Minimum Spanning Trees

2/3 points earned (66%)

Quiz passed!

1 / 1
points
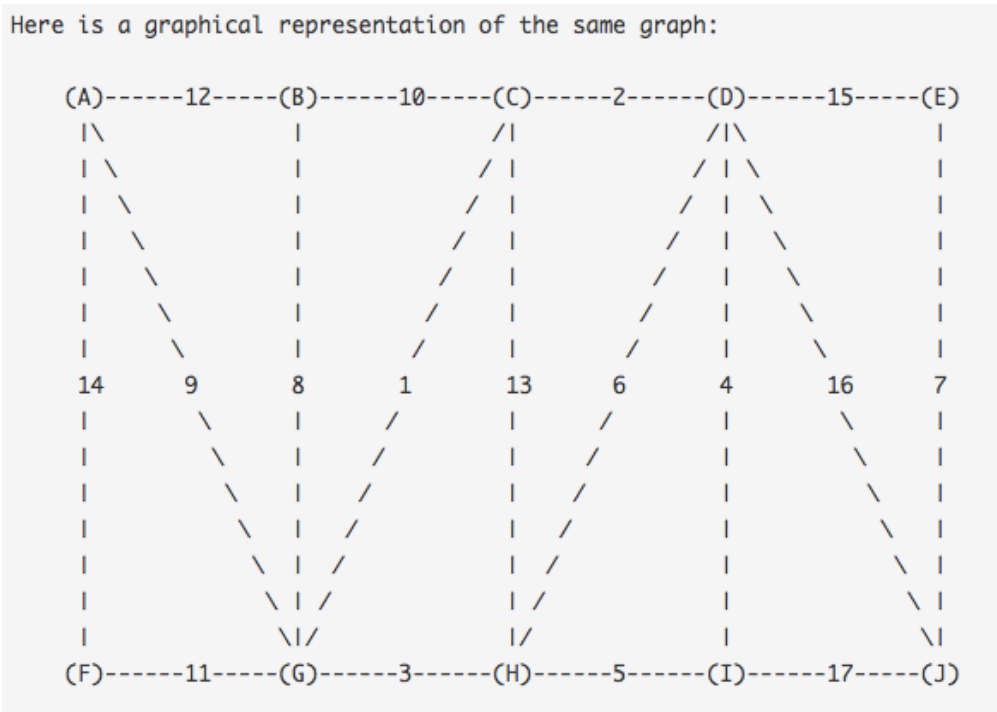
Consider the following edge-weighted graph with 10 vertices and 17 edges:

```
1        v-w  weight
2        -----------
3        F-A    14
4        A-B    12
5        G-A     9
6        B-C    10
7        G-B     8
8        C-H    13
9        C-D     2
10       G-C     1
11       J-D    16
12       D-E    15
13       D-H     6
14       D-I     4
15       E-J     7
16       F-G    11
17       H-G     3
18       I-H     5
19       J-I    17
20
```

Here is a graphical representation of the same graph:

```
Here is a graphical representation of the same graph:

   (A)------12-----(B)------10-----(C)------2------(D)------15-----(E)
    |\               |              /|              /|\              |
    | \              |             / |             / | \             |
    |  \             |            /  |            /  |  \            |
    |   \            |           /   |           /   |   \           |
    |    \           |          /    |          /    |    \          |
    |     \          |         /     |         /     |     \         |
    |      \         |        /      |        /      |      \        |
   14       9        8       1      13       6       4      16       7
    |        \       |      /        |      /        |       \       |
    |         \      |     /         |     /         |        \      |
    |          \     |    /          |    /          |         \     |
    |           \    |   /           |   /           |          \    |
    |            \   |  /            |  /            |           \   |
    |             \  | /             | /             |            \  |
    |              \ |/              |/              |             \ |
   (F)------11-----(G)------3------(H)------5------(I)------17-----(J)
```

Give the sequence of edges in the MST in the order that Kruskal's algorithm discovers them.

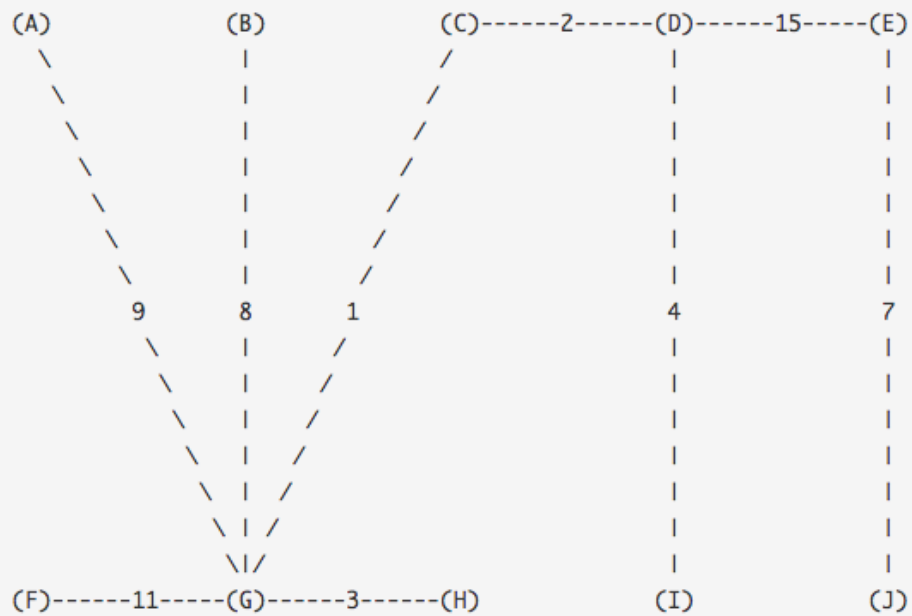1. To specify an edge, use its weight.

```
1 2 3 4 7 8 9 11 15
```

