Homework 02

Ming Lin

I pledge my honor that I have abided by the Stevens Honor System

Problem 1(i)

For, n = 20, P(N <= 8.5) = 0.5955987
For, n = 30, P(N <= 8.5) = 0.0940112
For, n = 50, P(N <= 8.5) = 2.3052286×10^{-4}
For, n = 75, P(N <= 8.5) = 1.826106×10^{-8}
For, n = 100, P(N <= 8.5) = $5.4311266 \times 10^{-13}$

Problem 1(ii)

For, n = 20, P(N <= 8.5) = 0.6331
For, n = 30, P(N <= 8.5) = 0.0606
For, n = 50, P(N <= 8.5) = 0.00058
For, n = 75, P(N <= 8.5) = 1.475e^-7
For, n = 100, P(N <= 8.5) = 8.91e^-11

Problem 1(iii)

Problem 1(iv)
As the errors downtrend to nearly but not 0, the N increases.

Problem 2

```
## [1] "N = 20"
```

```
## [1] "N = 30"
```

```
## [1] "N = 50"
```

```
## [1] "N = 75"
```

Problem 2(v)

After observing the plots, I notice that the values seem to peak in the middle and through sample size n that throughout each graph their slope and pattern are similar

Problem 2(vi)

After observing the scatter plots, I noticed that the values are uniformly randomly distributed in the middle of the scatter plot. Also, the scatter plot shows minimal correlation and increasing density in the middle.

Problem 3

$$E(N) = \sum_{N=0}^{n} N \binom{n}{N} p^N (1-p)^{n-N}$$

$$\sum_{N=0}^{n} n \cdot \left(\frac{n!}{n!\,(n-N)!} \right) p^N (1-p)^{n-N}$$

$$\sum_{N=1}^{n} \frac{n!}{(N-1)!\,(n-N)!} \cdot p^N (1-p)^{n-N}$$

$$\boxed{\begin{array}{l} y = N-1 \\ N = y+1 \\ m = n-1 \\ n = m+1 \end{array}}$$

$$\sum_{y=0}^{m} \frac{(m+1)!}{y!\,(m-y)!} \, p^{y+1} (1-p)^{m-y}$$

$$p(m+1) \sum_{y=0}^{m} \frac{m!}{y!\,(m-y)!} \cdot p^y (1-p)^{m-y}$$

$$\sum_{y=0}^{m} \frac{m!}{y!\,(m-y)!} \cdot \cancel{a^y b^{m-y}} \quad (a+b)^m$$

$$(p + 1 - p)^m = 1$$

binomial theorem

$a = p$ $b = 1-p$

therefore $E[N] = np$.

Problem 4

$$E(T) = \int_{-\infty}^{\infty} t \cdot f(t)\, dt$$

$$= \int_{-\infty}^{0} t \cdot f(t)\, dt + \int_{0}^{\infty} t f(t)\, dt$$

$$- \int_{\infty}^{0} t\ f(-t)\, dt + \int_{0}^{\infty} t \cdot f(t)\, dt$$

$$- \int_{0}^{\infty} t f(t)\, dt + \int_{0}^{\infty} t\ f(t)\, dt$$

$$\therefore f(t) - f(-t) = 0$$