# Supplementary Files

## S-1. Supporting Materials

### A. Definitions and Concepts

Generally, a multimodal multi-objective optimization problem (MMOP) can be formulated as

$$\begin{array}{ll} \text{Minimize} & \mathbf{F}(\mathbf{x}) = (f_1(\mathbf{x}), \cdots, f_m(\mathbf{x}))^T \\ \text{subject to} & \mathbf{x} \in \mathbb{S} \end{array}, \tag{1}$$

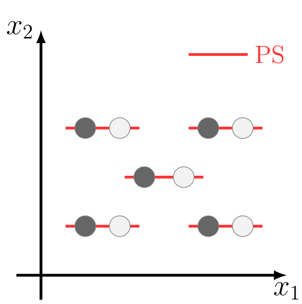
where $m$ denotes the number of objective functions; $\mathbf{x} = (x_1, \ldots, x_n)^T$ represents an $n$-dimensional decision vector; $n$ denotes the number of decision variables; $\mathbf{x} \in \mathbb{S}$ and $\mathbb{S} \subseteq \mathbb{R}^n$ represents the search space.

Some definitions and concepts in the multi-objective research field are as follows.

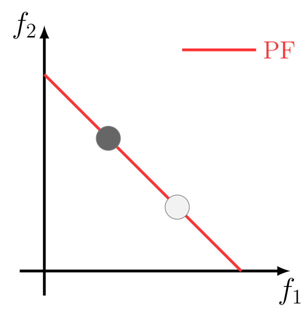
- *Pareto dominance*: for two solutions $\mathbf{x}^1$, $\mathbf{x}^2 \in \mathbb{S}$. $\mathbf{x}^1$ is said to Pareto dominate $\mathbf{x}^2$ (denoted as $\mathbf{x}^1 \prec \mathbf{x}^2$), if and only if $f_i(\mathbf{x}^1) \leq f_i(\mathbf{x}^2)$ for $\forall\ i \in \{1, \cdots, m\}$ and $f_j(\mathbf{x}^1) < f_j(\mathbf{x}^2)$ for $\exists\ j \in \{1, \cdots, m\}$;
- *Pareto set* (*PS*): which is the set $PS \subset \mathbb{S}$ that for $\forall\ \mathbf{x}^\diamond \in PS$, $\nexists\ \mathbf{x} \in \mathbb{S}$ such that $\mathbf{x} \prec \mathbf{x}^\diamond$;
- *Pareto front* (*PF*): $PF = \{\mathbf{F}(\mathbf{x}) | \mathbf{x} \in PS\}$;

### B. Illustration of the Feature of MMOPs

Fig. presents an artificial example to clearly illustrate the feature of MMOPs. The PS consists of five segments, where ten points are distributed. Five of them (marked in gray in Fig. 1(a)) correspond to the gray point on the PF in Fig. 1(b), while other five (marked in white in Fig. 1(a)) correspond to the white point on the PF in Fig. 1(b).




(a)




(b)

Fig. S-1. Illustration of the feature of MMOPs. The gray and white points on the PS in the decision space shown in 1(a) correspond to the gray and white point on the PF in the objective space shown in 1(b), respectively.

## S-2. Experimental Settings and Parameters

### A. Parameters and Features of MMF

The detailed introduction of MMF [1] test suite is given in Table S-I, including parameters and main features of all MMF instances.

TABLE S-I
Parameters and main features of MMF test suite.

| problem | $m$ | $n$ | number of PS segments | overlap in every dimension |
|---|---|---|---|---|
| MMF1 | 2 | 2 | 2 | × |
| MMF2 | 2 | 2 | 2 | × |
| MMF3 | 2 | 2 | 2 | ✓ |
| MMF4 | 2 | 2 | 4 | × |
| MMF5 | 2 | 2 | 4 | × |
| MMF6 | 2 | 2 | 4 | ✓ |
| MMF7 | 2 | 2 | 2 | × |
| MMF8 | 2 | 2 | 4 | × |

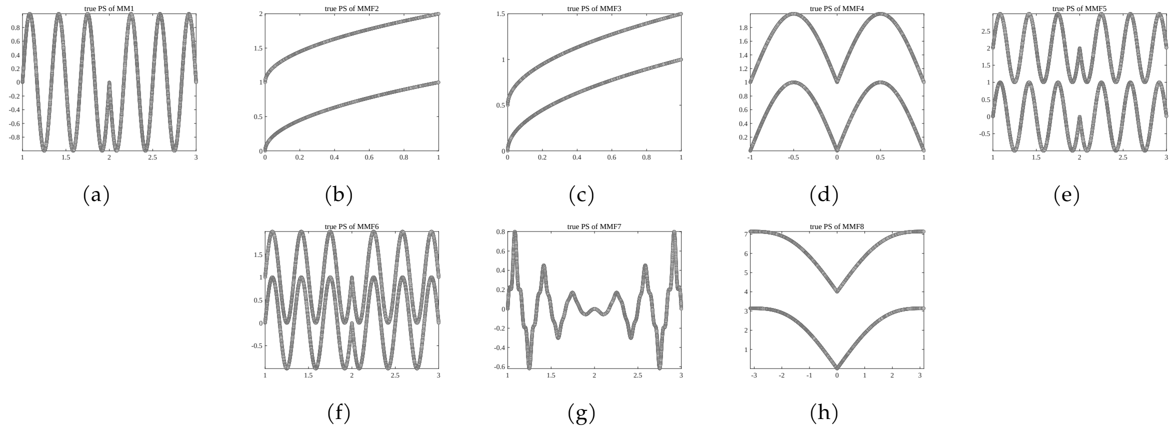



Fig. S-2. True PS of MMF test suite.

### *B. True PS of MMF test suite*

The true PS of MMF instances are shown in Fig. S-2. All eight instances are of two dimensions in the decision space, and 100,000 points are sampled on the true PS to show the distribution and shape of the PS of each instance.

### *C. Parameters and Features of MMMOP*

The detailed introduction of MMMOP [2] test suite is given in Table S-II, including all parameters and main features of all MMMOP instances.

TABLE S-II
PARAMETERS AND MAIN FEATURES OF MMMOP TEST SUITE.

| problems | parameters | | | | | | features | |
|---|---|---|---|---|---|---|---|---|
| | $m$ | $n$ | $K_A$ | $K_B$ | $c_i$ | $d_i$ | number of PS segments | challenges |
| MMMOP1-A | 2 | 3 | 1 | 1 | \ | \ | $5^{K_A}$ | relatively simple |
| MMMOP1-B | 3 | 7 | 1 | 4 | \ | \ | $5^{K_A}$ | higher objective and decision dimension than MMMOP1-A |
| MMMOP2-A | 2 | 3 | 1 | 1 | \ | \ | $6^{K_A}$ | biased in decision space and large decision space |
| MMMOP2-B | 3 | 7 | 1 | 4 | \ | \ | $6^{K_A}$ | higher objective and decision dimension than MMMOP2-A |
| MMMOP3-A | 2 | 2 | 0 | 1 | \ | 3 | $\prod_{i=m,\cdots,m+K_A-1} c_i \times \prod_{i=1,\cdots,m-1} d_i$ | massive Pareto optimal solutions and PS segments |
| MMMOP3-B | 3 | 7 | 0 | 5 | \ | 2 | $\prod_{i=m,\cdots,m+K_A-1} c_i \times \prod_{i=1,\cdots,m-1} d_i$ | massive Pareto optimal solutions and PS segments |
| MMMOP3-C | 2 | 6 | 1 | 4 | 3 | 3 | $\prod_{i=m,\cdots,m+K_A-1} c_i \times \prod_{i=1,\cdots,m-1} d_i$ | massive Pareto optimal solutions and PS segments |
| MMMOP3-D | 3 | 7 | 1 | 4 | 2 | 2 | $\prod_{i=m,\cdots,m+K_A-1} c_i \times \prod_{i=1,\cdots,m-1} d_i$ | massive Pareto optimal solutions and PS segments |
| MMMOP4-A | 2 | 2 | 0 | 1 | \ | 4 | $\prod_{i=m,\cdots,m+K_A-1} c_i \times \prod_{i=1,\cdots,m-1} d_i$ | same feature as MMMOP3 and local optima in the objective space |
| MMMOP4-B | 3 | 7 | 0 | 5 | \ | 3 | $\prod_{i=m,\cdots,m+K_A-1} c_i \times \prod_{i=1,\cdots,m-1} d_i$ | same feature as MMMOP3 and local optima in the objective space |
| MMMOP4-C | 2 | 6 | 1 | 4 | 2 | 4 | $\prod_{i=m,\cdots,m+K_A-1} c_i \times \prod_{i=1,\cdots,m-1} d_i$ | same feature as MMMOP3 and local optima in the objective space |
| MMMOP4-D | 3 | 7 | 1 | 4 | 2 | 3 | $\prod_{i=m,\cdots,m+K_A-1} c_i \times \prod_{i=1,\cdots,m-1} d_i$ | same feature as MMMOP3 and local optima in the objective space |
| MMMOP5-A | 2 | 2 | 0 | 1 | \ | 3 | $\prod_{i=m,\cdots,m+K_A-1} c_i \times \prod_{i=1,\cdots,m-1} d_i$ | same feature as MMMOP4 and PS of different densities |
| MMMOP5-B | 3 | 7 | 0 | 5 | \ | 1 | $\prod_{i=m,\cdots,m+K_A-1} c_i \times \prod_{i=1,\cdots,m-1} d_i$ | same feature as MMMOP4 and PS of different densities |
| MMMOP5-C | 2 | 6 | 1 | 4 | 2 | 2 | $\prod_{i=m,\cdots,m+K_A-1} c_i \times \prod_{i=1,\cdots,m-1} d_i$ | same feature as MMMOP4 and PS of different densities |
| MMMOP5-D | 3 | 7 | 1 | 4 | 2 | 1 | $\prod_{i=m,\cdots,m+K_A-1} c_i \times \prod_{i=1,\cdots,m-1} d_i$ | same feature as MMMOP4 and PS of different densities |
| MMMOP6-A | 2 | 2 | 0 | 1 | 2 | \ | $4^{K_A/2} \times \prod_{i=m+K_A,\cdots,n} c_i$ | variable linkages and large number of PS segments |
| MMMOP6-B | 3 | 3 | 0 | 2 | 2 | \ | $4^{K_A/2} \times \prod_{i=m+K_A,\cdots,n} c_i$ | variable linkages and large number of PS segments |
| MMMOP6-C | 2 | 4 | 2 | 1 | 2 | \ | $4^{K_A/2} \times \prod_{i=m+K_A,\cdots,n} c_i$ | variable linkages and large number of PS segments |
| MMMOP6-D | 3 | 5 | 2 | 1 | 2 | \ | $4^{K_A/2} \times \prod_{i=m+K_A,\cdots,n} c_i$ | variable linkages and large number of PS segments |

### *D. Features and Challenges of IDMP*

The IDMP [3] test suite mainly includes following two features and challenges brought by these features, which make it suitable for testing the performance of convergence-balanced MMOEAs:

- For a point on the PF, solutions close to one equivalent Pareto optimal solution are more likely to dominate solutions close to another equivalent Pareto optimal solution. Due to this condition, the solutions close to some equivalent Pareto optimal solutions have better convergence quality than those close to other equivalent Pareto optimal solutions.
- For a point on the PF, the complexity of searching for one equivalent Pareto optimal solution is lower than that of another equivalent Pareto optimal solution. Due to this condition, solutions with good convergence

quality are quickly found around some equivalent Pareto optimal solutions, while the convergence quality of the solutions found around other equivalent Pareto optimal solutions is poor.

Therefore, the population might quickly converges to some equivalent Pareto optimal solutions, since the solutions around them are preferred by the convergence-first selection criterion.

### E. Evolutionary Parameters Settings for Algorithms

Detailed evolutionary parameter settings of CMMO and other methods for experiments are introduced in Table S-III. In this table, $N$ denotes the population size, $G_{max}$ is the maximum number of iterations and $E_{max}$ is the maximum number of evaluations. It should be noted that in our experiments, $N$ and $E_{max}$, for obtaining a final solution set, are set to the same based on following two reasons for a fair comparison [4].

- Given the fact that some coevolutionary methods consumes more evaluations in one iteration, setting a same maximum number of evaluations could make the comparison fair.
- We set $N$ and $E_{max}$ to larger values than which of constrained multi-objective or many-objective optimization fields because in MMOPs, there are several PS segments need to be found. Consequently, a bigger population size and a larger maximum number of evaluations could ensure that all algorithms have the possibility of converging to the PF as well as the PS.

