1. Introduction

This paper aims to investigate the causal relationship between monetary policy and economic activities in Montana. Specifically, it explore the impact of the monetary policy interest rate on output growth and the unemployment rate. Understanding this relationship is crucial for comprehending the effectiveness of monetary policy in influencing economic outcomes, and this kind of investigation provides valuable insights into the effectiveness of monetary policy measures and facilitates informed decision-making in both policy and economic realms.

It is challenging to estimate the causal effect of monetary policy on economic activity due to endogeneity problems. The relationship between the monetary policy interest rate and real GDP growth/unemployment rate cannot be adequately captured by a single linear equation. Instead, we need to consider the simultaneous causality and dynamic interdependence between these variables, as changes in the interest rate can be influenced by lagged economic activity, and vice versa.

To address these challenges, the a Vector Autoregression (VAR) model is employed, based on Sims' (1980) solution, which allows for simultaneous estimation of the relationships while accounting for endogeneity. The model selection criteria indicate the optimal lag order for our analysis. By conducting the VAR analysis, we assess the causal effect of monetary policy on output growth and the unemployment rate.

The findings in this paper suggests a weak or negligible causal relationship between the monetary policy interest rate and output growth in Montana, as the impulse response analysis shows no immediate impact of the interest rate on output growth, and subsequent responses lack statistical significance. On the other hand, there could still be a potentially positive effect of the interest rate on the unemployment rate in Montana. In fact, the response appears over time, while it quickly diminishes and lacks statistical significance after an initial periods.

The remaining sections in this paper include a discussion of the data and VAR model estimation process, evidence of the effect of monetary policy on economic activity, limitations of the analysis, robustness checks, and potential extensions. By addressing these sections, we provide a comprehensive analysis of the causal relationship between monetary policy and economic activities in Montana, considering both its strengths and limitations.

2. Monetary Policy Effectiveness: Empirical Challenges

In real-life situations, estimating the causal effect of monetary policy on economic activity is challenging due to endogeneity problems. The relationship between real GDP growth/ unemployment rate in Montana and the monetary policy short-term nominal interest rates cannot be adequately expressed in a single linear equation with limited parameters. Instead, we need to consider the dynamic interdependence between these variables, which means our analysis should not only account for the dependence of real GDP/unemployment rate on lagged values of themselves and changes in the short-term nominal interest rate but also recognize that changes in the short-term nominal interest rate can be influenced by lagged real GDP/unemployment rates as well. Additionally, it is crucial to recognize that economic activity can have a simultaneous causal impact on monetary policy, further complicating the estimation process. Ignoring the interdependence and simultaneous causality would introduce bias into the estimates.

From an economic standpoint, to understand the effectiveness of monetary policy, it is essential to consider the various channels through which changes in the nominal interest rate can impact economic growth and employment. A higher interest rate set by monetary policy can increase the interbank interest rate, affecting both borrowers and savers. For borrowers, the higher interest rate makes borrowing more expensive, discouraging them from taking on additional debt. This decreased incentive to borrow can lead to reduced spending on economic activities, such as firm production and individual consumption, ultimately constraining GDP growth. However, higher interest rates work in the opposite direction for savers. With the same savings but higher returns, savers may be more willing to spend more income today, anticipating greater financial resources in the future. Consequently, increased demand from savers can boost GDP growth. The same logic applies to the unemployment rate as well, as higher interest rates can lead to reduced business investment, slower growth, and a decrease in job creation, potentially causing unemployment.

Regardless of the direction of interest rate changes, it is evident that monetary policy can potentially affect economic activities, and this effect can be captured by a parameter in the macroeconomic model. However, it is important to note the issue of endogeneity. The central bank aims to maintain the value of money (low inflation) while ensuring the economy operates near its potential level. This means that if the central bank observes a significant deviation of real GDP from its potential level, it is likely to control inflation by adjusting monetary policy through interest rates. Similarly, when the unemployment rate is high, indicating excess labor supply and downward pressure on wages and inflation, central banks may lower interest rates to stimulate borrowing, spending, and investment, thereby reducing unemployment. These circumstances establish a connection from real GDP growth/unemployment rate to the interest rate, which can be captured by another parameter in the model. As a result, simultaneous causality between GDP growth, unemployment, and monetary policy interest rates occurs, making it challenging to analyze the relationship using a single linear direction.

To address the endogeneity problem, my approach applies VAR models based on Sims' (1980) proposed solution. The VAR model allows for the simultaneous estimation of relationships, accounting for endogeneity. It helps us determine the number of propagation terms to include using model selection criteria and relevant statistical tests. However, before evaluating the effect of nominal interest rates on real GDP growth/unemployment, we need to determine the values of two parameters related to simultaneous causality. In this case, my approach assumes that the parameter capturing the effect of monetary policy on real GDP growth/unemployment rate is zero. This assumption implies that any current change in the interest rate affects output growth/unemployment rate with a lag, meaning contemporaneous changes in the interest rate have no immediate effect on output growth or unemployment rate.

This assumption is economically reasonable because the full impact of interest rate changes may take time to materialize as economic agents adjust their behavior, which can be influenced by factors such as the nature of contracts, decision-making processes, or institutional frameworks. In the short term, contemporaneous changes in interest rates may not be fully reflected in output growth or unemployment due to these time lags and adjustment processes.While this assumption has limitations in capturing the full complexity of the relationship between interest rates and the real economy, it allows us to measure the structural impulse response and subsequently compute the implied impulse response of output growth/unemployment rate to a shock in the interest rate, which serves our purpose of evaluating the causal effect of monetary policy interest rate on economic activities.

3. Data and VAR Model Estimation

Raw data obtained from the Federal Reserve Economic Data (FRED) are used to conduct our analysis. Specifically, we relied on the Federal Funds Effective Rates data spanning from 1954 to 2023 to represent the monetary policy interest rate. In addition, we incorporated FRED's raw data on real GDP in Montana from 1997 to 2022 and the unemployment rate in Montana from 1976 to 2023 as measures of economic activity for our investigation. And it is important to note that, in this analysis, we assume the effect of monetary policy is temporary. So instead of directly employing the output levels, I used the calculated the output growth by taking the first difference in the logarithm of output. This transformation was implemented to consider the stationarity of the variables when running the vector autoregression (VAR) model.

