Minimum Spanning Trees

1/3 points earned (33%)

You haven't passed yet. You need at least 66% to pass.
Review the material and try again! You have 3 attempts every 8 hours.

Review Related Lesson

0 / 1
points

(seed = 448267)

Consider the following edge-weighted graph with 10 vertices and 17 edges:

```
v-w  weight
-----------
F-A    16
G-A     5
A-B     1
B-C    12
B-G     6
D-C    15
C-H    13
G-C     7
D-J    11
D-E     8
H-D     3
D-I     2
J-E    10
F-G    17
G-H     9
H-I     4
J-I    14
```

```
Here is a graphical representation of the same graph:

   (A)------1------(B)------12-----(C)------15-----(D)------8------(E)
    |\              |              /|              /|\              |
    | \             |             / |             / | \             |
    |  \            |            /  |            /  |  \            |
    |   \           |           /   |           /   |   \           |
    |    \          |          /    |          /    |    \          |
    |     \         |         /     |         /     |     \         |
    |      \        |        /      |        /      |      \        |
    16      5       6       7       13      3       2       11      10
    |        \      |      /        |      /        |        \      |
    |         \     |     /         |     /         |         \     |
    |          \    |    /          |    /          |          \    |
    |           \   |   /           |   /           |           \   |
    |            \  |  /            |  /            |            \  |
    |             \ | /             | /             |             \ |
    |              \|/              |/              |              \|
   (F)------17-----(G)------9------(H)------4------(I)------14-----(J)
```

Give the sequence of edges in the MST in the order that Kruskal's algorithm discovers them.

1. To specify an edge, use its weight.

1 2 3 5 7 8 9 16

Incorrect Response

The correct answer is: 1 2 3 5 7 8 9 10 16

Here are the edges in the order considered by Kruskal's algorithm:

```
v-w  weight  action
--------------------------------------------------------------------
A-B     1    add to MST
D-I     2    add to MST
H-D     3    add to MST
H-I     4    discard (adding to MST would create cycle I-H-D-I)
G-A     5    add to MST
B-G     6    discard (adding to MST would create cycle G-B-A-G)
G-C     7    add to MST
D-E     8    add to MST
G-H     9    add to MST
J-E    10    add to MST
D-J    11    discard (adding to MST would create cycle J-D-E-J)
B-C    12    discard (adding to MST would create cycle C-B-A-G-C)
C-H    13    discard (adding to MST would create cycle H-C-G-H)
J-I    14    discard (adding to MST would create cycle I-J-E-D-I)
D-C    15    discard (adding to MST would create cycle C-D-H-G-C)
F-A    16    add to MST
```

```
Here is a graphical representation of the MST:

    (A)------1------(B)             (C)             (D)------8------(E)
     |\                             /               /|               |
     | \                           /               / |               |
     |  \                         /               /  |               |
     |   \                       /               /   |               |
     |    \                     /               /    |               |
     |     \                   /               /     |               |
     |      \                 /               /      |               |
     16      5               7               3       2               10
     |        \             /               /        |               |
     |         \           /               /         |               |
     |          \         /               /          |               |
     |           \       /               /           |               |
     |            \     /               /            |               |
     |             \   /               /             |               |
     |              \ /               /              |               |
    (F)             (G)------9------(H)             (I)             (J)
```

1 / 1
points

2.
(seed = 608030)

Consider the following edge-weighted graph with 10 vertices and 17 edges.

```
v-w   weight
-----------
B-A      10
F-A       6
A-G       2
B-G       9
H-B       5
C-B       3
C-D      14
C-H       8
I-C       4
D-E      11
D-I       7
E-J      17
E-I       1
G-F      16
G-H      15
I-H      12
I-J      13
```

```
Here is a graphical representation of the same graph:

    (A)------10-----(B)------3------(C)------14-----(D)------11-----(E)
     |\              |\              |\              |              /|
     | \             | \             | \             |             / |
     |  \            |  \            |  \            |            /  |
     |   \           |   \           |   \           |           /   |
     |    \          |    \          |    \          |          /    |
     |     \         |     \         |     \         |         /     |
     |      \        |      \        |      \        |        /      |
     6       2       9       5       8       4       7       1       17
     |        \      |        \      |        \      |      /        |
     |         \     |         \     |         \     |     /         |
     |          \    |          \    |          \    |    /          |
     |           \   |           \   |           \   |   /           |
     |            \  |            \  |            \  |  /            |
     |             \ |             \ |             \ | /             |
     |              \|              \|              \|/              |
    (F)------16-----(G)------15-----(H)------12-----(I)------13-----(J)
```

