Introduction to Theoretical Computer Science, Fall 2024
Assignment 3 (Due September 30 Monday 4:00 pm)

Only part I will be graded.

1 Part I

Q1. For any two regular expressions R_1 and R_2, we say $R_1 = R_2$ if $L(R_1) = L(R_2)$. Let R be a regular expression. Are the following statements true or false? Provide counterexamples for false statements.

(a) $R \cup \emptyset = R$

(b) $R\emptyset = R$

(c) $R \cup \emptyset^* = R$

(d) $R\emptyset^* = R$

Q2. Write a regular expression for the language

$$\{w \in \{a, b\}^* : \text{the number of } b\text{'s in } w \text{ is divisible by } 3\}.$$

Q3. Consider the NFA M in Figure 1. Construct a regular expression R such that $L(R) = L(N)$. You should strictly follow the algorithm we used in the class, and show all the intermediate steps. More precisely, you should first convert N into an equivalent NFA that satisfies certain conditions, and then eliminate state q_1 and q_2 in order.

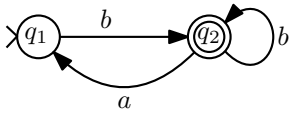

Figure 1: M

Q4. Use pumping theorem to show that the language $\{ww : w \in \{a, b\}^*\}$ is not regular.

2 Part II

Q5. Show that the language $\{0^m 1^n : m \neq n\}$ is not regular. (Hint: you may find that pumping theorem does not work well in this case. Try the closure property.)

Q6. Let A be a regular language. Let B be an arbitrary language. Show that the following language is regular.

$$A/B = \{w | wx \in A \text{ for some } x \in B\}$$