Exam 2: (Review 1)

Exam 2 Review

Problems from Devore (9^{th} Edition).

Chapter 5, Section 4.

53) Population: Rockwell hardness of pins.

$\mu = 50$, $\sigma = 1.2$

a) $n = 9$, assuming that popln is normally dist.

$$\bar{X} \sim N\left(\mu_{\bar{X}} = 50, \sigma_{\bar{X}}^2 = \frac{\sigma^2}{n}\right) = N\left(\mu_{\bar{X}} = 50, \sigma_{\bar{X}} = \frac{1.2}{\sqrt{n}}\right)$$

$$\boxed{\bar{X} \sim N\left(\mu_{\bar{X}} = 50, \sigma_{\bar{X}} = \frac{1.2}{3}\right)}$$

Want:

$$P(\bar{X} \geq 51) = P\Big(\underbrace{\frac{\bar{X}-50}{1.2/3}}_{\text{standard normal dist.}} \geq \frac{51-50}{1.2/3}\Big)$$

$$= P\left(Z \geq \frac{1}{0.4}\right) = P(Z \geq 2.5)$$

$$= 1 - P(Z \leq 2.5) = 1 - \Phi(2.5)$$

$$= 1 - 0.9938 = 0.0062$$

b) $n = 40 \Rightarrow \bar{X} \sim N\left(\mu_{\bar{X}} = 50, \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{40}}\right)$

$$\sim N\left(\mu_{\bar{X}} = 50, \sigma_{\bar{X}} = \frac{1.2}{\sqrt{40}}\right)$$

$$P(\bar{X} \geq 51) \approx P\left(\frac{\bar{X}-50}{1.2/\sqrt{40}} \geq \frac{51-50}{1.2/\sqrt{40}}\right)$$

$$= P\left(Z \geq \frac{\sqrt{40}}{1.2}\right)$$

$$= 1 - \underbrace{\Phi\left(\sqrt{40}/1.2\right)}$$

$$= 1 - \text{pnorm}(\sqrt{40}/1.2, \text{mean}=0, \text{sd}=1)$$

54) Popln $\sim N(\mu = 2.65, \sigma = 0.85)$

$n = 25$, (i) $P(\bar{X} \leq 3.00)$

(ii) $P(2.65 \leq \bar{X} \leq 3.00)$

(i) $\mu_{\bar{X}} = 2.65$ $\quad \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{0.85}{\sqrt{25}} = \frac{0.85}{5}$

$$\Rightarrow \bar{X} \sim N\left(\mu_{\bar{X}} = 2.65, \sigma_{\bar{X}} = \frac{0.85}{5}\right)$$

$$\therefore P(\bar{X} \leq 3.00) = P\left(\frac{\bar{X}-2.65}{0.85/5} \leq \frac{3-2.65}{0.85/5}\right) = P\left(Z \leq \left(\frac{0.35}{0.85}\right) \times 5\right) = P(Z \leq 2.058) \approx 0.9817$$

(ii) $P(2.65 \leq \bar{X} \leq 3.00)$

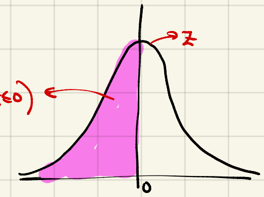
$$= P(\bar{X} \leq 3.00) - P(\bar{X} \leq \underbrace{2.65}_{\mu_{\bar{X}}})$$

$\downarrow$ 0.5

$$P\left(\frac{\bar{X}-2.65}{\sigma_{\bar{X}}} \leq \frac{2.65-2.65}{\sigma_{\bar{X}}}\right)$$

$$\Rightarrow P(Z \leq 0) = 0.5$$

$$= 0.9817 - 0.5 \approx$$



(b) What n is need so that

$$\boxed{P(\bar{X} \leq 3.00) = 0.99\ ?}$$

$$P\left(\frac{\bar{X}-2.65}{\sigma/\sqrt{n}} \leq \frac{3-2.65}{\sigma/\sqrt{n}}\right) = 0.99$$

$$\boxed{P\left(Z \leq \frac{0.35}{0.85}\sqrt{n}\right) = 0.99}$$

need to solve this "probability equation".