The correct answer is: 1 2 3 4 7 8 9 11 15

Here are the edges in the order considered by Kruskal's algorithm:

```
v-w   weight  action
-------------------------------------------------------------------
G-C    1      add to MST
C-D    2      add to MST
H-G    3      add to MST
D-I    4      add to MST
I-H    5      discard (adding to MST would create cycle H-I-D-C-G-H)
D-H    6      discard (adding to MST would create cycle H-D-C-G-H)
E-J    7      add to MST
G-B    8      add to MST
G-A    9      add to MST
B-C   10      discard (adding to MST would create cycle C-B-G-C)
F-G   11      add to MST
A-B   12      discard (adding to MST would create cycle B-A-G-B)
C-H   13      discard (adding to MST would create cycle H-C-G-H)
F-A   14      discard (adding to MST would create cycle A-F-G-A)
D-E   15      add to MST
```
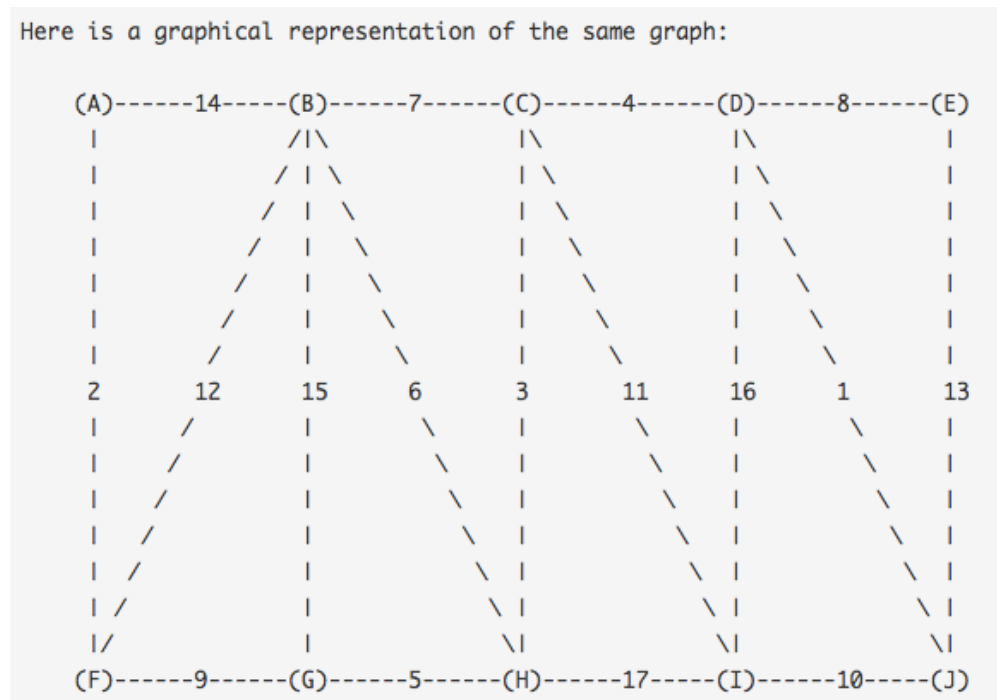
Here is a graphical representation of the MST:

```
    (A)                (B)            (C)------2------(D)------15-----(E)
      \                 |              /              |               |
       \                |             /               |               |
        \               |            /                |               |
         \              |           /                 |               |
          \             |          /                  |               |
           \            |         /                   |               |
            \           |        /                    |               |
             9          8       1                     4               7
              \         |      /                      |               |
               \        |     /                       |               |
                \       |    /                        |               |
                 \      |   /                         |               |
                  \     |  /                          |               |
                   \    | /                           |               |
                    \   |/                            |               |
                     \|/                              |               |
    (F)------11-----(G)------3------(H)              (I)             (J)
```

1 / 1
points

2.
(seed = 536089)

Consider the following edge-weighted graph with 10 vertices and 17 edges.

```
1        v-w  weight
2        -----------
3        A-B     14
4        A-F      2
5        B-G     15
6        F-B     12
7        C-B      7
8        H-B      6
9        C-I     11
10       D-C      4
11       H-C      3
12       D-I     16
13       D-E      8
14       D-J      1
15       E-J     13
16       F-G      9
17       H-G      5
18       H-I     17
19       I-J     10
```