Upon examining the plotted data of both output growth and the unemployment rate that we used in our analysis, it is noteworthy that both datasets exhibit a notable deviation from the expected trends starting from the beginning of 2020. More specifically, both indicators experienced a substantial and transient rise, followed by a rapid decline at an even faster rate after 2020 :

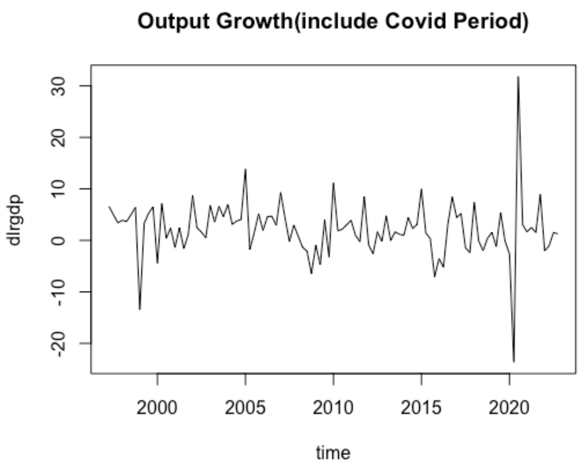

Figure 3.11 Computed FRED data for Output Growth in Montana

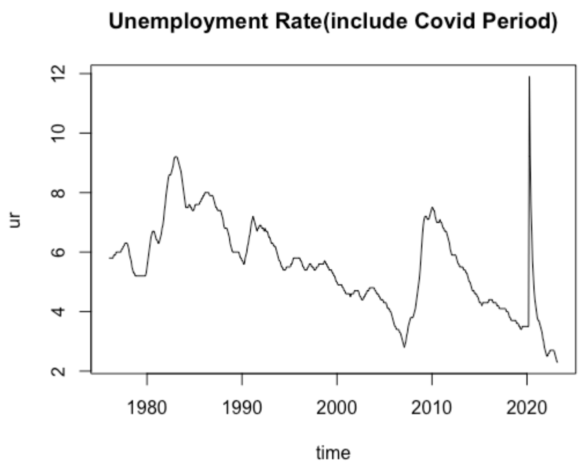

Figure 3.12 FRED raw data for Unemployment Rate in Montana

Taking into account that this period aligns with the onset of the Covid-19 pandemic, an extraordinary event, it is crucial to acknowledge that the data from this pandemic period may not accurately represent the broader trends and patterns observed over the longer term. Therefore, it is reasonable to consider the data from the pandemic period as potential outliers. In light of this, I made the decision to exclude the data from the Covid period in our analysis. By doing so, we aim to obtain a clearer understanding of the historical data, enabling a more precise analysis of the underlying trends and patterns that were present before the pandemic.

Monetary Policy on Output Growth

Upon combining the two time series datasets of monetary policy interest rates and real GDP growth, my first step was to conduct an autocorrelation function (ACF) analysis. This analysis, which examines correlations with lagged values, provided valuable insights into the potential dynamic interdependence between interest rates and real GDP growth, and the plotted output is indeed intuitive:

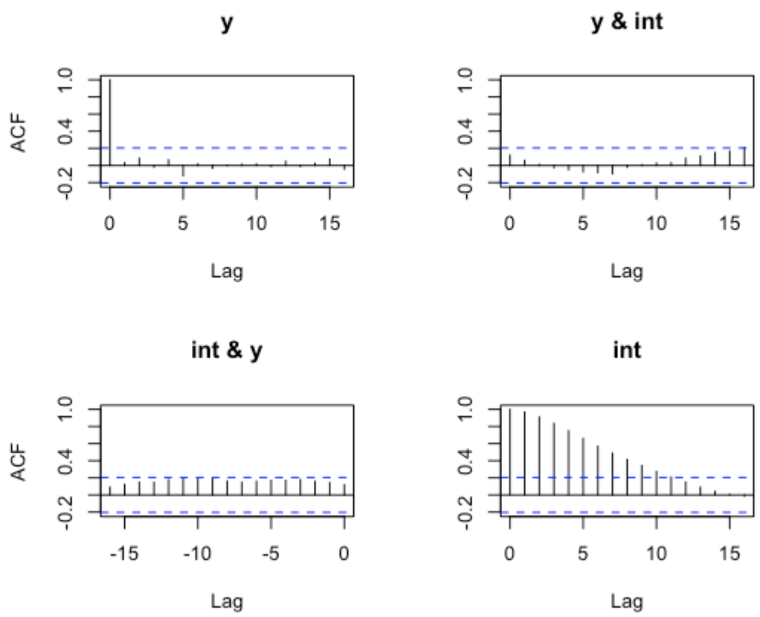

Figure 3.21 ACF for simulated bundle Data of interest rate and output growth

The ACF test conducted for output growth in relation to its own lagged values (left top corner) suggests that it may follow a moving average (MA) process, indicated by the absence of extended periods in which the correlation remains consistently high or low. On the other hand, the ACF test for the interest rate in relation to its own lagged values (right bottom corner) suggests that it is more likely to follow an autoregressive (AR) process, as there is a smooth decay in the function.

However, for our purpose of evaluating the impact of monetary policy on economic activities, the most notable finding from the ACF test is that neither the current output growth's correlation with past interest rates nor its current correlation with potential future interest rates appears to be statistically significant, because there’s no any lag exceeding the significance level. This finding raises the possibility that there might not be a meaningful relationship between the monetary policy interest rate and real GDP growth in Montana, and it is essential to keep this significant aspect in mind and revisit it after conducting subsequent VAR estimations.

My next step was to compute a structural VAR analysis for the interest rate and output growth in Montana. This allowed me to determine the optimal order of the VAR model for my analysis.:

```
$selection
AIC(n)  HQ(n)  SC(n) FPE(n) 
     2      2      2      2 

$criteria
               1         2         3         4         5         6         7
AIC(n) 0.8820480 0.2953030 0.3258888 0.3983183 0.3820636 0.3828306 0.4660291
HQ(n)  0.9555854 0.4178653 0.4974761 0.6189305 0.6517008 0.7014927 0.8337161
SC(n)  1.0660532 0.6019784 0.7552344 0.9503341 1.0567496 1.1801868 1.3860554
FPE(n) 2.4160406 1.3440449 1.3867127 1.4926475 1.4713171 1.4764247 1.6104710
               8         9        10        11        12        13        14
AIC(n) 0.4970285 0.5561651 0.5912527 0.6160446 0.5998861 0.5368266 0.3873559
HQ(n)  0.9137404 1.0219020 1.1060145 1.1798313 1.2126978 1.1986632 1.0982174
SC(n)  1.5397250 1.7215318 1.8792896 2.0267516 2.1332633 2.1928739 2.1660734
FPE(n) 1.6693190 1.7821427 1.8603137 1.9254777 1.9168561 1.8250207 1.5978606
              15        16
AIC(n) 0.4495721 0.4876131
HQ(n)  1.2094586 1.2965245
SC(n)  2.3509598 2.5116710
FPE(n) 1.7338179 1.8423953
```

Figure 3.22 Model selections for interest rate and output growth

Based on the model selection output, all of the selection criteria consistently indicate that the optimal order for our analysis should be "2". Therefore, we can confidently proceed with running the VAR model using 2 lags. Here is a summary of the regression results:

```
Estimation results for equation y:
==================================
y = y.l1 + int.l1 + y.l2 + int.l2 + const

       Estimate Std. Error t value Pr(>|t|)
y.l1   -0.02125    0.10966  -0.194  0.84678
int.l1  2.04992    1.16734   1.756  0.08268 .
y.l2    0.05057    0.10870   0.465  0.64297
int.l2 -1.97289    1.14331  -1.726  0.08806 .
const   1.93913    0.69983   2.771  0.00687 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1


Residual standard error: 4.222 on 85 degrees of freedom
Multiple R-Squared: 0.04264,    Adjusted R-squared: -0.00241
F-statistic: 0.9465 on 4 and 85 DF,  p-value: 0.4413
```

Figure 3.23 Estimation results for equation Output growth

```
Estimation results for equation int:
====================================
int = y.l1 + int.l1 + y.l2 + int.l2 + const

        Estimate Std. Error t value Pr(>|t|)
y.l1    0.003677   0.007641   0.481    0.632
int.l1  1.643052   0.081338  20.200  < 2e-16 ***
y.l2    0.001759   0.007574   0.232    0.817
int.l2 -0.679541   0.079664  -8.530 4.83e-13 ***
const   0.048959   0.048763   1.004    0.318
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1


Residual standard error: 0.2942 on 85 degrees of freedom
Multiple R-Squared: 0.981,      Adjusted R-squared: 0.9801
F-statistic:  1099 on 4 and 85 DF,  p-value: < 2.2e-16
```

Figure 3.24 Estimation results for equation Interest rate

In the equation for output growth in Montana, it is noteworthy that none of the estimated parameters are statistically significant, as all of the p-values are greater than 5%. However, there is a potential positive impact of the first lag of the interest rate on real GDP growth, and the second lag of the interest rate might have a negative effect on output growth, although these effects are only significant at a higher significance level of 10%. It is important to acknowledge that the p-value is not small enough to confidently rule out the possibility of no correlation between them.