Thus, $G_{max}$ can be obtained by $E_{max}/N$.

TABLE S-III
PARAMETERS SETTINGS FOR CMMO AND OTHER METHODS. $N$ DENOTES THE POPULATION SIZE, $G_{max}$ IS THE MAXIMUM NUMBER OF ITERATIONS AND $E_{max}$ IS THE MAXIMUM NUMBER OF EVALUATIONS.

| | MMF | MMMOP | | IDMP |
|---|---|---|---|---|
| | | m=2 | m=3 | |
| $N$ | 300 | 600 | 600 | 300 |
| $G_{max}$ | 200 | 200 | 300 | 300 |
| $E_{max}$ | 60000 | 120000 | 180000 | 90000 |

### F. Details for Variants

Table S-IV introduces the detailed information of different variants of CMMO for experiments in Section IV-B. These variants are used to conduct the ablation studies to evaluate the effectiveness of different proposed strategies on the chosen benchmarks.

TABLE S-IV
DETAILS FOR VARIANTS OF CMMO, INCLUDING THE NAME, FEATURE AND FUNCTION OF EACH VARIANT.

| name | feature | function |
|---|---|---|
| CMMOPop1 | use only $P_1$ | evaluate the effectiveness of $P_2$ |
| CMMOPop2 | use only $P_2$ | evaluate the effectiveness of $P_1$ |
| CMMOPop1Original | $P_1$ uses the original strategies of SPEA2 [5] | evaluate the environmental and mating selections for $P_1$ |
| CMMOMSOriginal | the mating selection of $P_1$ uses the original strategy of SPEA2 | evaluate the effectiveness of proposed mating selections |
| CMMOMSrand | random mating selections are used in $P_1$ and $P_2$ | evaluate the effectiveness of proposed mating selections |
| CMMOnoepsilon | $P_2$ has not used the proposed $\varepsilon$-Pareto dominance | evaluate the effectiveness of the proposed $\varepsilon$ |

### G. Details for Parameter Analysis

In order to test the parameter sensitivities of $\eta$, $\tau$ and $\theta$ in the proposed $\varepsilon$ ( *i.e.*, Equation (1)), the following variants are used as shown in Table S-V for experiments in Section IV-D.

TABLE S-V
DETAILS OF PARAMETER SETTINGS FOR PARAMETER ANALYSIS ON $\varepsilon$, INCLUDING THE NAME, FEATURE AND FUNCTION OF EACH VARIANT.

| name | parameter setting | function |
|---|---|---|
| CMMOeta1 | $\eta = 0.2$ | |
| CMMOeta2 | $\eta = 0.3$ | best $\eta$ settings for different benchmarks |
| CMMOeta3 | $\eta = 0$ | |
| CMMOtau1 | $\tau = 0.05$ | |
| CMMOtau2 | $\tau = 0.15$ | parameter sensitivities of $\tau$ |
| CMMOtau3 | $\tau = 0.2$ | |
| CMMOtheta1 | $\theta = 0.05$ | |
| CMMOtheta2 | $\theta = 0.15$ | parameter sensitivities of $\theta$ |
| CMMOtheta3 | $\theta = 0.2$ | |

## REFERENCES

[1] C. Yue, B. Qu, and J. Liang, "A multiobjective particle swarm optimizer using ring topology for solving multimodal multiobjective problems," *IEEE Transactions on Evolutionary Computation*, vol. 22, no. 5, pp. 805–817, 2018.
[2] Y. Liu, G. G. Yen, and D. Gong, "A multimodal multiobjective evolutionary algorithm using two-archive and recombination strategies," *IEEE Transactions on Evolutionary Computation*, vol. 23, no. 4, pp. 660–674, 2019.
[3] Y. Liu, H. Ishibuchi, G. G. Yen, Y. Nojima, and N. Masuyama, "Handling imbalance between convergence and diversity in the decision space in evolutionary multimodal multiobjective optimization," *IEEE Transactions on Evolutionary Computation*, vol. 24, no. 3, pp. 551–565, 2020.
[4] H. Ishibuchi, L. M. Pang, and K. Shang, "Difficulties in fair performance comparison of multi-objective evolutionary algorithms [research frontier]," *IEEE Computational Intelligence Magazine*, vol. 17, no. 1, pp. 86–101, 2022.
[5] E. Zitzler, M. Laumanns, and L. Thiele, "Spea2: Improving the strength pareto evolutionary algorithm," *TIK-Report*, vol. 103, 07 2001.

# S-3. Experimental Results

TABLE S-VI
Statistical results of IGDX obtained by CMMO and its variants on MMF benchmark problems. Best result in each row is highlighted.

| Problem | CMMOPop1 | CMMOPop2 | CMMOPop1Original | CMMOMSoriginal | CMMOMSrand | CMMOnoepsilon | CMMO |
|---|---|---|---|---|---|---|---|
| MMF1 | 2.2708e-2 (5.34e-4) − | 7.2087e-2 (9.32e-3) − | 3.5256e-2 (1.43e-3) − | 2.2198e-2 (4.96e-4) − | 2.2702e-2 (6.86e-4) − | 2.2779e-2 (7.85e-4) − | 2.1584e-2 (3.45e-4) |
| MMF2 | 8.5911e-3 (2.81e-3) ≈ | 6.1577e-2 (1.86e-2) − | 9.8318e-3 (2.59e-3) − | 1.5385e-2 (1.19e-2) − | 1.5447e-2 (5.22e-3) − | 8.7250e-3 (3.32e-3) ≈ | 7.6924e-3 (8.71e-4) |
| MMF3 | 7.4397e-3 (1.68e-3) − | 5.0539e-2 (1.67e-2) − | 7.4134e-3 (1.14e-3) − | 1.1427e-2 (7.26e-3) − | 1.0196e-2 (2.64e-3) − | 7.5067e-3 (1.81e-3) − | 6.4228e-3 (8.53e-4) |
| MMF4 | 1.2990e-2 (2.88e-4) + | 6.9678e-2 (1.59e-2) − | 1.9866e-2 (1.33e-3) − | 1.3090e-2 (3.21e-4) ≈ | 1.2933e-2 (3.06e-4) + | 1.2940e-2 (3.41e-4) + | 1.3294e-2 (4.34e-4) |
| MMF5 | 4.3836e-2 (1.52e-3) − | 1.1417e-1 (1.13e-2) − | 6.9266e-2 (4.12e-3) − | 4.2363e-2 (1.37e-3) − | 4.2982e-2 (1.21e-3) − | 4.3477e-2 (1.52e-3) − | 4.1204e-2 (7.36e-4) |
| MMF6 | 4.0855e-2 (1.68e-3) − | 8.6476e-2 (6.92e-3) − | 5.9257e-2 (3.06e-3) − | 3.9774e-2 (7.47e-4) − | 4.1156e-2 (1.11e-3) − | 4.0211e-2 (1.12e-3) − | 3.8727e-2 (6.93e-4) |
| MMF7 | 1.3131e-2 (3.67e-4) + | 4.5412e-2 (4.46e-3) − | 1.7772e-2 (1.51e-3) − | 1.3257e-2 (3.48e-4) + | 1.3270e-2 (5.17e-4) + | 1.3341e-2 (3.93e-4) + | 1.3707e-2 (4.04e-4) |
| MMF8 | 3.4904e-2 (1.08e-2) − | 3.4362e-1 (1.07e-1) − | 2.5472e-1 (5.69e-2) − | 4.2690e-2 (1.49e-2) − | 9.4346e-2 (3.88e-2) − | 3.3831e-2 (7.61e-3) − | 2.8146e-2 (2.09e-3) |
| +/ − / ≈ | 2/5/1 | 0/8/0 | 0/8/0 | 1/6/1 | 2/6/0 | 2/5/1 | |

TABLE S-VII
Statistical results of HV obtained by CMMO and its variants on MMF benchmark problems. Best result in each row is highlighted.

| Problem | CMMOPop1 | CMMOPop2 | CMMOPop1Original | CMMOMSoriginal | CMMOMSrand | CMMOnoepsilon | CMMO |
|---|---|---|---|---|---|---|---|
| MMF1 | 9.0705e-1 (7.57e-5) − | 9.0310e-1 (1.12e-3) − | 9.0767e-1 (6.43e-6) + | 9.0709e-1 (6.69e-5) − | 9.0709e-1 (8.84e-5) − | 9.0706e-1 (1.30e-4) − | 9.0722e-1 (1.69e-4) |
| MMF2 | 8.6063e-1 (1.27e-4) + | 8.3399e-1 (9.15e-3) − | 8.6066e-1 (1.03e-4) + | 8.6058e-1 (9.97e-5) ≈ | 8.6020e-1 (2.69e-4) − | 8.6065e-1 (9.31e-5) + | 8.6056e-1 (1.04e-4) |
| MMF3 | 8.1438e-1 (1.06e-4) ≈ | 7.8714e-1 (1.03e-2) − | 8.1450e-1 (1.28e-4) + | 8.1439e-1 (1.25e-4) ≈ | 8.1409e-1 (2.17e-4) − | 8.1442e-1 (1.16e-4) ≈ | 8.1443e-1 (1.06e-4) |
| MMF4 | 7.2313e-1 (7.33e-5) − | 7.1091e-1 (5.25e-3) − | 7.2376e-1 (1.42e-5) + | 7.2308e-1 (8.04e-5) − | 7.2308e-1 (1.13e-4) − | 7.2311e-1 (7.18e-5) − | 7.2323e-1 (5.70e-5) |
| MMF5 | 9.6897e-1 (7.10e-5) − | 9.6733e-1 (5.42e-4) − | 9.6922e-1 (2.77e-6) + | 9.6895e-1 (8.97e-5) − | 9.6898e-1 (7.37e-5) − | 9.6895e-1 (8.21e-5) − | 9.6903e-1 (4.47e-5) |
| MMF6 | 9.5342e-1 (1.12e-4) − | 9.5155e-1 (4.34e-4) − | 9.5383e-1 (4.45e-6) + | 9.5339e-1 (2.12e-4) − | 9.5344e-1 (1.30e-4) − | 9.5342e-1 (1.32e-4) − | 9.5356e-1 (6.60e-5) |
| MMF7 | 8.8456e-1 (8.44e-5) − | 8.8081e-1 (8.57e-4) − | 8.8514e-1 (9.24e-6) + | 8.8454e-1 (1.08e-4) − | 8.8459e-1 (6.97e-5) − | 8.8457e-1 (4.95e-5) − | 8.8469e-1 (4.09e-5) |
| MMF8 | 9.7052e-1 (2.53e-4) − | 9.6987e-1 (2.60e-4) − | 9.7097e-1 (4.18e-5) + | 9.7057e-1 (2.67e-4) − | 9.7065e-1 (1.93e-4) − | 9.7055e-1 (3.33e-4) − | 9.7078e-1 (1.96e-4) |
| +/ − / ≈ | 1/6/1 | 0/8/0 | 8/0/0 | 0/6/2 | 0/8/0 | 1/6/1 | |

TABLE S-VIII
Statistical results of IGDX obtained by CMMO and its variants on MMMOP benchmark problems. Best result in each row is highlighted.

