# MA331 Homework02

**Problem 1.** Assume that $N \sim \mathcal{B}(n, p)$, a Binomial distribution with number of trials $n$ and probability of success $p$. Set $p = 0.4$.

(i) For $n = 20, 30, 50, 75, 100$, accurately compute $P(N \leq 8.25)$ by using R function.

(ii) For $n = 20, 30, 50, 75, 100$, approximate $P(N \leq 8.25)$ by using Laplace theorem.

(iii) Evaluate and scatter plot errors of all approximations of (ii), i.e., the absolute difference between the accurate computation and the Laplace approximation.

(iv) What do you perceive based on the scatter plot of errors in (iii).

**Problem 2.** Check the instruction of R commands '`plot(density(x))`'. Generate a SRS of size $n$ for the population $X \sim \mathcal{N}(2, 3^2)$, and evaluate the samples of

$$\frac{\bar{X} - 2}{\sqrt{3^2/n}}, \qquad \frac{(n-1)S^2}{3^2}, \qquad \left(\frac{\bar{X} - 2}{\sqrt{3^2/n}}, \frac{(n-1)S^2}{3^2}\right),$$

respectively. Then, based on the corresponding samples, plot estimated density curves of $\frac{\bar{X}-2}{\sqrt{3^2/n}}$ and $\frac{(n-1)S^2}{3^2}$, respectively, and also make the scatter plot of $\left(\frac{\bar{X}-2}{\sqrt{3^2/n}}, \frac{(n-1)S^2}{3^2}\right)$.

(i) For $n = 20$, simulate $\frac{\bar{X}-2}{\sqrt{3^2/n}}$ and $\frac{(n-1)S^2}{3^2}$ for 100 times.

(ii) For $n = 30$, simulate $\frac{\bar{X}-2}{\sqrt{3^2/n}}$ and $\frac{(n-1)S^2}{3^2}$ for 100 times.

(iii) For $n = 50$, simulate $\frac{\bar{X}-2}{\sqrt{3^2/n}}$ and $\frac{(n-1)S^2}{3^2}$ for 100 times.

(iv) For $n = 75$, simulate $\frac{\bar{X}-2}{\sqrt{3^2/n}}$ and $\frac{(n-1)S^2}{3^2}$ for 100 times.

(v) Based on the plots of $\frac{\bar{X}-2}{\sqrt{3^2/n}}$ and $\frac{(n-1)S^2}{3^2}$ in (i) - (iv) describe your findings on probability distributions of $\frac{\bar{X}-2}{\sqrt{3^2/n}}$ and $\frac{(n-1)S^2}{3^2}$, respectively.

(vi) Based on the scatter plots of $\left(\frac{\bar{X}-2}{\sqrt{3^2/n}}, \frac{(n-1)S^2}{3^2}\right)$ in (i) - (iv) describe your findings on the statistical association between $\frac{\bar{X}-2}{\sqrt{3^2/n}}$ and $\frac{(n-1)S^2}{3^2}$.

**Problem 3.** Show that $\mathrm{E}[N] = np$ for $N \sim \mathcal{B}(n, p)$.

**Problem 4.** Show that $\mathrm{E}[T] = 0$ for $T \sim \mathcal{T}_n$.