define the price level such that

$$\int_0^1 P(j)C(j)dj = P_t C_t$$

we have

$$\int_0^1 P(j)C_t(j) = \left[\int_0^1 P_t(j)^{1-\varepsilon} dj\right]^{\frac{1}{1-\varepsilon}} C_t = P_t C_t$$

So

$$P_t = \left[\int_0^1 P_t(j)^{1-\varepsilon} dj\right]^{\frac{1}{1-\varepsilon}} = \lambda_t^{-1}$$

- Therefore, the demand equation is

$$C_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} C_t$$

Households Optimization

- Labor leisure decision

$$-\frac{U_n(C_t;N_t)}{U_c(C_t;N_t)} = \frac{W_t}{P_t}$$

- Intertemporal decision

$$Q_t = \beta E_t\left(\frac{U_{c,t+1}}{U_{c,t}}\frac{P_t}{P_{t+1}}\right)$$

- In log-linear terms

$$\begin{aligned} w_t - p_t &= \sigma c_t + \varphi n_t \\ c_t &= E_t c_{t+1} - \sigma^{-1}\left(i_t - E_t\pi_{t+1} - \rho\right) \end{aligned}$$

Firms

- Continuum of firms, indexed by j

- Each firm produces a differentiated good $Y_t(j)$

- Identical technology
$$Y_t(j) = A_t N_t(j)^{1-\alpha}$$

- Probability of resetting price in any given period: $1-\theta$, independent across firms.

- $\theta \in [0,1]$: index of price stickiness

- Implied average price duration $1/(1-\theta)$

Optimal Price Setting

- A firm reoptimizing in period t will choose the price P_t^* that maximizes the current market value of the profits generated while that price remains effective. The probability that this price will be affected at period $t+k$ is θ^k

$$\max \sum_k E_t \theta^k Q_{t,t+k} \left[P_t^* Y_{t+k,t} - \Psi\left(Y_{t+k,t}\right)\right]$$

  where:

  - $Q_{t,t+k}$ is the stochastic discount factor, given by $\beta^k \frac{U_{c,t+k}}{U_{c,t}} \frac{P_t}{P_{t+k}}$ (households own firms, and discount given their rate of marginal utility. When marginal utility in a given period is high relative to today, future profits are more valuable in utility terms, so firms are more patient)
  - $Y_{t+k,t}(j) = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\varepsilon} C_{t+k}$ (monopolistic competitive firm knows the form of its demand)
  - $\boldsymbol{\Psi}\left(Y_{t+k,t}\right)$ is the total cost (which in this case is given by $W_t N_t(j)$)

Optimal Price Setting

- Transformed problem:

$$\sum_k E_t \theta^k Q_{t,t+k} Y_{t+k,t} \left[P_t^* - \mathcal{M}\psi_{t+k,t}\right]$$

where $\mathcal{M} = \frac{\varepsilon}{1-\varepsilon}$ and $\psi_{t+k,t} = \Psi'_{t+k,t}$ is the marginal cost at period $t+k$ of firms that change their price at period t, so

$$\psi_{t+k,t} = W_{t+k}\left(Y_{t+k,t}\right)^{\alpha/1-\alpha}\left(A_{t+k}\right)^{-1/1-\alpha}$$

- The average marginal cost is

$$\psi_{t+k} = W_{t+k}\left(Y_{t+k}\right)^{\alpha/(1-\alpha)}\left(A_{t+k}\right)^{-1/(1-\alpha)}$$

So, given that

$$Y_{t+k,t} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon} Y_{t+k}$$

we have

$$\psi_{t+k,t} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon\alpha/(1-\alpha)} \psi_{t+k}$$

The Log-linear Pricing Equation

- Given that in steady state $P^* = P = \psi\mathcal{M}$ and $Q_k = \beta^k$, a Taylor expansion of the pricing equations

$$\sum_k YE_t(\theta\beta)^k \left[p_t^* - \log\left(\psi_{t+k,t}\right) - \mu\right] = 0$$

where $\mu \equiv \log(\mathcal{M})$.

- Alternatively, we can express it in terms of the average marginal cost

$$\sum_k YE_t(\theta\beta)^k \left[p_t^* - \Theta\left(\log\left(\psi_{t+k}\right) - p_{t+k}\right) - p_{t+k} - \Theta\mu\right] = 0$$

where $\Theta = \frac{1-\alpha}{1-\alpha+\alpha\epsilon}$.

