

Deep Learning

Mingming Li

First Created: September 29, 2020
Last Modified: February 27, 2023

Contents

Contents	iii
List of Figures	vii
List of Tables	ix
I Theory	1
1 Learning	3
2 Model	5
2.1 Capacity	5
2.1.1 The no free lunch theorem	5
2.1.2 Universal approximation theorem	6
2.2 Overfitting and underfitting	6
3 Training (optimization)	7
3.1 SGD	7
3.2 Momentum	8
3.3 Nesterov momentum	8
3.4 AdaGrad	8
3.5 Adam	9

4	Cost function	11
4.1	Regression cost function	11
4.1.1	Mean squared error	11
4.1.2	Mean absolute error	11
4.2	Classification cost function	11
5	Full Connected Networks	13
6	CNN	15
6.1	Convolution	15
6.2	Properties	16
6.2.1	Sparse interaction	16
6.2.2	Parameter sharing	16
6.2.3	Equivariant representations	16
6.3	Pooling	17
7	Metric	19
7.1	Precision	19
7.2	Recall	19
7.3	Accuracy	19
7.4	F-score	20
8	Regularization	21
8.1	Parameter norm penalties	21
8.1.1	L^2 Parameter Regularization	21
8.1.2	L^1 Regularization	21
8.2	Dataset Augmentation	22
8.3	Early stopping	22
8.4	Droupout	22

9	Activation functions	23
9.1	Desirable features	23
9.2	Sigmoid	23
9.3	Tanh	23
9.4	ReLU	23
9.5	Leaky ReLU	24
9.6	Swish	24
10	Machine learning process	25
II	Tools	27
11	PyTorch	29
12	NumPy	31
12.1	Load data	31
III	Practice	33
13	Preprocessing	35
13.1	Not valued based data	35
IV	Projects	37
	Bibliography	39

List of Figures

5.1 Full connected neural network	13
6.1 Convolution operation	16

List of Tables

7.1 Confusion matrix	19
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Part I

Theory

Chapter 1

Learning

A machine learning algorithm is an algorithm that is able to learn from data. But what do we mean by learning? Mitchell provides a succinct definition: “A computer program is said to learn from experience E with respect to some class of tasks T and performance measure P , if its performance at tasks in T , as measured by P , improves with experience E ”

Now, how does the learn happen? The model or algorithm learns by adjusting the parameters contained in it.

How to adjust the parameters? Refer to Chapter 3.

Chapter 2

Model

What is a model?

You can think a model as a function in math. For example,

$$ax + by = c$$

In this function, a, b and d are the parameters, x and y are input from data. Through adjusting the parameters a, b and d , we can implement the learning process. This is also called training in machine learning.

2.1 Capacity

The capacity is the function space. It defines the limitation that can learn from data.

For example,

$$ax + by = c$$

This function can only learn linear function from data.

$$ax_1^2 + bx_2 = c$$

This function can learn curve function from data.

2.1.1 The no free lunch theorem

For any algorithms (functions) a_1 and a_2 , at iteration step m

$$\sum P(d_m^y | f, m, a_1) = \sum P(d_m^y | f, m, a_2) \quad (2.1)$$

where d_m^y denotes the ordered set of size m of the cost values y associated to input values $x \in X$, $f : X \rightarrow Y$ is the function being optimized and $P(d_m^y | f, m, a)$ is the conditional probability of obtaining a given sequence of cost values from algorithm a run m times on function f .

The no free lunch theorem implies that we must design our machine learning algorithms to perform well on a specific task but not a universal task.

2.1.2 Universal approximation theorem

A feed-forward network with a single hidden layer containing a finite number of neurons can approximate continuous functions on compact subsets of R^n , under mild assumptions on the activation function.

