Reduction From Latin Square Completion To Sudoku Completion

1. Problem Definitions

The Latin Square Completion and Sudoku Completion problems are both combinatorial decision problems involving conditions on placing numbers in grids. Each has been shown to be NP-complete and is commonly referenced in the broader context of constraint satisfaction problems.

Latin Square Completion and **Sudoku** problems are defined as follows:

Latin Square Completion:
Given a partially filled n x n grid, where each cell may contain a number from 1 to n, the task is to determine whether the empty cells can be filled so that every number from 1 to n appears exactly once in each row and each column.

Sudoku Completion:
Given a partially filled n^2 x n^2 Sudoku grid divided into n x n blocks, the task is to determine whether the empty cells can be filled with numbers from 1 to n^2 such that each number appears exactly once in every row, column, and block.

2. Problem Instances with Explanation

Latin Square Example:

Suppose there is a 4 × 4 Latin square using the symbols 1 through 4:

Partially Filled Latin Square (Problem)

1			
		4	
3			**2**
		2	

In this incomplete grid, several values have already been placed. The task is to determine whether the remaining blank cells can be filled with numbers from 1 to 4 such that each number appears exactly once in every row and column. No number may repeat in any row or column. This is a typical instance of the problem.

Completed Latin Square (Solution)

1	**2**	**3**	**4**
2	**3**	**4**	**1**
3	**4**	**1**	**2**
4	**1**	**2**	**3**

In this completed grid, every number from 1 to 4 appears exactly once in each row and each column, fully satisfying the constraints of a Latin square. There are no duplicate values, and each symbol is used consistently and without conflict throughout the structure, ensuring a valid and complete solution.

Sudoku Example:

Let's say given a 9 × 9 partially filled Sudoku grid:

4	7	8	5	2	9	1	3	6
9	1	6	3	4	8	7	5	2
2	5	3	6	1	7	9	4	8
8	2	4	9	5	6	3	1	7
6	9	7	1	3	4	2	8	5
1	3	5	7	8	2	4	6	9
5	4	9	8	7	3	6	2	1
3	6	1	2	9	5	8	7	4
7	8	2	4	6	1	5	9	3

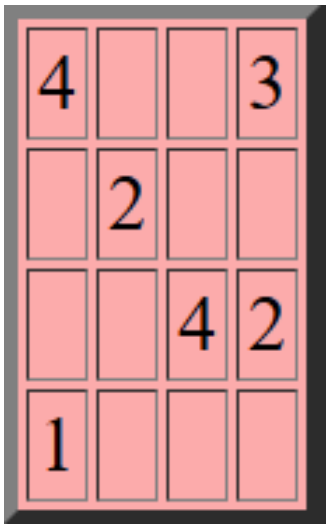
Source:
https://www.theguardian.com/science/2025/mar/03/did-you-solve-it-clueless-sudoku-a-genius-new-puzzle

In the example shown above, the grid follows the standard 9 × 9 structure with digits ranging from 1 to 9. The yellow-highlighted cells represent the initial values given at the start of the puzzle. The goal in a typical Sudoku Completion problem is to fill the remaining cells so that each row, column, and 3 × 3 block contains all digits from 1 to 9 exactly once. In this particular example, the puzzle has already been fully solved, illustrating a valid completion that satisfies all Sudoku conditions..

3. Reduction Diagram and Explanation (Latin Square to Sudoku)

The Latin Square Completion problem can be reduced to the Sudoku Completion problem by embedding the Latin square instance into a larger Sudoku grid that preserves its constraints while conforming to Sudoku's additional rules.

