GLG490/598 Numerical methods
Homework #8

Due 11:59pm, 3/25/2021

(100 points)

Question: Solve the following initial value problem of:

$$\begin{cases} y' = S(x,y) = \dfrac{x+y}{x} \\ y(2.0) = 2.0 \end{cases}$$

and find the solution at x=5.0 or $y(5.0)$.

Review:

The slope is defined as $S(x,y)$, which is a function of both x and y.
In this problem, $S(x,y) = x + y$ and we use the following notation:
x_i: the starting value of x, at the beginning step.
h: the step size.
$x_{i+1} = x_i + h$: the value of x at the end of the step.
$y_i = y(x_i)$: the starting value of y at the beginning of the step.
$y_{i+1} = y(x_{i+1})$: the value of y at the end of the step.
$S(x_i, y_i)$: the slope evaluated at (x_i, y_i).

Euler method:

$$K_0 = hS(x_i, y_i)$$

$$y_{i+1} = y_i + K_0$$

Backward Euler method:

$$K_0 = hS(x_i, y_i)$$

$$K_1 = hS(x_i + h, y_i + K_0)$$

$$y_{i+1} = y_i + K_1$$

Heun's method:

$$K_0 = hS(x_i, y_i)$$

$$K_1 = hS(x_i + h, y_i + K_0)$$

$$y_{i+1} = y_i + \frac{1}{2}(K_0 + K_1)$$

Midpoint method:

$$K_0 = hS(x_i, y_i)$$

$$K_1 = hS(x_i + \frac{1}{2}h, y_i + \frac{1}{2}K_0)$$

$$y_{i+1} = y_i + K_1$$

4th-order Runge-Kutta method:

$$K_0 = hS(x_i, y_i)$$

$$K_1 = hS(x_i + \frac{1}{2}h, y_i + \frac{1}{2}K_0)$$

$$K_2 = hS(x_i + \frac{1}{2}h, y_i + \frac{1}{2}K_1)$$

$$K_3 = hS(x_i + h, y_i + K_2)$$

$$y_{i+1} = y_i + \frac{1}{6}(K_0 + 2K_1 + 2K_2 + K_3)$$

Requirements:

- Write a C code to use Euler method, Backward Euler method, Heun's method and Midpoint method and 4th-order Runge-Kutta method to solve the differential equation.
- If the number of step is n, then the step size is calculated by $h = \frac{x_{final} - x_{initial}}{n}$, where $x_{final} = 5.0$ and $x_{initial} = 2.0$ in this homework. Here, you need to test the results of $y(5.0)$ for n varying from 1 to 10 with a step size of 1.
- Your results need to be **exactly** the same as below, where the first column is the number of n, and the 2-6 columns are the results for the Euler, Backward Euler, Heun's, Midpoint, and 4th-order Runge-Kutta methods, respectively.

```
$ ./a.exe
 1 8.000000 9.800000 8.900000 8.900000 9.524490
 2 8.642857 10.011224 9.327041 9.327041 9.574399
 3 8.916667 9.984722 9.450694 9.450694 9.579702
 4 9.067418 9.937120 9.502269 9.502269 9.580836
 5 9.162638 9.894320 9.528479 9.528479 9.581185
 6 9.228175 9.858958 9.543566 9.543566 9.581319
 7 9.276013 9.830044 9.553028 9.553028 9.581379
 8 9.312459 9.806235 9.559347 9.559347 9.581409
 9 9.341145 9.786402 9.563773 9.563773 9.581425
10 9.364308 9.769678 9.566993 9.566993 9.581435
```

- Note that this homework is similar to HW07, expect that you need to write a for loop to increase the number of steps this time and you print out the final answer for each method. You can modify you HW07 code for this homework.
- You may consider using different functions for each method. This way, the code can be more organized, and each method is independent.

How to submit your homework

1. Change the name of your C code as 'FirstName-LastName-HW08.c'.
2. Send your code file to Mingming.Li@asu.edu and enter the email subject title as "Numerical Methods Homework 08".