Tutorial 12: Assignment 5

Qu Xiaofeng, Teacher Asistant
quxiaofeng.at.polyu@gmail.com, PQ702

COMP210
Discrete Structure

December 7, 2011

THE HONG KONG POLYTECHNIC UNIVERSITY
香港理工大學

Table of Contents

Problem 15

Show that the maximum number of edges in a simple, disconnected graph with n vertices is (n-1)(n-2)/2.

Since the graph is disconnetced, we can partition it into two subgraphs with $m_1 + m_2 = n$ vertices.

The max number of edges is $\frac{m_1(m_1-1)}{2} + \frac{m_2(m_2-1)}{2}$

$$\frac{m_1(m_1-1)}{2} + \frac{(n-m_1)(n-m_1-1)}{2}$$

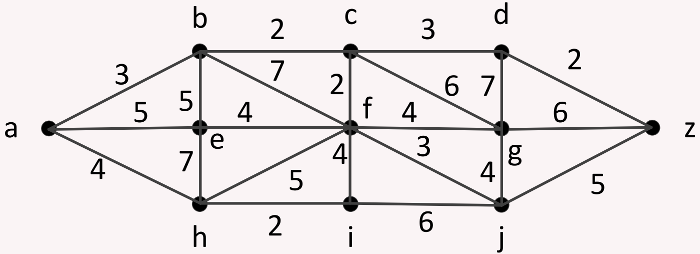
$$= \frac{m_1^2 - m_1 + n^2 - 2nm_1 + m_1^2 - n + m_1}{2}$$

$$= \frac{2m_1^2 - 2nm_1 + (n^2 - n)}{2}$$

$$= \frac{2(m_1 - \frac{n}{2})^2 - \frac{n^2}{2} + (n^2 - n)}{2}, \ \textit{where } 1 \le m_1 \le n-1$$

$$m_1 = 1 \textit{ or } m_1 = n-1 \textit{ maximize the sum}$$

$$= \frac{(n-1)(n-2)}{2}$$



Find the length of a shortest path and a shortest path between each pair of vertices in the weighted graph. 1) a,f.

The answer is 7;(a, b, c, f). □

Problem 21

Show that a tree is a bipartite graph.

Let T be a tree. Root T at some arbitrary vertex. Let V be the set of vertices on even levels and let W be the set of vertices on odd levels. Since each edge is incident on a vertex in V and a vertex in W, T is a bipartite graph. $\square$

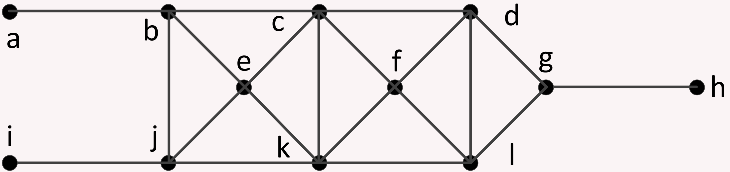
Problem 23

Show that a graph G with n vertices and fewer than $n-1$ edges is not connected.

Prove by contradiction.
If the graph is connected, we can find a spanning tree.
That consists n vertices and n-1 edges.
Contradiction. $\square$

Problem 26.1

Find a spanning tree for each graph using depth-first and breadth-first search.



Problem 27

Let T and T' be two spanning trees of a connected graph G. Suppose that an edge x is in T but not in T'. Show that there is an edge y in T' but not in T such that $(T - \{x\}) \cup \{y\}$ and $(T' - \{y\}) \cup \{x\}$ are spanning trees of G.

Suppose that x is incident on vertices a and b. Removing x form T produces a disconnected graph with two components, U and V. Vertices a and b belong to different components– say, $a \in U$ and $b \in V$. There is a path P from a to b in T'. As we move along P, at some point we encounter an edge $y = (v, w)$ with $v \in U$, $w \in V$. Since adding y to $T - \{x\}$ produces a connected graph, $(T - \{x\}) \cup \{y\}$ is a spanning tree. Clearly, $(T' - \{y\}) \cup \{x\}$ is a spanning tree. $\square$