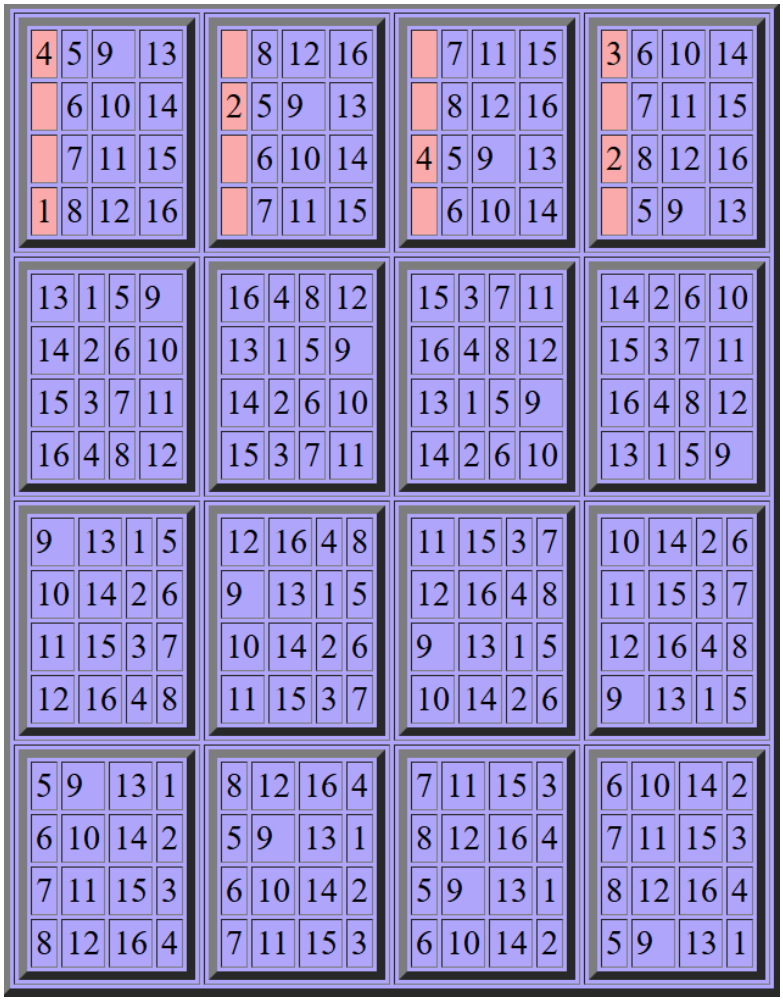
Partially Filled Latin Square (4 × 4 Instance)



Source: https://www.cs.ox.ac.uk/people/paul.goldberg/FCS/sudoku.html

This diagram above shows a partially filled 4 × 4 Latin square, where the goal is to complete the grid so that each number from 1 to 4 appears exactly once in every row and every column. This is a valid instance of the Latin Square Completion problem.

Sudoku Instance Derived from Latin Square (16 × 16 Grid)



4	5	9	13		8	12	16		7	11	15	3	6	10	14
	6	10	14	2	5	9	13		8	12	16		7	11	15
	7	11	15		6	10	14	4	5	9	13	2	8	12	16
1	8	12	16		7	11	15		6	10	14		5	9	13
13	1	5	9	16	4	8	12	15	3	7	11	14	2	6	10
14	2	6	10	13	1	5	9	16	4	8	12	15	3	7	11
15	3	7	11	14	2	6	10	13	1	5	9	16	4	8	12
16	4	8	12	15	3	7	11	14	2	6	10	13	1	5	9
9	13	1	5	12	16	4	8	11	15	3	7	10	14	2	6
10	14	2	6	9	13	1	5	12	16	4	8	11	15	3	7
11	15	3	7	10	14	2	6	9	13	1	5	12	16	4	8
12	16	4	8	11	15	3	7	10	14	2	6	9	13	1	5
5	9	13	1	8	12	16	4	7	11	15	3	6	10	14	2
6	10	14	2	5	9	13	1	8	12	16	4	7	11	15	3
7	11	15	3	6	10	14	2	5	9	13	1	8	12	16	4
8	12	16	4	7	11	15	3	6	10	14	2	5	9	13	1

Source: https://www.cs.ox.ac.uk/people/paul.goldberg/FCS/sudoku.html

The second diagram represents the corresponding 16 × 16 Sudoku instance that results from this reduction. Since the original Latin square is of size 4 × 4, we construct a Sudoku grid of size 16 × 16 (because 4^2 = 16). The partially filled Latin square is embedded into the top left 4 × 4 region of the Sudoku grid. The remainder of the grid is filled with values from 1 to 16 in a way that maintains all Sudoku rules. Each number appears exactly once in every row, column, and 4 × 4 block.

4. Web Source

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