

MA331 Homework03

Problem 1. Assume a SRS X_1, \dots, X_n from the population random variable X having uniform distribution $\mathcal{U}(0, \theta)$ with some $\theta > 0$.

- (i) Find the moment estimator $\hat{\theta}_M$;
- (ii) Find the maximum likelihood estimator $\hat{\theta}_L$;
- (iii) For the real data $(X_1, \dots, X_7) = (1.0, 2.4, 3.2, 1.2, 0.5, 3.1, 6.8)$, evaluate the observed values of $\hat{\theta}_M$ and $\hat{\theta}_L$. Which one is better? Why?
- (iv) For $\theta = 7$, generate 100 SRS's of size $n = 30$, evaluate the observed values of $\hat{\theta}_M$ and $\hat{\theta}_L$. Produce box plot and mark the sample mean of the corresponding observed values of $\hat{\theta}_M$ and $\hat{\theta}_L$, respectively.
- (v) For $\theta = 7$, generate a SRS of size $n = 20, 30, 50, 100, 150$, evaluate the observed values of $\hat{\theta}_M$ and $\hat{\theta}_L$. Plot the corresponding observed values of $\hat{\theta}_M$ and $\hat{\theta}_L$, respectively.

Problem 2. Assume a SRS X_1, \dots, X_n from the population random variable $X \sim \mathcal{N}(\mu, \sigma^2)$.

- (i) Find the moment estimator $(\hat{\mu}, \hat{\sigma}_2)$.
- (ii) Find the maximum likelihood estimator $(\tilde{\mu}, \tilde{\sigma}_2)$.

Problem 3. Finish the following problems in the textbook:

Exercises 6.17, 6.27 and 6.28.