| Problem | CMMOPop1 | CMMOPop2 | CMMOPop1Original | CMMOMSoriginal | CMMOMSrand | CMMOnoepsilon | CMMO |
|---|---|---|---|---|---|---|---|
| MMMOP1A | 2.1922e-3 (1.90e-5) + | 2.2798e-2 (6.66e-3) − | 5.4192e-2 (5.34e-2) − | 6.2626e-2 (7.85e-2) − | 6.7176e-2 (6.02e-2) − | 2.2012e-3 (2.38e-5) + | 3.5609e-3 (7.15e-3) |
| MMMOP1B | 5.3431e-2 (1.46e-2) ≈ | 1.2076e-1 (1.34e-2) − | 1.0749e-1 (6.09e-2) − | 1.3151e-1 (9.55e-2) − | 1.0836e-1 (6.59e-2) − | 4.5216e-2 (1.23e-2) + | 4.6663e-2 (1.20e-2) |
| MMMOP2A | 1.2363e-3 (9.92e-5) + | 2.0386e-2 (4.15e-3) − | 3.4064e-2 (4.20e-2) − | 8.6223e-3 (1.95e-2) − | 2.2834e-2 (3.99e-2) − | 1.2621e-3 (1.16e-4) + | 1.6587e-3 (8.90e-4) |
| MMMOP2B | 3.9923e-2 (3.52e-3) − | 1.9011e-1 (6.72e-2) − | 1.3200e-1 (6.94e-2) − | 5.0799e-2 (4.27e-3) − | 4.7250e-2 (4.13e-3) − | 4.2382e-2 (4.11e-3) − | 2.8087e-2 (4.12e-3) |
| MMMOP3A | 3.0038e-3 (9.23e-5) + | 2.2855e-2 (2.55e-3) − | 1.9874e-3 (1.35e-4) + | 3.1306e-3 (1.17e-4) + | 3.0066e-3 (1.31e-4) + | 3.1804e-3 (1.02e-4) + | 3.3665e-3 (1.73e-4) |
| MMMOP3B | 1.0128e-1 (2.14e-3) − | 2.7029e-1 (2.94e-2) − | 5.3502e-2 (1.65e-3) + | 1.0593e-1 (2.16e-3) − | 1.0195e-1 (2.27e-3) − | 1.0589e-1 (3.62e-3) − | 9.4685e-2 (2.74e-3) |
| MMMOP3C | 1.3939e-2 (3.45e-4) − | 2.1186e-1 (2.70e-2) − | 1.3025e-2 (1.78e-2) − | 1.4962e-2 (4.29e-4) − | 1.5027e-2 (3.49e-4) − | 1.4313e-2 (4.15e-4) − | 1.0900e-2 (4.49e-4) |
| MMMOP3D | 9.7982e-2 (2.54e-3) − | 2.5777e-1 (2.68e-2) − | 6.0041e-2 (1.36e-3) + | 1.0267e-1 (1.93e-3) − | 1.0033e-1 (2.09e-3) − | 1.0153e-1 (2.93e-3) − | 8.2873e-2 (2.74e-3) |
| MMMOP4A | 4.2898e-4 (7.67e-7) + | 4.5744e-3 (1.53e-3) − | 8.5537e-4 (4.82e-5) − | 4.3371e-4 (1.25e-6) ≈ | 4.3373e-4 (1.80e-6) ≈ | 4.3294e-4 (1.10e-6) + | 4.3422e-4 (1.32e-6) |
| MMMOP4B | 1.6646e-2 (6.38e-5) + | 1.0396e-1 (3.66e-2) − | 2.2213e-2 (4.31e-4) − | 1.6728e-2 (5.20e-5) + | 1.6731e-2 (5.33e-5) + | 1.6743e-2 (6.07e-5) + | 2.0129e-2 (3.53e-3) |
| MMMOP4C | 1.1454e-3 (2.90e-4) + | 6.5010e-2 (3.70e-2) − | 1.4569e-1 (1.11e-1) − | 1.9209e-1 (1.07e-1) − | 1.9821e-1 (9.23e-2) − | 1.3584e-3 (3.84e-4) + | 6.3111e-3 (2.27e-3) |
| MMMOP4D | 2.3825e-2 (1.03e-4) + | 1.0038e-1 (3.02e-2) − | 6.2518e-2 (6.75e-2) − | 1.4906e-1 (1.19e-1) ≈ | 7.0926e-2 (9.54e-2) − | 2.3956e-2 (9.75e-5) + | 2.5639e-2 (2.00e-4) |
| MMMOP5A | 4.2893e-4 (8.61e-7) + | 4.7170e-3 (1.24e-3) − | 6.9439e-4 (2.77e-5) − | 4.3316e-4 (1.14e-6) + | 4.3329e-4 (1.22e-6) + | 4.3288e-4 (9.70e-7) + | 5.8081e-4 (6.37e-6) |
| MMMOP5B | 1.6603e-2 (3.10e-5) + | 1.2547e-1 (4.51e-2) − | 2.3047e-2 (4.25e-4) − | 1.6749e-2 (5.64e-5) + | 1.6712e-2 (3.13e-5) + | 1.6704e-2 (3.81e-5) + | 1.8484e-2 (1.49e-4) |
| MMMOP5C | 9.9396e-4 (7.46e-5) + | 1.0269e-1 (4.34e-2) − | 2.1158e-1 (8.17e-2) − | 2.0867e-1 (9.45e-2) − | 2.4219e-1 (4.41e-2) − | 1.0079e-3 (6.84e-5) + | 3.0426e-3 (6.23e-4) |
| MMMOP5D | 2.3744e-2 (7.29e-5) + | 1.3062e-1 (3.36e-2) − | 9.4925e-2 (8.94e-2) − | 1.3344e-1 (1.19e-1) ≈ | 1.2559e-1 (1.18e-1) ≈ | 2.3907e-2 (8.56e-5) + | 2.6299e-2 (1.85e-4) |
| MMMOP6A | 2.3480e-3 (5.34e-5) − | 1.5421e-2 (1.90e-3) − | 3.0138e-3 (2.30e-4) − | 2.3455e-3 (3.74e-5) − | 2.3577e-3 (4.28e-5) − | 2.3730e-3 (7.88e-5) − | 2.3155e-3 (4.04e-5) |
| MMMOP6B | 3.0810e-2 (2.02e-4) + | 8.9523e-2 (3.99e-3) − | 4.2756e-2 (1.03e-3) − | 3.0956e-2 (2.23e-4) + | 3.0778e-2 (1.48e-4) + | 3.0879e-2 (2.08e-4) + | 3.1827e-2 (2.80e-4) |
| MMMOP6C | 3.1788e-2 (1.75e-2) + | 1.3531e-1 (1.45e-2) − | 2.4416e-1 (9.55e-2) − | 3.4683e-1 (6.56e-2) − | 3.6770e-1 (4.48e-2) − | 3.0875e-2 (8.31e-3) + | 3.6325e-2 (1.35e-2) |
| MMMOP6D | 8.2979e-2 (4.77e-3) + | 2.7648e-1 (1.62e-2) − | 2.3289e-1 (8.91e-2) − | 2.0714e-1 (9.18e-2) − | 3.1448e-1 (4.49e-2) − | 7.5944e-2 (3.83e-3) + | 8.8607e-2 (3.14e-3) |
| +/ − / ≈ | 14/5/1 | 0/20/0 | 3/17/0 | 5/12/3 | 5/13/2 | 15/5/0 | |

TABLE S-IX

STATISTICAL RESULTS OF IGDX OBTAINED BY CMMO AND ITS VARIANTS ON IDMP BENCHMARK PROBLEMS. BEST RESULT IN EACH ROW IS HIGHLIGHTED.

| Problem | CMMOPop1 | CMMOPop2 | CMMOPop1Original | CMMOMSoriginal | CMMOMSrand | CMMOnoepsilon | CMMO |
|---|---|---|---|---|---|---|---|
| IDMPM2T1 | 7.3452e-1 (3.58e-1) ≈ | 6.8880e-1 (3.39e-1) − | 2.0048e+0 (2.58e-1) − | 1.8636e+0 (4.88e-1) − | 1.9578e+0 (3.58e-1) − | 7.3448e-1 (3.58e-1) ≈ | 6.8751e-1 (2.58e-1) |
| IDMPM2T2 | 6.4032e-1 (2.36e-5) + | 6.2260e-1 (3.58e-2) + | 1.7313e+0 (5.91e-1) − | 1.4871e+0 (7.03e-1) − | 1.6754e+0 (6.35e-1) − | 6.4033e-1 (6.79e-5) + | 6.4035e-1 (5.84e-5) |
| IDMPM2T3 | 6.7180e-1 (4.73e-4) ≈ | 1.4365e+0 (6.50e-1) ≈ | 2.0518e+0 (1.23e-5) − | 8.1655e-1 (4.19e-1) − | 8.2600e-1 (4.16e-1) − | 6.7197e-1 (4.29e-4) − | 6.7169e-1 (4.71e-4) |
| IDMPM2T4 | 8.2855e-1 (4.88e-1) ≈ | 8.1746e-1 (5.18e-1) − | 1.7252e+0 (6.02e-1) − | 2.0048e+0 (2.58e-1) − | 2.0518e+0 (1.38e-5) − | 6.4037e-1 (1.61e-4) + | 6.9118e-1 (2.58e-1) |
| IDMPM3T1 | 1.0231e+0 (2.00e-3) ≈ | 9.7171e-1 (3.61e-2) + | 1.9676e+0 (5.16e-1) − | 1.9057e+0 (5.68e-1) − | 1.7121e+0 (6.00e-1) − | 1.0222e+0 (1.71e-3) ≈ | 1.0229e+0 (1.78e-3) |
| IDMPM3T2 | 1.0226e+0 (1.95e-3) ≈ | 9.7159e-1 (3.08e-2) + | 1.4327e+0 (5.24e-1) − | 1.6581e+0 (5.89e-1) − | 1.5948e+0 (5.47e-1) − | 1.0222e+0 (1.95e-3) ≈ | 1.0223e+0 (2.29e-3) |
| IDMPM3T3 | 1.0564e+0 (9.75e-2) ≈ | 1.0391e+0 (1.47e-1) − | 2.1337e+0 (4.90e-1) − | 1.2574e+0 (3.53e-1) − | 1.1560e+0 (2.70e-1) − | 1.0389e+0 (2.14e-3) ≈ | 1.0385e+0 (1.38e-3) |
| IDMPM3T4 | 1.0231e+0 (2.04e-3) ≈ | 1.0192e+0 (1.39e-1) + | 1.9002e+0 (5.48e-1) − | 1.9801e+0 (5.68e-1) − | 2.1609e+0 (4.71e-1) − | 1.0233e+0 (2.37e-3) ≈ | 1.0225e+0 (1.65e-3) |
| IDMPM4T1 | 1.7863e+0 (5.27e-1) ≈ | 1.8146e+0 (4.71e-1) ≈ | 3.0088e+0 (1.34e-1) − | 3.0334e+0 (1.55e-3) − | 3.0340e+0 (1.06e-3) − | 1.9162e+0 (5.06e-1) ≈ | 1.7511e+0 (6.03e-1) |
| IDMPM4T2 | 1.2950e+0 (5.72e-2) ≈ | 1.5760e+0 (4.42e-1) ≈ | 2.7162e+0 (5.13e-1) − | 2.8461e+0 (4.16e-1) − | 2.7751e+0 (3.71e-1) − | 1.3276e+0 (1.84e-1) ≈ | 1.3174e+0 (1.78e-1) |
| IDMPM4T3 | 1.3242e+0 (8.11e-2) ≈ | 1.5949e+0 (4.44e-1) ≈ | 2.6396e+0 (5.56e-1) − | 2.0207e+0 (6.49e-1) − | 2.0206e+0 (6.51e-1) − | 1.3346e+0 (1.75e-1) ≈ | 1.3451e+0 (1.83e-1) |
| IDMPM4T4 | 1.3604e+0 (2.51e-1) ≈ | 1.2900e+0 (9.01e-2) + | 2.5802e+0 (5.00e-1) − | 2.7360e+0 (5.15e-1) − | 2.8463e+0 (4.16e-1) − | 1.3058e+0 (7.84e-2) ≈ | 1.3051e+0 (7.80e-2) |
| +/ − / ≈ | 1/0/11 | 5/3/4 | 0/12/0 | 0/12/0 | 0/12/0 | 2/1/9 | |

