

Homework Set 1

BY MINGCEN WEI

Answer 1.

Part A.

Let $A = \{s \in S \mid \Pr[X = s] > \Pr[Y = s]\}$, then

$$\begin{aligned}
 \frac{1}{2} \sum_{s \in S} |\Pr[X = s] - \Pr[Y = s]| &= \frac{1}{2} \sum_{s \in A} (\Pr[X = s] - \Pr[Y = s]) + \\
 &\quad \frac{1}{2} \sum_{s \in S-A} (\Pr[Y = s] - \Pr[X = s]) \\
 &= \frac{1}{2} \sum_{s \in A} (\Pr[X = s] - \Pr[Y = s]) + \\
 &\quad \frac{1}{2} \left(\left(1 - \sum_{s \in A} \Pr[Y = s] \right) - \left(1 - \sum_{s \in A} \Pr[X = s] \right) \right) \\
 &= \frac{1}{2} \sum_{s \in A} (\Pr[X = s] - \Pr[Y = s]) + \frac{1}{2} \sum_{s \in A} (\Pr[X = s] - \Pr[Y = s]) \\
 &= \sum_{s \in A} (\Pr[X = s] - \Pr[Y = s]) \\
 &= \max_{T \subseteq S} (\Pr[X \in T] - \Pr[Y \in T]).
 \end{aligned}$$

Part B.

$$\begin{aligned}
 \Pr[D(X) = 1] - \Pr[D(Y) = 1] &= \Pr[X \in D^{-1}(1)] - \Pr[Y \in D^{-1}(1)] \\
 &\leq \max_{T \subseteq S} (\Pr[X \in T] - \Pr[Y \in T]) \\
 &= \Delta(X, Y).
 \end{aligned}$$

Part C.

$$\begin{aligned}
 \Delta(X, Y) &= \max_{T \subseteq S} (\Pr[X \in T] - \Pr[Y \in T]) \\
 &\geq \max_{\text{domain}(D) \subseteq S} (\Pr[X \in D^{-1}(1)] - \Pr[Y \in D^{-1}(1)]) \\
 &\geq \max_D (\Pr[D(X) = 1] - \Pr[D(Y) = 1]) \tag{*} \\
 &\geq \Pr[D(X) = 1] - \Pr[D(Y) = 1] \\
 &\quad \text{where } D(s) = 1 \text{ iff } \Pr[X = s] > \Pr[Y = s], \forall s \in S, \text{ and } 0 \text{ otherwise} \\
 &= \sum_{s \in A} (\Pr[X = s] - \Pr[Y = s]) \text{ where } A = \{s \in S \mid \Pr[X = s] > \Pr[Y = s]\} \\
 &= \Delta(X, Y) \text{ according to } \mathbf{Part A.}
 \end{aligned}$$

Answer 2.

This proposition is obviously true when $|\mathcal{K}| \geq |\mathcal{M}|$.

Now assume $|\mathcal{K}| < |\mathcal{M}|$. Given any $c \in \mathcal{C}$, there are at most $|\mathcal{K}|$ elements of $|\mathcal{M}|$ belonging to $\text{Dec}^{-1}(\mathcal{K}, c) = \{m \in \mathcal{M} \mid \exists k \in \mathcal{K} \text{ s.t. } m = \text{Dec}(k, c)\}$. Thus $\forall c \in \mathcal{C}, \Pr[M \in \text{Dec}^{-1}(\mathcal{K}, c)] \leq \frac{|\mathcal{K}|}{|\mathcal{M}|}$. $\therefore \forall k_0 \in \mathcal{K}, m_0 \in \mathcal{M}, \Pr[M \in \text{Dec}^{-1}(\mathcal{K}, \text{Enc}(k_0, m_0))] \leq \frac{|\mathcal{K}|}{|\mathcal{M}|}$. $\therefore \forall m_0 \in \mathcal{M}$,

$$\begin{aligned} \Pr[M \in \text{Dec}^{-1}(\mathcal{K}, \text{Enc}(K, m_0))] &= \sum_{k_0 \in \mathcal{K}} \Pr[M \in \text{Dec}^{-1}(\mathcal{K}, \text{Enc}(K, m_0)), K = k_0] \\ &= \sum_{k_0 \in \mathcal{K}} \Pr[M \in \text{Dec}^{-1}(\mathcal{K}, \text{Enc}(k_0, m_0)), K = k_0] \\ &= \sum_{k_0 \in \mathcal{K}} \Pr[M \in \text{Dec}^{-1}(\mathcal{K}, \text{Enc}(k_0, m_0))] \Pr[K = k_0] \\ &\leq \sum_{k_0 \in \mathcal{K}} \frac{|\mathcal{K}|}{|\mathcal{M}|} \frac{1}{|\mathcal{K}|} \\ &= \frac{|\mathcal{K}|}{|\mathcal{M}|}. \end{aligned}$$

$\therefore \exists m_1 \in \mathcal{M} \text{ s.t. } \Pr[m_1 \in \text{Dec}^{-1}(\mathcal{K}, \text{Enc}(K, m_0))] \leq \Pr[M \in \text{Dec}^{-1}(\mathcal{K}, \text{Enc}(K, m_0))] \leq \frac{|\mathcal{K}|}{|\mathcal{M}|}$. Since $\text{Enc}(\mathcal{K}, m_1) = \{c \in \mathcal{C} \mid \exists k \in \mathcal{K} \text{ s.t. } m_1 = \text{Dec}(k, c)\} \subseteq \mathcal{C}$, we have

$$\begin{aligned} \Delta(\text{Enc}(K, m_0), \text{Enc}(K, m_1)) &\geq \Pr[\text{Enc}(K, m_1) \in \text{Enc}(\mathcal{K}, m_1)] - \\ &\quad \Pr[\text{Enc}(K, m_0) \in \text{Enc}(\mathcal{K}, m_1)] \\ &= 1 - \Pr[m_1 \in \text{Dec}^{-1}(\mathcal{K}, \text{Enc}(K, m_0))] \\ &\geq 1 - \frac{|\mathcal{K}|}{|\mathcal{M}|}. \end{aligned}$$

Answer 3.