$P(Z \leq \eta_{0.99}) = 0.99 \Rightarrow$ calculating the 0.99^{th} percentile for the standard dist.

$$\boxed{\eta_{0.99} = \text{qnorm}(0.99, \text{mean}=0, \text{sd}=1, \text{lower.tail} = \text{True})}$$

$$\approx 2.33$$

$$\eta_{0.99} = \frac{0.35}{0.85}\sqrt{n} \Leftrightarrow 2.33 = \frac{0.35}{0.85} \cdot \sqrt{n}$$

$$\Leftrightarrow n \approx \left(\frac{2.33 \times 0.85}{0.35}\right)^2$$

$$\approx 32$$

$$X \sim \text{Gamma}(\alpha = 2, \beta = 5)$$

$$P(X \leq 3) = \text{pgamma}(3, \alpha = 2, \text{scale} = 5)$$

(i) What c will give

$$P(X \leq c) = 0.6$$

$c = 60^{th}$ percentile for $\text{Gamma}(\alpha = 2, \beta = 5)$

$$= \text{qgamma}(0.6, \text{shape} = 2, \text{scale} = 5)$$

(ii) $X \sim N(5, \sigma = 1.2)$

What a will give

Standard normal dist.

$$\boxed{P\left(-a \leq \frac{X-5}{\sigma} \leq a\right) = 0.99}$$

6.8) $\mu_1 = 520$ $\quad$ $\mu_2 = 500$

$\sigma_1 = 10$ $\quad$ $\sigma_2 = 10$

$X_1 \sim N(\mu_1 = 520, \sigma_1 = 10)$ $\quad$ $X_2 \sim N(\mu_2 = 500, \sigma_2 = 10)$

$X_1 \sim$ constant speed of plane 1 $\quad$ $X_2 \sim$ constant speed of plane 2

a) What is the distance covered by plane 1 after 2 hours

$\underline{2X_1}$ (because distance = time × speed.) $\quad$ time $= 2$, speed $= X_1$

b) What is the distance covered by plane 2 after 2 hours

$\underline{2X_2}$

Assuming that plane 1 is a 10 km ahead of plane 2

$P\left(\text{distance covered by plane 2} < \text{distance covered by plane 1} + 10\right)$

$$P(2X_2 \leq 2X_1 + 10) = P(X_2 - X_1 < 5)$$

∴ want to calculate the dist of $\underline{X_2 - X_1 = Y}$

Since X_1 and X_2 are normally dist

$$Y \sim N(\mu_Y, \sigma_Y)$$

$$\mu_Y = E(X_2 - X_1) = 500 - 520 = \underline{-20}$$

$$\sigma_Y = \sqrt{Var(X_2 - X_1)} = \sqrt{Var(X_1) + Var(X_2)}$$

$$= \sqrt{10^2 + 10^2}$$

$$= \underline{\sqrt{2} \cdot 10}$$

∴ Want

$$P(Y \leq 5) = P\left(\frac{Y - (-20)}{\sqrt{2} \cdot 10} \leq \frac{5 - (-20)}{\sqrt{2} \cdot 10}\right)$$

$$= P\left(Z \leq \frac{25}{\sqrt{2} \cdot 10}\right) = \underline{0.9616}$$

$$P(|2X_1 + 10 - 2X_2| \leq 10) =$$

$$\Updownarrow$$

$$P(-10 \leq 2X_1 - 2X_2 + 10 \leq 10) =$$

$$= P(-20 \leq 2X_1 - 2X_2 \leq 0)$$

$$= P(-10 \leq X_1 - X_2 \leq 0) = P(0 \leq X_2 - X_1 \leq 10)$$

$$= \underline{P(0 \leq Y \leq 10)}$$