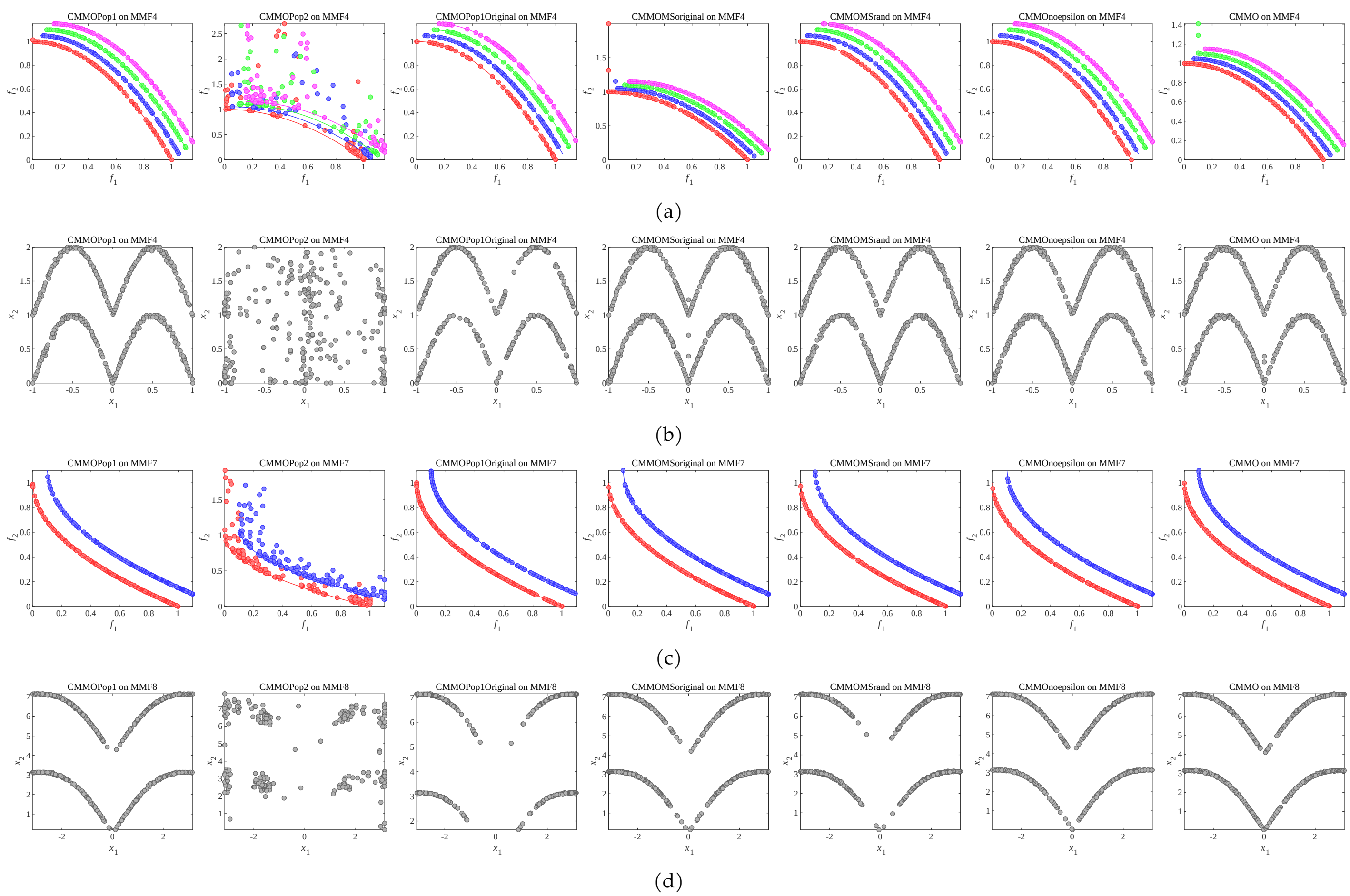



Fig. S-3. PFs and PSs obtained by CMMO and its variants with the median IGDX values among 30 runs on MMF4, MMF7 and MMF8.

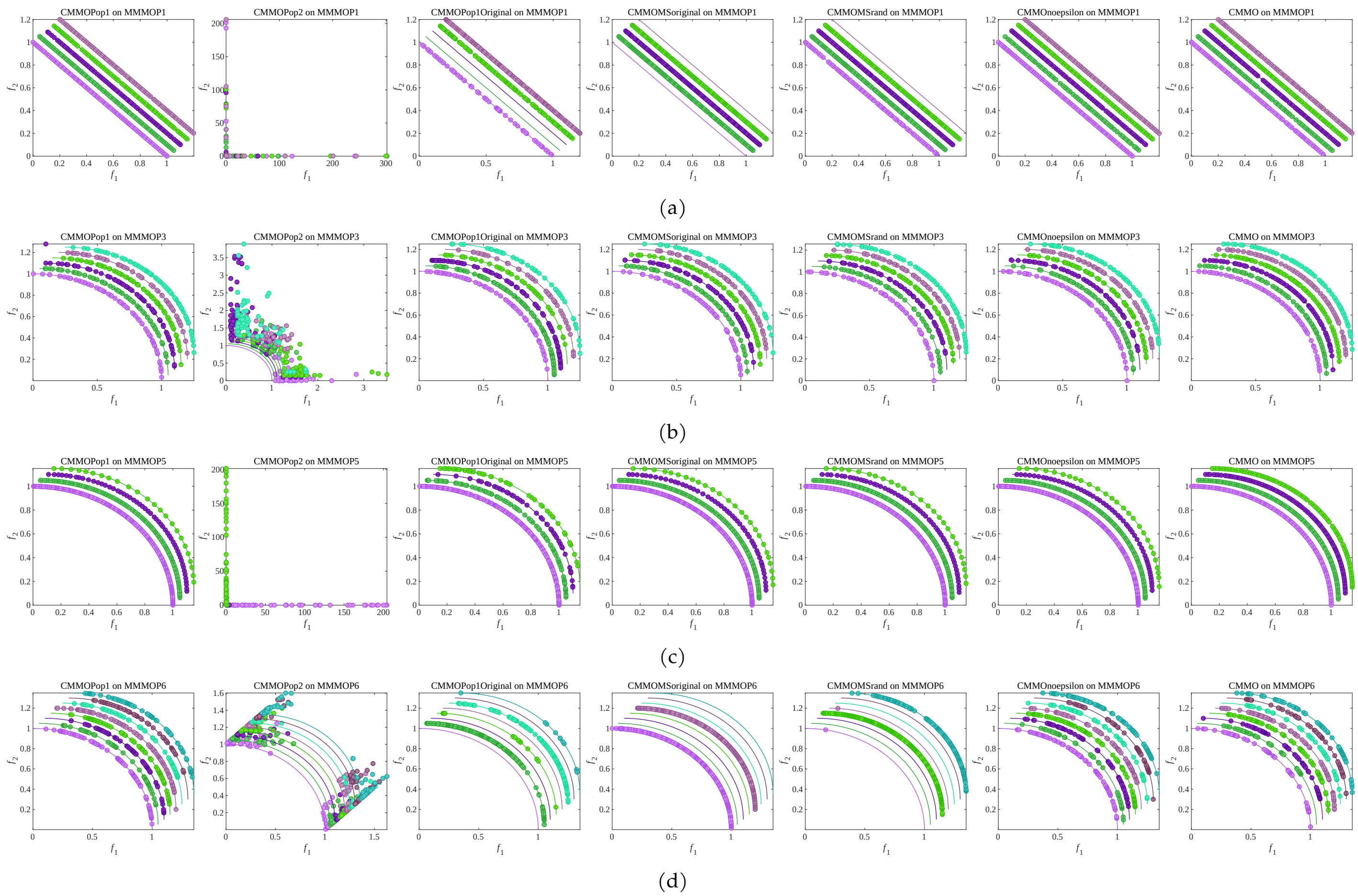



Fig. S-4. PFs obtained by CMMO and its variants with the median IGDX values among 30 runs on MMMOP1-A, MMMOP3-A, MMMOP5-C and MMMOP6-C.

TABLE S-X
STATISTICAL RESULTS OF HV OBTAINED BY CMMO AND ITS VARIANTS ON IDMP BENCHMARK PROBLEMS. BEST RESULT IN EACH ROW IS HIGHLIGHTED.

| Problem | CMMOPop1 | CMMOPop2 | CMMOPop1Original | CMMOMSoriginal | CMMOMSrand | CMMOnoepsilon | CMMO |
|---|---|---|---|---|---|---|---|
| IDMPM2T1 | 9.8337e-1 (1.71e-5) + | 9.8175e-1 (7.79e-4) − | 9.8340e-1 (5.63e-6) + | 9.8340e-1 (1.81e-5) + | 9.8340e-1 (1.02e-5) + | 9.8337e-1 (2.35e-5) + | 9.8333e-1 (3.92e-5) |
| IDMPM2T2 | 9.8337e-1 (7.88e-6) + | 9.8214e-1 (2.96e-4) − | 9.8340e-1 (8.20e-6) + | 9.8331e-1 (7.27e-5) − | 9.8321e-1 (1.29e-4) − | 9.8336e-1 (3.78e-5) ≈ | 9.8335e-1 (4.47e-5) |
| IDMPM2T3 | 9.8325e-1 (9.03e-5) ≈ | 9.8224e-1 (3.30e-4) − | 9.8340e-1 (5.23e-6) + | 9.8320e-1 (1.27e-4) ≈ | 9.8327e-1 (8.34e-5) ≈ | 9.8319e-1 (1.66e-4) ≈ | 9.8326e-1 (8.28e-5) |
| IDMPM2T4 | 9.8339e-1 (1.38e-5) + | 9.8237e-1 (2.55e-4) − | 9.8340e-1 (6.41e-6) + | 9.8341e-1 (5.34e-6) + | 9.8340e-1 (8.23e-6) + | 9.8339e-1 (7.84e-6) + | 9.8336e-1 (2.85e-5) |
| IDMPM3T1 | 9.6048e-1 (1.64e-3) ≈ | 9.5316e-1 (1.61e-3) − | 9.6188e-1 (1.55e-3) + | 9.6013e-1 (1.76e-3) ≈ | 9.5973e-1 (1.89e-3) − | 9.6040e-1 (1.64e-3) ≈ | 9.6077e-1 (1.63e-3) |
| IDMPM3T2 | 9.6065e-1 (1.88e-3) ≈ | 9.5443e-1 (1.64e-3) − | 9.6190e-1 (1.75e-3) + | 9.6073e-1 (1.52e-3) ≈ | 9.5968e-1 (1.40e-3) − | 9.6055e-1 (1.48e-3) ≈ | 9.6115e-1 (1.36e-3) |
| IDMPM3T3 | 9.5990e-1 (1.61e-3) − | 9.5231e-1 (2.13e-3) − | 9.6158e-1 (1.67e-3) + | 9.5900e-1 (2.03e-3) − | 9.5880e-1 (2.08e-3) − | 9.5875e-1 (2.39e-3) − | 9.6064e-1 (1.51e-3) |
| IDMPM3T4 | 9.6135e-1 (1.24e-3) ≈ | 9.5383e-1 (1.83e-3) − | 9.6157e-1 (1.40e-3) ≈ | 9.6048e-1 (1.72e-3) − | 9.5999e-1 (1.90e-3) − | 9.6049e-1 (1.69e-3) − | 9.6162e-1 (1.04e-3) |
| IDMPM4T1 | 9.3354e-1 (1.86e-3) + | 9.1387e-1 (4.71e-3) − | 9.3348e-1 (1.86e-3) ≈ | 9.3073e-1 (5.98e-3) ≈ | 9.3282e-1 (2.05e-3) ≈ | 9.3369e-1 (2.14e-3) ≈ | 9.3266e-1 (1.72e-3) |
| IDMPM4T2 | 9.3464e-1 (1.41e-3) + | 9.1762e-1 (3.11e-3) − | 9.3419e-1 (1.99e-3) ≈ | 9.3236e-1 (2.59e-3) ≈ | 9.3077e-1 (2.96e-3) − | 9.3390e-1 (2.40e-3) ≈ | 9.3357e-1 (1.89e-3) |
| IDMPM4T3 | 9.3248e-1 (1.98e-3) ≈ | 9.1573e-1 (3.43e-3) − | 9.3356e-1 (2.64e-3) + | 9.3035e-1 (2.08e-3) − | 9.2958e-1 (2.62e-3) − | 9.3280e-1 (2.60e-3) ≈ | 9.3237e-1 (1.98e-3) |
| IDMPM4T4 | 9.3542e-1 (1.67e-3) + | 9.1718e-1 (4.13e-3) − | 9.3459e-1 (1.93e-3) ≈ | 9.3256e-1 (2.03e-3) − | 9.3210e-1 (1.80e-3) − | 9.3453e-1 (2.14e-3) ≈ | 9.3382e-1 (1.75e-3) |
| +/ − / ≈ | 6/1/5 | 0/12/0 | 8/0/4 | 2/5/5 | 2/8/2 | 2/2/8 | |

