Algorithm 1 Monte Carlo Simulation for Discrete Choice Model

Input:
N: Number of individuals
D: Dimensions of product characteristics (≥ 3)
J: Number of alternatives (≥ 3)
T: Time periods (≥ 2)
B: Monte Carlo repetitions
β_0: True parameter vector
procedure DataGeneration
  for $b = 1$ to B **do**
    Generate latent variable $Z_i \sim \mathcal{N}(0, 1)$
    Initialize $X \in R^{N \times D \times J \times T}$:
      $X_{:,1,:,:} \sim \text{Uniform}(-1, 1)$
      $X_{:,2,:,:} \sim \mathcal{N}(Z_i, 2\sigma)$
      $X_{:,3:D,:,:} \sim \mathcal{N}(0, 1)$
    Generate A components:
      $A_{\text{scale}} \sim \text{Uniform}(2, 2.5)$
      $A_{\text{location}}(:, 1) = 0$
      $A_{\text{location}}(:, 2) = \max(Z_i, 0)$
      $A_{\text{location}}(:, 3 : J) \sim \text{Uniform}(-0.25, 0.25)$
    Compute utility: $U = A_{\text{scale}} \circ (X\beta_0 + A_{\text{location}}) + \epsilon$
    Generate choices $y = I[U = \max(U)]$
  end for
end procedure
procedure Estimation
  for $b = 1$ to B **do**
    Compute differences: $\Delta X = X_t - X_s$, $\Delta y = y_t - y_s$
    Estimate $E[\Delta y | \Delta X]$ using LASSO regression
    Initialize search space $\Theta = [-\pi/2, \pi/2] \times [-\pi, \pi]$
    while not converged **do**
      Discretize Θ into $M \times M$ grid
      Evaluate $Q(\theta) = \|\Delta y - \Delta X \beta(\theta)\|^2$
      Identify $\theta_{\min}$ minimizing $Q(\theta)$
      Refine Θ around $\theta_{\min}$
      Reduce tolerance $\tau \leftarrow \tau/2$
    end while
    Recover $\hat{\beta} = [\cos\theta_1 \cos\theta_2, \cos\theta_1 \sin\theta_2, \sin\theta_1]$
    Store estimates $\beta_B(b)$ and bounds
  end for
end procedure
procedure PerformanceEvaluation
  Compute bias: Mid_bias $= E[\hat{\beta} - \beta_0]$
  Compute MSE: Mid_MSE $= E[(\hat{\beta} - \beta_0)^2]$
  Compute bounds: dUL_mean $= E[\beta_{\text{upper}} - \beta_{\text{lower}}]$
  Output summary statistics
end procedure