Plaintext:

cryptographyisanindispensabletoolusedtoprotectinformationincomputingsystemsitisusedeverywhereandbybillionsofpeopleworldwideonadailybasisitisusedtoprotectdataatrestanddatainmotion

Cryptography is an indispensable tool used to protect information in computing systems. It is used everywhere and by billions of people worldwide on a daily basis. It is used to protect data at rest and data in motion.

— Oh! It seems that this is an excerpt from *A Quantum Leap in Cryptography: Interview with Grégoire Ribordy From ID Quantique*.

Cipher:

enjoy

Steps:

1. First I tried the *index of coincidence method* — computing sums of squared frequencies of characters in the spaced substrings with various interval lengths (see A.1). However, due to the short length of the ciphertext, this method didn't help much.
2. Then I tried KASISKI's *method*, and found that the substring `olh` appeared twice in the ciphertext (see A.2).
3. I guessed that `the` might be mapped to `olh`. However, when I mapped the ciphertext back, it seemed nonsense (see A.3).
4. Then I computed the distance between the two appearances and the result was $70 = 2 \times 5 \times 7$ (see A.4). Thus the cipher length was likely one in 5, 7, 10, 14, 35, 70 (1 and 2 are omitted since `olh` has 3 characters).

5. To ensure that I could find the cipher, I decided to use the *brute force method* to try all ciphers no longer than 4 characters (see A.6). The time complexity $T_1 = \sum_{1 \leq k \leq 4} 26^k = 475254$, and running the program took me several minutes (thus if I chose to try all 5 letter ciphers, this program would take me more than an hour).
6. To find meaningful strings from the output file obtained in 5, I used the Python library *Nostril: Nonsense String Evaluator*¹. It gave an empty output file (see A.7). In view of the low false positive rate of the library, I deduced that the cipher length was longer than 5 characters.
7. Then I started to try 5 letter ciphers. I calculated the frequencies of each letter in the spaced substrings (see A.8). Again, this information was of little use because of the limited size of the ciphertext.
8. The letter **e** appears very frequently throughout English language. Therefore I believed it would appear in every spaced substring. I used brute force to try every possibility (see A.5). The spaced substrings contained 14, 16, 13, 14, 16 distinct letters, respectively (see the output of A.8), so the time complexity $T_2 = 14 \times 16 \times 13 \times 14 \times 16 = 652288 \ll 11881376 = 26^5$. This time the program took me several minutes as well.
9. Once more, I used *Nostril* to find meaningful string from the output file obtained in 8. The result was a 41 line file (see A.7). Hooray! This time I finally found a meaningful string in line 19 of the file.
10. In the end, I used the plaintext and the ciphertext to compute the cipher (see A.9). It was enjoy, and I enjoyed this journey of cracking code very much!

Answer 4.

Definition. Fix an integer $\ell > 0$, $\mathcal{M} = \mathcal{C} = \{a, b, \dots, z\}^{\leq \ell}$, and $\mathcal{K} = S_\ell$ (the permutation group of order ℓ). Let $f: \{a, b, \dots, z\} \rightarrow \{0, 1, \dots, 25\}$ be the bijective ordinal mapping.

- Gen: choosing a key from \mathcal{K} uniformly. $k \leftarrow \text{Gen}(1^n)$.
- Enc: given a key $k \in \mathcal{K}$ and a message $m \in \mathcal{M}$, $c := \text{Enc}(k, m) = \text{map}(f^{-1}, \text{map}(k, \text{map}(f, m)))$, i.e. $\text{Enc}(k, \overline{\alpha_1 \alpha_2 \dots \alpha_j}) = \overline{f^{-1}(k(f(\alpha_1))) f^{-1}(k(f(\alpha_2))) \dots f^{-1}(k(f(\alpha_j)))}$.
- Dec: the same as Enc, except that it use k^{-1} instead of k .

Answer 5.

- a) No. For instance, $\Pr[M = 0 \mid C = 0] = \frac{1}{3}$, while $\Pr[M = 1 \mid C = 0] = \Pr[M = 2 \mid C = 0] = \Pr[M = 3 \mid C = 0] = \Pr[M = 4 \mid C = 0] = \frac{1}{6}$.

1. Its documentation says:

Nostril is the *Nonsense String Evaluator*: a Python module that infers whether a given short string of characters is likely to be random gibberish or something meaningful.

...

Nostril uses a combination of heuristic rules and a probabilistic assessment. It is not always correct (see below). It is tuned to reduce false positives: it is more likely to say something is *not* gibberish when it really might be. This is suitable for its intended purpose of filtering source code identifiers – a difficult problem, incidentally, because program identifiers often consist of acronyms and word fragments jammed together (e.g., “kBoPoMoFoOrderIdCID”, “ioFIXFndrInfo”, etc.), which can challenge even humans. Nevertheless, on the identifier strings from the [Loyola University of Delaware Identifier Splitting Oracle](#), Nostril classifies over 99% correctly.