Let $\varphi : R \rightarrow R$ be a nonconstant, bounded, and continuous functions. Let I_m denote the m -dimensional unit hypercube $[0, 1]^m$. The space of real-value continuous function on I_m is denoted by $C(I_m)$. Then, given any $\varepsilon > 0$ and function $f \in C(I_m)$, there exist an integer N , real constants $v_i, b_i \in R$ and real vectors $\omega_i \in R^m$ for $i = 1, \dots, N$, such that we may define:

$$F(x) = \sum_{i=1}^N v_i \varphi(\omega_i^T x + b_i) \quad (2.2)$$

as an approximate realization of the function f ; that is

$$|F(x) - f(x)| < \varepsilon \quad (2.3)$$

for all $x \in I_m$. In other words, functions of the form $F(x)$ are dense in $C(I_m)$.

This still holds when replacing I_m with any compact subset of R^m .

Kurt Hornik showed in 1991 that it is not the specific choice of the activation function, but rather the multi-layer feedforward architecture itself which gives neural networks the potential of being universal approximators. The output units are always assumed to be linear. For notational convenience, only the single output case will be shown. The general case can easily be deduced from the single output case.

In 2017 Lu et al. proved universal approximation theorem for width-bounded deep neural networks.

This is the base of deep learning.

2.2 Overfitting and underfitting

We train model on training data but use test data (not used to train the model) to test out model. The ability to perform on test data is called **generalization**. We can use model on test data because we assume that the train data and the test has the same probability distribution (i.e. they have relationship).

The error on training data is called **training error**. The error on test data is called **test error**. Underfitting occurs when the model is not able to obtain a sufficient low error value on the training set. Overfitting occurs when the gap between the training error and test error is too large.

We can control whether a model is more likely to overfit or underfit by altering its **capacity**. The overfitting and underfitting happen because of the mismatch of capacity and the data.

Training (optimization)

Training is the process of learning. By inputting the data, we adjust the parameters in the model to achieve better performance. In machine learning this is also called the optimization.

3.1 SGD

Stochastic gradient descent (SGD) is a very basic and important algorithm.

The cost function used by a machine learning algorithm often decomposes as a sum over training examples of some per-example loss function.

$$J(\theta) = \mathbb{E}_{\mathbf{x}, y \sim \hat{p}_{data}} L(\mathbf{x}, y, \theta) = \frac{1}{m} \sum_{i=1}^m L(\mathbf{x}^{(i)}, y^{(i)}, \theta), \quad (3.1)$$

Where L is the loss function, \mathbf{x} and y are the input data and labels, θ is the parameters in the model, m is the number of samples.

The gradient is

$$\nabla_{\theta} J(\theta) = \frac{1}{m} \sum_{i=1}^m \nabla_{\theta} L(\mathbf{x}^{(i)}, y^{(i)}, \theta). \quad (3.2)$$

The computational cost of this operation is $O(m)$. As the training set size grows to billions of examples, the time to take a single gradient step becomes prohibitively long.

The insight of SGD is that the gradient is an expectation. The expectation may be approximately estimated using a small set of samples. Specifically, on each step of the algorithm, we can sample a **minibatch** of examples $\mathbb{B} = \{\mathbf{x}^{(1)}, \dots, \mathbf{x}^{(m')}\}$ drawn uniformly from the training set. The minibatch size m' is typically chosen to be a relatively small number of examples, ranging from one to a few hundred.

The estimator of the gradient is formed as

$$\mathbf{g} = \frac{1}{m'} \nabla_{\theta} \sum_{i=1}^{m'} L(\mathbf{x}^{(i)}, y^{(i)}, \theta) \quad (3.3)$$

using examples from the minibatch \mathbb{B} . The stochastic gradient descent algorithm then follows the estimated gradient downhill:

$$\theta \leftarrow \theta - \epsilon g, \quad (3.4)$$

where ϵ is the learning rate.

3.2 Momentum

The method of momentum is designed to accelerate learning in SGD. The momentum algorithm accumulates an exponentially decaying moving average of past gradients and continues to move in their direction.

$$v \leftarrow \alpha v - \epsilon g \quad (3.5)$$

$$\theta \leftarrow \theta + v. \quad (3.6)$$

Where g is the gradient, $\alpha \in [0, 1]$ determines how quickly the contributions of previous gradients exponentially decay.

