Note: problems in red color will be discussed during tutorial sessions and need NOT to be handed in.

Problem 1: (Page 185, Q14) Write an algorithm that reverses a string $s_1, \ldots, s_n$. Example: If the sequence is
AMY BRUNO ELIE,
the reversed sequence is
ELIE BRUNO AMY

Problem 2: (Page 195, Q19) Write an algorithm that receives an input the matrix of a relation R and tests whether R is transitive.

Problem 3: (Page 192, Q13) Write an algorithm that returns the index of the first occurrence of the value key in the sequence $s_1, \ldots, s_n$. If key is not in the sequence, the algorithm returns the value 0. Example: If the sequence is

12 11 12 23

and key is 12, the algorithm returns the value 1.

Please give the execution trace of your algorithm step-by-step for the input sequence 11 23 5 6, key = 4.

Problem 4: (Page 207, Q 1 - 15) Select a theta notation from Table 4.3.3 for each express in the following:

1) $3n^2 + 2n \lg n$
2) $2 \lg n + 4n + 3n \lg n$
3) $\frac{(n+1)(n+3)}{n+1}$
4) $\frac{(n^2+lgn)(n+1)}{n+n^2}$
5) $2 + 4 + 8 + \cdots 2^n$.
6) $f(n) + g(n)$, where $f(n) = 6n^3 + 2n^2 + 4$ and $g(n) = \Theta(n \lg n)$.

Problem 5: (Page 207, Q 18, 24) Express in theta notation the number of times the statement $x = x + 1$ is executed.

1)
```
for i = 1 to n
  for j = 1 to n
    x = x + 1
```
2)
```
j = n
while (j >= 1) {
  for i = 1 to j
    x = x + 1
  j = floor(j/3)
}
```
where $floor(x) = \lfloor x \rfloor$

3)
```
i = 2
while (i < n) {
  i = i*i
  x = x + 1
}
```

Problem 6: (Page 209, 37(Show that $\lg(n^k + c) = \Theta(\lg n)$ for every fixed $k > 0$ and $c > 0$.

Problem 7: (Page 209, 38) Show that if n is a power of 2, say $n = 2^k$, then

$$\sum_{i=0} k \lg(n/2^i) = \Theta(\lg^2 n).$$

Problem 8: (Page 209, 62)

1) Show, by consulting the figure, that

$$1/2 + 1/3 + \cdots 1/n < \log_e n.$$

2) Show, by consulting the figure that

$$\log_e n < 1 + 1/2 + \cdots + 1/n.$$

3) Use parts (a) and (b) to show that

$$1 + 1/2 + \cdots 1/n) = \theta(\lg n).$$

Problem 8: A robot can take steps of 1 meter, 2 meters, or 3 meters. Write an algorithm to list all the ways that the robot can walk n meters.

Problem 9: How many different car license plates can be constructed if the licenses contain three letters followed by two digits if repetitions are allowed? if repetitions are not allowed.

Problem 10: (Page 274, 16) Two dice are rolled, one blue and one red. How many outcomes give the sum of 2 or the sum 12.

Problem 11: (Page 274, 17) A committee composed or Morgan, Tyler, Max, and Leslie is to select a president and secretary. How many selections are there in which Max is president or secretary.

Problem 12: (Page 274, 52 – 62) For integers from 5 to 200, inclusive

1) How many are greater than 101 and do not contain the digit 6?
2) How many do not contain the digit 0?
3) How many have the digits in strictly increasing order? (Examples are 13, 147, 8)
4) How many are of the form xyz, where $0 \neq x < y$ and $y > zZ$.

Problem 13: (Page 276, 72) How many terms are there in the expansion of

$$(x + y)(a + b + c)(e + f + g)(h + i).$$

Problem 14: (Page 276, 81, 82) How many symmetric and antisymmetric relations are there on an n-element set? How many reflexive, symmetric, and antisymmetric relations are there on an n-element set?