

# CS 202

## Design and Analysis of Algorithms

### Assignment 2

#### [Question 1] Code Debugging Exercise

(10 marks)

Study the code provided in `A2Q1_coin_change_limited.py` and answer the following questions:

1. Provide an example input that causes the code to fail. (3 marks)
2. Generalize the issue by identifying the common characteristic(s) of input examples that lead to failure. (3 marks)
3. debug this code and justify the changes you made. (4 marks)

Please write your answer in a word document `A2.docx`, and keep your answer within 1 page for this question.

#### [Question 2] Longest Common Valley Sub-sequence

(10 marks)

Given two sequences  $A = \{a_1, a_2, \dots, a_{|A|}\}$  and  $B = \{b_1, b_2, \dots, b_{|B|}\}$ , a sequence  $C = \{c_1, c_2, \dots, c_{|C|}\}$  is called a common valley sub-sequence of  $A$  and  $B$ , if (i) there exist two strictly increasing functions  $f$  and  $g$  (meaning  $f(x_1) < f(x_2)$  if  $x_1 < x_2$ ), such that  $c_i = a_{f(i)} = b_{g(i)}$  for all  $1 \leq i \leq |C|$ ; and (ii) there exists an index  $k$ , such that  $c_1 > c_2 > \dots > c_k < c_{k+1} < \dots < c_{|C|}$ , for some  $k$  in  $1 \leq k \leq |C|$ . For example, with functions  $f(1) = 2, f(2) = 3, f(3) = 5$ , and  $g(1) = 2, g(2) = 5, g(3) = 6$ ,  $\{c_1, c_2, c_3\}$  is a common sub-sequence to  $A$  and  $B$  if  $c_1 = a_2 = b_2, c_2 = a_3 = b_5$ , and  $c_3 = a_5 = b_6$ . Additionally, if  $c_2 < c_1$  and  $c_2 < c_3$ ,  $\{c_1, c_2, c_3\}$  forms a common valley sub-sequence of  $A$  and  $B$ .

There are many such common valley sub-sequences between  $A$  and  $B$ , however, there is a longest subsequence among them. In this question, you are to write a program which computes the length of a longest common valley subsequence.

In simpler terms, you need to find the longest sub-sequence that is common to both  $A$  and  $B$ . However, this common sub-sequence must also have a special "valley" shape, which means (1) the numbers in the sub-sequence must first decrease and then increase (think of it like a "V" shape), and (2) The numbers in the sub-sequence must appear in both  $A$  and  $B$  in the same order, but they do not have to be next to each other.

Implement an algorithm in the function LCVS. Please also state and justify the time complexity of your algorithm as comments in your code.

Test inputs begin with the number of pairs of sequences. Each sequence is described by one line and contains a list of positive integers between 1 and 16777216. The length of each sequence is denoted as  $n$ , and  $1 \leq n \leq 5000$ . For each pair of sequences, output the length of a longest common valley sub-sequence.

Your code should be submitted to the course's **online judge** for evaluation against the 6 additional test inputs, besides the sample input below. (7 marks)

Please write the time complexity (including its justification) of your algorithm in the word document A2.docx, and keep your answer within 0.5 page for this question. (3 marks)

### Sample Input

Please refer to the file A2Q2.in.

### Sample Output

4  
5  
4  
5  
10

## **[Question 3] Asymptotic Analysis**

**(10 marks)**

Using any of the techniques for solving recurrence in class, provide an asymptotic bound for  $T(n)$  for each of the following recurrences:

i.  $T(n) = 3 \cdot T\left(\frac{n}{4}\right) + \frac{n}{\log n}$  (3 marks)

ii.  $T(n) = 2 \cdot T(n - 1) + n$  (7 marks)

Please write your answer in the word document A2.docx, and keep your answer within 1 page for this question.

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You may use command "python A2Q2\_soln.py < A2Q2.in" in anaconda command prompt to test your own solution.