

Introduction to Theoretical Computer Science, Fall 2024  
Assignment 5 (Due October 21 Monday 4:00 pm)

Only part I will be graded.

## 1 Part I

- Q1. Show that if  $L$  is context-free, so is

$$L^R = \{w^R : w \in L\}.$$

- Q2. Show that the following language is not context free.

$$L = \{w \in \{1, 2, 3, 4\}^* : \text{the number of 1's equals the number of 2's,}\\ \text{and the number of 3's equals the number of 4's}\}$$

## 2 Part II

- Q3. Let  $A$  be a context-free language. Let  $B$  be a regular language. Prove that  $A \cap B$  is context-free. You may assume that  $A$  and  $B$  are defined over the same alphabet  $\Sigma$ . (Hint: let  $P_A$  be a PDA accepting  $A$ . Let  $M_B$  be an NFA accepting  $B$ . Construct a PDA  $P_{\cap}$  that conceptually runs  $P_A$  and  $M_B$  in parallel.)