In-class exercise 11

2nd-order ODE:

A second order ODE can be easily transformed into two, first order ODEs:

$$y'' = S(x, y, y')$$

Can be transformed into:

$$y' = g$$

$$g' = S(x, y, g)$$

Note we need two pieces of information to solve this problem. If both pieces of information are provided at the same value of the independent variable, we call this problem an "initial value problem", and we can use our previously-derived methods to solve for both y and y'.

Initial values: $y'(x_0) = g(x_0) = g_0, y(x_0) = y_0$

Euler Method: $K_0 = hg_i$ $L_0 = hS(x_i, y_i, g_i)$ $y_{i+1} = y_i + K_0$ $g_{i+1} = g_i + L_0$	**Midpoint method:** $K_0 = hg_i$ $L_0 = hS(x_i, y_i, g_i)$ $K_1 = h(g_i + \frac{1}{2}L_0)$ $L_1 = hS(x_i + \frac{1}{2}h, y_i + \frac{1}{2}K_0, g_i + \frac{1}{2}L_0)$ $y_{i+1} = y_i + K_1$ $g_{i+1} = g_i + L_1$
Backward Euler Method: $K_0 = hg_i$ $L_0 = hS(x_i, y_i, g_i)$ $K_1 = h(g_i + L_0)$ $L_1 = hS(x_i + h, y_i + K_0, g_i + L_0)$ $y_{i+1} = y_i + K_1$ $g_{i+1} = g_i + L_1$ **Heun's Method:** $K_0 = hg_i$ $L_0 = hS(x_i, y_i, g_i)$ $K_1 = h(g_i + L_0)$ $L_1 = hS(x_i + h, y_i + K_0, g_i + L_0)$ $y_{i+1} = y_i + \frac{1}{2}(K_0 + K_1)$ $g_{i+1} = g_i + \frac{1}{2}(L_0 + L_1)$	**4th order Runge-Kutta method:** $K_0 = hg_i$ $L_0 = hS(x_i, y_i, g_i)$ $K_1 = h(g_i + \frac{1}{2}L_0)$ $L_1 = hS(x_i + \frac{1}{2}h, y_i + \frac{1}{2}K_0, g_i + \frac{1}{2}L_0)$ $K_2 = h(g_i + \frac{1}{2}L_1)$ $L_2 = hS(x_i + \frac{1}{2}h, y_i + \frac{1}{2}K_1, g_i + \frac{1}{2}L_1)$ $K_3 = h(g_i + L_2)$ $L_3 = hS(x_i + h, y_i + K_2, g_i + L_2)$ $y_{i+1} = y_i + \frac{1}{6}(K_0 + 2K_1 + 2K_2 + K_3)$ $g_{i+1} = g_i + \frac{1}{6}(L_0 + 2L_1 + 2L_2 + L_3)$

Solve the following differential equation using 4th-order Runge-Kutta method:

$$y'' = x, y(0) = 0, y'(0) = 0$$

Find the value of y when x=1.0.