PhD Macro Core Part I:
Lecture 11 – Job Search and Matching Problems

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Today

- Third Application of Stochastic Dynamic Programming
- Job Search and Matching
- Applications to Labor Markets

Search Frictions

- In a standard labor supply/demand models
- firm can hire as much labor as it wants at prevailing wage
- workers can find employment at the prevailing wage
- In the search model, neither of these is immediately true
- Unemployed workers need to find jobs
- Firms with vacancies need to find workers, and these activities take time and resources
- Search friction is not the only reason for unemployment (But our focus today!)

Search Models of the Labor Market

- Tractable alternative to labor supply/demand models
- Emphasizes labor market flows: (e.g., transitions in/out employment, in/out labor force, etc.)
- Natural connection to data on job creation and job destruction
- We will consider two distinct examples:
  (i) Individual decision problem, sequential search in the spirit of McCall
  (ii) General equilibrium, random matching in the spirit of Diamond, Mortensen, and Pissarides

McCall Approach

ECONOMICS OF INFORMATION AND JOB SEARCH *

J. J. McCall

I. Introduction

In the recent literature A. A. Alchian and W. R. Allen,[1] G. J. Stigler,[2] and probably others have suggested that unemployed resources may be productive in a world where uncertainty prevails and information is costly. The activity that renders these resources productive is search. The search activities that unemployed human resources undertake have been discussed by Stigler.[3] This paper, following suggestions from the preceding sources, applies some well-known results from the theory of optimal stopping rules to the unemployment phenomenon.[4] These applications are all performed at the individual or micro level. The implications of this analysis for the macro problems of unemployment are not investigated.[5]

Setup

- Time $t = 0, 1, 2, \ldots$
- Single agent with risk-neutral preferences

$$\mathbb{E}\left\{\sum_{t=0}^{\infty} \beta^t c_t\right\}, \quad 0 < \beta < 1$$

- Each period, unemployed worker draws IID wage offer $w \sim F(w)$
- Two actions
- accept offer: become employed and have $c_t = w$ forever
- reject offer: remain unemployed, receive benefits $c_t = b$ this period and draw new w' next period

Dynamic Programming Problem

- Bellman equation can be written

$$v(w) = \max_{\text{accept, reject}} \left[\frac{1}{1-\beta} w, b + \beta \int_0^\infty v\left(w'\right) dF\left(w'\right) \right]$$

- Accept wage offer if

$$\frac{1}{1-\beta} w > b + \beta \int_0^\infty v\left(w'\right) dF\left(w'\right)$$

- Reject wage offer if

$$\frac{1}{1-\beta} w < b + \beta \int_0^\infty v\left(w'\right) dF\left(w'\right)$$

- RHS of these inequalities is a constant, independent of current w

Reservation Wage

- Let $\bar{w}$ be such that

$$\frac{1}{1-\beta}\bar{w} = b + \beta \int_0^\infty v\left(w'\right) dF\left(w'\right)$$

- Then value function has the piecewise linear form

$$v(w) = \begin{cases} \frac{1}{1-\beta}\bar{w} & w \leqslant \bar{w} \\ \frac{1}{1-\beta}w & w \geqslant \bar{w} \end{cases}$$

- This $\bar{w}$ is known as the reservation wage; unemployed worker will not work for $w < \bar{w}$

- But still need to determine $\bar{w}$

Determining $\bar{w}$

- Reservation wage $\bar{w}$ solves indifference condition

$$\frac{1}{1-\beta}\bar{w} = b + \beta \int_0^\infty v\left(w'\right) dF\left(w'\right)$$

or

$$\frac{1}{1-\beta}\bar{w} = b + \frac{\beta}{1-\beta}\left(\int_0^{\bar{w}} \bar{w} dF\left(w'\right) + \int_{\bar{w}}^\infty w' dF\left(w'\right)\right)$$

- Collecting terms and rearranging gives

$$\bar{w} - b = \frac{\beta}{1-\beta}\int_{\bar{w}}^\infty \left(w' - \bar{w}\right) dF\left(w'\right)$$

- LHS is the opportunity cost of searching again with $w = \bar{w}$ in hand, RHS is the expected discounted benefit of searching again given that only $w' > \bar{w}$ will be accepted

Determining $\bar{w}$

- Single equation in unknown scalar $\bar{w}$

$$\bar{w} - b = \frac{\beta}{1-\beta} \int_{\bar{w}}^{\infty} (w' - \bar{w})\, dF(w')$$