Nostril is reasonably fast: once the module is loaded, on a 4 Ghz Apple OS X 10.12 computer, calling the evaluation function returns a result in 30–50 microseconds per string on average.

b) Yes. Because for each $m \in \mathcal{M}, c \in \mathcal{C}$, $\Pr[M = m \mid C = c] \equiv \frac{1}{|\mathcal{M}|}$.

Answer 6.

Example. Given $\varepsilon > 0$, there exists an integer $n > \frac{1}{\varepsilon} + 1$. Let $\mathcal{M} = \mathcal{C} = \{0, 1, 2, \dots, n\}$, $\mathcal{K} = \{0, 1, 2, \dots, (n-1)\}$, $|\mathcal{K}| < |\mathcal{M}|$. Let $\text{Enc}(m, k) = [(m+k) \bmod (n+1)]$, $\text{Dec}(c, k) = [(c-k) \bmod (n+1)]$. Then given any distinct $m_0, m_1 \in \mathcal{M}$, if $k \in \mathcal{K} - \{[(m_0+n) \bmod (n+1)], [(m_1+n) \bmod (n+1)]\}$, \mathcal{A} has a chance $\frac{1}{2}$ of success; otherwise if $k = [(m_0+n) \bmod (n+1)]$, \mathcal{A} can deduce that $M = m_1$, and if $k = [(m_1+n) \bmod (n+1)]$, \mathcal{A} can deduce that $M = m_0$. Thus

$$\Pr[\text{PrivK}_{\mathcal{A}, \Pi}^{\text{eav}} = 1] = \frac{n-3}{n-1} \times \frac{1}{2} + \frac{2}{n-1} \times 1 = \frac{1}{2} + \frac{1}{n-1} \leq \frac{1}{2} + \varepsilon.$$

Proposition. (Lower Bound of the Cardinality of the Key Space)

If $\Pr[\text{PrivK}_{\mathcal{A}, \Pi}^{\text{eav}} = 1] \leq \frac{1}{2} + \varepsilon$, then $|\mathcal{K}| \geq |\mathcal{M}|(1 - \varepsilon)$.

Proof. According to **Answer 2**, if $\forall m_0, m_1 \in \mathcal{M}$, $\Delta(\text{Enc}(K, m_0), \text{Enc}(K, m_1)) \leq \varepsilon$, then $1 - \frac{|\mathcal{K}|}{|\mathcal{M}|} \leq \varepsilon$, i.e. $|\mathcal{K}| \geq |\mathcal{M}|(1 - \varepsilon)$. Therefore we only need to prove that $\forall m_0, m_1 \in \mathcal{M}$, $\Delta(\text{Enc}(K, m_0), \text{Enc}(K, m_1)) \leq \varepsilon$.

$\forall m_0, m_1 \in \mathcal{M}$, let $S = \{c \in \mathcal{C} \mid \Pr[\text{Enc}(K, m_0) = c] > \Pr[\text{Enc}(K, m_1) = c]\}$. \mathcal{A} has a chance no greater than $(\frac{1}{2} + \varepsilon)$ to successfully distinguish $\text{Enc}(K, m_0)$ and $\text{Enc}(K, m_1)$. If whenever $c \in S$, \mathcal{A} will output m_0 , and m_1 otherwise, then \mathcal{A} can perform best on average. Thus

$$\begin{aligned} \frac{1}{2} + \varepsilon &\geq \sum_{c \in S} \Pr[\text{Enc}(K, m_0) = c] \Pr[m_0 \text{ is chosen}] + \sum_{c \in \mathcal{C} - S} \Pr[\text{Enc}(K, m_1) = c] \Pr[m_1 \text{ is chosen}] \\ &= \frac{1}{2} \sum_{c \in S} \Pr[\text{Enc}(K, m_0) = c] + \frac{1}{2} \left(1 - \sum_{c \in \mathcal{C}} \Pr[\text{Enc}(K, m_1) = c] \right) \\ &= \frac{1}{2} + \sum_{c \in S} (\Pr[\text{Enc}(K, m_0) = c] - \Pr[\text{Enc}(K, m_1) = c]) \\ &= \frac{1}{2} + (\Pr[\text{Enc}(K, m_0) \in S] - \Pr[\text{Enc}(K, m_1) \in S]) \\ &= \frac{1}{2} + \max_{T \subseteq \mathcal{C}} (\Pr[\text{Enc}(K, m_0) \in T] - \Pr[\text{Enc}(K, m_1) \in T]) \\ &= \frac{1}{2} + \Delta(\text{Enc}(K, m_0), \text{Enc}(K, m_1)) \end{aligned}$$

Then we have $\Delta(\text{Enc}(K, m_0), \text{Enc}(K, m_1)) \leq \varepsilon$. □

Appendix

A Python Programs and Outputs

For the source code and some of the results files, visit https://github.com/mingcenwei/pku_course_fundamentals_of_cryptography.

