Introduction to Theoretical Computer Science, Fall 2024
Assignment 9

Q1. Prove that if A is in $\mathcal{P}$, so is $\overline{A}$.

Q2. Let A be some language in P. Show that A^* is also in P. (Hint: Use dynamic programming. On input $y = y_1 \cdots y_n$ for $y_i \in \Sigma$, build a table indicating for each $i \leq j$ whether the substring $y_i \cdots y_j \in A^*$.)

Q3. Define co-$\mathcal{NP}$ to be the following set of languages.

$$\text{co-}\mathcal{NP} = \{A : \overline{A} \in \mathcal{NP}\}$$

Prove that $\mathcal{P} \subseteq \mathcal{NP} \cap \text{co-}\mathcal{NP}$.

Q4. Let $\text{DOUBLE} - \text{SAT}$ be

$$\{\text{“}\varphi\text{”} : \varphi \text{ is a cnf formular that has at least two satisfying assignments}\}.$$

Show that $\text{DOUBLE} - \text{SAT}$ is NP-complete. (Hint: you may reduce SAT to $\text{DOUBLE} - \text{SAT}$)

Q5. A subset of the nodes of a graph G is a dominating set if every other node of G is adjacent to some node in the subset. Let

$$\text{DOMINATING} - \text{SET} = \{\text{“}G\text{”}\ \text{“}k\text{”} — G \text{ has a dominating set with } k \text{ nodes}\}.$$

Show that it is NP-complete. (Hint: you may reduce VERTEX-COVER to DOMINATING-SET.)