The Log-linear Pricing Equation

- Therefore, defining the real marginal cost as $mc_t \equiv \log(\psi_t) - p_t$
- And given that in steady state $mc = -\mu$

$$p_t^* = (1 - \beta\theta) \sum_k E_t (\theta\beta)^k \left[\Theta \widehat{mc}_{t+k} + p_{t+k}\right]$$

- or

$$p_t^* - p_t = (1 - \beta\theta)\Theta\widehat{mc}_t + \theta\beta E_t \left(p_{t+1}^* - p_t\right)$$

Aggregate Price Dynamics

- The aggregate price is

$$P_t = \left[\theta\left(P_{t-1}\right)^{1-\varepsilon} + (1-\theta) P_t^{*\,1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

- Defining $\pi \equiv \log P_t/P_{t-1}$ and log-linerizing around the zero-inflation steady state

$$\pi_t = (1-\theta)\left(p_t^* - p_{t-1}\right)$$

- Turning again to the pricing equation, we can write

$$\pi_t = \lambda \widehat{mc}_t + \beta E_t \pi_{t+1}$$

where $\lambda \equiv \frac{(1-\beta\theta)(1-\theta)}{\theta}\Theta$

The Log-linear Phillips Curve

- To write the above pricing equation in terms of output, we need to derive some conditions for the actual marginal cost and market clearing

- Goods market clearing $y_t = c_t$

- Labor market clearing

$$N_t = \int_0^1 N_t(j)dj = \int_0^1 \left(\frac{Y_t(j)}{A_t}\right)^{1/(1-\alpha)} dj$$
$$= \left(\frac{Y_t}{A_t}\right)^{1/(1-\alpha)} \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon/(1-\alpha)} dj$$

or given that to a first-order approximation, the price dispersion term $\int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon/(1-\alpha)} dj$, is eliminated (See Gali, ch. 3 Appendix)

$$(1-\alpha)n_t = y_t - a_t$$

The Log-linear Phillips Curve

- The real marginal cost

$$MC_t = \frac{W_t}{P_t} \frac{Y_t^{\alpha/(1-\alpha)}}{(1-\alpha)A_t^{1/(1-\alpha)}}$$

  or

$$mc_t = w_t - p_t + (1-\alpha)^{-1}(\alpha y_t - a_t) - \log(1-\alpha)$$

- And recalling the labor leisure decision:

$$w_t - p_t = \sigma c_t + \varphi n_t$$

  and market clearing, we can write

$$mc_t = \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t - \left(\frac{\varphi+1}{1-\alpha}\right) a_t - \log(1-\alpha)$$

The Log-linear Phillips Curve

- The Phillips curve

$$\lambda^{-1}\pi_t = \left[\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t - \left(\frac{\varphi+1}{1-\alpha}\right) a_t - \log(1-\alpha) + \mu\right] + \lambda^{-1}\beta E_t \pi_{t+1}$$

- With flexible prices $\lambda^{-1} \to 0$ (denoted with a superscript n)

$$y_t^n = \psi_{ya} a_t - \delta_y$$

where $\psi_{ya} = \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)^{-1} \left(\frac{\varphi+1}{1-\alpha}\right)$ and $\delta_y = \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)^{-1} (\mu - \log(1-\alpha))$

- So, we can write the Phillips curve in terms of the output gap $\widetilde{y}_t = y_t - y_t^n$

$$\pi_t = \kappa \widetilde{y}_t + \beta E_t \pi_{t+1}$$

where $\kappa = \lambda\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)$

The Three Equation New Keynesian Model

1. 1) *PC* curve
$$\pi_t = \kappa \widetilde{y}_t + \beta E_t \pi_{t+1}$$

2. 2) IS curve
$$\widetilde{y}_t = E_t \widetilde{y}_{t+1} - \sigma^{-1} \left(i_t - E_t \pi_{t+1} - r_t^n \right)$$
where the natural rate of interest is defined as $r_t^n = \rho + \sigma \psi_{ya} E_t \Delta a_{t+1}$

3. 3) A monetary policy rule! (This will be the last lecture)

The Rotemberg Model

The Rotemberg Model

- Monopolistic competition firms (The KEY)
- Sticky prices (staggered price setting)
- Competitive labor markets, Closed economy, No capital accumulation
- But this model assumes that firms can always opt to change their price, but doing so involves paying a quadratic adjustment cost of the form

$$AC_t(j) = \frac{\lambda}{2}\left(\frac{\widetilde{P}_t^*}{\widetilde{P}_{t-1}^*} - 1\right)^2 Y_t$$

  where $\lambda > 0$ captures the degree of price stickiness and $\widetilde{P}_t^*$ denotes the optimal reset price.