3.3 Nesterov momentum

Nesterov Momentum is a variant of the Momentum algorithm. The difference between Nesterov momentum and standard momentum is where the gradient is evaluated. With Nesterov momentum the gradient is evaluated after the current velocity is applied.

$$g = \frac{1}{m'} \nabla_{\theta} \sum_{i=1}^{m'} L(x, y, \theta + \alpha v) \quad (3.7)$$

3.4 AdaGrad

AdaGrad is designed to converge rapidly when applied to a convex function. Comparing to SGD, AdaGrad algorithm individually adapts the learning rates of all model parameters by scaling them inversely proportional to the square root of the sum of all of their historical squared values.

$$\theta = \theta - \frac{\epsilon}{\sqrt{\delta \mathbf{I} + \text{diag}(G)}} \odot g \quad (3.8)$$

where θ is the parameter to be updated, ϵ is the initial learning rate, δ is some small quantity that used to avoid the division of zero, \mathbf{I} is the identity matrix, g is the gradient estimate.

$$G = \sum_{\tau=1}^t g_{\tau} g_{\tau}^T \quad (3.9)$$

AdaGrad shrinks the learning rate according to the entire history of the squared gradient and may have made the learning rate too small before arriving at such a convex structure.

3.5 Adam

The RMSProp algorithm modifies AdaGrad to perform better in the non-convex setting by changing the gradient accumulation into an exponentially weighted moving average. RMSProp uses an exponentially decaying average to discard history from the extreme past so that it can converge rapidly after finding a convex bowl.

$$\mathbf{r} \leftarrow \mathbf{r} + \mathbf{g} \odot \mathbf{g} \tag{3.10}$$

$$\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} - \frac{\epsilon}{\delta + \sqrt{\mathbf{r}}} \odot \mathbf{g} \tag{3.11}$$

Where \mathbf{g} is the gradient, $\boldsymbol{\theta}$ is the parameters in a model, \mathbf{r} is initialized to 0..

Cost function

Loss is the difference between the predicted value and the actual value. The Function used to quantify this loss during the training phase in the form of a single real number is known as **Loss Function**. **Cost function** refers to an average of the loss functions over an entire training dataset. Cost function helps us reach the optimal solution. The cost function is the technique of evaluating “the performance of our algorithm/model”.

There are many cost functions in machine learning and each has its use cases depending on whether it is a regression problem or classification problem.

4.1 Regression cost function

Regression models deal with predicting a continuous value. They are calculated on the distance-based error.

$$\text{Error} = y - y' \quad (4.1)$$

4.1.1 Mean squared error

$$\text{MSE} = \frac{\sum_{i=0}^n (y - y')^2}{n} \quad (4.2)$$

4.1.2 Mean absolute error

$$\text{MSE} = \frac{\sum_{i=0}^n |y - y'|}{n} \quad (4.3)$$

4.2 Classification cost function

In classification, we usually use one hot to encode the labels. This can eliminate the affect of the distance.

The cross entropy of the distribution q relative to distribution p over a given set is defined as

$$H(p, q) = -E_p[\log q] \quad (4.4)$$

Where the E is the expected value operator respected to the probability q .

