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Interview Questions: Linear Programming

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1 / 1
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1.
Feasibility detection. Suppose that you want to solve a linear program but you don't have a starting initial basic feasible solution—perhaps the all 0 vector is not feasible. Design a related linear program whose solution will be a basic feasible solution to the original linear program (assuming the original linear program has a basic feasible solution).

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Thank you for your response.
Hint: add new artificial variables to each row and let the artificial variables be the initial basis. Select the objective function to drive the artificial variables out of the basis. This is known as *Phase I* of the simplex algorithm.

1 / 1
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2.
Detecting unboundedness. Describe how to modify the simplex algorithm to detect an unbounded linear program—a linear program in which there is a feasible solution that makes the objective function arbitrarily large.

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Thank you for your response.
Hint identify a vector $d \neq 0$ such that if x is a feasible solution, then so is $x + \alpha\, d$ for any $\alpha > 0$. To identify such a vector d, consider what happens when the simplex algorithm has an entering column but no leaving row (because all entries in that column are nonpositive).

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1 / 1
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3.
Birkhoff-von Neumann theorem. Consider the polyhedron P defined by $\sum_i x_{ij} = 1$, $\sum_j x_{ij} = 1$, and $x_{ij} \geq 0$. Prove that all extreme points of P have integer coordinates (0 or 1).

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Thank you for your response.
Hint: let $x \in P$ be a vector that has some fractional entries. Prove that x is not an extreme point by expressing it as the convex combination of two vectors $x + \epsilon y$ and $x - \epsilon y$. To identify y, find a cycle in the bipartite graph

that has a vertex for each i and j and an edge (i, j) if and only if $0 < x_{ij} < 1$.