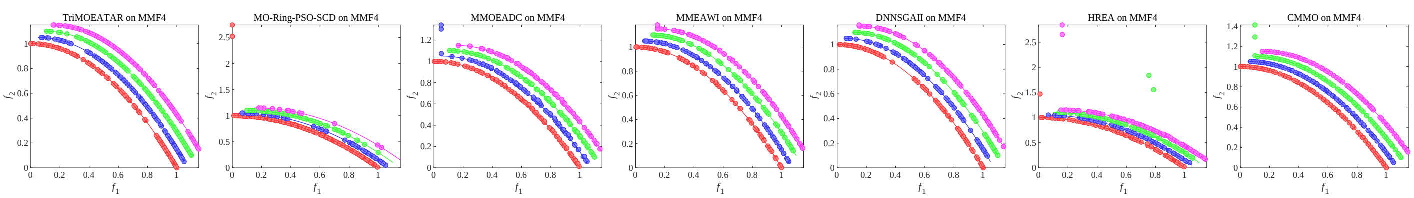
TABLE S-XI
STATISTICAL RESULTS OF IGDX OBTAINED BY CMMO AND OTHER METHODS ON MMF BENCHMARK PROBLEMS. BEST RESULT IN EACH ROW IS HIGHLIGHTED.

| Problem | TriMOEA-TA&R | MO-Ring-PSO-SCD | MMOEADC | MMEAWI | DN-NSGA-II | HREA | CMMO |
|---|---|---|---|---|---|---|---|
| MMF1 | 4.2194e-2 (2.22e-3) − | 5.0283e-2 (5.78e-3) − | 3.1634e-2 (1.03e-3) − | 2.9642e-2 (1.37e-3) − | 2.8840e-2 (7.89e-4) − | 3.1110e-2 (1.74e-3) − | 2.1501e-2 (3.79e-4) |
| MMF2 | 3.5549e-2 (2.55e-2) − | 4.3472e-2 (2.02e-2) − | 5.3414e-3 (1.23e-3) + | 8.4701e-3 (3.41e-3) ≈ | 2.0768e-2 (1.93e-2) − | 8.1799e-3 (2.72e-3) ≈ | 7.3586e-3 (9.15e-4) |
| MMF3 | 2.2079e-2 (8.87e-3) − | 3.5519e-2 (1.28e-2) − | 4.9046e-3 (4.74e-4) + | 7.5709e-3 (4.42e-3) ≈ | 2.2931e-2 (2.19e-2) − | 6.7633e-3 (1.80e-3) ≈ | 6.2426e-3 (6.22e-4) |
| MMF4 | 1.8256e-2 (6.69e-3) − | 4.3592e-2 (1.05e-2) − | 2.0290e-2 (1.96e-3) − | 1.7052e-2 (5.16e-4) − | 1.8592e-2 (9.62e-4) − | 2.0880e-2 (1.32e-3) − | 1.3392e-2 (4.20e-4) |
| MMF5 | 6.6297e-2 (9.93e-3) − | 9.9974e-2 (1.11e-2) − | 5.7871e-2 (2.11e-3) − | 5.1288e-2 (2.13e-3) − | 6.0487e-2 (2.92e-3) − | 5.4488e-2 (2.48e-3) − | 4.0969e-2 (8.63e-4) |
| MMF6 | 5.0886e-2 (4.55e-3) − | 8.4912e-2 (1.07e-2) − | 5.2339e-2 (1.63e-3) − | 4.7707e-2 (1.45e-3) − | 5.3242e-2 (1.92e-3) − | 5.0790e-2 (2.73e-3) − | 3.8460e-2 (5.16e-4) |
| MMF7 | 1.9342e-2 (2.91e-3) − | 4.0007e-2 (7.91e-3) − | 2.0293e-2 (8.49e-4) − | 1.7115e-2 (6.71e-4) − | 1.5974e-2 (7.74e-4) − | 1.6914e-2 (7.80e-4) − | 1.3798e-2 (3.50e-4) |
| MMF8 | 3.2554e-1 (9.71e-2) − | 1.2324e-1 (6.43e-2) − | 3.7442e-2 (2.92e-3) − | 3.7825e-2 (5.37e-3) − | 5.1661e-2 (1.77e-2) − | 4.3178e-2 (3.11e-3) − | 2.7783e-2 (1.94e-3) |
| +/ − / ≈ | 0/8/0 | 0/8/0 | 2/6/0 | 0/6/2 | 0/8/0 | 0/6/2 | |

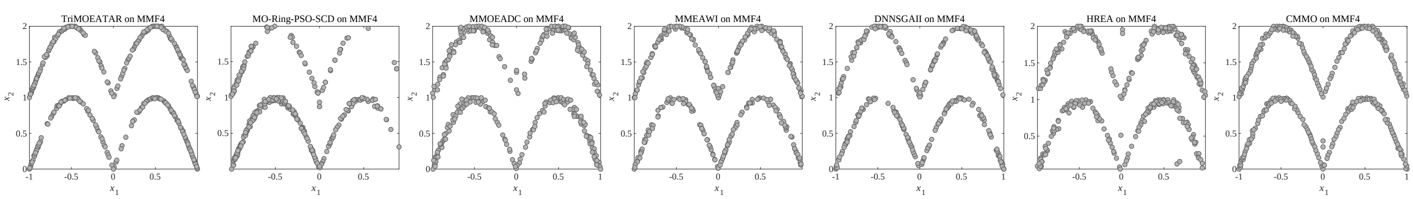
TABLE S-XII

Statistical results of HV obtained by CMMO and other methods on MMF benchmark problems. Best result in each row is highlighted.

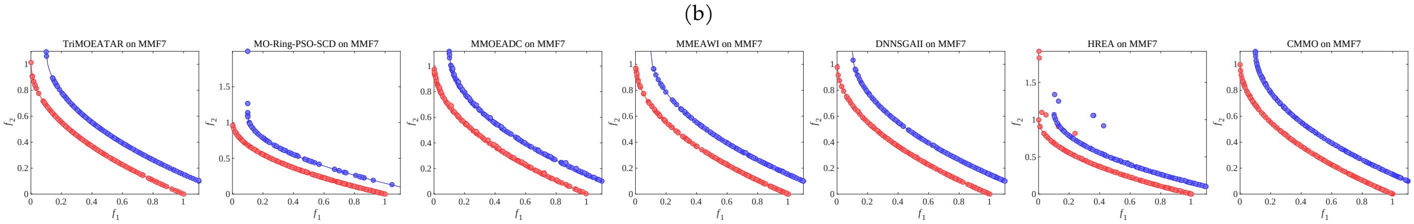
| Problem | TriMOEA-TA&R | MO-Ring-PSO-SCD | MMOEADC | MMEAWI | DN-NSGA-II | HREA | CMMO |
|---|---|---|---|---|---|---|---|
| MMF1 | 9.0684e-1 (5.46e-4) − | 9.0652e-1 (2.88e-4) − | 9.0638e-1 (7.33e-4) − | 9.0691e-1 (1.31e-4) − | 9.0728e-1 (4.53e-5) ≈ | 9.0667e-1 (6.06e-4) − | 9.0722e-1 (1.69e-4) |
| MMF2 | 8.5730e-1 (3.22e-3) − | 8.5020e-1 (4.06e-3) − | 8.6008e-1 (3.23e-4) − | 8.6039e-1 (2.10e-4) − | 8.6037e-1 (3.65e-4) ≈ | 8.5999e-1 (4.94e-4) − | 8.6056e-1 (1.04e-4) |
| MMF3 | 8.1155e-1 (1.58e-3) − | 8.0464e-1 (2.34e-3) − | 8.1365e-1 (4.56e-4) − | 8.1371e-1 (2.10e-3) − | 8.1385e-1 (9.82e-4) − | 8.1347e-1 (6.28e-4) − | 8.1443e-1 (1.06e-4) |
| MMF4 | 7.2285e-1 (1.16e-4) − | 7.2302e-1 (7.75e-5) − | 7.2292e-1 (2.20e-4) − | 7.2291e-1 (1.10e-4) − | 7.2288e-1 (6.39e-4) − | 7.2222e-1 (4.22e-4) − | 7.2323e-1 (5.70e-5) |
| MMF5 | 9.6902e-1 (4.95e-5) ≈ | 9.6890e-1 (9.14e-5) − | 9.6862e-1 (3.83e-4) − | 9.6898e-1 (5.95e-5) − | 9.6890e-1 (1.43e-4) − | 9.6880e-1 (3.02e-4) − | 9.6903e-1 (4.47e-5) |
| MMF6 | 9.5357e-1 (5.48e-5) ≈ | 9.5329e-1 (2.02e-4) − | 9.5302e-1 (4.71e-4) − | 9.5351e-1 (6.53e-5) − | 9.5342e-1 (2.31e-4) − | 9.5303e-1 (4.40e-4) − | 9.5356e-1 (6.60e-5) |
| MMF7 | 8.8443e-1 (3.69e-5) − | 8.8435e-1 (2.05e-4) − | 8.8352e-1 (7.66e-4) − | 8.8432e-1 (1.06e-4) − | 8.8463e-1 (5.68e-5) − | 8.8413e-1 (9.18e-4) − | 8.8469e-1 (4.09e-5) |
| MMF8 | 9.7087e-1 (4.49e-5) ≈ | 9.7066e-1 (1.96e-4) − | 9.7049e-1 (3.69e-4) − | 9.7051e-1 (3.31e-4) − | 9.7060e-1 (2.55e-4) − | 9.7037e-1 (3.54e-4) − | 9.7078e-1 (1.96e-4) |
| +/ − / ≈ | 0/5/3 | 0/8/0 | 0/8/0 | 0/8/0 | 0/6/2 | 0/8/0 | |



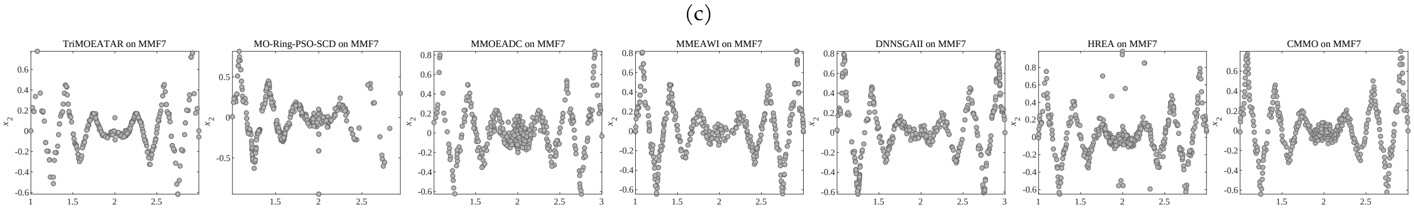

(a)




(b)

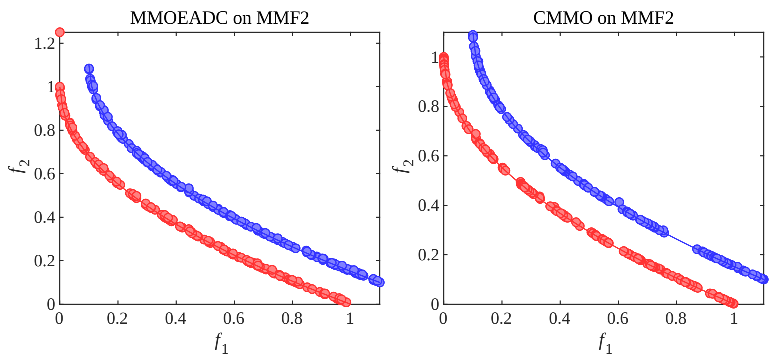



(c)

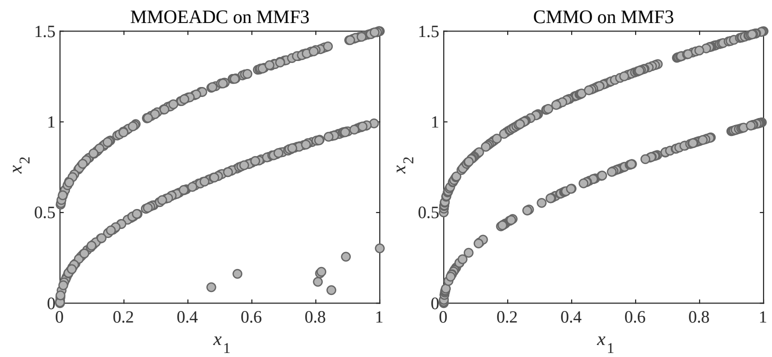



(d)

Fig. S-5. PFs and PSs obtained by CMMO and other methods with the median IGDX values among 30 runs on MMF4 and MMF7.




(a)




(b)

Fig. S-6. PFs and PSs obtained by CMMO and MMOEADC with the median IGDX values among 30 runs on MMF2 and MMF3.