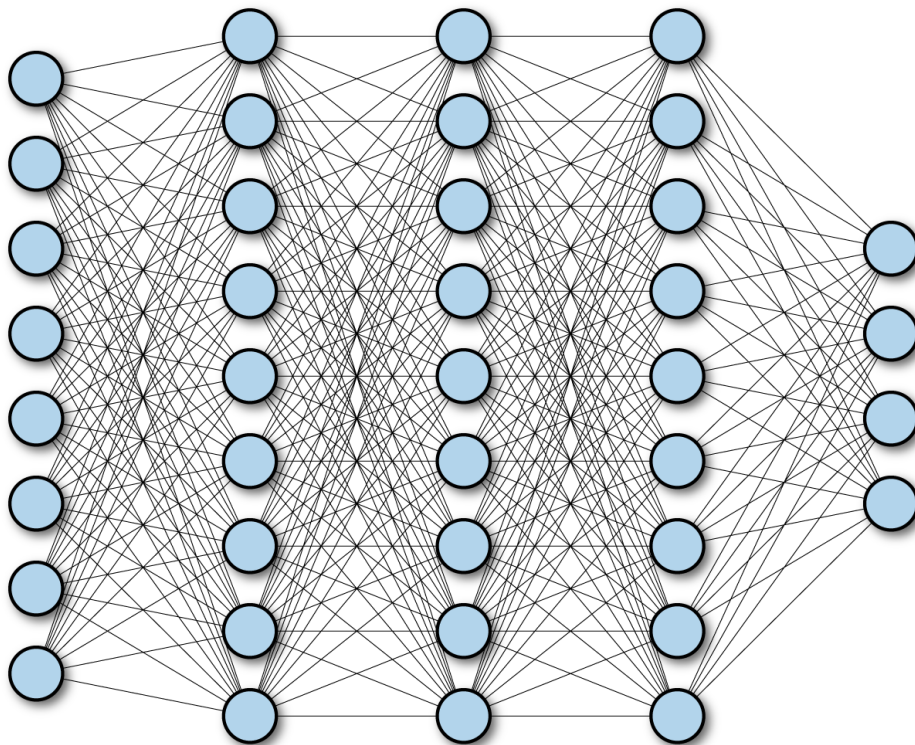
For discrete probability distribution p and q with the same support \mathcal{X} this means

$$H(P, Q) = - \sum_{x \in \mathcal{X}} p(x) \log q(x) \quad (4.5)$$

Full Connected Networks

A fully connected neural network consists of a series of fully connected layers that connect every neuron in one layer to every neuron in the other layer. Full connected neural network layers use matrix multiplication by a matrix of parameters with a separate parameter describing the interaction between each input unit and each output unit. This means that every output unit interacts with every input unit.

Figure 5.1 show the full connects neural network.



Figures 5.1: Full connected neural network

Chapter 6

CNN

CNN stands for convolutional neural network. Convolutional networks are neural networks that have convolutional layers. A typical convolutional layer consists of three stages:

- 1 convolution stage: affine transform
- 2 detector stage: nonlinearty
- 3 pooling stage

6.1 Convolution

$$s(t) = \int x(a)w(t-a)da. \quad (6.1)$$

This operation is called **convolution**. The convolution operation is typically denoted with an asterisk:

$$s(t) = (x * w)(t). \quad (6.2)$$

In convolutional network terminology, the first argument (in this example, the function x) to the convolution is often referred to as the **input**, and the second argument (int this example, the function w) as the **kernel**. The output is sometimes referred to as the **feature map**.

If we assume that x and w are defined only on integer t , we can define the discrete convolution:

$$s(t) = (x * w)(t) = \sum_{a=-\infty}^{\infty} x(a)w(t-a). \quad (6.3)$$

We often use convolutions over more than one axis at a time. For example, if we use a two-dimensional image I as our input, we probably also want to use a two-dimensional kernel K :

