

# Introduction to Theoretical Computer, Fall 2024

## Assignment 1 (Due September 23 Monday 4:00 pm)

Only the problems in part I will be graded.

### 1 Part I

Q1. Are the following statements true or false? No explanation is required.

- (a) Let  $w$  be a string. The number of distinct symbols that appear in  $w$  must be finite.
- (b) Let  $L$  be a language. The number of distinct symbols that appear in  $L$  must be finite.
- (c) Every finite automaton accepts one and only one string.
- (d) Every finite automaton accepts one and only one language.
- (e) If  $(q_1, w_1) \vdash_M^* (q_2, w_2)$  and  $(q_2, w_2) \vdash_M (q_3, w_3)$ , then  $(q_1, w_1) \vdash_M^* (q_3, w_3)$ .
- (f) If a language is finite, it must be regular.
- (g) Let  $L$  be a language over some alphabet  $\Sigma$ .  $L$  is also a language over any alphabet that is a superset of  $\Sigma$ .

Q2. Prove that the following languages are regular.

$$L = \{w \in \{0, 1\}^* : w \text{ ends with } 00.\}$$

Q3. Let  $A$  and  $B$  be two regular languages over some alphabet  $\Sigma$ . Define

$$A \cap B = \{w : w \in A \wedge w \in B\}.$$

Show that  $A \cap B$  is also regular. (Hint: think about how we proved  $A \cup B$  is regular in class.)

Q4. Let  $A$  be a regular language over  $\Sigma$ . Consider the following language.

$$\overline{A} = \{w \in \Sigma^* : w \notin A\}$$

Show that  $\overline{A}$  is regular.

### 2 Part II

Q5. Read Chapter 2 (Computation and Representation) of the book Introduction to Theoretical Computer Science.

Q6. Prove that the following language is regular.

$$L = \{w \in \{0, 1\}^* : w \text{ starts with } 1 \text{ and is a multiple of } 3 \text{ when interpreted as a binary integer}\}$$

(Hint: use states to maintain the remainder of number read so far, when divided by 3.)

Q7. Let  $M = (K, \Sigma, \delta, s, F)$  be a finite automaton. Let  $\Sigma'$  be some superset of  $\Sigma$ . Extend the input alphabet of  $M$  to  $\Sigma'$  without changing the language it accepts. In other words, you need to construct a finite automaton  $M' = (K', \Sigma', \delta', s', F')$  such that  $L(M') = L(M)$ .

Q8. Let  $A$  and  $B$  be two regular languages over some alphabet  $\Sigma$ . Let  $\#$  be a symbol not in  $\Sigma$ . Consider the following language.

$$L = \{w_1 \# w_2 : w_1 \in A \wedge w_2 \in B\}$$

Show that  $L$  is also regular.