A.1 sums_of_squared_frequencies.py

Code

```
from itertools import takewhile, count
from string import ascii_lowercase

CIPHERTEXT = "GEHDRSTAONLLRGYRVWRGWCNBQEUSRSBUIQIQCCNVBCSA" + \
    "XVWTMVZJHGSRBASZYIRMAPGWGNAQMGRGSRMSTIEH" + \
    "KFIENOLHOHPGPYRCLWBODCSCUSUSEURUMQNCLEQJWJCOJ" + \
    "GGWVCWQYFNRRSCACRIPCXYXNJHPIFCOLHQBHYMAVCRMBW"
MAX_CIPHER_LENGTH = 10

CIPHERTEXT = CIPHERTEXT.lower()

for cipherLength in range(1, MAX_CIPHER_LENGTH + 1):
    substrings = ''.join(
        CIPHERTEXT[start_i + j * cipherLength] for j in takewhile(lambda j:
start_i + j * cipherLength < len(CIPHERTEXT), count())
    ) for start_i in range(cipherLength)]
    sumsOfSquaredFrequencies = [round(sum(
        substring.count(letter) ** 2 for letter in ascii_lowercase
    ) / (len(substring) ** 2), 3) for substring in substrings]
    print(sumsOfSquaredFrequencies)
```

Output

```
[0.05]
[0.054, 0.056]
[0.069, 0.058, 0.057]
[0.054, 0.07, 0.063, 0.06]
[0.113, 0.079, 0.103, 0.091, 0.092]
[0.084, 0.087, 0.069, 0.078, 0.082, 0.07]
[0.074, 0.086, 0.083, 0.085, 0.12, 0.085, 0.075]
[0.07, 0.081, 0.083, 0.087, 0.07, 0.091, 0.087, 0.079]
[0.115, 0.095, 0.105, 0.105, 0.095, 0.085, 0.08, 0.069, 0.091]
[0.123, 0.086, 0.154, 0.117, 0.16, 0.142, 0.117, 0.123, 0.093, 0.08]
```

A.2 find_repeated_substrings.py

Code

```

CIPHERTEXT = "GEHDRSTAONLLRGYRVWRGWCNBQEUSRSBUIQIQCCNVBCSA" + \
    "XVWTMVZJHGSARBASZYIRMAPGWWGNAQMGRGSRMSTIEH" + \
    "KFIENOLHOHPGYPYRCLWBODCSCUSUSEURUMQNCLEQJWJCOJ" + \
    "GGWVCWQYFNRRSCACRIPCXYXNJHPICOLHQJHYMAVCRMBW"
TARGET_WORD = "the"

CIPHERTEXT = CIPHERTEXT.lower()
TAEGET_WORD = TARGET_WORD.lower()

stringFrequencies = dict()

for i1 in range(len(CIPHERTEXT) - len(TAEGET_WORD) + 1):
    word = CIPHERTEXT[i1: i1 + len(TAEGET_WORD)]
    if word in stringFrequencies.keys():
        continue
    else:
        count = 1
        for i2 in range(i1 + 1, len(CIPHERTEXT) - len(TAEGET_WORD) + 1):
            if word == CIPHERTEXT[i2: i2 + len(TAEGET_WORD)]:
                count += 1
            else:
                continue
        if count == 1:
            continue
        else:
            stringFrequencies[word] = count

sortedList = sorted(stringFrequencies.items(), key=(lambda kv: kv[1]),
reverse=True)
print(sortedList)

```

Output

```
[('olh', 2)]
```

A.3 known_substrings_plaintext_attack.py

Code

```

CIPHERTEXT = "GEHDRSTAONLLRGYRVWRGWCNBQEUSRSBUIQIQCCNVBCSA" + \
    "XVWTMVZJHGSARBASZYIRMAPGWWGNAQMGRGSRMSTIEH" + \
    "KFIENOLHOHPGYPYRCLWBODCSCUSUSEURUMQNCLEQJWJCOJ" + \
    "GGWVCWQYFNRRSCACRIPCXYXNJHPICOLHQJHYMAVCRMBW"
TARGET_WORD = "the"
ENCRYPTED_STRING = "olh"
CIPHER_LENGTH = 5

CIPHERTEXT = CIPHERTEXT.lower()
TAEGET_WORD = TARGET_WORD.lower()
ENCRYPTED_STRING = ENCRYPTED_STRING.lower()

forwardShifts = [(ord(ENCRYPTED_STRING[i]) - ord(TAEGET_WORD[i])) % 26 for i
in range(len(TAEGET_WORD))]
encryptedStringIndex = CIPHERTEXT.find(ENCRYPTED_STRING)
partiallyDecryptedString = ""
for i in range(len(CIPHERTEXT)):
    offset = (i - encryptedStringIndex) % CIPHER_LENGTH
    if offset < len(TARGET_WORD):

```

```

        o1 = ord(CIPHERTEXT[i])    forwardShifts[offset]
        decryptedChar = chr(o1) if o1 >= ord("a") else chr(o1 + 26)
        partiallyDecryptedString += decryptedChar
    else:
        partiallyDecryptedString += chr(ord(CIPHERTEXT[i]) + ord("A"))
ord("a"))
print(partiallyDecryptedString)

```

Output

```

dEHinpTAtjiLRluoVWwctCNgmbOUxnpBUnmfQChjsBCxwuVWyisZJmcpARgwpZYnnjAPlst
GNfmjGRlotRMxpfEHpbfENtheOHucmYRhhtBOiypCUxqpEUwqjQNhhbQJbfzOJlctVCbmVFN
wnpCAhnfPCwuunJmlfFCtheQJmujAVhnjBW