- Rotemburg (1982), "Sticky Prices in the United States", Journal of Political Economy.

The Rotemberg Model

- The choice of a new price today affects our profits today.
- But it can also affect our profits directly tomorrow. Why?
- Because in $t+1$, the reset price $\widetilde{P}_t^*$ will feature in the adjustment cost function

$$AC_{t+1}(j) = \frac{\lambda}{2}\left(\frac{\widetilde{P}_{t+1}^*}{\widetilde{P}_t^*} - 1\right)^2 Y_{t+1}$$

- Any other future periods where today's price choice will have a direct effect?
- What about time $t+2$? Nope. The price chosen at $t+1$ will affect that (as in the previous slide).

Firm Objective

- Again, the firm seeks to maximize the discounted expected value of future profits for its shareholders

$$\widetilde{\Gamma}_t(j) = \mathbb{E}_t \left\{ \sum_{k=0}^{\infty} Q_{t,t+k} \widetilde{V}_{t+k}(j) \right\}$$

where

$$\widetilde{V}_{t+k}(j) = \widetilde{P}_t^* Y_{t+k,t} - \Psi\left(Y_{t+k,t}\right) - P_{t+k} AC_{t+k}(j)$$

and

$$Y_{t+k}(j) = \left(\frac{\widetilde{P}_t^*}{P_{t+k}} \right)^{-\epsilon} Y_{t+k}$$

Optimal Price

- FOC

$$\frac{\partial \widetilde{\Gamma}_t(j)}{\partial \widetilde{P}_t^*} = 0 \Rightarrow \mathbb{E}_t \left\{ \sum_{k=0}^{\infty} Q_{t,t+k} \frac{\partial \widetilde{V}_{t+k}(j)}{\partial \widetilde{P}_t^*} \right\} = 0$$

where

$$\frac{\partial \widetilde{V}_t(j)}{\partial \widetilde{P}_t^*} = Y_t(j) + \widetilde{P}_t^* \frac{\partial Y_t(j)}{\partial \widetilde{P}_t^*} + \Psi' \left(Y_t(j) \right) \frac{\partial Y_t(j)}{\partial \widetilde{P}_t^*} - \lambda \left(\frac{\widetilde{P}_t^*}{\widetilde{P}_{t-1}^*} - 1 \right) Y_t \frac{\widetilde{P}_t^*}{\widetilde{P}_{t-1}^*}$$

$$\frac{\partial \widetilde{V}_{t+1}(j)}{\partial \widetilde{P}_t^*} = \lambda \frac{\widetilde{P}_{t+1}^*}{\left(\widetilde{P}_t^* \right)^2} \left(\frac{\widetilde{P}_{t+1}^*}{\widetilde{P}_t^*} - 1 \right) P_{t+1} Y_{t+1}$$

Optimal Price

- Putting it all together yields

$$\left(\frac{\left(\widetilde{P}_t^*\right)}{P_t}\right)^{-\epsilon} Y_t - \frac{\widetilde{P}_t^*}{P_t}\epsilon\left(\frac{\left(\widetilde{P}_t^*\right)}{P_t}\right)^{-\epsilon-1} Y_t + \Psi'\left(Y_t(j)\right)\epsilon\left(\frac{\left(\widetilde{P}_t^*\right)}{P_t}\right)^{-\epsilon-1} Y_t\frac{1}{P_t} -$$

$$\lambda\left(\frac{\left(\widetilde{P}_t^*\right)}{\left(\widetilde{P}_{t-1}^*\right)} - 1\right)\frac{Y_tP_t}{\left(\widetilde{P}_{t-1}^*\right)} + \lambda\mathbb{E}_t\left[Q_{t,t+1}\frac{\left(\widetilde{P}_{t+1}^*\right)}{\left(\widetilde{P}_t^*\right)^2}\left(\frac{\left(\widetilde{P}_{t+1}^*\right)}{\left(\widetilde{P}_t^*\right)} - 1\right)Y_{t+1}P_{t+1}\right] = 0$$

- You may get homework here to derive this above

- Also, would you get the same Phillips curve?

Comparing the Two

Cross-Sectional Price Dispersion

- Is there a cross-section of different prices across the varieties?

- Rotemburg: none. Firms will just re-adjust each period. All the same, so they set the same price.