- Let $R(x)$ denote the function on the RHS

$$R(x) \equiv \frac{\beta}{1-\beta} \int_{x}^{\infty} (w' - x)\, dF(w')$$

which has the properties

$$R(0) = \frac{\beta}{1-\beta} \mathbb{E}\{w\} > 0, \quad \text{and} \quad R(\infty) = 0$$

with

$$R'(x) = -\frac{\beta}{1-\beta}[1 - F(x)] < 0, \quad \text{and} \quad R''(x) = +\frac{\beta}{1-\beta} F'(x) > 0$$

Comparative Statics of $\bar{w}$

- Hence there is indeed a unique $\bar{w}(> b)$ such that

$$\bar{w} - b = \frac{\beta}{1-\beta} \int_{\bar{w}}^{\infty} (w' - \bar{w})\, dF(w') \tag{*}$$

- Implicitly determines $\bar{w}$ in terms of parameters b, β, $F(\cdot)$

- Comparative statics of reservation wage $\bar{w}$

- higher benefits b increase reservation wage

- higher discount factor β increases reservation wage

- mean-preserving spread in $F(w')$ increases reservation wage

Mean-preserving Spread in $F(w')$

- To see the effect of a mean-preserving spread, use integration-by-parts to rewrite $(*)$ as

$$\bar{w} - b = \beta(\mathbb{E}\{w\} - b) + \beta \int_0^{\bar{w}} F(w')\, dw' \tag{**}$$

- Now consider two distributions $F_1(w')$ and $F_2(w')$ where $F_2(w')$ is a mean-preserving spread of $F_1(w')$. Then $F_1(w')$ second-order stochastically dominates $F_2(w')$ in the sense that

$$\int_0^x F_1(w')\, dw' < \int_0^x F_2(w')\, dw', \qquad \text{for any } x$$

- Hence reservation wage $\bar{w}_1$ for $F_1(w')$ is less than $\bar{w}_2$ for $F_2(w')$

- Key intuition is that more dispersion in $F(w')$ creates option value

Job Loss

- Suppose employed worker loses job with exogenous probability δ
- Bellman equation can be written

$$v(w) = \max \left\{ \begin{array}{c} w + \beta \left((1-\delta) v(w) + \delta \left(b + \beta \int_0^\infty v(w')\, dF(w') \right) \right) \\ b + \beta \int_0^\infty v(w')\, dF(w') \end{array} \right\}$$

- Solution again characterized by a reservation wage $\bar{w}$, can show $\bar{w}$ lower than previous case with $\delta = 0$

Unemployment Flows

- Let u_t denote aggregate unemployment rate
- Employed workers become unemployed with probability δ
- Unemployed workers become employed with probability $1 - F(\bar{w})$
- Hence, unemployment evolves according to

$$u_{t+1} = \delta\,(1 - u_t) + F(\bar{w})u_t$$

- Steady-state unemployment rate

$$u = \frac{\delta}{\delta + 1 - F(\bar{w})}$$

(increasing in δ and increasing in $\bar{w}$)

Diamond-Mortensen-Pissarides Approach

The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 2010

Summary

Laureates

- Peter A. Diamond
- Dale T. Mortensen
- Christopher A. Pissarides

Prize announcement

Press release

Popular information

Advanced information

Illustrated information

Speed read

Award ceremony video

Award ceremony speech

Banquet video

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The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 2010

Peter A. Diamond

Prize share: 1/3

Dale T. Mortensen

Prize share: 1/3

Christopher A. Pissarides

Prize share: 1/3

The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 2010 was awarded jointly to Peter A. Diamond, Dale T. Mortensen and Christopher A. Pissarides "for their analysis of markets with search frictions"

Matching Function

- Let $L > 0$ denote size of the labor force
- Let mL denote number of job matches, uL number of unemployed, and vL number of vacant jobs
- Assume number matches given by matching function

$$mL = M(uL, vL)$$

that is increasing, concave and has constant returns to scale so that

$$m = M(u, v)$$

Matching Function

- Job finding rate f

$$fu = m = M(u,v) \quad \Rightarrow \quad f = \frac{M(u,v)}{u}$$

- The Other side of this is vacancy filling rate q

$$qv = m = M(u,v), \quad \Rightarrow \quad q = \frac{M(u,v)}{v} = \frac{fu}{v}$$

- With constant returns to scale

$$f = M\left(1, \frac{v}{u}\right) \equiv f(\theta)$$
$$q = M\left(\frac{u}{v}, 1\right) \equiv q(\theta) = f(\theta)/\theta$$

where $\theta \equiv v/u$ is known as 'labor market tightness'