```

A.4 get_gaps_between_repeated_substrings.py

Code

```

CIPHERTEXT = "GEHDRSTAONLLRGYRVWRGWCNBQEUSRSBUIQIQCCNVBCSA" + \
    "XVWTMVZJHGSARBASZYIRMAPGWWGNAQMGRGSRMSTIEH" + \
    "KFIENOLHOHPGPYRCLWBODCSCUSUSEURUMQNCLEQJWJCOJ" + \
    "GGWVCWQYFNRRSCACRIPCXYXNJHPFCOLHQJHYMAVCRMBW"
POSSIBLE_ENCRYPTED_STRINGS = ("olh", )

CIPHERTEXT = CIPHERTEXT.lower()
POSSIBLE_ENCRYPTED_STRINGS = [string.lower() for string in
POSSIBLE_ENCRYPTED_STRINGS]

for string in POSSIBLE_ENCRYPTED_STRINGS:
    possible_encrypted_string_indices = list()
    start_i = CIPHERTEXT.find(string)
    while start_i != -1:
        possible_encrypted_string_indices.append(start_i)
        start_i = CIPHERTEXT.find(string, start_i + 1)

    for i in range(len(possible_encrypted_string_indices) - 1):
        print(possible_encrypted_string_indices[i + 1]
possible_encrypted_string_indices[i], end=" ")
    print()

```

Output

```

70

```

A.5 shift_text_backward.py

Code

```

from itertools import product

def shiftBack(ciphertext, step, plain_char, cipher_char, start_i):
    shift = (ord(cipher_char.lower()) - ord(plain_char.lower())) % 26

    decryptedText = ""
    for i in range(len(ciphertext)):
        if (i - start_i) % step == 0:
            o1 = ord(ciphertext[i].lower()) - shift

```

```

        char = chr(o1) if o1 >= ord("a") else chr(o1 + 26)
        decryptedText += char
    else:
        decryptedText += ciphertext[i]

    return decryptedText

if __name__ == "__main__":
    FILENAME = "brute_force_results.txt"
    CIPHERTEXT = "GEHDRSTAONLLRGYRVWRGWCNBQEOUSRSBUIQIQCCNVBCSA" + \
        "XVWTMVZJHGSARBASZYIRMAPGWWGNAQMGRGSRMSTIEH" + \
        "KFIENOLHOHPGPYRCLWBODCSCUSUSEURUMQNCLEQJWJCOJ" + \
        "GGWVCWQYFNRRSCACRIPCRYXNJHPICOLHQJHYMAVCRMBW"
    CIPHER_LENGTH = 5
    # FIRST_N_TO_TRY = 4
    FIRST_N_TO_TRY = 2
    # MOST_COMMON_LETTERS = ("e", "t", "a", "i")
    MOST_COMMON_LETTERS = ("e", )
    FREQUENCY_LISTS = [
        [('s', 0.194), ('i', 0.139), ('m', 0.139), ('w', 0.139), ('e',
0.056), ('h', 0.056), ('v', 0.056), ('x', 0.056), ('c', 0.028), ('g',
0.028), ('l', 0.028), ('p', 0.028), ('r', 0.028), ('y', 0.028)],
        [('b', 0.111), ('e', 0.111), ('q', 0.111), ('a', 0.083), ('c',
0.083), ('o', 0.083), ('v', 0.083), ('f', 0.056), ('g', 0.056), ('z',
0.056), ('l', 0.028), ('n', 0.028), ('p', 0.028), ('r', 0.028), ('t',
0.028), ('y', 0.028)],
        [('c', 0.139), ('j', 0.139), ('n', 0.139), ('r', 0.111), ('u',
0.111), ('h', 0.083), ('w', 0.083), ('a', 0.056), ('m', 0.028), ('o',
0.028), ('p', 0.028), ('v', 0.028), ('y', 0.028)],
        [('c', 0.143), ('g', 0.114), ('r', 0.114), ('s', 0.114), ('h',
0.086), ('o', 0.086), ('b', 0.057), ('d', 0.057), ('i', 0.057), ('w',
0.057), ('a', 0.029), ('k', 0.029), ('p', 0.029), ('t', 0.029)],
        [('r', 0.171), ('g', 0.114), ('l', 0.114), ('q', 0.114), ('y',
0.086), ('a', 0.057), ('n', 0.057), ('u', 0.057), ('c', 0.029), ('f',
0.029), ('j', 0.029), ('m', 0.029), ('p', 0.029), ('s', 0.029), ('t',
0.029), ('w', 0.029)]]

    CIPHERTEXT = CIPHERTEXT.upper()

    with open(FILENAME, "w") as file:
        # for indexTuple in product(range(FIRST_N_TO_TRY),
repeat=CIPHER_LENGTH):
            ranges = [range(len(l)) for l in FREQUENCY_LISTS]
            for indexTuple in product(*ranges):
                for indexTuple2 in product(MOST_COMMON_LETTERS,
repeat=CIPHER_LENGTH):
                    decryptedText = CIPHERTEXT
                    for i in range(CIPHER_LENGTH):
                        decryptedText = shiftBack(decryptedText, CIPHER_LENGTH,
indexTuple2[i], FREQUENCY_LISTS[i][indexTuple[i]][0], i)

                    print(decryptedText, file=file)

```

Output

A 652288 line large file (size: 116,759,552 bytes).