$$S(i, j) = (I * K)(i, j) = \sum_m \sum_n I(m, n)K(i-m, j-n). \quad (6.4)$$

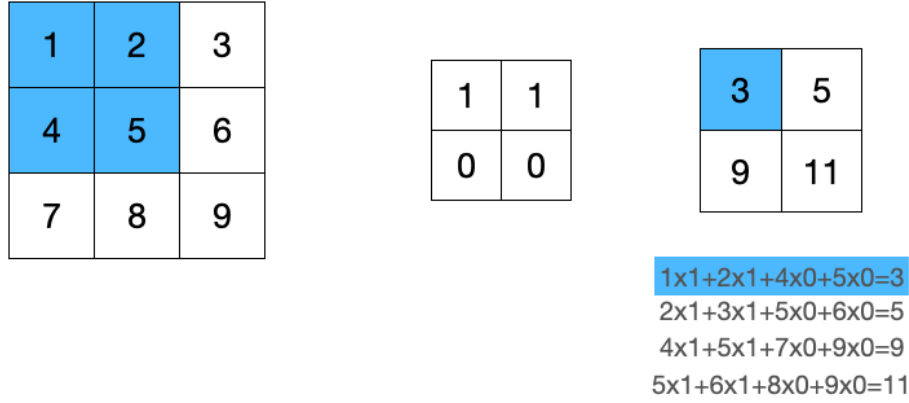
The following formula can be used to calculate the output dimension.

$$h_o = \frac{h_i - h_k}{h_s} + 1 \quad (6.5)$$

$$w_o = \frac{w_i - w_k}{w_s} + 1 \quad (6.6)$$

where h_o is the output height, h_i is the input height, h_k is the kernel height, h_s is the stride height, w_o is the output width, w_i is the input width, w_k is the kernel width, w_s is the stride width.

The convolution operation is shown in Figure 6.1.



Figures 6.1: Convolution operation

6.2 Properties

CNN leverages three important ideas:

- ♥ sparse interaction.
- ♥ parameter sharing.
- ♥ equivariant representations.

6.2.1 Sparse interaction

This is accomplished by making the kernel smaller than the input.

6.2.2 Parameter sharing

In convolutional layers, the same parameter defined in one kernel are used at every position of the input.

6.2.3 Equivariant representations

In the case of convolution, the particular form of a parameter sharing causes the layer to have a property called **equivariance** to translation. To say a function is equivariant means that if the input changes, the output changes in the same way.

6.3 Pooling

A pooling function replaces the output of the net at a certain location with a summary statistic of the nearby outputs. For example, the max pooling operation reports the maximum output within a rectangular neighborhood. Pooling helps to make the representation approximately invariant to small translations of the input. Invariant to translation means that if we translate the input by a small amount, the values of most of the pooled outputs do not change.

The following formula can be used to calculate the output dimension.

$$h_o = \frac{h_i - h_k}{h_s} + 1 \quad (6.7)$$

$$w_o = \frac{w_i - w_k}{w_s} + 1 \quad (6.8)$$

where h_o is the output height, h_i is the input height, h_k is the pooling height, h_s is the stride height, w_o is the output width, w_i is the input width, w_k is the pooling width, w_s is the stride width.

Metric

		Real value	
		positive	negative
Predict value	positive	true positive	false positive
	negative	false negative	true negative

Tables 7.1: Confusion matrix

7.1 Precision

$$\text{precision} = \frac{\text{TP}}{\text{TP} + \text{FP}} \quad (7.1)$$

Of all the predicted positive values, the ratio of the true positive values (the real value is positive and the predicted value is positive).

7.2 Recall

$$\text{recall} = \frac{\text{TP}}{\text{TP} + \text{FN}} \quad (7.2)$$

Of all the real positive values, the ratio of the true positive values.

7.3 Accuracy

$$\text{accuracy} = \frac{\text{TP} + \text{TN}}{\text{TP} + \text{TN} + \text{FP} + \text{FN}} \quad (7.3)$$

Of all the values, the ratio of correctly predicted values. The disadvantage is that it is not suitable for unbalanced data.

7.4 F-score

$$F = \frac{(\alpha^2 + 1) \times \text{precision} \times \text{recall}}{\alpha^2 \times \text{precision} + \text{recall}} \quad (7.4)$$

When determining the value of the parameter α , if we pay more attention to recall (compared to precision), we should choose a larger α . The F-1 score is the expression when $\alpha = 1$

Regularization

We train model on training data but use test data (not used to train the model) to test out model. The ability to perform on test data is called **generalization**. We can use model on test data because we assume that the train data and the test has the same probability distribution (i.e. they have relationship).

Regularization is any modification we make to a learning algorithm that is intended to reduce its generalization error.