On the other hand, in the equation for the interest rate, there is stronger evidence of the first lag having a positive impact on the interest rate, and the second lag having a negative impact, as indicated by the extremely small p-values for these coefficients. However, there is limited statistical inference regarding the correlation between the interest rate and lagged output growth, as the p-value is large, and we cannot reject the null hypothesis.

Both of these results suggest a weak or potentially no correlation between the monetary policy interest rate and output growth in Montana. The normalized covariance matrix of residuals between the two variables is approximately 0.14 (refer to the appendix). This indicates that there may still be some contemporaneous correlation between the monetary policy interest rate and output growth in Montana. However, even if there is a correlation, the impact is not substantial given the magnitude of this value.

Monetary Policy on Unemployment Rate

The same analytical steps were followed to assess the impact of the monetary policy interest rate on the unemployment rate in Montana. After combining the time series datasets of monetary policy interest rates and the unemployment rate, I conducted the ACF test to gain insights into the potential dynamic interdependence between these two variables:

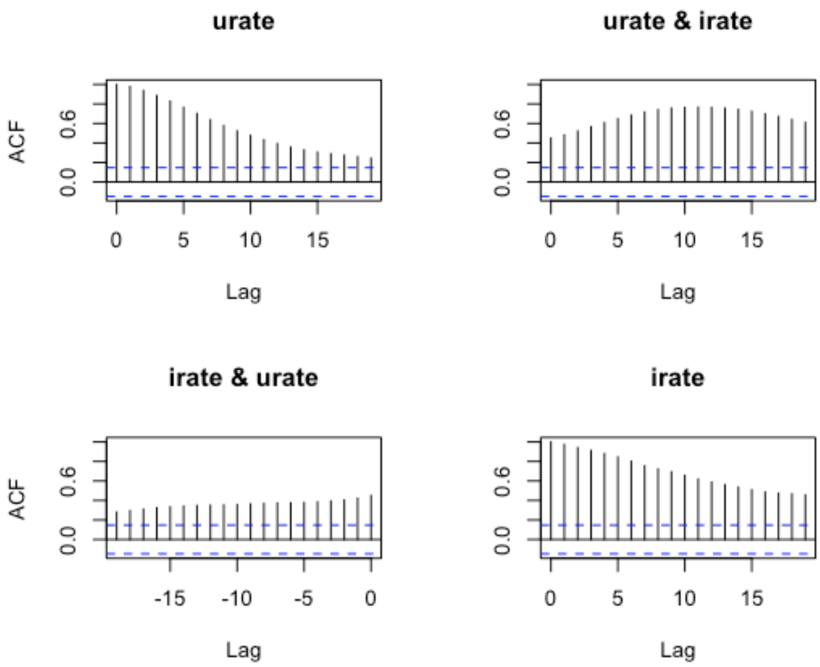

Figure 3.25 ACF for simulated bundle Data of interest rate and unemployment rate

Both the interest rate and unemployment rate exhibit an AR process in relation to their own lag values, indicated by the smooth decay observed in the ACF test. However, unlike the findings from the ACF test for output growth and interest rates, the test results for the unemployment rate and interest rates suggest a positive correlation. This correlation reaches its peak at lag 10 and then smoothly decays. This implies a potential relationship between monetary policy, the unemployment rate in Montana, and interest rates. And in order to check if this relationship exists, we need to construct the VAR analysis for these two variables :

```
$selection
AIC(n)  HQ(n)  SC(n) FPE(n)
    10      6      4     10

$criteria
                 1          2           3           4           5           6
AIC(n) -3.34088298 -4.0622000 -4.12914319 -4.23799247 -4.22172259 -4.33399616
HQ(n)  -3.29425533 -3.9844872 -4.02034533 -4.09810952 -4.05075453 -4.13194300
SC(n)  -3.22604804 -3.8708084 -3.86119499 -3.89348764 -3.80066113 -3.83637806
FPE(n)  0.03540599  0.0172118  0.01609843  0.01443991  0.01467961  0.01312428
                 7           8           9          10          11          12
AIC(n) -4.28758265 -4.33004621 -4.31299973 -4.35660872 -4.32243306 -4.31449935
HQ(n)  -4.05444438 -4.06582285 -4.01769127 -4.03021515 -3.96495439 -3.92593558
SC(n)  -3.71340792 -3.67931486 -3.58571175 -3.55276411 -3.44203181 -3.35754147
FPE(n)  0.01375299  0.01318776  0.01342294  0.01286018  0.01331988  0.01344116
                13          14          15          16
AIC(n) -4.28032562 -4.27011139 -4.26873451 -4.23743427
HQ(n)  -3.86067675 -3.81937741 -3.78691543 -3.72453009
SC(n)  -3.24681112 -3.16004025 -3.08210674 -2.97424988
FPE(n)  0.01392696  0.01409178  0.01413645  0.01461577
```

Figure 3.26 Model selections for interest rate and unemployment rate

The model selection output reveals an interesting observation: the AIC and SC suggest different optimal orders. Typically, in structural VAR analysis, the AIC criterion is favored as it imposes a smaller penalty on models with more parameters. In normal cases, we tend to choose the larger model that is richer in terms of structure. However, in this particular analysis, the AIC criterion suggests 10 lags, which would result in estimating 40 parameters. As I believe this number of parameters is a bit excessive for our analysis, I decided to initially consider the SC criterion, incorporate 4 lags it infers, and assess its results (I will go back to more lags to see if anything changes, and this will be covered in section 5, where I discuss cuss the robustness) :

```
Estimation results for equation urate:
======================================
urate = urate.l1 + irate.l1 + urate.l2 + irate.l2 + urate.l3 + irate.l3 + urate.l4 + irate.l4 + const

          Estimate Std. Error t value Pr(>|t|)
urate.l1  1.917093   0.073666  26.024  < 2e-16 ***
irate.l1 -0.023420   0.013291  -1.762  0.07991 .
urate.l2 -1.513936   0.150857 -10.036  < 2e-16 ***
irate.l2  0.055955   0.019583   2.857  0.00483 **
urate.l3  0.888652   0.148832   5.971 1.42e-08 ***
irate.l3 -0.028360   0.019793  -1.433  0.15381
urate.l4 -0.333228   0.070706  -4.713 5.18e-06 ***
irate.l4  0.009098   0.013922   0.653  0.51435
const     0.166170   0.050272   3.305  0.00116 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1


Residual standard error: 0.1401 on 164 degrees of freedom
Multiple R-Squared: 0.9904,     Adjusted R-squared: 0.9899
F-statistic:  2115 on 8 and 164 DF,  p-value: < 2.2e-16
```

Figure 3.27   Estimation results for equation unemployment rate

In the equation for the unemployment rate in Montana, it is notable that all lagged values of the unemployment rate exhibit a significant impact on the current rate, as indicated by the extremely small p-values. However, the effect of higher lagged unemployment rates, referring to those further back in time, appears to be weaker, as the coefficients become larger with more recent lags.