- Calvo: in general, there will be dispersion. E.g., consider the initial price index P_0.

$$\Rightarrow P_1 = \left[\theta\left(P_0\right)^{1-\epsilon} + (1-\theta)\left(P_1^*\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$$

$$\Rightarrow P_2 = \left[\theta\left\{\theta\left(P_0\right)^{1-\epsilon} + (1-\theta)\left(P_1^*\right)^{1-\epsilon}\right\} + (1-\theta)\left(P_2^*\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$$

$$= \left[\theta^2\left(P_0\right)^{1-\epsilon} + \theta(1-\theta)\left(P_1^*\right)^{1-\epsilon} + (1-\theta)\left(P_2^*\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$$

$$\Rightarrow P_3 = \left[\theta^3\left(P_0\right)^{1-\epsilon} + \theta^2(1-\theta)\left(P_1^*\right)^{1-\epsilon} + \theta(1-\theta)\left(P_2^*\right)^{1-\epsilon} + (1-\theta)\left(P_3^*\right)^{1-\epsilon}\right] \text{ and so on.}$$

Induced Distortions

- Adjustment costs in Rotemburg are goods that come out of the resource constraint

$$
\begin{aligned}
Y_t &= C_t + \int_0^1 AC_t(j)dj \\
&= C_t + \int_0^1 \frac{\lambda}{2}\left(\frac{\left(\widetilde{P}_t^*\right)}{\left(\widetilde{P}_{t-1}^*\right)} - 1\right)^2 Y_t dj \\
&= C_t + Y_t \frac{\lambda}{2}\int_0^1 \left(\frac{P_t}{P_{t-1}} - 1\right)^2 dj \\
&= C_t + Y_t \frac{\lambda}{2}\left(\frac{P_t}{P_{t-1}} - 1\right)^2 \\
\Rightarrow Y_t &= \left\{1 - \frac{\lambda}{2}\left(\frac{P_t}{P_{t-1}} - 1\right)^2\right\}^{-1} C_t
\end{aligned}
$$

i.e., there is an "inefficiency wedge" between output and consumption.

Induced Distortions

- Under the Calvo model, price dispersion creates distortions.

- Recall from the production function that

$$Y_t(j) = A_t N_t(j)^{1-\alpha}$$

$$\Rightarrow N_t(j) = \left(\frac{Y_t(j)}{A_t}\right)^{\frac{1}{1-\alpha}}$$

which is labor demand for firm j. Aggregation gives

$$N_t = \int_0^1 N_t(j)dj = \int_0^1 \left(\frac{Y_t(j)}{A_t}\right)^{\frac{1}{1-\alpha}} dj$$

$$= \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{(P_t^*)}{P_t}\right)^{\frac{-\epsilon}{1-\alpha}} dj$$

where the last line comes from plugging in j 's demand function.

Induced Distortions

- Notice that if there is perfect price flexibility, then

$$\int_0^1 \left(\frac{(P_t^*)}{P_t}\right)^{\frac{-\epsilon}{1-\alpha}} dj = \int_0^1 \left(\frac{P_t}{P_t}\right)^{\frac{-\epsilon}{1-\alpha}} dj = 1$$

- With rigidities, though, see that

$$N_t^{1-\alpha} = \left(\frac{Y_t}{A_t}\right) \left\{\int_0^1 \left(\frac{(P_t^*)}{P_t}\right)^{\frac{-\epsilon}{1-\alpha}} dj\right\}^{1-\alpha}$$

$$\Rightarrow Y_t = A_t N_t^{1-\alpha} \left\{\int_0^1 \left(\frac{(P_t^*)}{P_t}\right)^{\frac{-\epsilon}{1-\alpha}} dj\right\}^{\alpha-1}$$

where $\left\{\int_0^1 \left(\frac{(P_t^*)}{P_t}\right)^{\frac{-\epsilon}{1-\alpha}} dj\right\}^{\alpha-1} < 1$

- You can interpret the last equality as an aggregate production function.

Big Picture

- We want a model with price rigidity so we can think about a non-neutral monetary policy.
- But what does price stickiness itself imply about welfare?
- It's a bad thing.
- It's A friction: firms cannot update their prices freely, even if they want to.
- This can only hurt our economy relative to a benchmark without price rigidity.
- The setup of these two models allows us to think a bit about the welfare cost of this friction.
- In Calvo, the dispersion hurts welfare.
- In Rotemburg, the adjustment costs hurt welfare.