Matching Function

- Job finding rate $f(\theta)$, increasing in labor market tightness.
- Expected duration unemployment $1/f(\theta)$, decreasing in θ
- Vacancy filling rate $q(\theta)$, decreasing in labor market tightness.
- Expected duration vacancy $1/q(\theta)$, increasing in θ
- Example: if $M(u,v) = u^{\alpha}v^{1-\alpha}$ for $0 < \alpha < 1$ then

$$f(\theta) = \theta^{1-\alpha}, \quad q(\theta) = \theta^{-\alpha}$$

Unemployment Flows Revisited

- Employed workers become unemployed with probability δ
- Unemployed workers become employed with probability $f(\theta)$
- Hence, unemployment evolves according to

$$u_{t+1} = \delta\,(1-u_t) + (1-f(\theta))u_t$$

- Steady-state unemployment rate in this setting

$$u = \frac{\delta}{\delta + f(\theta)}$$

Beveridge Curve

- Now write steady-state unemployment condition

$$u = \frac{\delta}{\delta + f(\theta)}, \quad \theta = v/u \tag{1}$$

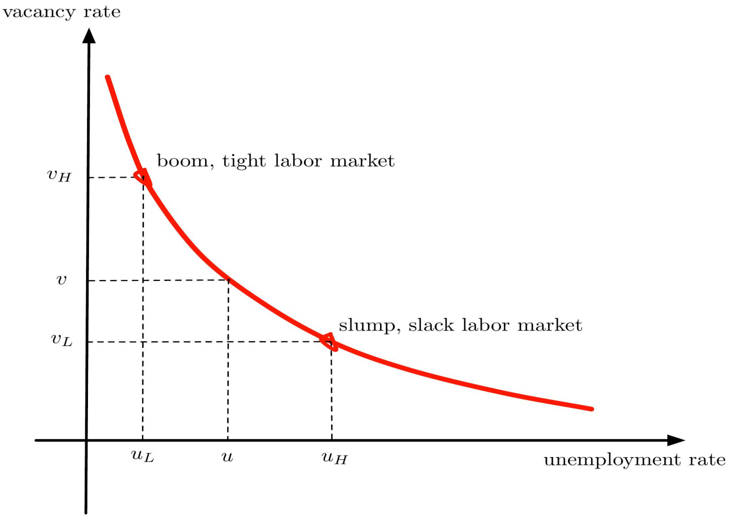
- Set of (v, u) satisfying (1) is known as the 'Beveridge curve'

- An inverse relationship between v and u. Shifted by changes in the job destruction rate δ or the matching technology $f(\cdot)$

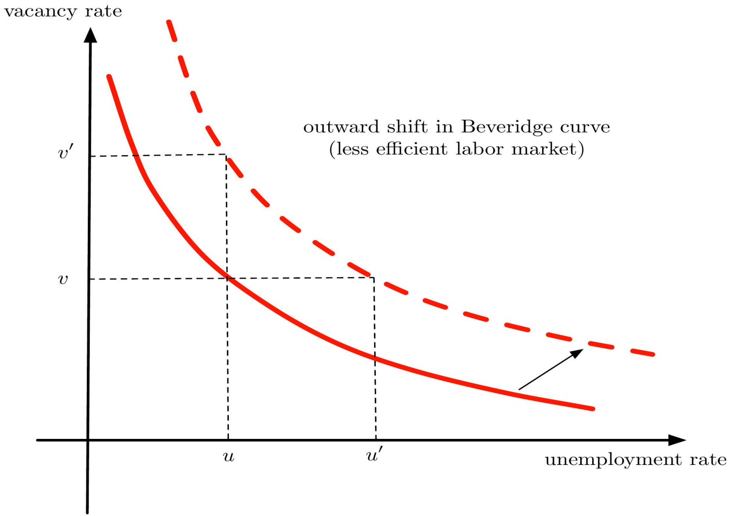
- Example: if $M(u,v) = Au^{\alpha}v^{1-\alpha}$ for $0 < \alpha < 1$ and $A > 0$ then

$$v = \left(\left(\frac{\delta}{A}\right)\left(\frac{1-u}{u^{\alpha}}\right)\right)^{1/(1-\alpha)}$$

Beveridge Curve



Shifts in the Beveridge Curve



Setup

- Risk neutral workers and firms, discount factor $\beta \in (0, 1)$
- Unemployed workers and firms with vacancies matched via $M(u, v)$
- Workers and firms bargain over wages w
- Free-entry into vacancy creation
- Focus on steady states

Job Creation and Destruction

- Firms can employ one worker
- Output from match $y = z > 0$
- Wage w paid to the employed worker (no wage distribution)
- Jobs destroyed with probability $\delta \in (0, 1)$
- Jobs created by posting vacancies, cost $\kappa z > 0$
- Vacancy filled with probability $q(\theta)$