Moreover, a crucial observation is that the second lag of the interest rate has a positive impact on the current unemployment rate, as reflected by its low p-value. However, this impact does not appear to be substantial, as the coefficient is close to zero.

```
Estimation results for equation irate:
======================================
irate = urate.l1 + irate.l1 + urate.l2 + irate.l2 + urate.l3 + irate.l3 + urate.l4 + irate.l4 + const

         Estimate Std. Error t value Pr(>|t|)
urate.l1 -1.24894    0.43172  -2.893  0.00433 **
irate.l1  1.13873    0.07789  14.619  < 2e-16 ***
urate.l2  1.26438    0.88410   1.430  0.15458
irate.l2 -0.35689    0.11476  -3.110  0.00221 **
urate.l3  0.26936    0.87223   0.309  0.75785
irate.l3  0.36657    0.11600   3.160  0.00188 **
urate.l4 -0.32015    0.41437  -0.773  0.44086
irate.l4 -0.15479    0.08159  -1.897  0.05956 .
const     0.20163    0.29462   0.684  0.49471
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1


Residual standard error: 0.8213 on 164 degrees of freedom
Multiple R-Squared: 0.9607,     Adjusted R-squared: 0.9588
F-statistic: 501.3 on 8 and 164 DF,  p-value: < 2.2e-16
```

Figure 3.28   Estimation results for equation interest rates

In the equation for the interest rate, the most recent unemployment rate (first lag) seems to have a negative impact on the current interest rate, while the impact of more distant lags is not statistically significant, similar to the previous equations.

The normalized covariance matrix of residuals between the two variables is approximately -0.15 (see appendix). This suggests the presence of some contemporaneous negative correlation between the monetary policy interest rate and the unemployment rate in Montana. However personally, the magnitude of this correlation is not that larger.

4. Evidence of Effect of Monetary Policy on Economic Activity

Monetary Policy on Output Growth

To assess whether monetary policy causes changes in economic activity, it is crucial to employ the Sims (1980) solution for structural impulse responses using Cholesky decomposition. After conducting the computation, the residuals appear satisfactory and well-fitting, as they do not surpass the significance level in the partial autocorrelation function and the shock to the residuals of output growth also follows a regular pattern, with both output growth and interest rate initially increasing and gradually returning to zero (these graphs can be found in the appendix section). However, for our purpose of investigating the impact of monetary policy, the key evidence lies in the following graph, which illustrates the response of output growth to a one-time increase in the interest rate, depicted in the upper section :

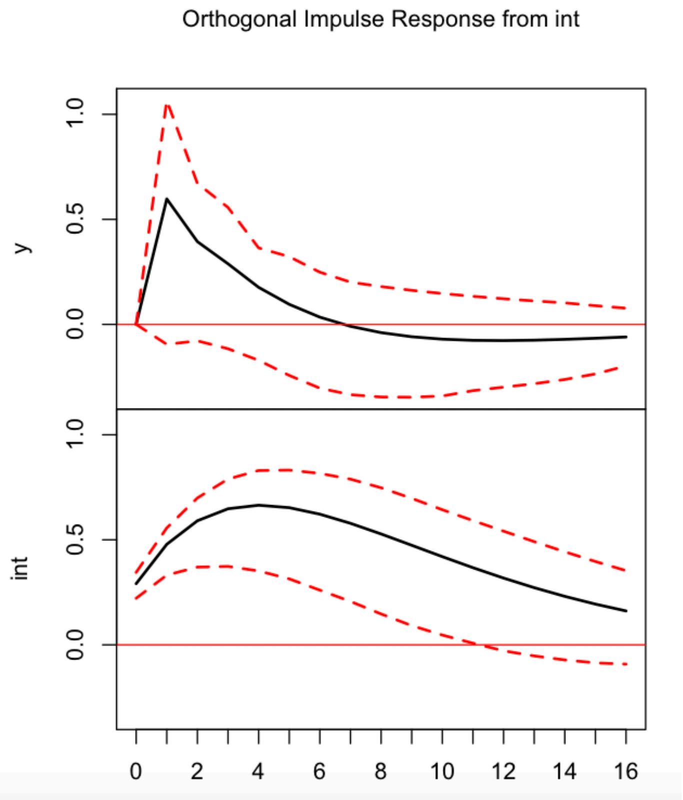

Figure 4.1 Structural Impulse Response evaluating monetary policy on output growth (upper)

The upper section of the graph represents the response of output growth to a shock in the interest rate, capturing the causal effect of the monetary policy interest rate on real GDP growth. Upon examining the response depicted in the upper section, we observe that there is no immediate response at period 0, as our assumption implies that contemporaneous changes in the interest rate do not have an immediate impact on output growth.

At period 1, the response becomes positive and reaches its peak, but gradually declines and turns negative by period 7. Subsequently, the response appears to converge back to zero. However, it is crucial to note that the confidence intervals do not extend beyond the zero line throughout the entire time period. This implies that although the initial point is positive, it is not statistically significant enough to establish a nonzero impact.

In this case, it is reasonable to argue that there is no causal effect of the imposed monetary policy interest rate on real GDP growth in Montana based on the given structural VAR analysis. This finding aligns with the results obtained from the previous section on the ACF test, which raised the possibility of there being no poetntail relationship between the monetary policy interest rate and real GDP growth in Montana.

Monetary Policy on Unemployment Rate

Similarly, we employ the same procedures to examine whether monetary policy influences changes in the unemployment rate. The residuals in the analysis also look well-fitting, with most of them not exceeding the significance level in the partial autocorrelation function, except for just a few labs that might have measurement errors. Additionally, the shock to the residuals of the unemployment rate follows a regular pattern, initially increasing and gradually returning to zero, while the interest rates move in the opposite direction(these graphs can be found in the appendix section).

