Tutorial 4

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Problem 1.4



The statement is true.

$$X\cap (Y-Z)=(X\cap Y)-(X\cap Z)$$

RHS:

$$\begin{split} (X \cap Y) - (X \cap Z) &= (X \cap Y) \cap \overline{(X \cap Z)} \\ &= (X \cap Y) \cap (\overline{X} \cup \overline{Z}) \\ &= Y \cap (X \cap (\overline{X} \cup \overline{Z})) \\ &= Y \cap (X \cap \overline{Z}) \end{split}$$

LHS:

$$X \cap (Y - Z) = X \cap Y \cap \overline{Z}$$



Problem 2



- Assume the conclusion is false, i.e., no two bags have the same number of coins.
- Let $x_i, i = 1, \dots, 9$ be the numbers of coins in the i^{th} bag. By hypothesis, we have

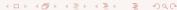
$$\left\{\begin{array}{l} x_i \ge 1, i = 1, \cdots, 9 \\ \sum_{i=1}^n x_i = 40 \end{array}\right\}$$

- By negation of the conclusion we have $x_i \neq x_i, \forall i \neq j, i, j = 1, \dots, 9$.
- We reorder the bags, s.t. the numbers of coins contained follow acending order, namely, $x_i < x_j$, if i < j
- Then $x_1 \ge 1, x_2 \ge 2, x_3 \ge 3, \dots, x_9 \ge 9$

$$\sum_{i=1}^{9} x_i \ge 45 > 40$$

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- contradition here!
- then the original statement is correct.



Problem 4.1



- The statement is true
- Prove by contradiction
- the negation of the original statement $\forall i, s_i > A$
- then

$$\frac{s_1 + \dots + s_n}{n} > A$$

- contradition
- then the original statement is true

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Problem 7 direct proof



$$\frac{1}{(2n-1)(2n+1)} = \left(\frac{1}{2n-1} - \frac{1}{2n+1}\right) \cdot \frac{1}{2}$$

$$\sum_{i=1}^{n} \frac{1}{(2i-1)(2i+1)} = \sum_{i=1}^{n} \left(\frac{1}{2i-1} - \frac{1}{2i+1} \right) \cdot \frac{1}{2}$$
$$= \frac{1}{2} \cdot \left(1 - \frac{1}{2n+1} \right)$$
$$= \frac{n}{2n+1}$$

Problem 7 induction



Basis step: for n=1 $\frac{1}{1\cdot 3}=\frac{1}{3}=\frac{1}{2\cdot 1+1}$ Induction step, assume the statment is true for n=k for n=k+1

$$\sum_{i=1}^{k+1} \frac{1}{(2i-1)(2i+1)} = \sum_{i=1}^{k} \left(\frac{1}{2i-1} - \frac{1}{2i+1}\right) - \left(\frac{1}{2k+1} - \frac{1}{2k+3}\right)$$

$$= \frac{k}{2k+1} + \left(\frac{1}{2k+1} - \frac{1}{2k+3}\right)$$

$$= \frac{2k^2 + 3k + 1}{(2k+1)(2k+3)}$$

$$= \frac{k+1}{2k+3} = \frac{(k+1)}{2(k+1)+1}$$

By the principle of math induction, the statement is true.



Questions about the problems?

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