Register Allocation

Lecture 16

The Memory Hierarchy

Register	0.3 ns	1-16 KB
Cache L1	0.5 ns	8-64 KB
Cache L2	7 ns	256-512 KB
Cache L3	12 ns	1-8 MB
Main memory	100 ns	1-32 GB
HD seek	10M ns (10ms)	up to 10 TB
SSD seek	0.1M ns (0.1ms)	up to 1 TB
Network	150M ns (150 ms)	

Managing the Memory Hierarchy

- Most programs are written as if there are only two kinds of memory: main memory and disk
  - Programmer is responsible for moving data from disk to memory (e.g., file I/O)
  - Hardware is responsible for moving data between memory and caches
  - Compiler is responsible for moving data between memory and registers

Current Trends

- Power usage limits
  - Size and speed of registers/caches
  - Speed of processors
- But
  - The cost of a cache miss is very high
  - Typically requires 2-3 caches to bridge fast processor with large main memory
- It is very important to:
  - Manage registers properly
  - Manage caches properly
- Compilers are good at managing registers

The Register Allocation Problem

- Intermediate code uses unlimited temporaries
  - Simplifies code generation and optimization
  - Complicates final translation to assembly
- Typical intermediate code uses too many temporaries

The Register Allocation Problem (Cont.)

- The problem:

  Rewrite the intermediate code to use no more temporaries than there are machine registers

- Method:
  - Assign multiple temporaries to each register
  - But without changing the program behavior

History

- Register allocation is as old as compilers
  - Register allocation was used in the original FORTRAN compiler in the '50s
  - Very crude algorithms
- A breakthrough came in 1980
  - Register allocation scheme based on graph coloring
  - Relatively simple, global and works well in practice

An Example

- Consider the program

```
a := c + d
e := a + b
f := e - 1
```

- Assume a and e dead after use
  - Temporary a can be "reused" after e := a + b
  - So can temporary e
- Can allocate a, e, and f all to one register (r_1):

$$r_1 := r_2 + r_3$$
$$r_1 := r_1 + r_4$$
$$r_1 := r_1 - 1$$

- A dead temporary is not needed
  - A dead temporary can be reused

The Idea

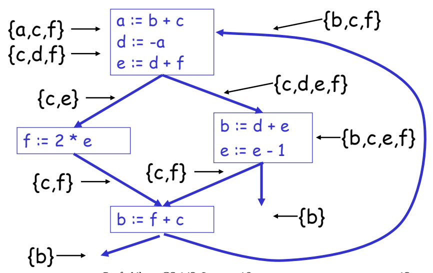
Temporaries t_1 and t_2 can share the same register if <u>at any point in the program at most one</u> of t_1 or t_2 is live .

Or

If t_1 and t_2 are live at the same time, they cannot share a register

Algorithm: Part I

- Compute live variables for each point:

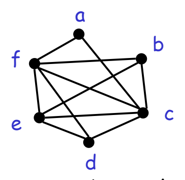


The Register Interference Graph

- Construct an undirected graph
  - A node for each temporary
  - An edge between t_1 and t_2 if they are live simultaneously at some point in the program
- This is the *register interference graph* (RIG)
  - Two temporaries can be allocated to the same register if there is no edge connecting them

Example

- For our example:



- E.g., b and c cannot be in the same register
- E.g., b and d could be in the same register

Notes on Register Interference Graphs

- Extracts exactly the information needed to characterize legal register assignments
- Gives a global (i.e., over the entire flow graph) picture of the register requirements
- After RIG construction the register allocation algorithm is architecture independent

Definitions

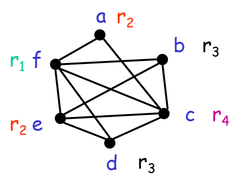
- A <u>coloring of a graph</u> is an assignment of colors to nodes, such that nodes connected by an edge have different colors
- A graph is <u>k-colorable</u> if it has a coloring with k colors

Register Allocation Through Graph Coloring

- In our problem, colors = registers
  - We need to assign colors (registers) to graph nodes (temporaries)
- Let k = number of machine registers
- If the RIG is k-colorable then there is a register assignment that uses no more than k registers

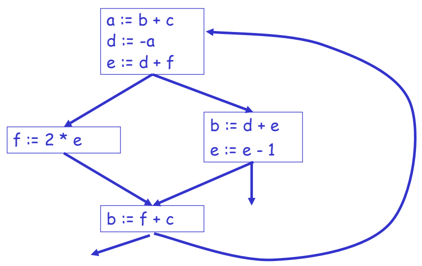
Graph Coloring Example

- Consider the example RIG



- There is no coloring with less than 4 colors
- There are 4-colorings of this graph