In practice, it is very difficult to find a model with the right number of parameters. Indeed, we often use a larger model that has been regularized appropriately.

8.1 Parameter norm penalties

Parameter norm penalties ($\Omega(\theta)$) can be added to the object function J to limit the capacity of the model.

$$\tilde{J}(\theta; X, y) = J(\theta, X, y) + \alpha\Omega(\theta) \quad (8.1)$$

8.1.1 L^2 Parameter Regularization

The L^2 parameter norm penalty commonly known as **weight decay**.

$$\Omega(\theta) = \frac{1}{2}\|w\|_2^2 \quad (8.2)$$

Where w is the model parameter matrix.

8.1.2 L^1 Regularization

L^1 regularization on the model parameter w is defined as

$$\Omega(\theta) = \|w\|_1 \quad (8.3)$$

8.2 Dataset Augmentation

The best way to make a machine learning model generalize better is to train it on more data. Of course, in practice, the amount of data we have is limited. One way to get around this problem is to create fake data and add it to the training set. Dataset augmentation has been a particular effective technique for a specific classification problem: object recognition.

8.3 Early stopping

Early stopping is used to avoid overfit.

The algorithm terminates when no parameters have improved over the best recorded validation error for some pre-specified number of iterations. This strategy is known as **early stopping**. It is probably the most commonly used form of regularization in deep learning.

8.4 Droupout

Activation functions

Activation functions are usually added into neural network in order to help the network learn complex patterns. Because many of the patterns we want to learn are non-linear, we usually add non-linear activation functions into neural network to add the ability to learn non-linear patterns. Its function is to determine what information can be passed to the next neuron.

9.1 Desirable features

- ♥ Differentiable. This is needed for optimization.
- ♥ Low computational expense.
- ♥ Zero-centered.

9.2 Sigmoid

$$\text{sigmoid}(x) = \frac{1}{1 + e^{-x}} \quad (9.1)$$

It is no longer used in recent deep learning models because it is computationally expensive, causes vanishing gradient problem and not zero-centered.

9.3 Tanh

$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{e^{2x} - 1}{e^{2x} + 1} \quad (9.2)$$

Comparing to sigmoid, it solve the not zero-centered problem.

9.4 ReLU

$$\text{relu}(x) = \max(0, x) \quad (9.3)$$

This is a widely used activation function.

9.5 Leaky ReLU

$$\text{lrelu}(x) = \max(\alpha x, x) \tag{9.4}$$

9.6 Swish

$$\text{swish}(x) = x * \text{sigmoid}(x) = x * (1 + e^{-x})^{-1} \tag{9.5}$$

Chapter 10

Machine learning process

- 1 confirm your target
- 2 observe the data
- 3 preprocess the data if necessary
- 4 select the model
- 5 select the cost function
- 6 select the optimizer
- 7 train the model
- 8 fine-tune the model
- 9 predict the data using the model
- 10 deploy the model

Part II

Tools

Chapter 1

PyTorch

Chapter 12

NumPy

12.1 Load data

```
1 import numpy as np
2
3 data = np.load('foo.npz')
```


Part III

Practice

Chapter 13

Preprocessing

Preprocessing is not necessary. We preprocess the data because we can not get what we want from the data directly.

13.1 Not valued based data

The computer can only process number-based data. For not number-based data, we need to convert them into number-based data. For example, convert a string into a number list. For null values, we can drop them or compute them with other not null values.

```
1 from sklearn.preprocessing import LabelBinarizer
2
3 data = ['dog', 'cat', 'dog', 'horse']
4 print('data: ', data)
5 binarizer = LabelBinarizer()
6 binarizer.fit(data)
7 print(binarizer.transform(data))
8 print(binarizer.transform(['dog']))
9
10 """
11 data:  ['dog', 'cat', 'dog', 'horse']
12 [[0 1 0]
13  [1 0 0]
14  [0 1 0]
15  [0 0 1]]
16 [[0 1 0]]
17 """
```


Part IV

Projects

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