

(100 points)

**Question:** Solve the following initial value problem of:

$$\begin{cases} y'' + 16y = 24\cos(4x) \\ y(0.0) = 0.0 \\ y'(0.0) = 0.0 \end{cases}$$

and find the solution at  $x=10.0$  or  $y(10.0)$  using the 4<sup>th</sup> order Runge-Kutta method.**Review:****4<sup>th</sup> order Runge-Kutta method:**

$$\begin{aligned} K_0 &= hg_i \\ L_0 &= hS(x_i, y_i, g_i) \\ K_1 &= h(g_i + \frac{1}{2}L_0) \\ L_1 &= hS(x_i + \frac{1}{2}h, y_i + \frac{1}{2}K_0, g_i + \frac{1}{2}L_0) \\ K_2 &= h(g_i + \frac{1}{2}L_1) \\ L_2 &= hS(x_i + \frac{1}{2}h, y_i + \frac{1}{2}K_1, g_i + \frac{1}{2}L_1) \\ K_3 &= h(g_i + L_2) \\ L_3 &= hS(x_i + h, y_i + K_2, g_i + L_2) \\ y_{i+1} &= y_i + \frac{1}{6}(K_0 + 2K_1 + 2K_2 + K_3) \\ g_{i+1} &= g_i + \frac{1}{6}(L_0 + 2L_1 + 2L_2 + L_3) \end{aligned}$$

**Requirements:**

As we discussed in class, the precision of the result depends on the number of steps or iterations used to solve the equation. In this homework, your code needs to start with 1 step ( $n=1$ ), and successively double the number of steps until your final answer converges (e.g., the value do not change significantly any more), based on the following criteria:

$$|y(10.0)_{2n \text{ steps}} - y(10.0)_{n \text{ steps}}| < 0.00001$$

In C, the function of `fabs()` is used to get the absolute value. For example, `fabs(-1.0) = 1.0`. You need to use this function in your code.

Your code needs to print out the same information as `y(10.0)=22.353395`

**How to submit your homework**

1. Change the name of your C code as 'FirstName-LastName-HW09.c'.
2. Send your code file to [Mingming.Li@asu.edu](mailto:Mingming.Li@asu.edu) and enter the email subject title as "Numerical Methods Homework 09".