COLUMBIA UNIVERSITY
Statistical Inference and Modeling
QUIZ 1

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Name:

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You have 20 minutes to answer the following 10 multiple choice questions. Good luck!

Question 1

Let $X_1, \ldots, X_n \overset{iid}{\sim} Poisson(\lambda)$. Define $\hat{\lambda}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$. Which one the following statements is NOT correct?

(a) $\text{MSE}(\hat{\lambda}_n, \lambda) = \text{var}(\hat{\lambda}_n)$

(b) $\text{MSE}(\hat{\lambda}_n, \lambda) \underset{n\to\infty}{\to} 0$.

(c) $\hat{\lambda}$ is the Maximum Likelihood Estimator of λ.

(d) All of the above statements are correct.

(e) None of the above statements are correct.

Question 2

Which of the following statements is the most accurate?

(a) Unbiased estimators have lower MSE than biased estimators.

(b) Given two estimators, the one with lower variance is more efficient.

(c) Maximum likelihood estimators have the lowest MSE among estimators.

(d) All of the above statements are correct.

(e) None of the above statements are correct.

Question 3

Which of the following statements is the most accurate?

(a) Unbiased estimators are always consistent.

(b) Method of moments estimators are always normally distributed.

(c) Among unbiased estimators, higher variance implies higher MSE.

(d) All of the above statements are correct.

(e) None of the above statements are correct.

Question 4

Let X and Y be two random variables such that $Y^2 = .5\log(X) - 0.3$. Which one the following statements is correct?

(a) $corr(\sqrt{X}, e^{Y^2}) = 1$

(b) $corr(\log X, Y^2) = .5$

(c) $corr(\log X, Y) = 1$

(d) None of the above statements are correct.

Question 5

Let $X_1, \ldots, X_{2n} \overset{iid}{\sim} N(0, \sigma^2)$ and consider two estimators of σ^2 defined as $T = \frac{1}{n}\sum_{i=1}^{n} X_i^2$ and $\tilde{T} = \frac{1}{2n}\sum_{i=1}^{2n} X_i^2$. Which of the following statements is NOT correct?

(a) T and $\tilde{T}$ are both unbiased estimators.

(b) T and $\tilde{T}$ are both consistent estimators.

(c) $\text{MSE}(T)/\text{MSE}(\tilde{T}) = 2$.

(d) All of the above statements are correct.

(e) None of the above statements are correct.

Question 6

A scientist collected 15 observations in their lab. Assuming they are i.i.d. normally distributed, they computed a t-statistic to test $H_0 : \mu = 0$. They also obtained the 0.95 confidence interval $[.11, 2.13]$. Under their working assumptions, which of the following statements is correct?

(a) We do not have enough evidence to reject H_0.

(b) The probability that $\mu = 0$ is less than 0.95.

(c) The estimated mean was 1.12.

(d) All of the above statements are correct.

(e) None of the above statements are correct.

Question 7

Let $X_1, \ldots, X_n \overset{iid}{\sim} N(\mu, \sigma^2)$, $\overline{X_n} = \frac{1}{n}\sum_{i=1}^n X_i$ and $s^2 = \frac{1}{n}\sum_{i=1}^n (X_i - \overline{X_n})^2$. We are interested in testing the hypothesis $H_0 : \mu = 0$ versus $H_1 : \mu \neq 0$. Which of the following statements is correct?

(a) Under $H_0 : \mu = 0$ we have that $\sqrt{n} \cdot \overline{X_n} \sim t_{n-1}$

(b) Under H_0 we have that $\overline{X_n} \sim N(0, \sigma/n)$

(c) s^2 is an unbiased and consistent estimator of σ^2

(d) Under $H_1 : \mu \neq 0$ we have that $\sqrt{n}\frac{\overline{X_n}-\mu}{\sqrt{s^2}} \xrightarrow[n\to\infty]{\mathcal{D}} N(0,1)$.

Question 8

Let $X_1, \ldots, X_n \overset{iid}{\sim} N(\mu, \sigma^2)$. Which of the following statistics is not a pivot

(a) $T_1 = \sum_{i=1}^n (X_i - \mu)^2$

(b) $T_2 = \frac{\overline{X_n}-\mu}{\sqrt{\frac{1}{n-1}\sum_{i=1}^n (X_i - \overline{X_n})^2}}$

(c) $T_3 = \frac{\overline{X_n}-\mu}{\sigma}$

(d) All of the above 3 statistics are pivots.

(e) None of the above 3 statistics are pivots.

Question 9

Let $X_1, \ldots, X_n \overset{iid}{\sim} Ber(p)$, hence $\mathbb{E}[X_1] = p$ and $\mathbb{E}[X_1^2] = p$. We consider the empirical moments $\overline{X_n} = \frac{1}{n}\sum_{i=1}^n X_i$ and $\overline{X_n^2} = \frac{1}{n}\sum_{i=1}^n X_i^2$. Which one the following statements is NOT correct?

(a) $\frac{1}{n}\sum_{i=1}^n X_i^2 \xrightarrow[n\to\infty]{\mathcal{P}} p^2$.

(b) $\sqrt{n}(\overline{X_n} - p)/\sqrt{\overline{X_n}(1-\overline{X_n})} \xrightarrow[n\to\infty]{\mathcal{D}} N(0,1)$.

(c) $\overline{X_n^2} - (\overline{X_n})^2 \xrightarrow[n\to\infty]{\mathcal{P}} p(1-p)$.

(d) $\sqrt{\overline{X_n^2}} \xrightarrow[n\to\infty]{\mathcal{P}} \sqrt{p}$.

Question 10

Let $X_1, \ldots, X_n$ be i.i.d. exponential random variables with expectation $\lambda > 0$ and let the statistic $\hat{\lambda}_n = \frac{1}{n}\sum_{i=1}^n X_i$ be an estimator of λ. Which of the following statements is correct?

(a) $\sqrt{n}(\hat{\lambda}_n - \lambda) \xrightarrow[n\to\infty]{\mathcal{D}} N(0, \lambda^2)$.

(b) $\sqrt{n}(\log(\hat{\lambda}_n) - \log(\lambda)) \xrightarrow[n\to\infty]{\mathcal{D}} N(0,1)$.

(c) $\sqrt{n}(\hat{\lambda}_n^2 - \lambda^2) \xrightarrow[n\to\infty]{\mathcal{D}} N(0, 4\lambda^4)$.

(d) All of the above statements are correct.

(e) None of the above statements are correct.