The first 10 lines:


```

1 shjfeewcqaxotildyitfipddqrwueewkduteeaeunjjyvzhcljtedtdnecakpydriljpcdyjtifiuouguhjmsuhpqytrjrtbbteyl
2 shjfpewcqlxotidwyteifpddqrwueewkouteelheeuujyvvhcljtedtdnecakpydriljpcdyjtifiuouguhjmduhpqtrjrtbbteyl
3 shjfkewcqxotidyytziifpddqrwueewkjuteegheeutjjyvvhcljtedtdnecakpydriljpcdyjtifiuouguhjmyuhpqtrjrtbbteyl
4 shjffewcqxotidyytziifpddqrwueewkuteeheeujyvvhcljtedtdnecakpydriljpcdyjtifiuouguhjmtuhpqtrjrtbbteyl
5 shjffewcqxotidyytziifpddqrwueewkwuteeheeujyvvhcljmedtdgecakpydriljpcdyjtifiuouguhjmluhpqtrjrtbbteyl
6 shjfvewcqxotidyytziifpddqrwueewkwuteeheeujyvvhcljmedtdgecakpydriljpcdyjtifiuouguhjmuhpqtrjrtbbteyl
7 shjfiwcqxotidyytziifpddqrwueewkwuteeheeujyvvhcljtedtdnecakpydriljpcdyjtifiuouguhjmwuhpqtrjrtbbteyl
8 shjfbewcqxotidyytziifpddqrwueewkwuteeheeujyvvhcljtedtdnecakpydriljpcdyjtifiuouguhjmuuhpqtrjrtbbteyl
9 shjftewcqxotidyytziifpddqrwueewkwuteeheeujyvvhcljtedtdnecakpydriljpcdyjtifiuouguhjmuhpqtrjrtbbteyl
10 shjfgewcqxotidyytziifpddqrwueewkwuteeheeujyvvhcljtedtdnecakpydriljpcdyjtifiuouguhjmeuhpqtrjrtbbteyl

```

The last 10 lines:

```

652279 mknoizgzerrxprxbcccitmkuaadyhatowinebhidbcebfpsyxgmryfetisgvrncmtlsmxrcjcxsdokknvoktzcunavexnnc
652280 mknoybzgzerrxprxbcccitmkuaadyhatowinebhidbcebfpsyxgmryfetisgvrncmtlsmxrcjcxsdokknvoktzcunavexnnc
652281 mknotygzerrxprxbcccitmkuaadyhatowinebhidbcebfpsyxgmryfetisgvrncmtlsmxrcjcxsdokknvoktzcunavexnnc
652282 mknoygzerrxprxbcccitmkuaadyhatowinebhidbcebfpsyxgmryfetisgvrncmtlsmxrcjcxsdokknvoktzcunavexnnc
652283 mknoygzerrxprxbcccitmkuaadyhatowinebhidbcebfpsyxgmryfetisgvrncmtlsmxrcjcxsdokknvoktzcunavexnnc
652284 mknoygzerrxprxbcccitmkuaadyhatowinebhidbcebfpsyxgmryfetisgvrncmtlsmxrcjcxsdokknvoktzcunavexnnc
652285 mknoygzerrxprxbcccitmkuaadyhatowinebhidbcebfpsyxgmryfetisgvrncmtlsmxrcjcxsdokknvoktzcunavexnnc
652286 mknoygzerrxprxbcccitmkuaadyhatowinebhidbcebfpsyxgmryfetisgvrncmtlsmxrcjcxsdokknvoktzcunavexnnc
652287 mknoygzerrxprxbcccitmkuaadyhatowinebhidbcebfpsyxgmryfetisgvrncmtlsmxrcjcxsdokknvoktzcunavexnnc
652288 mknoygzerrxprxbcccitmkuaadyhatowinebhidbcebfpsyxgmryfetisgvrncmtlsmxrcjcxsdokknvoktzcunavexnnc

```

A.6 brute_force.py

Code

For `shift_text_backward.py`, see A.5.

```

from shift_text_backward import shiftBack
from string import ascii_lowercase
from itertools import product

FILENAME = "brute_force_results.txt"
CIPHERTEXT = "GEHDRSTAONLLRGYRVWRGWCNBQEUSRSBUIQIQCCNVBCSA" + \
    "XVWTMVZJHGSARBASZYIRMAPGWWGNAQMGRGSRMSTIEH" + \
    "KFIEENOLHOHPGYPYRCLWBODCSCUSUSEURUMQNCLEQJWJCOJ" + \
    "GGWVCWQYFNRRSCACRIPCRYXNJHPIFCOLHQJHYMAVCRMBW"
CIPHER_LENGTHS = list(range(1, 5))
ALPHABET = ascii_lowercase

CIPHERTEXT = CIPHERTEXT.upper()

with open(FILENAME, "w") as file:
    for cipher_length in CIPHER_LENGTHS:
        for shiftTuple in product(range(len(ALPHABET)),
            repeat=cipher_length):
            decryptedText = CIPHERTEXT
            for i in range(cipher_length):
                decryptedText = shiftBack(decryptedText, cipher_length,
                    ALPHABET[0], ALPHABET[shiftTuple[i]], i)

            print(decryptedText, file=file)

```

Output

A 475254 line large file (size: 85,070,466 bytes).

