Tutorial 4

Qu Xiaofeng

COMP210
Discrete Structure

October 7, 2011

The Hong Kong Polytechnic University
香港理工大學

Table of Contents

Problem 1.4

The statement is true.

$$X \cap (Y - Z) = (X \cap Y) - (X \cap Z)$$

RHS:

$$\begin{aligned}(X \cap Y) - (X \cap Z) &= (X \cap Y) \cap \overline{(X \cap Z)} \\ &= (X \cap Y) \cap (\overline{X} \cup \overline{Z}) \\ &= Y \cap (X \cap (\overline{X} \cup \overline{Z})) \\ &= Y \cap (X \cap \overline{Z})\end{aligned}$$

LHS:

$$X \cap (Y - Z) = X \cap Y \cap \overline{Z}$$

Problem 2

- Assume the conclusion is false, i.e., no two bags have the same number of coins.
- Let $x_i, i = 1, \cdots, 9$ be the numbers of coins in the i^{th} bag. By hypothesis, we have

$$\left\{ \begin{array}{l} x_i \geq 1, i = 1, \cdots, 9 \\ \sum_{i=1}^{n} x_i = 40 \end{array} \right\}$$

- By negation of the conclusion we have $x_i \neq x_j, \forall i \neq j, i, j = 1, \cdots, 9$.
- We reorder the bags, s.t. the numbers of coins contained follow acending order, namely, $x_i < x_j,\ if\ i < j$
- Then $x_1 \geq 1, x_2 \geq 2, x_3 \geq 3, \cdots, x_9 \geq 9$

$$\sum_{i=1}^{9} x_i \geq 45 > 40$$

- contradition here!
- then the original statement is correct.

Problem 4.1

- The statement is true
- Prove by contradiction
- the negation of the original statement $\forall i,\ s_i > A$
- then

$$\frac{s_1 + \cdots + s_n}{n} > A$$

- contradition
- then the original statement is true

Problem 7 direct proof

$$\frac{1}{(2n-1)(2n+1)} = \left(\frac{1}{2n-1} - \frac{1}{2n+1}\right) \cdot \frac{1}{2}$$

$$\begin{aligned}
\sum_{i=1}^{n} \frac{1}{(2i-1)(2i+1)} &= \sum_{i=1}^{n} \left(\frac{1}{2i-1} - \frac{1}{2i+1}\right) \cdot \frac{1}{2} \\
&= \frac{1}{2} \cdot \left(1 - \frac{1}{2n+1}\right) \\
&= \frac{n}{2n+1}
\end{aligned}$$

Problem 7 induction

Basis step: for $n = 1$ $\quad \frac{1}{1\cdot 3} = \frac{1}{3} = \frac{1}{2\cdot 1+1}$
Induction step, assume the statment is true for $n = k$
for $n = k + 1$

$$\begin{aligned}
\sum_{i=1}^{k+1} \frac{1}{(2i-1)(2i+1)} &= \sum_{i=1}^{k} \left(\frac{1}{2i-1} - \frac{1}{2i+1}\right) - \left(\frac{1}{2k+1} - \frac{1}{2k+3}\right) \\
&= \frac{k}{2k+1} + \left(\frac{1}{2k+1} - \frac{1}{2k+3}\right) \\
&= \frac{2k^2+3k+1}{(2k+1)(2k+3)} \\
&= \frac{k+1}{2k+3} = \frac{(k+1)}{2(k+1)+1}
\end{aligned}$$

By the principle of math induction, the statement is true.

Q & A

Questions about the problems?