Problem #1

i) $x \sim \text{uniform}(0, \theta) \quad \therefore f(x) = \frac{1}{\theta}$

moment estimator θ_m

$\int_0^\theta x f(x)\,dx$ pop. moment

$\frac{x_1 + x_2 + x_3 + \dots + x_n}{n} =$

$\frac{x_1^2 + \dots + x_n^2}{n}$

$\int_0^\theta \frac{x}{\theta}\,dx = \frac{\theta}{2}$

$\frac{\theta}{2} = \frac{x_1 + \dots + x_n}{n} \Rightarrow \underline{\frac{2}{n}\sum_{i=1}^{n} x_i = \theta_m}$

ii) $L(\theta | x_n) = \{ \frac{1}{\theta^n}, \quad 0 \leq x_y \leq \theta \quad y = 1(n)$

base on likelihood of the function

$\underline{\underline{\theta_n = X_n}}$

iii) $\theta_m = 5.2$

$x = 6.8$

θ_m is better because it is closer to the average value and is a greater representation

Problem #2

$M_1 = E(x) = M$

i)

$$E(x^2) = V(x) + E(x)^2$$

$$m_1 = \frac{1}{n}\sum_{i=1}^{n} x_i^2 = \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \mu_x^2$$

?

$$\bar{x} \ \& \ \sqrt{\frac{1}{n}\Sigma(x_i-\bar{x})^2}$$

$$\bar{x}, \ \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2 \ ?$$

ii)

$$\lambda = \left(\frac{1}{\sigma\sqrt{2\pi}}\right)^n \exp - \sum_{i=1}^{n}\frac{(x_i-M)^2}{2\sigma^2}$$

$$Y \sim N(M, \sigma^2)$$

$$\log\lambda = \frac{-n}{2}\log(2\pi)$$

$$-\frac{n}{2}\log(\sigma^2) - \frac{1}{2\sigma^2}$$

$$\frac{\partial \log L}{\partial M} = 0 \quad n\bar{x} = nM \quad \& \ M_{MLE} = \bar{x}$$

$$\frac{\partial \log L}{\partial \sigma} = 0 \rightarrow \frac{-n}{2}\cdot\frac{1}{\sigma^2}\cdot 2\sigma = \frac{1}{2}\sum_{i=1}^{n}(x_i - M)^2 \cdot \frac{x}{3}$$

$$\sigma^2 = \frac{1}{n}\Sigma(x_i - M)^2 = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i-M)^2}$$

$$\hat{M}_{MLE} = \bar{x} \quad \sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2}$$

$\leftarrow \theta_{MLE}$

Problem #3

6.17a)

$1.96 \cdot \frac{2.3}{\sqrt{344}} = .24$, $\underline{5.156, 5.644}$

6.17b) $\underline{\text{err} = 0.321}$

6.27a) $11.5 \pm 1.96 \frac{83}{\sqrt{1200}} = 11$

6.27b) No, it represents the average value

6.27c) yes, a greater sample size would be more accurate

6.28a) $\theta \rightarrow$ $\swarrow$ 6.28b

$690 \pm 1.96 \frac{4.98}{\sqrt{1200}} = \underline{661.87, 718.17}$

x

6.28c) multiply the std by 60 to calculate directly