GLG490/598 Numerical methods
Homework #7

Due 11:59pm, 3/18/2021

(100 points)

Question: Solve the following initial value problem of:

$$\begin{cases} y' = S(x,y) = x + y \\ y(0) = 1.0 \end{cases}$$

Use a step-size of h=0.5, and find the solution at x=0.5 and x=1.0.

Review:

The slope is defined as $S(x,y)$, which is a function of both x and y.
In this problem, $S(x,y) = x + y$ and we use the following notation:
x_i: the starting value of x, at the beginning step.
h: the step size.
$x_{i+1} = x_i + h$: the value of x at the end of the step.
$y_i = y(x_i)$: the starting value of y at the beginning of the step.
$y_{i+1} = y(x_{i+1})$: the value of y at the end of the step.
$S(x_i, y_i)$: the slope evaluated at (x_i, y_i).

Euler method:

$$K_0 = hS(x_i, y_i)$$

$$y_{i+1} = y_i + K_0$$

Backward Euler method:

$$K_0 = hS(x_i, y_i)$$

$$K_1 = hS(x_i + h, y_i + K_0)$$

$$y_{i+1} = y_i + K_1$$

Heun's method:

$$K_0 = hS(x_i, y_i)$$

$$K_1 = hS(x_i + h, y_i + K_0)$$

$$y_{i+1} = y_i + \frac{1}{2}(K_0 + K_1)$$

Midpoint method:

$$K_0 = hS(x_i, y_i)$$

$$K_1 = hS(x_i + \frac{1}{2}h, y_i + \frac{1}{2}K_0)$$

$$y_{i+1} = y_i + K_1$$

Requirements:

- Write a C code to use Euler method, Backward Euler method, Heun's method and Midpoind method to solve the differential equation.
- Write the code step-by-step. If you are not sure how to use for loop iteration, you can first write out all the iterations (e.g., the same as when you solve the problem by hand), and then find a way to simplify the code with loops. Luckily, there are only 2 iterations in this problem. Compile your code frequently after you add a few lines to make sure there is no bug. You may also want to define a function named as S to calculate $S(x, y)$. This way, your code can be modified very easily to solve other differential equations by simply changing the expression of $S(x, y)$.
- You code needs to output the **exact** following information:

```
$ ./a.exe
Euler method: x=0.500000 K0=0.500000 y=1.500000
Euler method: x=1.000000 K0=1.000000 y=2.500000
Backward Euler method: x=0.500000 K0=0.500000 K1=1.000000 y=2.000000
Backward Euler method: x=1.000000 K0=1.250000 K1=2.125000 y=4.125000
Heun' method: x=0.500000 K0=0.500000 K1=1.000000 y=1.750000
Heun' method: x=1.000000 K0=1.125000 K1=1.937500 y=3.281250
Midpoint method: x=0.500000 K0=0.500000 K1=0.750000 y=1.750000
Midpoint method: x=1.000000 K0=1.125000 K1=1.531250 y=3.281250
```

How to submit your homework

1. Change the name of your C code as 'FirstName-LastName-HW07.c'.
2. Send your code file to Mingming.Li@asu.edu and enter the email subject title as "Numerical Methods Homework 07".