TABLE S-XIII
STATISTICAL RESULTS OF IGDX OBTAINED BY CMMO AND OTHER METHODS ON MMMOP BENCHMARK PROBLEMS. BEST RESULT IN EACH ROW IS HIGHLIGHTED.

| Problem | TriMOEA-TA&R | MO-Ring-PSO-SCD | MMOEADC | MMEAWI | DN-NSGA-II | HREA | CMMO |
|---|---|---|---|---|---|---|---|
| MMMOP1A | 2.1631e-2 (1.26e-2) − | 1.9040e-1 (4.46e-2) − | 4.3185e-3 (3.06e-4) − | 1.8381e-2 (1.89e-2) − | 1.7039e-1 (8.64e-2) − | 6.3349e-3 (9.25e-4) − | 3.5609e-3 (7.15e-3) |
| MMMOP1B | 2.0140e-1 (6.23e-2) − | 4.9947e-1 (2.96e-2) − | 2.8763e-1 (1.48e-1) − | 1.1806e-1 (3.44e-2) − | 2.2827e-1 (5.78e-2) − | 3.9691e-1 (2.59e-2) − | 4.6663e-2 (1.20e-2) |
| MMMOP2A | 2.2301e-2 (1.71e-2) − | 1.1320e-1 (6.64e-2) − | 2.2222e-3 (1.77e-4) − | 1.9820e-3 (3.43e-4) − | 1.0342e-1 (7.29e-2) − | 2.4586e-3 (2.84e-4) − | 1.6587e-3 (8.90e-4) |
| MMMOP2B | 5.2852e-2 (3.82e-2) − | 5.6372e-1 (1.15e-1) − | 9.9357e-2 (1.99e-2) − | 4.5716e-2 (1.13e-2) − | 1.3905e-1 (5.98e-2) − | 4.4001e-1 (6.18e-2) − | 2.8087e-2 (4.12e-3) |
| MMMOP3A | 1.0221e-3 (2.78e-5) + | 7.5246e-3 (6.43e-4) − | 5.2799e-3 (9.42e-4) − | 3.9199e-3 (1.27e-4) − | 3.6680e-3 (2.31e-4) − | 4.8349e-3 (2.80e-4) − | 3.3665e-3 (1.73e-4) |
| MMMOP3B | 3.0162e-2 (4.62e-4) + | 2.5871e-1 (1.81e-2) − | 1.9164e-1 (1.16e-2) − | 6.8943e-2 (1.49e-3) + | 8.8886e-2 (2.53e-3) + | 3.9353e-1 (2.68e-2) − | 9.4685e-2 (2.74e-3) |
| MMMOP3C | 2.0098e-2 (1.67e-2) − | 1.7767e-1 (2.35e-2) − | 5.0820e-2 (4.23e-3) − | 1.0174e-1 (4.03e-2) − | 6.5746e-2 (3.70e-2) − | 3.1668e-1 (2.91e-2) − | 1.0900e-2 (4.49e-4) |
| MMMOP3D | 4.4422e-2 (3.75e-3) + | 2.9486e-1 (1.81e-2) − | 1.9449e-1 (1.52e-2) − | 7.3854e-2 (1.62e-3) + | 9.2969e-2 (3.49e-3) − | 3.7651e-1 (2.15e-2) − | 8.2873e-2 (2.74e-3) |
| MMMOP4A | 1.0073e-3 (1.17e-5) − | 8.4217e-3 (3.07e-3) − | 1.6956e-3 (5.65e-4) − | 7.0416e-4 (2.54e-5) − | 5.6223e-4 (1.02e-5) − | 8.1721e-4 (8.89e-5) − | 4.3422e-4 (1.32e-6) |
| MMMOP4B | 2.3850e-2 (3.72e-3) − | 4.6953e-1 (4.06e-2) − | 4.5773e-1 (4.93e-2) − | 1.9585e-2 (2.05e-4) + | 2.2386e-2 (4.30e-4) − | 4.0675e-1 (3.62e-2) − | 2.0129e-2 (3.53e-3) |
| MMMOP4C | 1.3562e-1 (8.62e-2) − | 4.3010e-1 (4.85e-2) − | 4.2965e-2 (5.37e-2) ≈ | 3.0243e-2 (2.16e-2) − | 2.1357e-1 (7.05e-2) − | 3.0120e-1 (2.16e-2) − | 6.3111e-3 (2.27e-3) |
| MMMOP4D | 9.3682e-2 (5.18e-2) − | 4.8626e-1 (3.83e-2) − | 5.5031e-1 (5.35e-2) − | 2.8604e-2 (4.16e-4) − | 1.5593e-1 (6.23e-2) − | 3.8519e-1 (1.89e-2) − | 2.5639e-2 (2.00e-4) |
| MMMOP5A | 1.0155e-3 (1.31e-5) − | 6.0907e-3 (2.36e-3) − | 8.5192e-4 (6.41e-5) − | 7.0831e-4 (2.35e-5) − | 5.6278e-4 (6.73e-6) + | 8.1390e-4 (1.05e-4) − | 5.8081e-4 (6.37e-6) |
| MMMOP5B | 2.6054e-2 (6.51e-4) − | 4.5556e-1 (4.16e-2) − | 1.4515e-1 (1.34e-1) − | 1.9646e-2 (1.86e-4) − | 2.2770e-2 (5.86e-4) − | 4.3916e-1 (3.79e-2) − | 1.8484e-2 (1.49e-4) |
| MMMOP5C | 1.3977e-1 (9.57e-2) − | 4.2228e-1 (6.43e-2) − | 5.8412e-2 (8.18e-2) − | 1.0309e-1 (9.29e-2) − | 1.9001e-1 (9.57e-2) − | 2.9112e-1 (2.52e-2) − | 3.0426e-3 (6.23e-4) |
| MMMOP5D | 1.5010e-1 (6.27e-2) − | 4.7263e-1 (3.34e-2) − | 2.3009e-1 (1.94e-1) − | 2.8604e-2 (2.17e-4) − | 1.5807e-1 (7.21e-2) − | 4.0120e-1 (2.60e-2) − | 2.6299e-2 (1.85e-4) |
| MMMOP6A | 4.5951e-3 (2.04e-3) − | 5.6094e-3 (6.37e-4) − | 3.9107e-3 (1.19e-4) − | 3.1502e-3 (9.14e-5) − | 3.1556e-3 (2.00e-4) − | 3.2349e-3 (1.51e-4) − | 2.3155e-3 (4.04e-5) |
| MMMOP6B | 5.0574e-2 (7.97e-4) − | 4.7294e-2 (1.83e-3) − | 4.8955e-2 (2.08e-3) − | 3.7825e-2 (5.78e-4) − | 4.2627e-2 (1.17e-3) − | 3.7968e-2 (1.25e-3) − | 3.1827e-2 (2.80e-4) |
| MMMOP6C | 2.7706e-1 (8.20e-2) − | 8.4807e-2 (1.48e-2) − | 1.4173e-2 (1.57e-3) + | 1.9437e-1 (3.28e-2) − | 3.6690e-1 (4.81e-2) − | 1.5102e-2 (1.04e-3) + | 3.6325e-2 (1.35e-2) |
| MMMOP6D | 3.1059e-1 (5.79e-2) − | 1.2049e-1 (9.15e-3) − | 1.0680e-1 (1.87e-3) − | 1.3180e-1 (1.74e-2) − | 3.0609e-1 (4.70e-2) − | 1.5066e-1 (1.32e-2) − | 8.8607e-2 (3.14e-3) |
| +/−/≈ | 3/17/0 | 0/20/0 | 1/18/1 | 3/17/0 | 2/18/0 | 1/19/0 | |

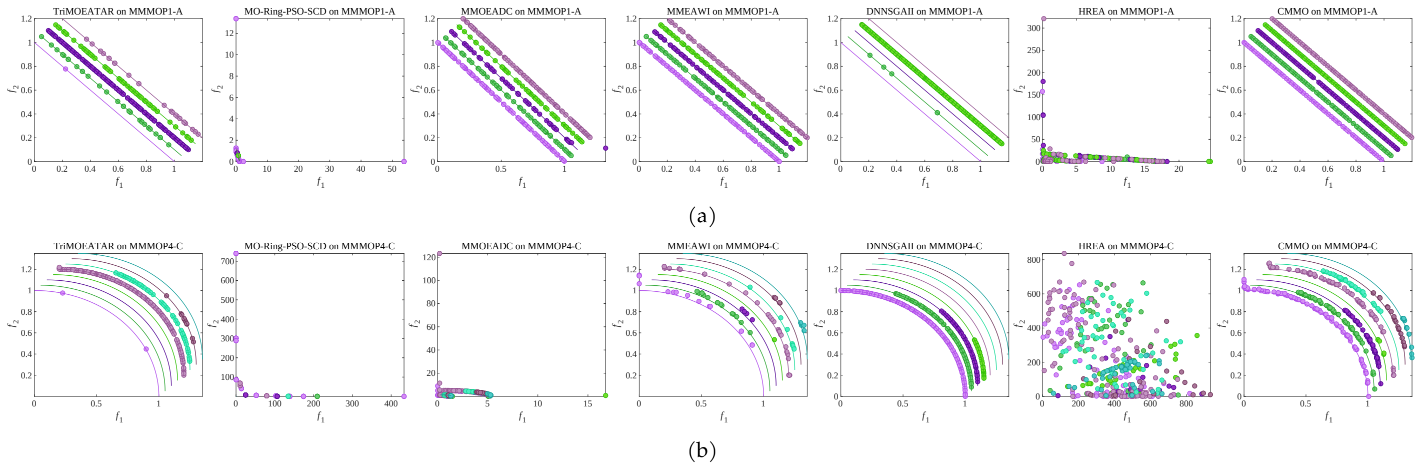



Fig. S-7. PFs obtained by CMMO and other methods with the median IGDX values among 30 runs on MMMOP1-A and MMMOP4-C.

TABLE S-XIV
Statistical results of IGDX obtained by CMMO and other methods on IDMP benchmark problems. Best result in each row is highlighted.

| Problem | TriMOEA-TA&R | MO-Ring-PSO-SCD | MMOEADC | MMEAWI | DN-NSGA-II | HREA | CMMO |
|---|---|---|---|---|---|---|---|
| IDMPM2T1 | 2.0071e+0 (2.45e-1) − | 7.2631e-1 (2.53e-1) − | 6.4050e-1 (3.93e-4) ≈ | 6.4043e-1 (3.38e-4) ≈ | 1.9578e+0 (3.58e-1) − | 6.4070e-1 (5.28e-4) + | 6.8751e-1 (2.58e-1) |
| IDMPM2T2 | 1.6296e+0 (6.56e-1) − | 6.4403e-1 (3.30e-3) − | 6.4046e-1 (2.42e-4) ≈ | 6.4050e-1 (5.25e-4) ≈ | 1.1579e+0 (6.92e-1) − | 6.3590e-1 (2.56e-2) ≈ | 6.4035e-1 (5.84e-5) |
| IDMPM2T3 | 1.1894e+0 (6.68e-1) − | 6.7256e-1 (9.86e-4) − | 6.7138e-1 (6.87e-4) ≈ | 6.7188e-1 (6.50e-4) ≈ | 9.1468e-1 (5.17e-1) − | 6.7124e-1 (9.40e-4) ≈ | 6.7169e-1 (4.71e-4) |
| IDMPM2T4 | 2.0518e+0 (7.99e-6) − | 6.4483e-1 (5.49e-3) + | 8.2869e-1 (4.88e-1) ≈ | 6.4046e-1 (3.53e-4) ≈ | 2.0048e+0 (2.58e-1) − | 6.2765e-1 (7.05e-2) ≈ | 6.9118e-1 (2.58e-1) |
| IDMPM3T1 | 2.2060e+0 (3.56e-1) − | 1.0937e+0 (1.68e-1) − | 9.7580e-1 (7.61e-2) + | 1.0223e+0 (2.18e-3) ≈ | 1.6065e+0 (5.93e-1) − | 9.3664e-1 (1.45e-1) + | 1.0229e+0 (1.78e-3) |
| IDMPM3T2 | 1.6345e+0 (4.85e-1) − | 1.0293e+0 (6.18e-3) − | 9.7174e-1 (7.60e-2) + | 1.0221e+0 (1.73e-3) ≈ | 1.5431e+0 (5.23e-1) − | 8.7395e-1 (1.48e-1) + | 1.0223e+0 (2.29e-3) |
| IDMPM3T3 | 1.3275e+0 (3.83e-1) − | 1.0386e+0 (2.66e-3) ≈ | 9.9390e-1 (7.06e-2) + | 1.0381e+0 (1.55e-3) ≈ | 1.5216e+0 (4.45e-1) − | 1.0096e+0 (7.70e-2) + | 1.0385e+0 (1.38e-3) |
| IDMPM3T4 | 2.1118e+0 (4.62e-1) − | 1.0295e+0 (7.63e-3) − | 9.5952e-1 (8.84e-2) + | 1.0762e+0 (1.65e-1) ≈ | 1.8069e+0 (5.61e-1) − | 8.4737e-1 (1.45e-1) + | 1.0225e+0 (1.65e-3) |
| IDMPM4T1 | 3.0341e+0 (9.39e-5) − | 1.9051e+0 (5.82e-1) − | 1.1491e+0 (1.85e-1) + | 1.3053e+0 (7.82e-2) + | 3.0342e+0 (2.70e-3) − | 5.4763e-1 (1.93e-1) + | 1.7511e+0 (6.03e-1) |
| IDMPM4T2 | 2.8566e+0 (3.27e-1) − | 1.3176e+0 (5.77e-2) − | 1.1521e+0 (1.51e-1) + | 1.3890e+0 (2.10e-1) ≈ | 2.6144e+0 (4.61e-1) − | 5.3637e-1 (2.27e-1) + | 1.3174e+0 (1.78e-1) |
| IDMPM4T3 | 1.9363e+0 (7.03e-1) − | 1.3031e+0 (3.95e-3) ≈ | 1.1827e+0 (2.00e-1) + | 1.3215e+0 (7.80e-2) ≈ | 2.3054e+0 (5.92e-1) − | 6.1836e-1 (1.82e-1) + | 1.3451e+0 (1.83e-1) |
| IDMPM4T4 | 2.6641e+0 (5.50e-1) − | 1.3090e+0 (6.08e-2) − | 1.2236e+0 (2.42e-1) + | 1.5022e+0 (2.07e-1) − | 2.7890e+0 (5.02e-1) − | 5.0227e-1 (1.60e-1) + | 1.3051e+0 (7.80e-2) |
| +/ − / ≈ | 0/12/0 | 1/9/2 | 8/0/4 | 1/1/10 | 0/12/0 | 9/0/3 | |