After performing the Cholesky decomposition, we obtain a graph that depicts the response from the structural VAR analysis, focusing on the causal effect of the interest rate on the unemployment rate in Montana in the upper section:

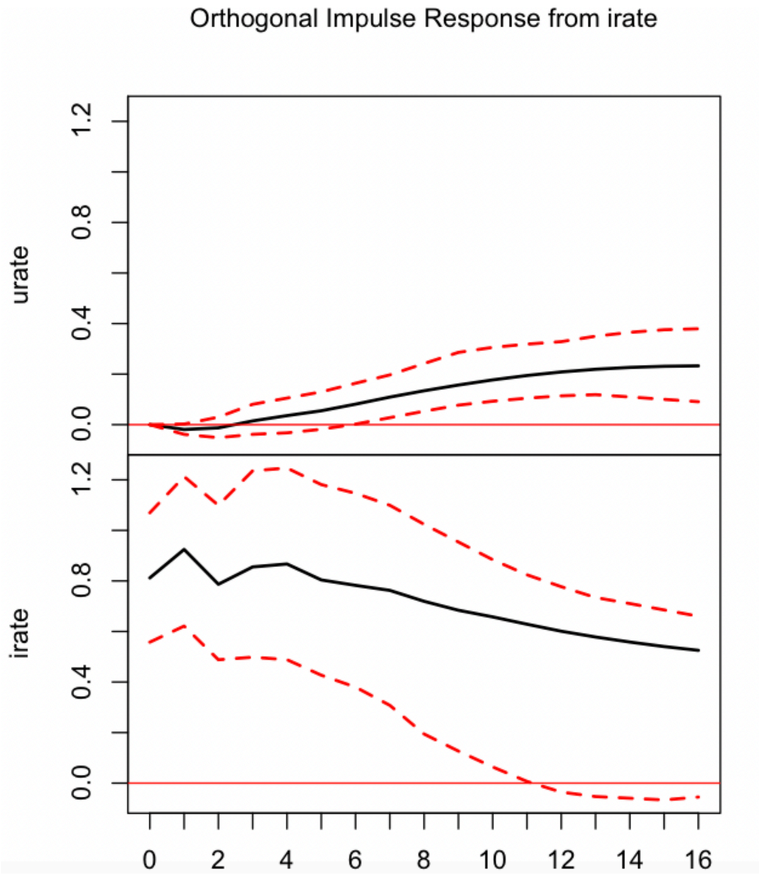

Figure 4.2 Structural VAR evaluating monetary policy on unemployment rate (upper)

The upper section of the graph illustrates the response of the unemployment rate to a shock in the interest rate, capturing the causal effect of the monetary policy interest rate on the unemployment rate in Montana. As our assumption suggests that contemporaneous changes in the interest rate do not have an immediate impact on economic activities, there is no immediate response observed at period 0. Although the point estimate at period 1 appears to be negative, the confidence interval includes the zero line, indicating that it is not statistically different from zero. The response gradually becomes positive and steadily increases from period 2 to period 14. However, it is important to note that the confidence interval encompasses the zero line until period 6, providing evidence that after the first 5 quarters, the monetary policy interest rate begins to exert an influence on the unemployment rate in Montana. Over time, the response weakens, and a decreasing trend becomes apparent after period 12, eventually converging back to zero.

Given the assumption of no contemporaneous effect, it is reasonable to argue that the impact of the monetary policy interest rate on the unemployment rate in Montana emerges after 5 quarters, with a continuously increasing effect in a positive direction. After 12 quarters, the effect gradually dissipates, returning to zero.

In summary, the empirical VAR analysis reveals that the monetary policy interest rate has no significant causal effect on output growth in Montana, as the response of output growth to a shock in the interest rate shows no immediate impact, with subsequent responses lacking statistical significance. On the other hand, there could be a positive effect of the monetary policy interest rate on the unemployment rate in Montana. While the positive response does appear over time, it quickly diminishes and lacks statistical significance after an initial period. Overall, the empirical evidence suggests a weak or negligible relationship between the monetary policy interest rate and economic activities in terms of output growth and unemployment rate in Montana.

5. Discussion of Limitations, Robustness, and Extensions

The analysis presented in this study has provided valuable insights into the causal effect of monetary policy on economic activities. However, it is important to acknowledge certain limitations in the interpretation of the findings. One significant limitation is the exclusion of data from the Covid-19 pandemic period, which could potentially impact the relationship between monetary policy and economic variables. To address this, additional analysis incorporating the Covid-19 period data has been included in the appendix. It is interesting to note that while the inclusion of this data does not significantly alter the analysis results for the causal effect of monetary policy on output growth in Montana, it does have some impact on the analysis of the unemployment rate, as evidenced by the change in the suggested optimal order in the models. This suggests that accounting for the Covid-19 period data could influence the analysis of monetary policies, although the specific impact may not be pronounced in the case of Montana.

Another limitation of this study is the assumption made regarding the immediate effect of contemporaneous changes in interest rates on economic activities. While this assumption was justified based on the complexity and context-dependency of the relationship between interest rates and the real economy, it should be exercised with caution. Economic conditions, policy frameworks, and other factors can significantly influence the outcomes and introduce variations in the results. Therefore, a comprehensive analysis considering the specific economic context is necessary to draw accurate conclusions about the impact of interest rate changes on output growth and unemployment, particularly in different cases and scenarios.

In the robustness analysis, I conducted additional checks to ensure the reliability of the findings by incorporating more lags in the analysis of both output growth and the unemployment rate. This was done for both the dataset including the Covid period and the dataset excluding the Covid period. The results demonstrate robustness in the relationship between monetary policy and economic activities. For instance, in the VAR analysis of the causal effect of monetary policy on real GDP growth, I doubled the number of computed lags. The resulting output graph (included in the appendix section) exhibits a similar pattern to the previous analysis. Although it takes longer to return to zero, this is a natural outcome of incorporating more lags into the VAR structure, which leads to larger standard errors. The crucial point is that the shape of the impulse response remains consistent, providing robust evidence for the addition of more lags. By conducting the analysis with different time periods and variances, the robustness of the findings has been confirmed.

An important avenue for potential extension for this analysis would be to investigate the possibility of nonlinear relationships between monetary policy and economic activities. While the current analysis primarily focuses on linear relationships, it is plausible that the relationship between these variables may exhibit nonlinear dynamics. To explore this, an interesting direction would be to employ advanced methods such as threshold models or regime-switching models which would capture potential nonlinearities and regime-specific effects, leading to a more nuanced and comprehensive understanding of the relationship between monetary policy and output growth/unemployment rate in Montana. By considering nonlinear relationships, I can uncover hidden patterns and dynamics that may exist beyond linear associations, thus enhancing the richness of the analysis.

Appendix

Figure 1 (Normalized) Covariance Matrix of residuals between growth rate and interest rate

```
Covariance matrix of residuals:
          y     int
y   17.8268 0.17289
int  0.1729 0.08655

Correlation matrix of residuals:
         y    int
y   1.0000 0.1392
int 0.1392 1.0000
```