Value Functions

- Let J denote the value of a filled job to a firm. Satisfies the steady-state Bellman equation

$$J = z - w + \beta(\delta V + (1-\delta)J)$$

hence

$$J = \frac{1}{1-\beta(1-\delta)}(z - w + \beta\delta V)$$

- Let V denote the value of a vacancy to a firm. Satisfies the steady-state Bellman equation

$$V = -\kappa z + \beta(q(\theta)J + (1-q(\theta))V)$$

Job Creation

- Free-entry into job creation drives V to $V = 0$, so

$$0 = -\kappa z + \beta q(\theta) J \quad \Rightarrow \quad J = \frac{\kappa z}{\beta q(\theta)}$$

- Plugging this into first Bellman equation and collecting terms gives

$$w = z - (1 - \beta(1 - \delta)) \frac{\kappa z}{\beta q(\theta)} \tag{2}$$

- Wage equated to marginal product of labor less expected discounted search costs. Plays the role of a labor demand schedule

- For given wage w, this will determine labor market tightness θ.

Workers

- Let W denote the value of a job to a worker. Satisfies the steady-state Bellman equation

$$W = w + \beta(\delta U + (1-\delta)W)$$

hence

$$W = \frac{1}{1-\beta(1-\delta)}(w + \beta\delta U)$$

- Let U denote the value of being unemployed. Satisfies the steady-state Bellman equation

$$U = b + \beta(f(\theta)W + (1-f(\theta))U)$$

where $b \leqslant w$ denotes unemployment benefits etc

Wage Determination

- A Match between an unemployed worker and a firm with a vacancy creates a mutual profit opportunity. How should these profits be split?

- Payments $z - w$ to firm, w to worker

- Wage w determined by bargaining between worker and firm

- Choice of w affects job value to individual firm $J(w)$ and to individual worker $W(w)$ taking as given aggregate market conditions U, V etc

- At a wage of w, the firm's surplus from a match is $J(w) - V$ and the worker's surplus is $W(w) - U$

Generalized Nash Bargaining

- Wage w maximizes the Nash product

$$(W(w)-U)^{\phi}(J(w)-V)^{1-\phi}, \quad 0 \leqslant \phi \leqslant 1$$

where the parameter ϕ denotes the workers' bargaining power

- First order condition for this problem can be written

$$\phi \frac{W'(w)}{W(w)-U} = -(1-\phi)\frac{J'(w)}{J(w)-V}$$

Now note that, treating aggregate U, V as given,

$$W'(w) = \frac{1}{1-\beta(1-\delta)}, \quad J'(w) = -\frac{1}{1-\beta(1-\delta)}$$

- So we can write

$$W = U + \phi S$$

where $S = W - U + J$ is the total match surplus (given $V = 0$)

Wages and the Value of Unemployment

- Recall that

$$W = \frac{1}{1-\beta(1-\delta)}(w + \beta\delta U), \quad J = \frac{1}{1-\beta(1-\delta)}(z - w)$$

- Then given surplus splitting $W - U = \phi(W - U + J)$ we have

$$w - (1-\beta)U = \phi(w - (1-\beta)U + z - w)$$

- Collecting terms and simplifying

$$w = \phi z + (1-\phi)(1-\beta)U$$

- Wage is bargaining-weighted average of productivity z and flow value of unemployment $(1-\beta)U$

Wage Curve

- From the Bellman equation for U

$$(1-\beta)U = b + \beta f(\theta)(W-U)$$

- But from the Nash bargain, worker surplus is proportional to firm surplus, which is pinned down by free entry

$$W - U = \frac{\phi}{1-\phi}J = \frac{\phi}{1-\phi}\left(\frac{\kappa z}{\beta q(\theta)}\right)$$

  Hence

$$(1-\beta)U = b + \beta f(\theta)\frac{\phi}{1-\phi}\left(\frac{\kappa z}{\beta q(\theta)}\right) = b + \frac{\phi}{1-\phi}\kappa z\theta$$

- Plugging this into our expression for wages and collecting terms

$$w = (1-\phi)b + \phi(1+\kappa\theta)z \tag{3}$$

  This 'wage curve' plays the role of a labor supply schedule

Steady State Equilibrium

- To summarize, in a steady state equilibrium, we solve for w, θ simultaneously from (i) the wage curve

$$w = (1-\phi)b + \phi(1+\kappa\theta)z$$

and (ii) the marginal product condition

$$w = z - (1-\beta(1-\delta))\frac{z\kappa}{\beta q(\theta)}$$

- Given w, θ from these two equations, we can back out the unemployment rate u from the Beveridge curve

$$u = \frac{\delta}{\delta + f(\theta)}$$

and then determine $v = \theta u$ and the present values W, U, J etc

Solving for w, θ