MA331 Quiz 01 Section __ Name_____

Printout the necessary formulae and report the numerical results with 2 decimals.

Assume a SRS of body length: $170, 177, 168, 169, 184, 180, 175, 165, 176, 178, 182, 174, 179, 166, 185$.
*** The first 4 problems are based on this data.

Problem 1. Provide the 5-number summary of the sample and construct the box plot. (10pts)

Problem 2. Evaluate mean, variance and standard deviation of the sample, and identify the tail skewness. (10pts)

Assume further the body length $X \sim \mathcal{N}(\mu, \sigma^2)$ with $\mu = 174$ and $\sigma^2 = 38$.

Problem 3. (i) Let $T_1 = \sum_{i=1}^{n} (X_i - \mu)^2/\sigma^2$, compute the observed value t_1 of T_1 based on the above data, and then determine $\mathrm{P}(T_1 > t_1)$. (5pts)

(ii) Let $T_2 = \sum_{i=1}^{n} (X_i - \bar{X})^2/\sigma^2$, compute the observed value t_2 of T_2 based on the above data, and then determine $\mathrm{P}(T_2 > t_2)$. (5pts)

(iii) Let the statistic $T_3 = \frac{\bar{X}-\mu}{\sqrt{S^2/n}}$, compute the observed value t_3 of T_3 based on the above data, and then determine $\mathrm{P}(T_3 > t_3)$. (5pts)

Problem 4. For the confidence level 0.89, construct (i) the μ-interval for μ. (5pts)

(ii) the t-interval for μ. (5pts)

(iii) the confidence interval of σ^2. (5pts)

*** The following 3 problems are Not based on the real data.

Problem 5. Assume a simple and random sample $X_1, \cdots, X_n$ from the population $X \sim \mathcal{U}(0, 2\theta+1)$, the uniform distribution on the interval $(0, 2\theta+1)$. Find the maximum likelihood estimate $\hat{\theta}$. (20pts)

Problem 6. Assume a SRS $X_1, \cdots, X_n$ from the population $X \sim \mathcal{G}(p)$, Geometric distribution with probability of success $p \in (0, 1)$. Derive the moment estimator $\hat{p}$. (20pts)

Problem 7. Assume that random variables $X \sim \mathcal{N}(-1, 3^2)$ and $Y \sim \mathcal{N}(1, 2^2)$ are mutually independent. Determine the probability distribution of the following random variables. (i) $2X + 3Y$; (5pts) (ii) $\left(\frac{X+1)^2}{9}\right) \Big/ \left(\frac{(Y-1)^2}{4}\right)$. (5pts)