TABLE S-XV
Statistical results of HV obtained by CMMO and other methods on IDMP benchmark problems. Best result in each row is highlighted.

| Problem | TriMOEA-TA&R | MO-Ring-PSO-SCD | MMOEADC | MMEAWI | DN-NSGA-II | HREA | CMMO |
|---|---|---|---|---|---|---|---|
| IDMPM2T1 | 9.8330e-1 (7.93e-6) − | 9.8237e-1 (3.49e-4) − | 9.8322e-1 (1.48e-4) − | 9.8321e-1 (1.01e-4) − | 9.8334e-1 (1.41e-5) ≈ | 9.8293e-1 (4.19e-4) − | 9.8333e-1 (3.92e-5) |
| IDMPM2T2 | 9.8330e-1 (1.36e-5) − | 9.8307e-1 (1.33e-4) − | 9.8321e-1 (1.36e-4) − | 9.8316e-1 (1.27e-4) − | 9.8336e-1 (1.22e-5) ≈ | 9.8284e-1 (4.41e-4) − | 9.8335e-1 (4.47e-5) |
| IDMPM2T3 | 9.8329e-1 (1.64e-5) ≈ | 9.8298e-1 (1.88e-4) − | 9.8305e-1 (3.07e-4) − | 9.8305e-1 (1.93e-4) − | 9.8338e-1 (1.31e-5) + | 9.8275e-1 (4.36e-4) − | 9.8326e-1 (8.28e-5) |
| IDMPM2T4 | 9.8329e-1 (9.78e-6) − | 9.8301e-1 (1.39e-4) − | 9.8312e-1 (2.78e-4) − | 9.8316e-1 (1.51e-4) − | 9.8336e-1 (1.57e-5) ≈ | 9.8292e-1 (3.37e-4) − | 9.8336e-1 (2.85e-5) |
| IDMPM3T1 | 9.6195e-1 (3.08e-4) + | 9.5070e-1 (2.90e-3) − | 9.5753e-1 (1.96e-3) − | 9.5997e-1 (1.39e-3) − | 9.6069e-1 (1.61e-3) ≈ | 9.5633e-1 (2.30e-3) − | 9.6077e-1 (1.63e-3) |
| IDMPM3T2 | 9.6218e-1 (3.25e-4) + | 9.5631e-1 (2.37e-3) − | 9.5757e-1 (1.72e-3) − | 9.6024e-1 (1.61e-3) − | 9.6114e-1 (9.03e-4) ≈ | 9.5619e-1 (3.03e-3) − | 9.6115e-1 (1.36e-3) |
| IDMPM3T3 | 9.6163e-1 (9.49e-4) + | 9.5564e-1 (1.91e-3) − | 9.5686e-1 (2.28e-3) − | 9.5890e-1 (1.35e-3) − | 9.6062e-1 (1.48e-3) ≈ | 9.5445e-1 (3.17e-3) − | 9.6064e-1 (1.51e-3) |
| IDMPM3T4 | 9.6190e-1 (3.24e-4) ≈ | 9.5546e-1 (2.11e-3) − | 9.5828e-1 (3.00e-3) − | 9.6015e-1 (1.30e-3) − | 9.6078e-1 (1.30e-3) − | 9.5668e-1 (2.46e-3) − | 9.6162e-1 (1.04e-3) |
| IDMPM4T1 | 9.2833e-1 (1.34e-3) − | 8.9324e-1 (4.99e-3) − | 9.2802e-1 (2.79e-3) − | 9.3178e-1 (1.57e-3) − | 9.2300e-1 (3.90e-3) − | 9.1496e-1 (8.27e-3) − | 9.3266e-1 (1.72e-3) |
| IDMPM4T2 | 9.2884e-1 (9.61e-4) − | 9.1667e-1 (3.32e-3) − | 9.2986e-1 (2.44e-3) − | 9.3253e-1 (1.25e-3) − | 9.2581e-1 (3.28e-3) − | 9.1699e-1 (6.81e-3) − | 9.3357e-1 (1.89e-3) |
| IDMPM4T3 | 9.2580e-1 (2.71e-3) − | 9.1472e-1 (4.61e-3) − | 9.2783e-1 (2.55e-3) − | 9.3067e-1 (1.76e-3) − | 9.2574e-1 (3.25e-3) − | 9.1451e-1 (7.31e-3) − | 9.3237e-1 (1.98e-3) |
| IDMPM4T4 | 9.2825e-1 (1.42e-3) − | 9.1424e-1 (4.37e-3) − | 9.3033e-1 (2.04e-3) − | 9.3165e-1 (1.93e-3) − | 9.2651e-1 (2.82e-3) − | 9.1183e-1 (1.05e-2) − | 9.3382e-1 (1.75e-3) |
| +/ − / ≈ | 3/7/2 | 0/12/0 | 0/12/0 | 0/12/0 | 1/5/6 | 0/12/0 | |

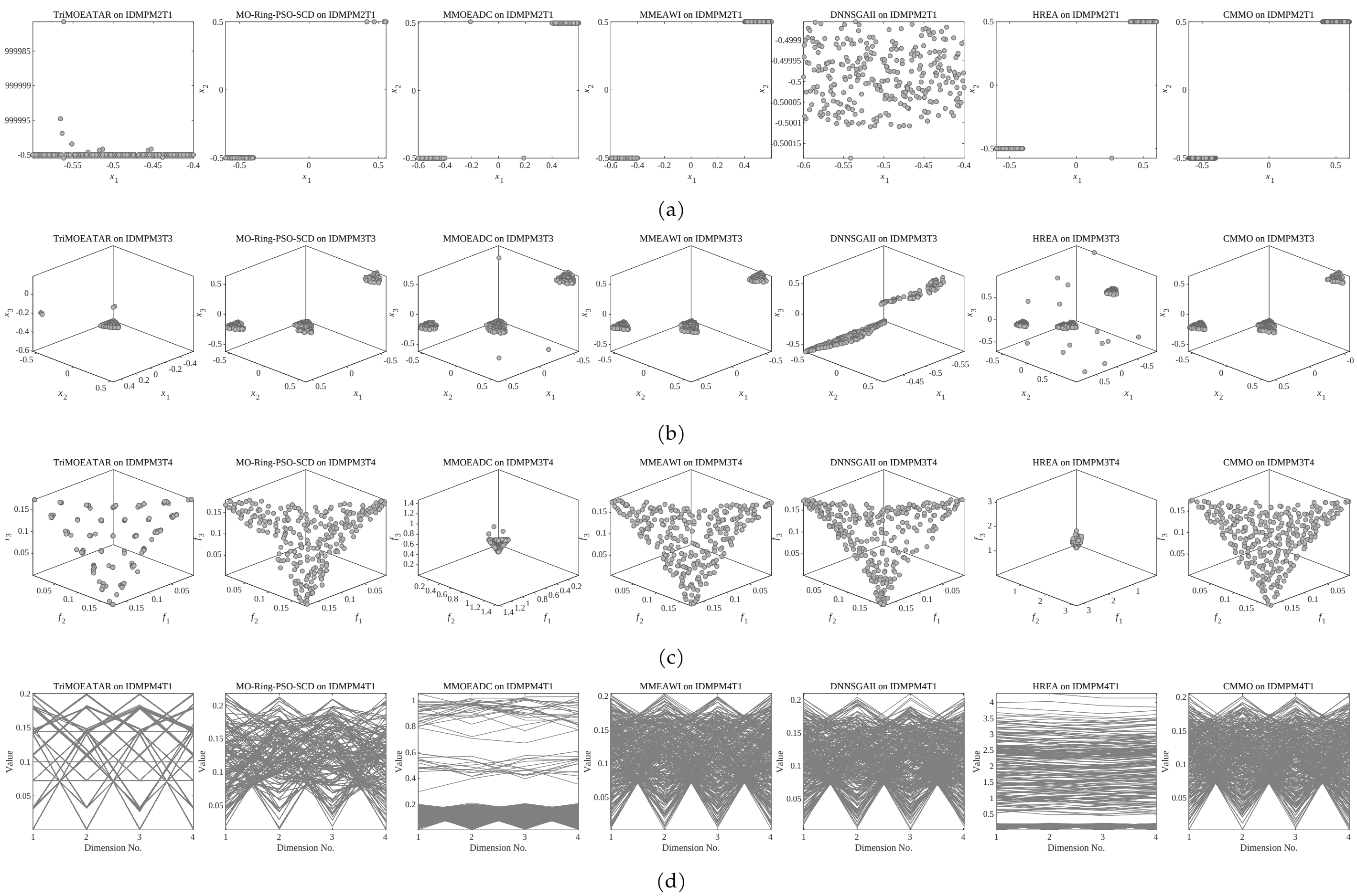



Fig. S-8. PSs obtained by CMMO and other methods with the median IGDX values among 30 runs on IDMPM2T1 and IDMPM3T3. PFs obtained by CMMO and other methods with the median HV values among 30 runs on IDMPM3T4 and IDMPM4T1.

