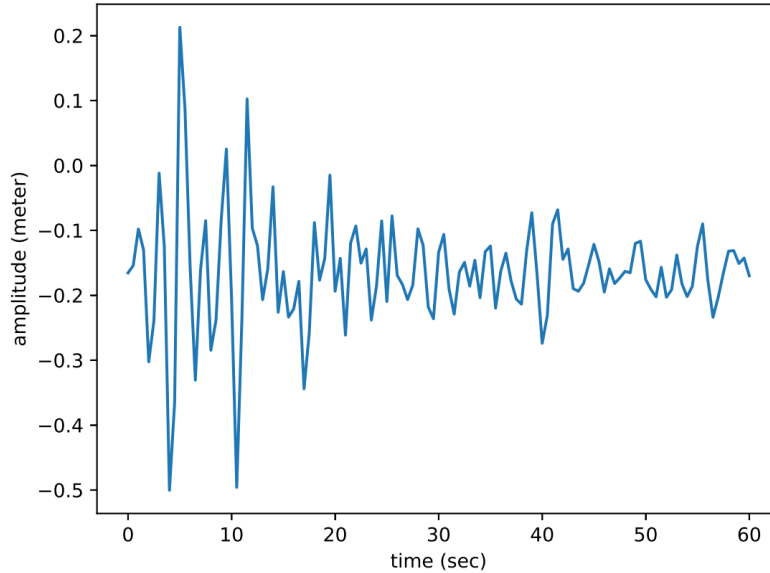(100 points)

Question: In this homework, you will need to write a code to do Fourier transformation on a time-series data and filter the data within a range of period. The data is named "HW11.input.dat" under the homework folder. A plot of the data is shown below:

You may notice that this is the same data for HW06 in which we practiced the method of interpolation:)

Review:

If $f(x)$ is a function defined on $[0,2L]$, and is represented by a list of equally spaced discrete data points of $(x_i, f(x_i))$ for $i = 1, 2, 3, \dots, np$, where np is the number of data points, $x_1 = 0$ is x value for the first data point, $x_{np} = 2L$ is the x value for the last data points, Δx is the increment of x between every two neighboring data points, then $f(x)$ can be represented by:

$$f(x) = a_0 + \sum_{n=1}^{\frac{np-1}{2}} [a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right)], (1)$$

where,

$$a_0 = \frac{\Delta x}{2L} \sum_{i=1}^{np-1} f(x_i), (2)$$

$$a_n = \frac{\Delta x}{L} \sum_{i=1}^{np-1} f(x_i)\cos\left(\frac{n\pi x_i}{L}\right), (3)$$

$$b_n = \frac{\Delta x}{L} \sum_{i=1}^{np-1} f(x_i) \sin\left(\frac{n\pi x_i}{L}\right), (4)$$

The amplitude at period of $T_n = 2L/n$ is given by $A_n = \sqrt{a_n^2 + b_n^2}$, for n ranges from 1 to $(np - 1)/2$.

Because $(np - 1)/2$ needs to be an integer, therefore, np needs to be an odd number. If np is an even number in the data, you need to reduce 1 last point of the data, by $np = np - 1$.

Requirements:

- You need to output the amplitude for the period in the range of [1.0, 60.0] into a file named "dft.dat", then make a plot of this file, same as below (or hw11-1.pdf):
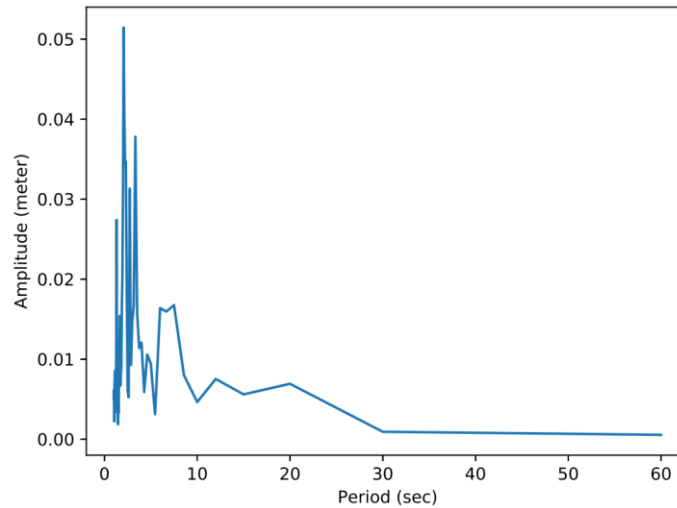
- You need to output the x values (1st column), the observed y values (2nd column), and the approximated y values recovered by dft (3rd column) in a single file named "compare.dat", and then plot the 1st column vs the 2nd and the 3rd column. An example plot is shown below (or hw11-2.pdf):
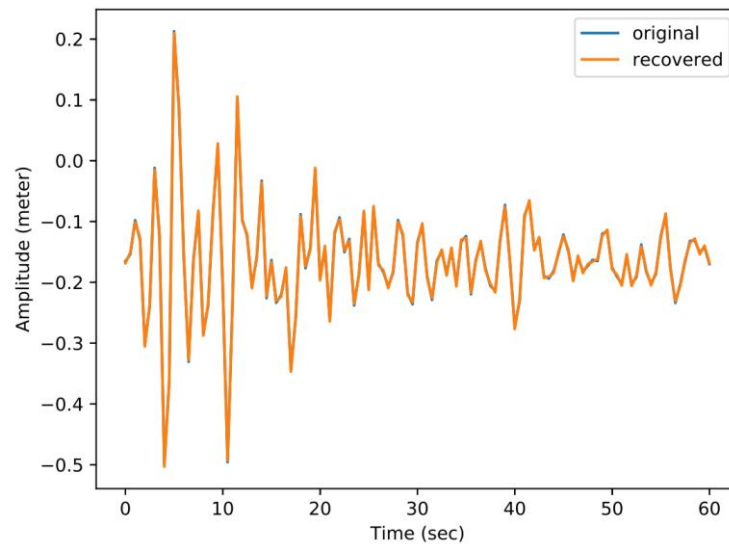
- Filter the data in the period range of [5,10], save it in a file named "filter.dat", and then plot the filtered data and the original data in the same plot. See an example below (or hw11-3.pdf):
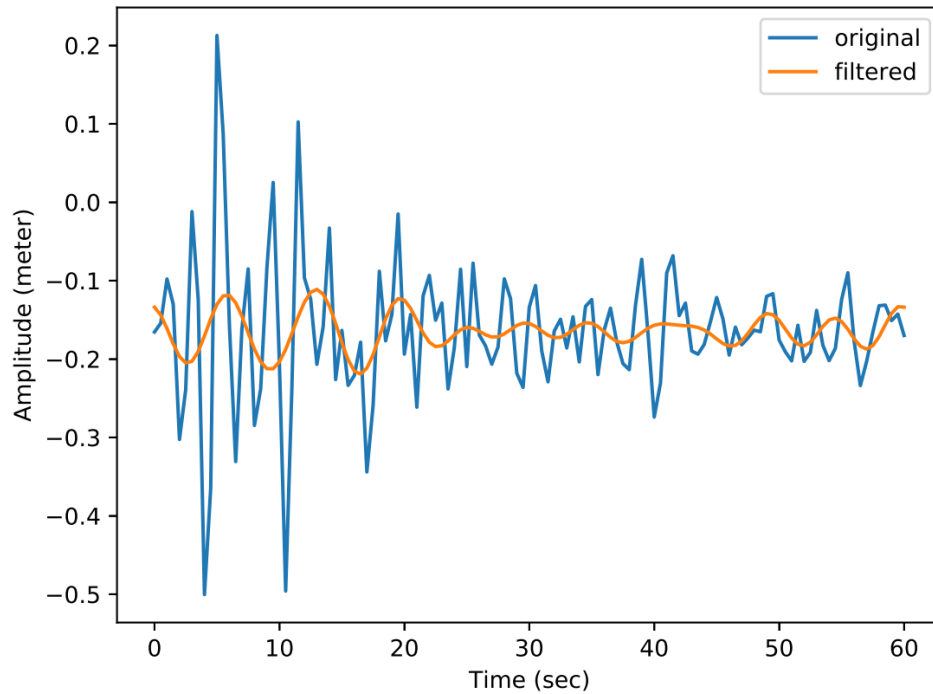
- For your comparison, I have provided my output of dft.dat, compare.dat, and filter.dat in the homework folder.

How to submit your homework

1. Change the name of your C code as 'FirstName-LastName-HW11.c'.
2. Send your code file to Mingming.Li@asu.edu and enter the email subject title as "Numerical Methods Homework 11".