The first 10 lines:

```

1 gehdrstaonllrgyrvwrwcneousrsbuiqiqccnvbcsvxwtmvzjhgsarbaszyirmagpwwgnaqmrgsrmsstiehkfiolenohpgpyrcw
2 fdgcqrsznmkqfxquvfvbmnpdntqrathphbbmuabrzvuvsluyifrgzqzryxhqlzofvfmzplfqfrqlrshdghdmnkgofoxqbkv
3 ecfbpgqrmlyjpewptupeualzocmsqgqzsgogoaaltzaqyvturktxhfeqpyzqywgpkynueulyokepeupkargcfidgclmjfmfnenwpaju
4 dbaepqxlkiidovostodtzkynblpopyrfnfnzksyzpxustqjswgedpxoyxpwfjxmdtdkxjndodtpojpqfbehcfbklielmdmvozt
5 cadznopqkjhncunrncsyjxmakqonoxgememyjryxowtrspirvfdcowxwovueniwlcssjwmiccosnioeadgbeajkhdkdclunys
6 bzcymnovjigmbtmqrmbrxiwlzjpnmmwpdldlxixqwnvsqrohquecbnmvnmvntdmhvkbrbivlhbmbnrmhnodzcfadziigcjkbbtmxgr
7 aybklmnuhfflaslpqlaqwhvkiomlmvockckwhpvmvurnpqnqgptdbamulvumtscgluajaqahukgalamqgmncybezcyhifbijajslwq
8 zxaawlthgeekzrkopkzpvgujxhnlklunbjbvvgouvltpomfoscazltkutlsrbkftizppzgtjfkzlpkflmbxadybxgheahaizirkvpe
9 ywzvjklsgfdjyqnoyoutfiwgmktmaiaiuufntuksnolenrbzyksjtskrqajeshyoofysieyjkokeklawzcxawfgdzgzhjqjudo
10 xvuyjkrfeccixpimninteshvfljijslzhzhttemstjromnkdqayxjrisrjqzidrgxnnxrhdxixjnidjkzyvbwzvefcyfygxpitcn

```

The last 10 lines:

```
475245 instukpomvshzbwxsqdolrfpetstlvjrsrddxwcdbywgunwjkihcbsctazssnbzhxxqobrwhshcxsnucjfrlgjoopmrpiqqzsmmxcy
475246 imstujpomushzawxspdxokrfpdtstkvjrrrddwcdbywfunwikihbsctazrsnbhxxpobrvshbxsnbujfqlgjnompqiopqzslmxcx
475247 ilstuipomtshzzwxsoxdofrfpctstjvjrqddvwcdabyweunwhkihabscitazsqsnbxxoobruhsaxsnaujflgjmopmpipiqqzskmxcw
475248 ikstuhpomsshzywxsnxdofrfpbtstjvjprdduwcdzbywdunwgkihbzscitazpsnbwhxxnobrthshxsnzujfolgjlompipiqnqzsjmxcv
475249 ijstugpomrshzxwxsmxdohrfpatsthvjrorddwcdbywfunwfkhiybsegtazosnbvxxmobrshshxsnuyfngljjkopmnpimqzsimxcu
475250 iistufpomqshzwxslxdogrfpztstgvjrnrdswcdxywbunwekihbzscitaznsnbuhxxlobrrshxsnxujfmlgjjopmmpipiqzshmxct
475251 ihstuepomshzwxskxdofrfpytstfvjrmrddrwcdbwywunwdkihbzscitazmsnbthxxkobrqshxsnwujflgjiopmpliqqzsgmxcv
475252 igstudpomoshzuwxsjxdoerfpxtstevjrlrddqwcdbvywunwckihvbsctazlsnbshxxjobrphshxsnvujfklgjhompkipiqzsfmxcv
475253 ifstucpomshzwtwxslxdodrfpwtstdvjrkrdpwcdbwywunwbkihbzscitazksnbhxxiobrohshxsnvujfklgjjopmpliqqzsemxcv
475254 iestubpomshzwxshxdocrfpvtstcvjrrddowcdbywunwakihtbsctazjsnbqhxhobrnhshtxsnvujfklgjjopmpliqqzsdmxcv
```

A.7 find_meaningful_strings.py

Code

```
from nostril import nonsense

INPUT_FILENAME = "brute_force_results.txt"
OUTPUT_FILENAME = "meaningful_results.txt"

with open(INPUT_FILENAME) as input_file, open(OUTPUT_FILENAME, "w") as output_file:
    for line in input_file:
        if not nonsense(line):
            output_file.write(line)
```

Output

An empty file when inputting the output file of A.6.

A 41 line file when inputting the output file of A.5. Below are 10 lines chosen from the file.

```
15  cryppogralhyiswnindespeneablepooluoedtolroteytinfkrmateoninyompupingsustemoitisq
    sedererywdeareajdybelliojsofpaoplesorlsideojadaihybasesitiouslyusedpopropectdwt
    aatnestajddatwinmopion
16  crypfograbhyismninduspeneablefoolueedtoroteotinformatuoninoompufingskstemeitisg
    sedeherywtereazdbybulliozsofpqopleiorldiideozadaixybasusitiouslyusedfoprofectdmt
    aatdestazddatminmofion
17  crypxograithyisenindmspenwablexooluwedtotrotegtinfrmatmoningompuxingscstemwitisy
    sedezywlerearabybmlliorsofpiopleaorldaideoradaipybasmsitiwusedxopropectdet
    aatvestarddateinmoxion
18  crypvograrhyiscnindkspenuablevoooluedtorroteetinfrmatkonineompuvingsastemuitisw
    sedexerywjereapdybklloipsofpgopleyorldeopadainybasksitiuusedvopropectdct
    aattestapddatcinmovion
19  cryptographyisanindispensabletoolusedtoprotectinformationincomputingsystemsitisu
    sedeverywhereandbybillionsofpeopleworldwideonadailybasisitisusedtoproctdta
    aatrestanddatainmotion
20  crypmograihyistnindbspenlablemooluledtoirotevtinfhrmatboninvompumingsrstemlitisn
    sedeorywaereagdybblliogsofpxopleorldeogadaieybasbsitilusedmopromectdt
    aatkestagddatinnomion
21  crypjograithyisqnindyspeniablejooluiedtofrotestinformatyoninsompupjingsostemiitisk
    sedelerywxereaddybylliodsofpuopleworldmideodadaibybasysitiiouslyusedjoprojectdqt
    aathestaddatqinmojion
22  crypdograzhyisknindsspencabledoolucedtozrotemtinfrmationinmompudingsistemcitise
    sedeferywreexdybsllioxsofpooplegorldgideoxadaivybasssiticuseddopropectdkt
    aatbestaxddatkinmodion
23  crypcograyhyisjnindrspenbablecoolubedtoyroteltinfxmatroninlompucingshstembitisd
    sedeerywqereawdybrlliwsofnpopleforldfideowadaiuybasrsitibusedcopropectdjt
    aataestawddatjinmocion
24  crypzogravhyisgnindospnyablezooluyedtovroteitinfurmatooniniompuzingsestemyitisa
    sedeberywnereatdyblliotsofpkoplecorldcideotadairybasositiyusedzopropectdgt
    aatxestatddatginmozion
```