TABLE S-XVI

Statistical results of IGDX obtained by CMMO and CMMO with different parameter settings of $\eta$ on MMF benchmark problems. Best result in each row is highlighted.

| Problem | CMMO$\eta$1 | CMMO$\eta$2 | CMMO$\eta$3 | CMMO |
|---|---|---|---|---|
| MMF1 | 2.2076e-2 (4.46e-4) $-$ | 2.2389e-2 (5.08e-4) $-$ | 2.2204e-2 (5.34e-4) $-$ | 2.1501e-2 (3.79e-4) |
| MMF2 | 7.7578e-3 (1.53e-3) $\approx$ | 8.1042e-3 (1.66e-3) $\approx$ | 7.5972e-3 (9.28e-4) $\approx$ | 7.3586e-3 (9.15e-4) |
| MMF3 | 6.4861e-3 (9.57e-4) $\approx$ | 6.4263e-3 (1.00e-3) $\approx$ | 6.6763e-3 (9.06e-4) $-$ | 6.2426e-3 (6.22e-4) |
| MMF4 | 1.3084e-2 (2.67e-4) $+$ | 1.3042e-2 (3.19e-4) $+$ | 1.3046e-2 (3.33e-4) $+$ | 1.3392e-2 (4.20e-4) |
| MMF5 | 4.2631e-2 (1.46e-3) $-$ | 4.2649e-2 (1.58e-3) $-$ | 4.2152e-2 (1.11e-3) $-$ | 4.0969e-2 (8.63e-4) |
| MMF6 | 3.9758e-2 (1.06e-3) $-$ | 3.9671e-2 (9.42e-4) $-$ | 3.9739e-2 (1.15e-3) $-$ | 3.8460e-2 (5.16e-4) |
| MMF7 | 1.3480e-2 (3.23e-4) $+$ | 1.3475e-2 (2.72e-4) $+$ | 1.3392e-2 (4.26e-4) $+$ | 1.3798e-2 (3.50e-4) |
| MMF8 | 2.7344e-2 (1.80e-3) $\approx$ | 2.7741e-2 (2.14e-3) $\approx$ | 2.8613e-2 (3.22e-3) $\approx$ | 2.7783e-2 (1.94e-3) |
| $+/-/\approx$ | 2/3/3 | 2/3/3 | 2/4/2 | |

TABLE S-XVII

Statistical results of HV obtained by CMMO and CMMO with different parameter settings of $\eta$ on MMF benchmark problems. Best result in each row is highlighted.

| Problem | CMMO$\eta$1 | CMMO$\eta$2 | CMMO$\eta$3 | CMMO |
|---|---|---|---|---|
| MMF1 | 9.0707e-1 (7.57e-5) $-$ | 9.0704e-1 (1.08e-4) $-$ | 9.0704e-1 (8.15e-5) $-$ | 9.0722e-1 (1.69e-4) |
| MMF2 | 8.6057e-1 (1.36e-4) $\approx$ | 8.6054e-1 (1.31e-4) $\approx$ | 8.6045e-1 (1.43e-4) $-$ | 8.6056e-1 (1.04e-4) |
| MMF3 | 8.1440e-1 (1.09e-4) $\approx$ | 8.1437e-1 (1.11e-4) $\approx$ | 8.1419e-1 (1.72e-4) $-$ | 8.1443e-1 (1.06e-4) |
| MMF4 | 7.2311e-1 (6.14e-5) $-$ | 7.2312e-1 (5.31e-5) $-$ | 7.2308e-1 (7.56e-5) $-$ | 7.2323e-1 (5.70e-5) |
| MMF5 | 9.6894e-1 (9.21e-5) $-$ | 9.6894e-1 (9.38e-5) $-$ | 9.6893e-1 (1.14e-4) $-$ | 9.6903e-1 (4.47e-5) |
| MMF6 | 9.5344e-1 (1.14e-4) $-$ | 9.5346e-1 (8.36e-5) $-$ | 9.5345e-1 (8.59e-5) $-$ | 9.5356e-1 (6.60e-5) |
| MMF7 | 8.8451e-1 (1.09e-4) $-$ | 8.8451e-1 (6.42e-5) $-$ | 8.8452e-1 (7.22e-5) $-$ | 8.8469e-1 (4.09e-5) |
| MMF8 | 9.7057e-1 (3.06e-4) $-$ | 9.7053e-1 (3.02e-4) $-$ | 9.7065e-1 (3.28e-4) $\approx$ | 9.7078e-1 (1.96e-4) |
| $+/-/\approx$ | 0/6/2 | 0/6/2 | 0/7/1 | |

TABLE S-XVIII

Statistical results of IGDX obtained by CMMO and CMMO with different parameter settings of $\tau$ on MMF benchmark problems. Best result in each row is highlighted.

| Problem | CMMO$\tau$1 | CMMO$\tau$2 | CMMO$\tau$3 | CMMO |
|---|---|---|---|---|
| MMF1 | 2.2065e-2 (4.94e-4) $-$ | 2.2033e-2 (4.28e-4) $-$ | 2.2261e-2 (5.35e-4) $-$ | 2.1501e-2 (3.79e-4) |
| MMF2 | 7.8364e-3 (1.03e-3) $\approx$ | 7.5718e-3 (1.13e-3) $\approx$ | 6.9498e-3 (6.29e-4) $\approx$ | 7.3586e-3 (9.15e-4) |
| MMF3 | 6.5805e-3 (1.34e-3) $\approx$ | 6.3795e-3 (7.83e-4) $\approx$ | 6.3186e-3 (9.94e-4) $\approx$ | 6.2426e-3 (6.22e-4) |
| MMF4 | 1.2997e-2 (3.60e-4) $+$ | 1.3071e-2 (4.03e-4) $+$ | 1.3167e-2 (3.62e-4) $+$ | 1.3392e-2 (4.20e-4) |
| MMF5 | 4.2063e-2 (9.00e-4) $-$ | 4.2029e-2 (9.66e-4) $-$ | 4.2645e-2 (1.61e-3) $-$ | 4.0969e-2 (8.63e-4) |
| MMF6 | 3.9976e-2 (8.59e-4) $-$ | 3.9743e-2 (7.42e-4) $-$ | 3.9872e-2 (8.89e-4) $-$ | 3.8460e-2 (5.16e-4) |
| MMF7 | 1.3473e-2 (3.88e-4) $+$ | 1.3363e-2 (2.72e-4) $+$ | 1.3584e-2 (3.24e-4) $+$ | 1.3798e-2 (3.50e-4) |
| MMF8 | 2.8773e-2 (3.41e-3) $\approx$ | 2.7798e-2 (2.60e-3) $\approx$ | 2.7938e-2 (2.27e-3) $\approx$ | 2.7783e-2 (1.94e-3) |
| $+/-/\approx$ | 2/3/3 | 2/3/3 | 2/3/3 | |

TABLE S-XIX

STATISTICAL RESULTS OF HV OBTAINED BY CMMO AND CMMO WITH DIFFERENT PARAMETER SETTINGS OF $\tau$ ON MMF BENCHMARK PROBLEMS. BEST RESULT IN EACH ROW IS HIGHLIGHTED.

| Problem | CMMO$\tau$1 | CMMO$\tau$2 | CMMO$\tau$3 | CMMO |
|---|---|---|---|---|
| MMF1 | 9.0705e-1 (8.91e-5) $-$ | 9.0705e-1 (1.02e-4) $-$ | 9.0701e-1 (1.69e-4) $-$ | 9.0722e-1 (1.69e-4) |
| MMF2 | 8.6052e-1 (1.12e-4) $\approx$ | 8.6056e-1 (9.02e-5) $\approx$ | 8.6064e-1 (1.00e-4) $+$ | 8.6056e-1 (1.04e-4) |
| MMF3 | 8.1433e-1 (1.67e-4) $-$ | 8.1435e-1 (1.08e-4) $-$ | 8.1438e-1 (1.10e-4) $\approx$ | 8.1443e-1 (1.06e-4) |
| MMF4 | 7.2312e-1 (5.47e-5) $-$ | 7.2311e-1 (6.03e-5) $-$ | 7.2309e-1 (5.78e-5) $-$ | 7.2323e-1 (5.70e-5) |
| MMF5 | 9.6896e-1 (6.82e-5) $-$ | 9.6895e-1 (8.75e-5) $-$ | 9.6895e-1 (6.25e-5) $-$ | 9.6903e-1 (4.47e-5) |
| MMF6 | 9.5344e-1 (9.84e-5) $-$ | 9.5344e-1 (1.02e-4) $-$ | 9.5344e-1 (8.54e-5) $-$ | 9.5356e-1 (6.60e-5) |
| MMF7 | 8.8452e-1 (9.51e-5) $-$ | 8.8455e-1 (3.98e-5) $-$ | 8.8449e-1 (1.27e-4) $-$ | 8.8469e-1 (4.09e-5) |
| MMF8 | 9.7060e-1 (2.09e-4) $-$ | 9.7054e-1 (2.65e-4) $-$ | 9.7045e-1 (4.05e-4) $-$ | 9.7078e-1 (1.96e-4) |
| $+/-/\approx$ | 0/7/1 | 0/7/1 | 1/6/1 | |

TABLE S-XX

STATISTICAL RESULTS OF IGDX OBTAINED BY CMMO AND CMMO WITH DIFFERENT PARAMETER SETTINGS OF $\theta$ ON MMF BENCHMARK PROBLEMS. BEST RESULT IN EACH ROW IS HIGHLIGHTED.

| Problem | CMMO$\theta$1 | CMMO$\theta$2 | CMMO$\theta$3 | CMMO |
|---|---|---|---|---|
| MMF1 | 2.2141e-2 (4.31e-4) $-$ | 2.2320e-2 (8.24e-4) $-$ | 2.2123e-2 (3.83e-4) $-$ | 2.1501e-2 (3.79e-4) |
| MMF2 | 7.5824e-3 (1.35e-3) $\approx$ | 7.2367e-3 (9.38e-4) $\approx$ | 8.0311e-3 (1.91e-3) $\approx$ | 7.3586e-3 (9.15e-4) |
| MMF3 | 6.3977e-3 (8.84e-4) $\approx$ | 6.4072e-3 (7.80e-4) $\approx$ | 6.2173e-3 (7.09e-4) $\approx$ | 6.2426e-3 (6.22e-4) |
| MMF4 | 1.2969e-2 (3.54e-4) $+$ | 1.2980e-2 (3.11e-4) $+$ | 1.3003e-2 (3.59e-4) $+$ | 1.3392e-2 (4.20e-4) |
| MMF5 | 4.2418e-2 (1.04e-3) $-$ | 4.2693e-2 (1.56e-3) $-$ | 4.2751e-2 (1.54e-3) $-$ | 4.0969e-2 (8.63e-4) |
| MMF6 | 3.9697e-2 (8.38e-4) $-$ | 3.9857e-2 (1.13e-3) $-$ | 3.9457e-2 (8.93e-4) $-$ | 3.8460e-2 (5.16e-4) |
| MMF7 | 1.3509e-2 (4.47e-4) $+$ | 1.3405e-2 (4.11e-4) $+$ | 1.3540e-2 (4.39e-4) $+$ | 1.3798e-2 (3.50e-4) |
| MMF8 | 2.9514e-2 (3.97e-3) $\approx$ | 2.8397e-2 (2.97e-3) $\approx$ | 2.7795e-2 (2.03e-3) $\approx$ | 2.7783e-2 (1.94e-3) |
| $+/-/\approx$ | 2/3/3 | 2/3/3 | 2/3/3 | |

TABLE S-XXI

STATISTICAL RESULTS OF IGDX OBTAINED BY CMMO AND CMMO WITH DIFFERENT PARAMETER SETTINGS OF $\theta$ ON MMF BENCHMARK PROBLEMS. BEST RESULT IN EACH ROW IS HIGHLIGHTED.

| Problem | CMMO$\theta$1 | CMMO$\theta$2 | CMMO$\theta$3 | CMMO |
|---|---|---|---|---|
| MMF1 | 9.0702e-1 (1.41e-4) $-$ | 9.0706e-1 (9.53e-5) $-$ | 9.0704e-1 (1.10e-4) $-$ | 9.0722e-1 (1.69e-4) |
| MMF2 | 8.6059e-1 (9.33e-5) $\approx$ | 8.6056e-1 (1.08e-4) $\approx$ | 8.6053e-1 (1.19e-4) $\approx$ | 8.6056e-1 (1.04e-4) |
| MMF3 | 8.1432e-1 (1.14e-4) $-$ | 8.1437e-1 (1.30e-4) $\approx$ | 8.1438e-1 (1.01e-4) $\approx$ | 8.1443e-1 (1.06e-4) |
| MMF4 | 7.2310e-1 (6.13e-5) $-$ | 7.2310e-1 (7.13e-5) $-$ | 7.2310e-1 (6.99e-5) $-$ | 7.2323e-1 (5.70e-5) |
| MMF5 | 9.6895e-1 (9.22e-5) $-$ | 9.6897e-1 (6.94e-5) $-$ | 9.6894e-1 (9.04e-5) $-$ | 9.6903e-1 (4.47e-5) |
| MMF6 | 9.5341e-1 (1.52e-4) $-$ | 9.5343e-1 (1.59e-4) $-$ | 9.5346e-1 (1.08e-4) $-$ | 9.5356e-1 (6.60e-5) |
| MMF7 | 8.8452e-1 (7.58e-5) $-$ | 8.8454e-1 (5.42e-5) $-$ | 8.8455e-1 (4.50e-5) $-$ | 8.8469e-1 (4.09e-5) |
| MMF8 | 9.7058e-1 (1.91e-4) $-$ | 9.7059e-1 (3.05e-4) $-$ | 9.7059e-1 (2.09e-4) $-$ | 9.7078e-1 (1.96e-4) |
| $+/-/\approx$ | 0/7/1 | 0/6/2 | 0/6/2 | |