Here is a graphical representation of the same graph:

```
(A)------14-----(B)------7------(C)------4------(D)------8------(E)
 |              /I\              I\              I\              |
 |             / I \             I \             I \            |
 |            /  I  \            I  \            I  \           |
 |           /   I   \           I   \           I   \          |
 |          /    I    \          I    \          I    \         |
 |         /     I     \         I     \         I     \        |
 |        /      I      \        I      \        I      \       |
 2       12     15       6       3       11      16       1      13
 |      /        I        \      I        \      I        \      |
 |     /         I         \     I         \     I         \     |
 |    /          I          \    I          \    I          \    |
 |   /           I           \   I           \   I           \   |
 |  /            I            \  I            \  I            \  |
 | /             I             \ I             \ I             \ |
 |/              I              \I              \I              \I
(F)------9------(G)------5------(H)------17-----(I)------10-----(J)
```

Give the sequence of edges in the MST in the order that Prim's algorithm adds them to the MST, when starting Prim's algorithm from vertex B. To specify an edge, use its weight.
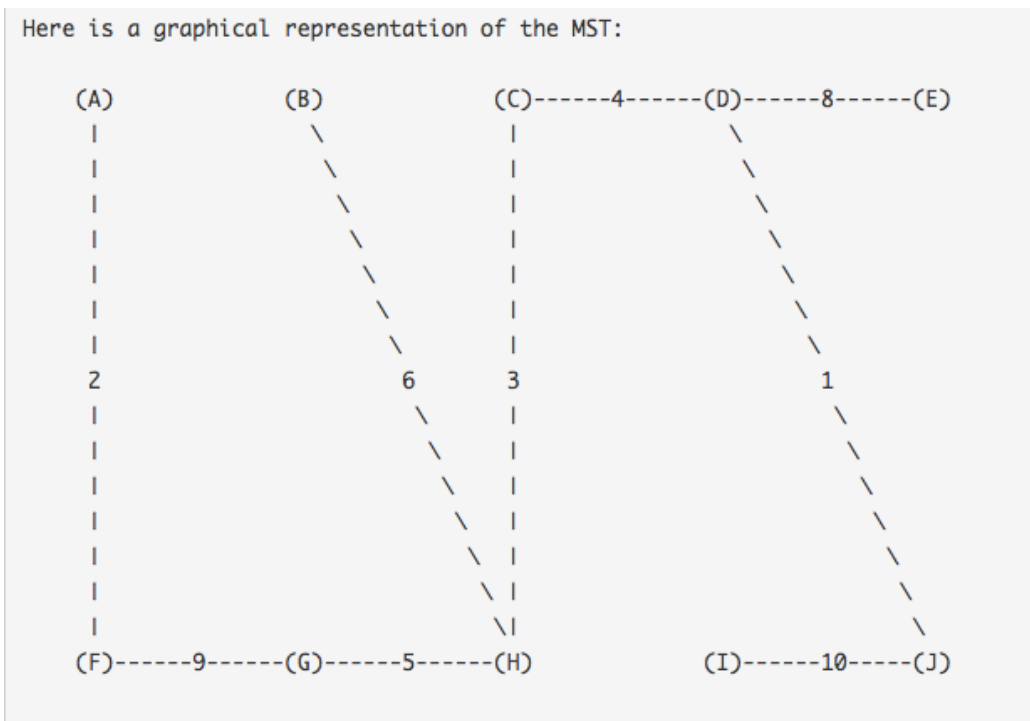
```
6 3 4 1 5 8 9 2 10
```

Correct Response

The correct answer is: 6 3 4 1 5 8 9 2 10

Here are the vertices and edges added to the tree, in the order considered by Prim's algorithm:

```
 1   tree vertices          v-w  weight
 2   -------------------------------
 3   B                      H-B     6
 4   B H                    H-C     3
 5   B H C                  D-C     4
 6   B H C D                D-J     1
 7   B H C D J              H-G     5
 8   B H C D J G            D-E     8
 9   B H C D J G E          F-G     9
10   B H C D J G E F        A-F     2
11   B H C D J G E F A      I-J    10
12   B H C D J G E F A I
```

Here is a graphical representation of the MST:

```
   (A)              (B)            (C)------4------(D)------8------(E)
    |                \              |               \
    |                 \             |                \
    |                  \            |                 \
    |                   \           |                  \
    |                    \          |                   \
    |                     \         |                    \
    2                      6        3                     1
    |                       \       |                      \
    |                        \      |                       \
    |                         \     |                        \
    |                          \    |                         \
    |                           \   |                          \
    |                            \  |                           \
    |                             \ |                            \
   (F)------9------(G)------5------(H)            (I)------10-----(J)
```

✖ 0 / 1
   points

3.
(seed = 603748)

Which of the following statements about minimum spanning trees (MSTs) are guaranteed to be true in any edge-weighted graph G? Assume that G is connected and has no parallel edge or self-loops. Do not assume the edge weights are distinct unless this is specifically stated. Check all that apply.

☐   Let T be a MST of G. Suppose that we decrease the weight of some edge e in T from w(e) to w'(e). Then, the weight of the MST in the modified edge-weighted graph G' will decrease by exactly w(e) - w'(e) units.

This should be selected