Figure 2 ***Fit and residuals for real GDP growth in Montana***

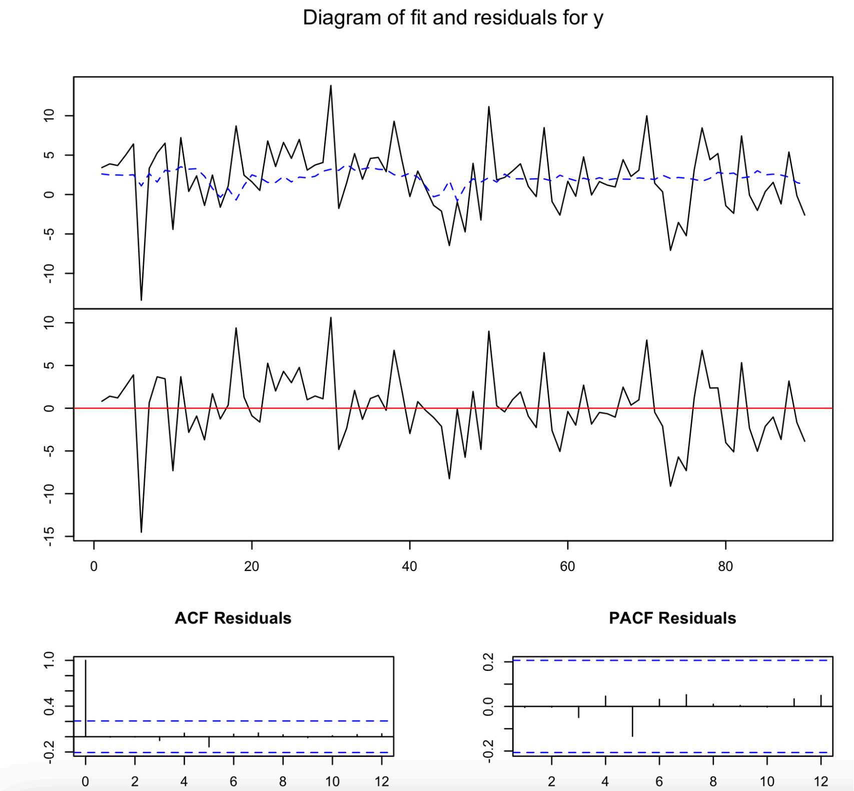

Figure 3 *Fit and residuals for interest rate*

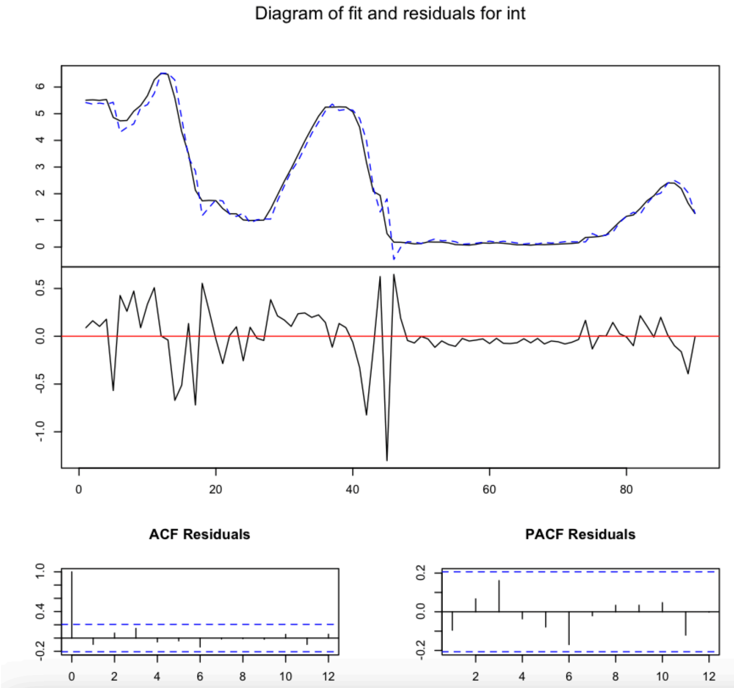

Figure 4

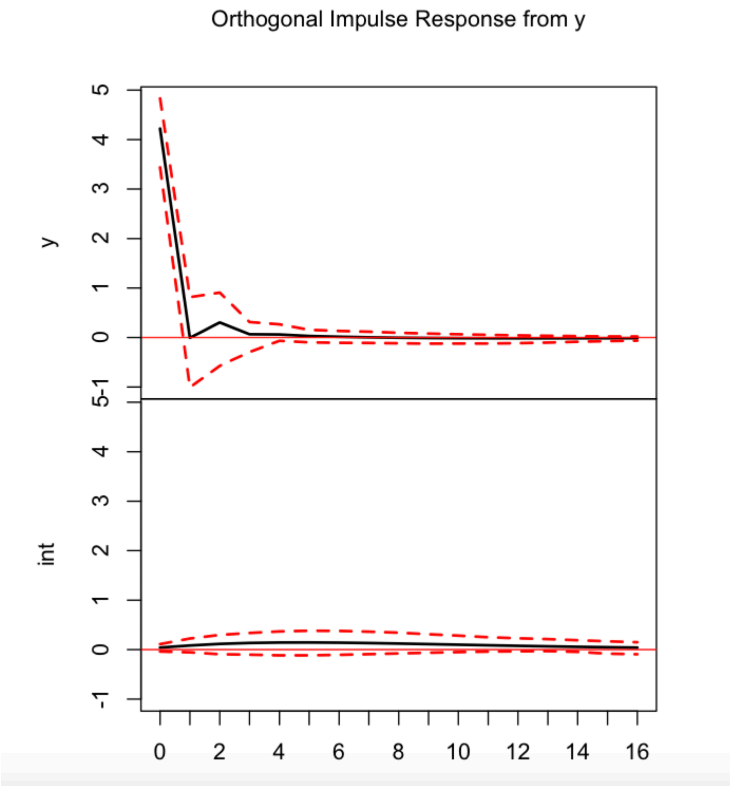

Figure 5 Structural Impulse Response for output growth at 4 lags (robustness check example)

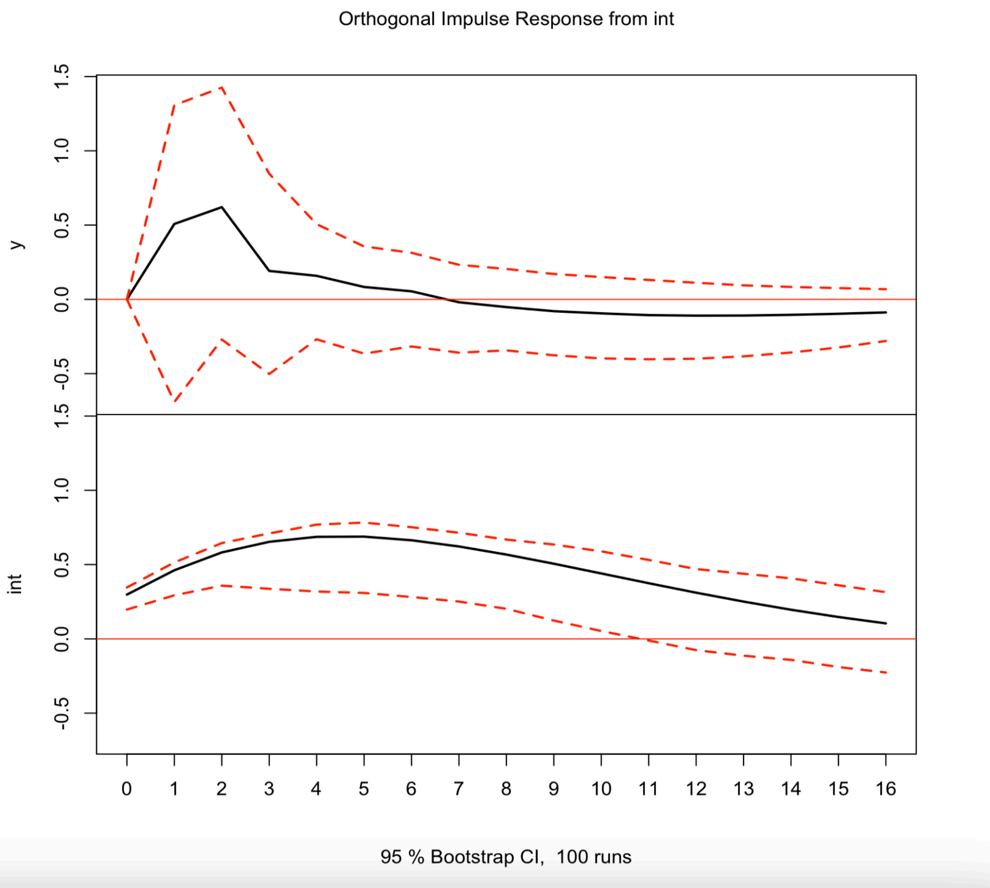

Figure 6 (Normalized) Covariance Matrix of residuals between unemployment rate and interest rate

```
Covariance matrix of residuals:
         urate    irate
urate  0.01964 -0.01725
irate -0.01725  0.67446

Correlation matrix of residuals:
        urate   irate
urate  1.0000 -0.1499
irate -0.1499  1.0000
```

