Review

① when is $E(X)$ undefined.

X is geometric with parameter $p=\frac{1}{2}$

$\mathcal{X}=\{1,2,3,4,\dots,\infty\}$

$p_X(x)=p\cdot(1-p)^{x-1}=\left(\frac{1}{2}\right)^x \quad x=1,2,3,\dots$

$=\left(\frac{1}{2}\right)\left(1-\frac{1}{2}\right)^{x-1}=\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^{x-1}=\left(\frac{1}{2}\right)^{1+(x-1)}=\left(\frac{1}{2}\right)^x$

x	1	2	3	4	5	6	....
$p_X(x)$	$1/2$	$1/2^2$	$1/2^3$	$1/2^4$	$1/2^5$	$1/2^6$	

$E(X)=\frac{1}{1/2}=\underline{\underline{2}}$

$\sum_{x\in\mathcal{X}} p_X(x)=\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\dots$

$=\sum_{k=1}^{\infty}\left(\frac{1}{2}\right)^k=1$

$\sum_{x\in\mathcal{X}} x\,p_X(x)=\sum_{k=1}^{\infty}k\left(\frac{1}{2^k}\right)$

Get a r.v $Y=h(X)$ $\quad h:\mathcal{X}\to\mathbb{R}$.

$h(x)=2^x \qquad \mathcal{Y}=\{2,2^2,2^3,2^4,\dots\dots\}$

y	2^1	2^2	2^3	2^4	2^5	2^6	.
$p_Y(y)$	$1/2$	$1/2^2$	$1/2^3$	$1/2^4$	$1/2^5$	-	

$\sum_{y\in\mathcal{Y}} p_Y(y)=\sum_{k=1}^{\infty}\left(\frac{1}{2^k}\right)$

$E(Y)=\sum_{y\in\mathcal{Y}} y\cdot p_Y(y)=\sum_{k=1}^{\infty}2^k\cdot p_Y(y)=\sum_{k=1}^{\infty}2^k\left(\frac{1}{2^k}\right)$

$=\sum_{k=1}^{\infty}1$ divergent!

$I=\int_{-\infty}^{\infty}\frac{x}{1+x^2}\,dx \overset{?}{=}$ does this exist ?

$I=\underbrace{\int_{-\infty}^{0}\frac{x}{1+x^2}\,dx}_{I_1}+\underbrace{\int_{0}^{\infty}\frac{x}{1+x^2}\,dx}_{I_2}$

I converges $\Leftrightarrow$ both I_1 and I_2 converge.

$\int_0^{\infty}\frac{x}{1+x^2}\,dx$ does not converge

$\underline{\lim_{a\to\infty}\int_0^a\frac{x}{1+x^2}\,dx}$. does not exist.

$V(X)\geq 0$

non-negative $\rightsquigarrow$ ≥ 0

positive $\longrightarrow$ >0

$Y=h(X)=\underline{\underline{aX+b}}$

$X_1\sim \mathrm{Po}(\lambda)$ $\qquad X_2\sim \mathrm{N.B}(r,p)$

$\mathcal{X}_1=\{0,1,2,\dots\}$ $\qquad \mathcal{X}_2=\{0,1,2,\dots\}$

$X \rightsquigarrow h(X)=aX+b$

$E(h(X))=aE(X)+b$

Cauchy Distribution

$f(x)=\frac{1}{\pi}\cdot\left(\frac{1}{1+x^2}\right)$

$\int_{-\infty}^{\infty}f(x)\,dx=1$; $\int_{-\infty}^{\infty}\frac{x}{(1+x^2)}\,dx$ not convergent integral.

$g(x)=\frac{x}{1+x^2}$ $\quad x\in(-\infty,\infty)$

odd function

$\int_{-\infty}^{\infty}g(x)\,dx\overset{?}{=}0$ ✗

Exam - Related

Intro to stat ⇝

Sampling → Simple Random Sample
→ Stratified Sampling
→ Cluster Sampling

Sample data = $\{x_1, x_2, \dots, x_n\}$

Measures Location → mean ⇝ sample mean.
→ median
→ percentile
→
⋮

⇝ population parameters.

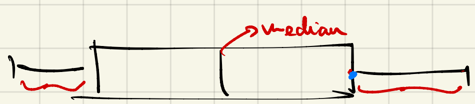
Measures of Variability → sample variance, s.d
→ Interquartile range
→ range

$$\boxed{\frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2}$$

Given $\{a_1, a_2, \dots, a_n\}$

Variance of $\{a_1, a_2, \dots, a_n\}$ $= \frac{1}{n}\sum_{i=1}^{n}(a_i-\bar{a})^2$

Box plots



Outlier : data point outsides of the fences.

Chapter 2) Prob and Counting:

① Axioms of probability

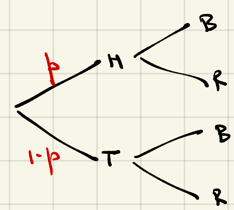
Examples → all outcomes are equally likely ⇝ use counting principles to calculate $P(A)$

$$P(A) = \frac{n(A)}{n(S)}$$

→ Example: Coin Toss, Draw ball from urns } get comfortable drawing the tree diagram.

Example:

Toss a coin, draw a ball from one of two bags depending on heads or tails. (B, R)



Independence and Conditional Probability

$$P(A\cap B) = P(A)\cdot P(B)$$

$$P(B|A) = \frac{P(A\cap B)}{P(A)} \qquad P(A)>0$$

Bayes Theorem

$P(B|A)$ ⇝ $P(A)$; $P(A|B)$

Random Variables

Experiments

Sample spaces of experiments

$X: S \to \mathbb{R}$

⇓

Want to know X, $P_X(x)$ ⇝ pmf of X

$E(X)$ ⇝ expected

$V(X)$ ⇝ variance

Example Discrete

Finite sample: Uniform, Binomial, Hypergeometric, Bernoulli

Infinite sample: Negative Binomial, Poisson, Geometric.

→ # of events in a time interval.