Example Review



```
a := b + c
d := -a
e := d + f
```

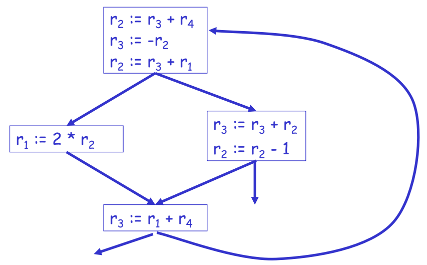
```
f := 2 * e
```

```
b := d + e
e := e - 1
```

```
b := f + c
```

Example After Register Allocation

- Under this coloring the code becomes:



$r_2 := r_3 + r_4$
$r_3 := -r_2$
$r_2 := r_3 + r_1$

$r_1 := 2 * r_2$

$r_3 := r_3 + r_2$
$r_2 := r_2 - 1$

$r_3 := r_1 + r_4$

Computing Graph Colorings

- How do we compute graph colorings?
- It isn't easy:
  1. This problem is very hard (NP-hard). No efficient algorithms are known.
     - *Solution: use heuristics*
  2. A coloring might not exist for a given number of registers
     - *Solution: later*

Graph Coloring Heuristic

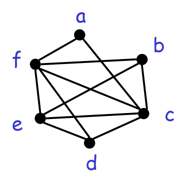
- Observation:
  - Pick a node t with fewer than k neighbors in RIG
  - Eliminate t and its edges from RIG
  - If resulting graph is k-colorable, then so is the original graph
- Why?
  - Let $c_1,\ldots,c_n$ be the colors assigned to the neighbors of t in the reduced graph
  - Since n < k we can pick some color for t that is different from those of its neighbors

Graph Coloring Heuristic

- The following works well in practice:
  - Pick a node t with fewer than k neighbors
  - Put t on a stack and remove it from the RIG
  - Repeat until the graph has one node
- Assign colors to nodes on the stack
  - Start with the last node added
  - At each step pick a color different from those assigned to already colored neighbors

Graph Coloring Example (1)

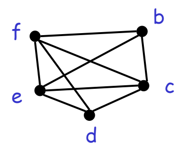
- Start with the RIG and with k = 4:



Stack: {}

- Remove a

Graph Coloring Example (2)

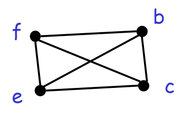


Stack: {a}

- Remove d

Graph Coloring Example (3)

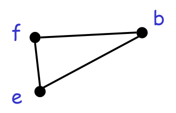
- Note: all nodes now have fewer than 4 neighbors



Stack: {d, a}

- Remove c

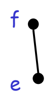
Graph Coloring Example (4)



Stack: {c, d, a}

- Remove b

Graph Coloring Example (5)



Stack: {b, c, d, a}

- Remove e

Graph Coloring Example (6)

Stack: {e, b, c, d, a}

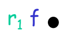
- Remove f

Graph Coloring Example (7)

- Now start assigning colors to nodes, starting with the top of the stack

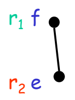
Stack: {f, e, b, c, d, a}

Graph Coloring Example (8)



Stack: {e, b, c, d, a}

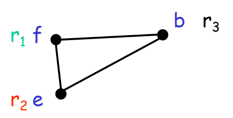
Graph Coloring Example (9)



Stack: {b, c, d, a}

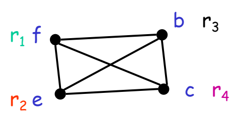
- e must be in a different register from f

Graph Coloring Example (10)



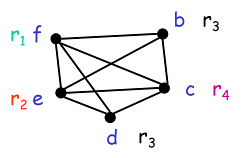
Stack: {c, d, a}

Graph Coloring Example (11)



Stack: {d, a}

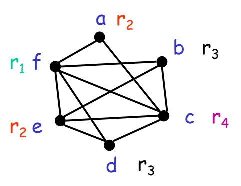
Graph Coloring Example (12)



Stack: {a}

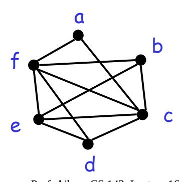
- d can be in the same register as b

Graph Coloring Example (13)



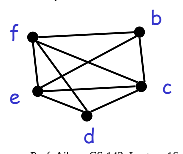
What if the Heuristic Fails?

- What if all nodes have k or more neighbors ?
- Example: Try to find a 3-coloring of the RIG:



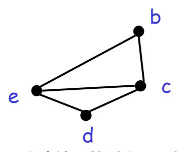
What if the Heuristic Fails?

- Remove a and get stuck (as shown below)
- Pick a node as a candidate for *spilling*
  - A spilled temporary "lives" in memory
  - Assume that f is picked as a candidate



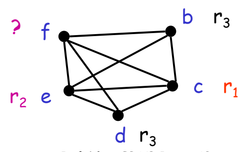
What if the Heuristic Fails?