Figure 7 Fit and residuals for unemployment rate in Montana

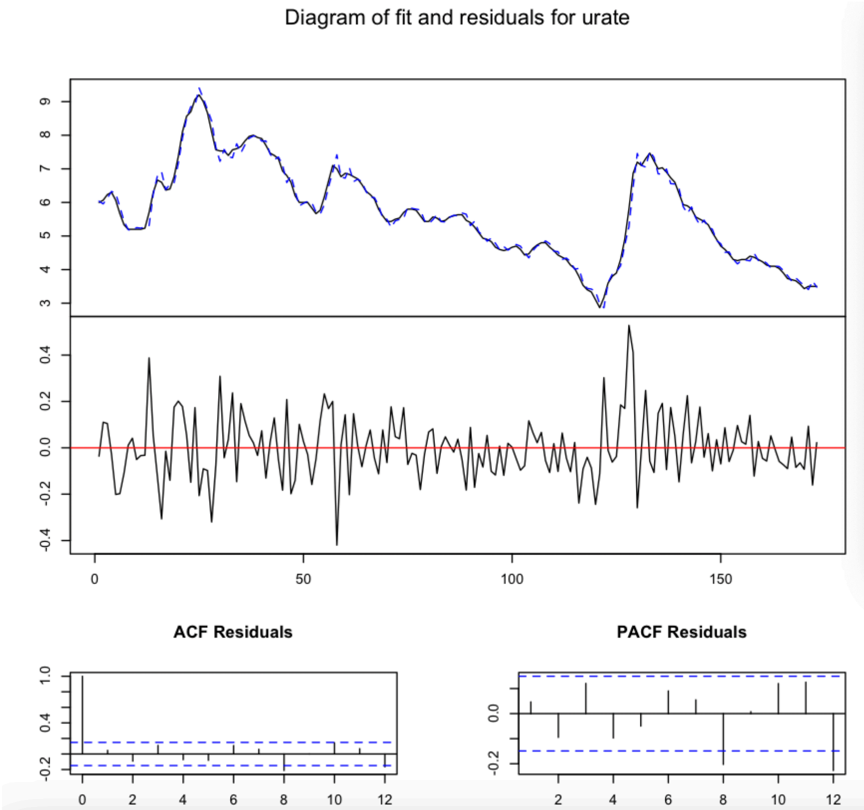

Figure 8

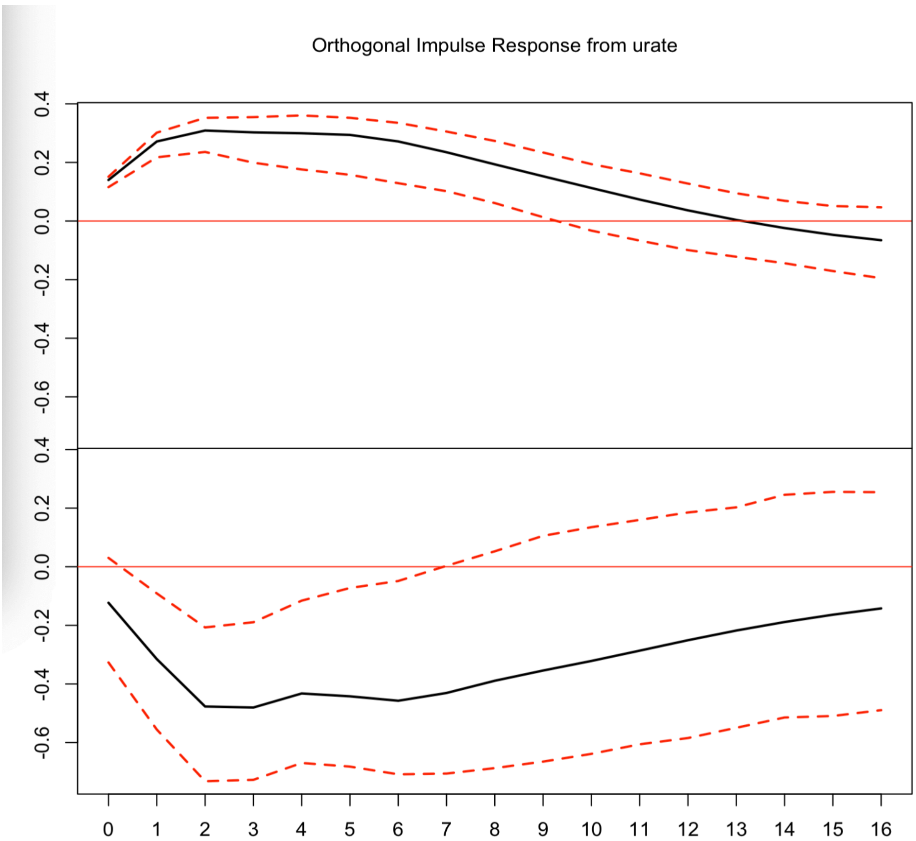

Figure 9 Plotted graph of interest rate (blue) and output growth (black) including Covid period

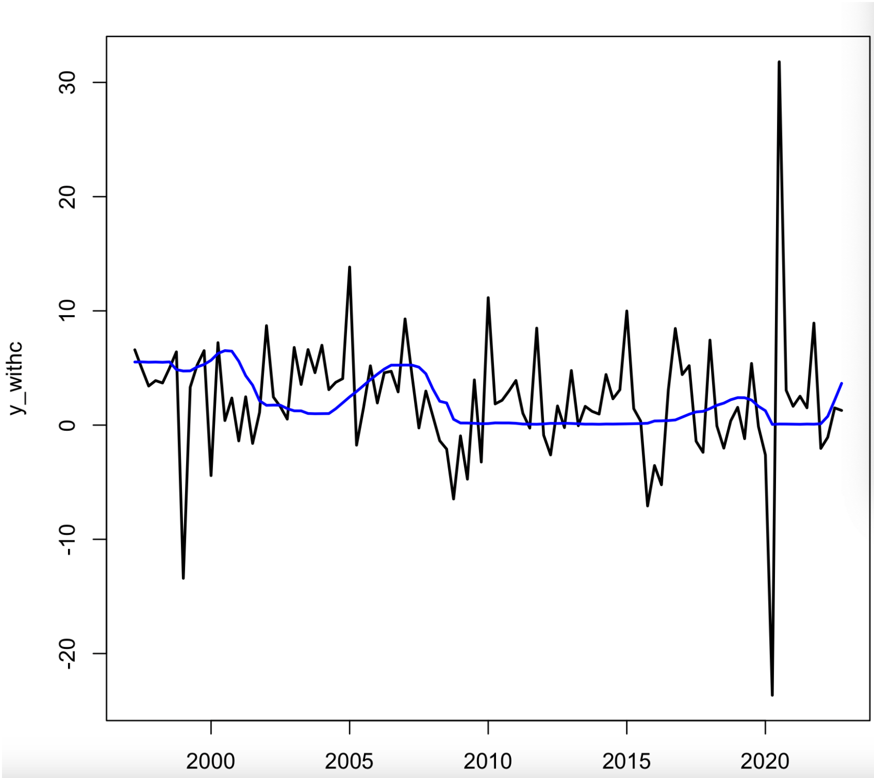

Figure 10 Plotted graph of interest rate (blue) and unemployment rate (black) including Covid period

