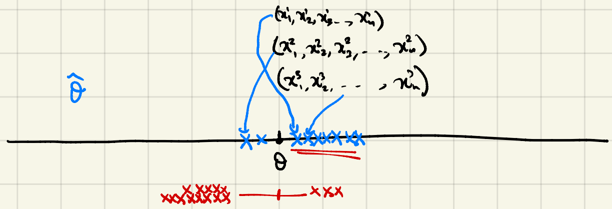
# Point Estimators: Examples.

# Point Estimators

$\hat{\theta}$ is an estimator for $\theta$.

$\hat{\theta}$ is statistic that depends on a random sample. $\rightsquigarrow \{X_1, X_2, \ldots, X_n\}$




Bias:

$$E(\hat{\theta} - \theta) = \text{Bias}(\hat{\theta}).$$

$$\hat{\theta} = \underline{\theta} + \text{Error}.$$

$\hat{\theta} - \theta \rightsquigarrow$ "error in the estimate"

---

$\hat{\theta}$ is unbiased if $\text{Bias}(\hat{\theta}) = 0$ for all possible values of $\theta$.

principle of unbiased estimation $\rightsquigarrow$ $\{\theta_1, \theta_2, \ldots, \theta_n\}$ each estimating $\theta$ then choose an estimator that is unbiased (if there is one).

---

$k \in (0, 50)$.

If $\bar{X} \rightsquigarrow$ sample.

$\bar{X}_{(k)} \rightsquigarrow k^{th}$ trimmed mean.

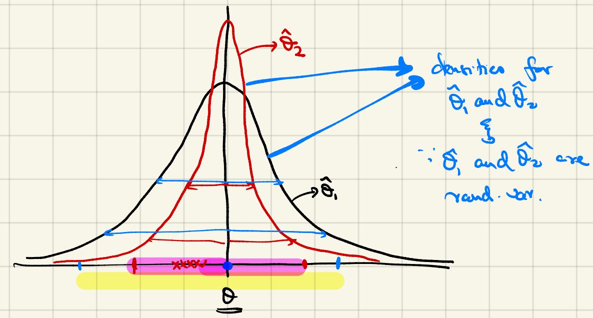
gets rid of outlying data.

If population has a symmetric unimodal distribution (think normal distribution)

$\bar{X}_{(k)}$ is an unbiased estimator for $\mu$.

every $k \in (0, 50) \rightsquigarrow$ gives an unbiased estimator for $\mu$.

Need the variance.



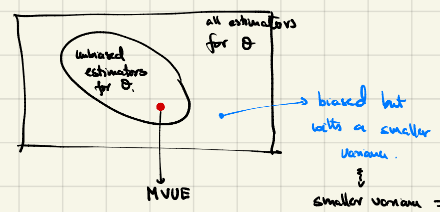

Variance of the estimator $\hat{\theta}$ $\rightsquigarrow$ $\sigma_{\hat{\theta}}^2 = V(\hat{\theta})$.

need the 'sampling dist" of $\hat{\theta}$

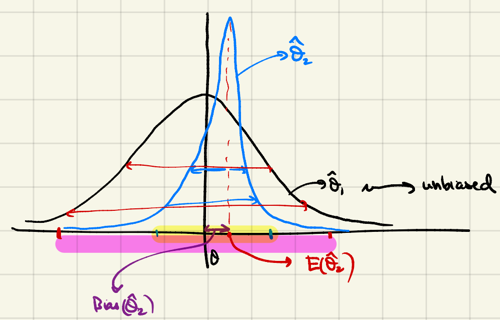
difficult to get a handle on this.

Principle of MVUE

" Among all unbiased estimators, choose the estimator that has the least variance. "

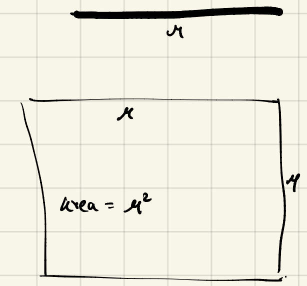



smaller variance $\Rightarrow$ good predic




"Bias-Variance" trade off.

Devore 10)




$\{X_1, X_2, \dots, X_n\}$

$X_i \sim N(\mu, \sigma^2)$

$E(\bar{X}) = \mu \Rightarrow \bar{X}$ is unbiased est for $\mu$.

But:

a) $E(\bar{X}^2) = ?$

$Y = \bar{X} \qquad V(Y) = E(Y^2) - (E(Y))^2$

$V(\bar{X}) = E(\bar{X}^2) - (E(\bar{X}))^2$

But:

$V(\bar{X}) = \dfrac{\sigma^2}{n} \qquad E(\bar{X}) = \mu.$

$\therefore E(\bar{X}^2) = V(\bar{X}) + (E(\bar{X}))^2$

$= \dfrac{\sigma^2}{n} + \mu^2 \neq \mu^2$

$\Rightarrow \bar{X}^2$ is not an unbiased estimator for $\mu^2$.

b) $\hat{\theta} = \bar{X}^2 - kS^2 \qquad S^2 = \dfrac{1}{(n-1)} \displaystyle\sum_{i=1}^{n} (X_i - \bar{X})^2$

$E(S^2) = \sigma^2$ (can show this).

$E(\hat{\theta}) = E(\bar{X}^2) - E(kS^2)$

$= E(\bar{X}^2) - kE(S^2)$

$= \left(\dfrac{\sigma^2}{n} + \mu^2\right) - k\sigma^2$

$E(\hat{\theta}) = \mu^2 + \sigma^2\left(\dfrac{1}{n} - k\right)$

For $\hat{\theta}$ to be unbiased we want

$E(\hat{\theta}) = \mu^2 \Rightarrow \cancel{\mu^2} + \sigma^2\left(\dfrac{1}{n} - k\right) = \cancel{\mu^2}$

$\Rightarrow \sigma^2\left(\dfrac{1}{n} - k\right) = 0$

$\sigma^2 \neq 0$ then $\boxed{\dfrac{1}{n} = k}$

$\therefore \boxed{\hat{\theta} = \bar{X}^2 - \dfrac{1}{n}S^2}$

unbiased estimator for $\mu^2$.

---

$\bar{X}$ unbiased est for $\mu$.

$S^2$ unbiased est for $\sigma^2$.