- Remove f and continue the simplification
  - Simplification now succeeds: b, d, e, c



What if the Heuristic Fails?

- Eventually we must assign a color to f
- We hope that among the 4 neighbors of f we use less than 3 colors ⇒ optimistic coloring

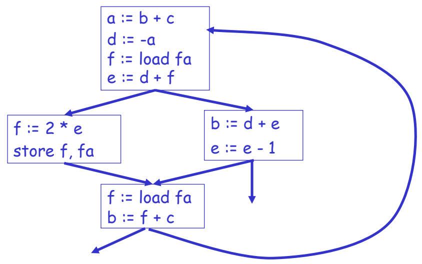


Spilling

- If optimistic coloring fails, we spill f
  - Allocate a memory location for f
    - Typically in the current stack frame
    - Call this address fa
- Before each operation that reads f, insert
  f := load fa
- After each operation that writes f, insert
  store f, fa

Spilling Example

- This is the new code after spilling f



```
a := b + c
d := -a
f := load fa
e := d + f
```

```
f := 2 * e
store f, fa
```

```
b := d + e
e := e - 1
```

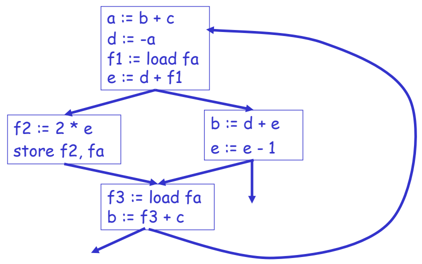
```
f := load fa
b := f + c
```

A Problem

- This code reuses the register name f
- Correct, but suboptimal
  - Should use distinct register names whenever possible
  - Allows different uses to have different colors

Spilling Example

- This is the new code after spilling f



```
a := b + c
d := -a
f1 := load fa
e := d + f1
```

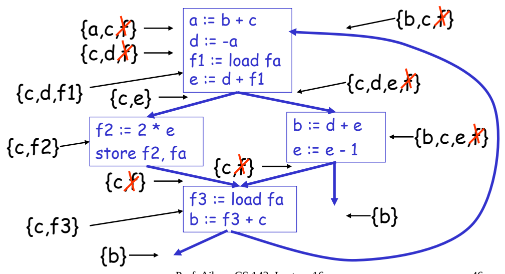
```
f2 := 2 * e
store f2, fa
```

```
b := d + e
e := e - 1
```

```
f3 := load fa
b := f3 + c
```

Recomputing Liveness Information

- The new liveness information after spilling:



```
a := b + c
d := -a
f1 := load fa
e := d + f1
```

```
f2 := 2 * e
store f2, fa
```

```
b := d + e
e := e - 1
```

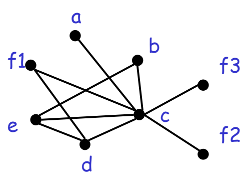
```
f3 := load fa
b := f3 + c
```

Recomputing Liveness Information

- New liveness information is almost as before
  - Note f has been split into three temporaries
- fi is live only
  - Between a fi := load fa and the next instruction
  - Between a store fi, fa and the preceding instr.
- Spilling reduces the live range of f
  - And thus reduces its interferences
  - Which results in fewer RIG neighbors

Recompute RIG After Spilling

- Some edges of the spilled node are removed
- In our case f still interferes only with c and d
- And the resulting RIG is 3-colorable



Spilling Notes

- Additional spills might be required before a coloring is found
- The tricky part is deciding what to spill
  - But any choice is correct
- Possible heuristics:
  - Spill temporaries with most conflicts
  - Spill temporaries with few definitions and uses
  - Avoid spilling in inner loops

Caches

- Compilers are very good at managing registers
  - Much better than a programmer could be
- Compilers are not good at managing caches
  - This problem is still left to programmers
  - It is still an open question how much a compiler can do to improve cache performance
- Compilers can, and a few do, perform some cache optimizations

Cache Optimization

- Consider the loop

```
for(j := 1; j < 10; j++)
    for(i=1; i<1000000; i++)
        a[i] *= b[i]
```

- This program has terrible cache performance
  - Why?

Cache Optimization (Cont.)

- Consider the program:

```
for(i=1; i<1000000; i++)
  for(j := 1; j < 10; j++)
        a[i] *= b[i]
```

  - Computes the same thing
  - But with much better cache behavior
  - Might actually be more than 10x faster

- A compiler can perform this optimization
  - called loop interchange

Conclusions

- Register allocation is a "must have" in compilers:
  - Because intermediate code uses too many temporaries
  - Because it makes a big difference in performance

- Register allocation is more complicated for CISC machines