Give the sequence of edges in the MST in the order that Prim's algorithm adds them to the MST, when starting Prim's algorithm from vertex H. To specify an edge, use its weight.

5 3 4 1 7 9 2 6 13

Correct Response
The correct answer is: 5 3 4 1 7 9 2 6 13

Here are the vertices and edges added to the tree, in the order considered by Prim's algorithm:

```
tree vertices        v-w  weight
--------------------------------
H                    H-B       5
H B                  C-B       3
H B C                I-C       4
H B C I              E-I       1
H B C I E            D-I       7
H B C I E D          B-G       9
H B C I E D G        A-G       2
H B C I E D G A      F-A       6
H B C I E D G A F    I-J      13
H B C I E D G A F J

```

```
Here is a graphical representation of the MST:

    (A)             (B)------3------(C)             (D)             (E)
     |\              |\               \              |              /
     | \             | \               \             |             /
     |  \            |  \               \            |            /
     |   \           |   \               \           |           /
     |    \          |    \               \          |          /
     |     \         |     \               \         |         /
     |      \        |      \               \        |        /
     6       2       9       5               4       7       1
     |        \      |        \               \      |      /
     |         \     |         \               \     |     /
     |          \    |          \               \    |    /
     |           \   |           \               \   |   /
     |            \  |            \               \  |  /
     |             \ |             \               \ | /
     |              \|              \               \|/
    (F)             (G)             (H)             (I)------13-----(J)
```

Give the sequence of edges in the MST in the order that Prim's algorithm adds them to the MST, when starting Prim's algorithm from vertex H. To specify an edge, use its weight.

---

0 / 1
points

3.
(seed = 699940)

Which of the following statements about minimum spanning trees (MSTs) are guaranteed to be true in any edge-weighted graph G? Assume that G is connected and has no parallel edge or self-loops. Do not assume the edge weights are distinct unless this is specifically stated. Check all that apply.

If edge e is a heaviest edge in some cycle C, then there exists a MST which does not contain e.

Correct

Let T be any MST of G. If edge e is in T, then we are done. Otherwise, assume that edge e is not in T. Consider the cut (A, B) defined by deleting edge e from T. There is (at least) one other edge f in C that crosses the cut. By assumption w(f) ≤ w(e). Thus, replacing edge e with edge f in T yields a MST which contains edge e.

- [x] Let (A, B) be a cut in G with at least two crossing edges, and whose heaviest crossing edge e is unique. Then, edge e is not in any MST.

This should not be selected

Consider an edge-weighted graph with these edges and capacities: v-w (1), v-x (2). The heaviest edge in the cut A = { v }, B = { w, x } is in the MST.

- [ ] Let e be a lightest edge that crosses some cut (A, B). Then, edge e is in some MST.

This should be selected

- [ ] Let T be a MST of G. Suppose that we increase the weight of some edge e in T from w(e) to w'(e). Then, the weight of the MST in the modified edge-weighted graph G' will either remain the same or increase by exactly w'(e) - w(e) units.

Un-selected is correct

- [x] Let T and T' be two different spanning trees in an undirected graph G. Let e' be any edge in T' that is not in T. Then, there exists an edge e that is in T but not T' such that replacing edge e with edge e' in T is also a spanning tree of G.

Correct

Adding e' to T results in a unique cycle C. There must be some edge e in this cycle C that is not in T' (since otherwise T' would be cyclic). Thus, replacing edge e with edge e' in T yields a spanning tree of G. This is known as the exchange property: it implies that we can "walk" between any two spanning trees via a sequence of spanning trees, by exchanging edges one at a time.