$$E(N) = \sum_{N=0}^{n} N\binom{n}{N} p^N (1-p)^{n-N}$$

$$\sum_{N=0}^{n} n \cdot \left(\frac{n!}{n!\,(n-N)!}\right) p^N (1-p)^{n-N}$$

$$\sum_{N=1}^{n} \frac{n!}{(N-1)!\,(n-N)!} \cdot p^N (1-p)^{n-N}$$

$$\boxed{\begin{array}{l} y = N-1 \\ N = y+1 \\ m = n-1 \\ n = m+1 \end{array}}$$

$$\sum_{y=0}^{m} \frac{(m+1)!}{y!\,(m-y)!} \cdot p^{y+1} (1-p)^{m-y}$$

$$p(m+1) \sum_{y=0}^{m} \frac{m!}{y!\,(m-y)!} \cdot p^y (1-p)^{m-y}$$

$$\sum_{y=0}^{m} \frac{m!}{y!\,(m-y)!} \cdot \cancel{a^y b^{m-y}} \quad (a+b)^m$$

$$(p + 1 - p)^m = 1$$

binomial theorem

$a = p \quad b = 1-p$

therefore $E[N] = np$.