

First-order ODE

Solution of the 1st order ODE with the following format:

$$y' = S(x, y)$$

The slope is defined as $S(x, y)$, which is a function of both x and y .

In this problem, $S(x, y) = x + y$ and we use the following notation:

x_i : the starting value of x , at the beginning step.

h : the step size.

$x_{i+1} = x_i + h$: the value of x at the end of the step.

$y_i = y(x_i)$: the starting value of y at the beginning of the step.

$y_{i+1} = y(x_{i+1})$: the value of y at the end of the step.

$S(x_i, y_i)$: the slope evaluated at (x_i, y_i) .

Euler method: $K_0 = hS(x_i, y_i)$ $y_{i+1} = y_i + K_0$	Midpoint method: $K_0 = hS(x_i, y_i)$ $K_1 = hS(x_i + \frac{1}{2}h, y_i + \frac{1}{2}K_0)$ $y_{i+1} = y_i + K_1$
Backward Euler method: $K_0 = hS(x_i, y_i)$ $K_1 = hS(x_i + h, y_i + K_0)$ $y_{i+1} = y_i + K_1$	4th-order Runge-Kutta method: $K_0 = hS(x_i, y_i)$ $K_1 = hS(x_i + \frac{1}{2}h, y_i + \frac{1}{2}K_0)$ $K_2 = hS(x_i + \frac{1}{2}h, y_i + \frac{1}{2}K_1)$ $K_3 = hS(x_i + h, y_i + K_2)$ $y_{i+1} = y_i + \frac{1}{6}(K_0 + 2K_1 + 2K_2 + K_3)$
Heun's method: $K_0 = hS(x_i, y_i)$ $K_1 = hS(x_i + h, y_i + K_0)$ $y_{i+1} = y_i + \frac{1}{2}(K_0 + K_1)$	