A.8 get_frequencies.py

Code

```
from itertools import takewhile, count
from string import ascii_lowercase

CIPHERTEXT = "GEHDRSTAONLLRGYRVWRGWCNBQEUSRSBUIQIQQCNVBCSA" + \
    "XVWTMVZJHGSARBASZYIRMAPGWWGNAQMGRGSRMSTIEH" + \
    "KFIENOLHOHPGYPYRCLWBODCSCUSUSEURUMQNCLEQJWJCOJ" + \
    "GGWVCWQYFNRRSCACRIPCXYXNJHPIFCOLHQJHYMAVCRMBW"
CIPHER_LENGTH = 5

CIPHERTEXT = CIPHERTEXT.lower()

substrings = ''.join(
    CIPHERTEXT[start_i + j * CIPHER_LENGTH] for j in takewhile(lambda j:
start_i + j * CIPHER_LENGTH < len(CIPHERTEXT), count())
    ) for start_i in range(CIPHER_LENGTH)]

frequencyLists = [
    sorted(((l, round(substring.count(l) / len(substring), 3)) for l
in ascii_lowercase if substring.count(l) > 0), key=(lambda t: t[1]),
reverse=True)
    for substring in substrings]

for list1 in frequencyLists:
    print(list1)
```

Output

```
[('s', 0.194), ('i', 0.139), ('m', 0.139), ('w', 0.139), ('e', 0.056),
('h', 0.056), ('v', 0.056), ('x', 0.056), ('c', 0.028), ('g', 0.028), ('l',
0.028), ('p', 0.028), ('r', 0.028), ('y', 0.028)]

[('b', 0.111), ('e', 0.111), ('q', 0.111), ('a', 0.083), ('c', 0.083),
('o', 0.083), ('v', 0.083), ('f', 0.056), ('g', 0.056), ('z', 0.056),
('l', 0.028), ('n', 0.028), ('p', 0.028), ('r', 0.028), ('t', 0.028), ('y',
0.028)]

[('c', 0.139), ('j', 0.139), ('n', 0.139), ('r', 0.111), ('u', 0.111),
('h', 0.083), ('w', 0.083), ('a', 0.056), ('m', 0.028), ('o', 0.028), ('p',
0.028), ('v', 0.028), ('y', 0.028)]

[('c', 0.143), ('g', 0.114), ('r', 0.114), ('s', 0.114), ('h', 0.086),
('o', 0.086), ('b', 0.057), ('d', 0.057), ('i', 0.057), ('w', 0.057), ('a',
0.029), ('k', 0.029), ('p', 0.029), ('t', 0.029)]

[('r', 0.171), ('g', 0.114), ('l', 0.114), ('q', 0.114), ('y', 0.086),
('a', 0.057), ('n', 0.057), ('u', 0.057), ('c', 0.029), ('f', 0.029),
('j', 0.029), ('m', 0.029), ('p', 0.029), ('s', 0.029), ('t', 0.029), ('w',
0.029)]
```

A.9 compute_cipher.py

Code

```
CIPHERTEXT = "GEHDRSTAONLLRGYRVWRGWCNBQEUSRSBUIQIQQCNVBCSA" + \
```

```

        "XVWTMVZJHGSARBASZYIRMAPGWWGNAQMGRGSRMSTIEH" + \
        "KFIENOLHOHPGYPYRCLWBODCSCUSUSEURUMQNCLEQJWJCOJ" + \
        "GGWVCWQYFNRRSCACRIPCXYXNJHPIFCOLHQBHYMAVCRMBW"
PLAINTEXT = "cryptographyisanindispensabletoolusedtoprotec" + \
            "tinformationincomputingsystemsitisusedevery" + \
            "whereandbybillionsofpeopleworldwideonadailyba" + \
            "sisitisusedtoprotectdataatrestanddatainmotion"
CIPHER_LENGTH = 5

CIPHERTEXT = CIPHERTEXT.lower()
PLAINTEXT = PLAINTEXT.lower()

cipher = ""
for i in range(CIPHER_LENGTH):
    oc = ord(CIPHERTEXT[i])
    op = ord(PLAINTEXT[i])
    cipher += chr(ord("a") + (oc - op) % 26)

print(cipher)

```

Output

```
enjoy
```