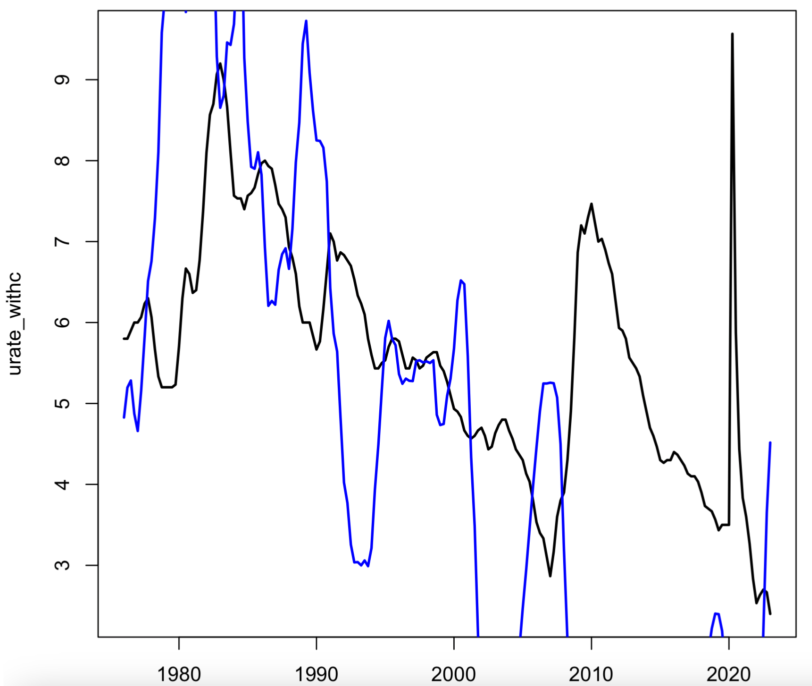

Figure 11 *Model selections for interest rate and output growth (including Covid period)*

```
$selection
AIC(n)  HQ(n)  SC(n) FPE(n)
     2      2      2      2

$criteria
              1        2        3        4        5        6        7        8        9       10
AIC(n) 1.904509 1.172139 1.205257 1.291071 1.341615 1.385690 1.436566 1.437803 1.508282 1.543253
HQ(n)  1.972988 1.286271 1.365041 1.496508 1.592705 1.682432 1.778961 1.825851 1.941983 2.022606
SC(n)  2.074571 1.455577 1.602070 1.801259 1.965178 2.122628 2.286880 2.401492 2.585346 2.733691
FPE(n) 6.716474 3.229711 3.339944 3.642087 3.835641 4.015648 4.235598 4.254535 4.584046 4.771633
             11       12       13       14       15       16
AIC(n) 1.555531 1.563958 1.581193 1.556473 1.539398 1.584722
HQ(n)  2.080536 2.134616 2.197504 2.218436 2.247014 2.337991
SC(n)  2.859344 2.981147 3.111757 3.200412 3.296712 3.455411
FPE(n) 4.861048 4.939490 5.070918 4.999980 4.976746 5.283010
```

Figure 12 *Model selections for interest rate and unemployment rate (including Covid period)*

```
$selection
AIC(n)  HQ(n)  SC(n) FPE(n)
     3      2      1      3

$criteria
               1          2         3          4          5          6          7          8
AIC(n) -1.4286718 -1.4608655 -1.464228 -1.4373988 -1.4012046 -1.4461175 -1.4028869 -1.4177377
HQ(n)  -1.3843040 -1.3869192 -1.360703 -1.3042954 -1.2385227 -1.2538570 -1.1810480 -1.1663202
SC(n)  -1.3193091 -1.2785943 -1.209048 -1.1093107 -1.0002080 -0.9722124 -0.8560733 -0.7980157
FPE(n)  0.2396287  0.2320428  0.231277  0.2375895  0.2463847  0.2356165  0.2461009  0.2425699
               9         10         11         12         13         14          15          16
AIC(n) -1.3781249 -1.3520093 -1.3106787 -1.2834728 -1.2509851 -1.2094665 -1.16510593 -1.12770188
HQ(n)  -1.0971289 -1.0414348 -0.9705257 -0.9137412 -0.8516749 -0.7805778 -0.70663876 -0.63965617
SC(n)  -0.6854944 -0.5864703 -0.4722313 -0.3721168 -0.2667206 -0.1522936 -0.03502455  0.07528798
FPE(n)  0.2524996  0.2593432  0.2704922  0.2782047  0.2876988  0.3002686  0.31433906  0.32685450
```

Figure 13 Structural VAR evaluating monetary policy on output growth(Covid period included)

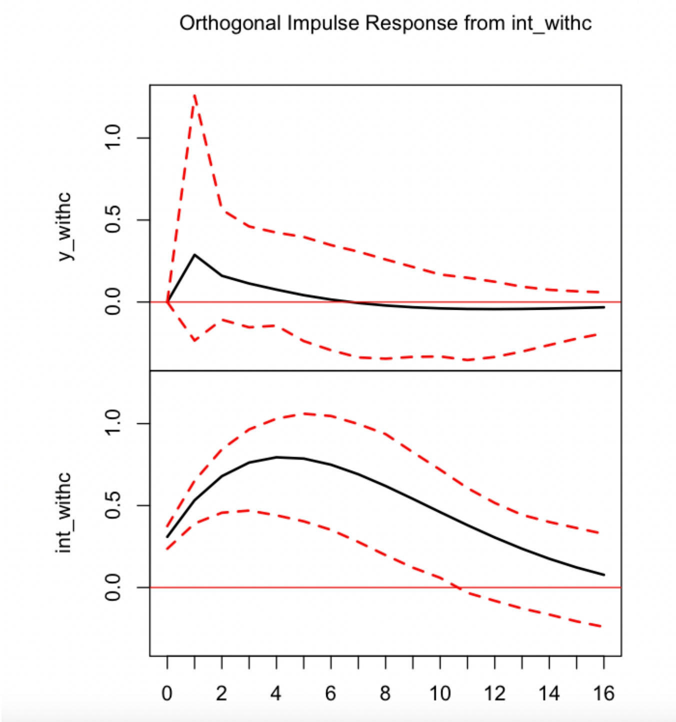

Figure 14 Structural VAR evaluating monetary policy on unemployment rate (Covid period included)