Dynamic Modeling for Optimal Cryptoeconomic Policies

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Research question

How should we design dynamic staking and burning policies for Proof-of-Stake cryptoeconomies?

- Current protocol-coded policies are static even though crypto-economies face dynamic shocks. "Staking" and "burning" rates are the primary coded policy rules in the underlying blockchain system
  - **Staking (increases "token supply", decreases "tokens in circulation")**: mint new tokens and give to those who stake their existing tokens as interest. Similar to interest on reserves.
  - **Burning (decreases "token supply")**: burn (remove from circulation) a portion of the transaction fee paid by users during blockchain transactions
- Policy goal: maximize welfare for good actors: users (increase in token price & consumption) and validators (transaction fees)

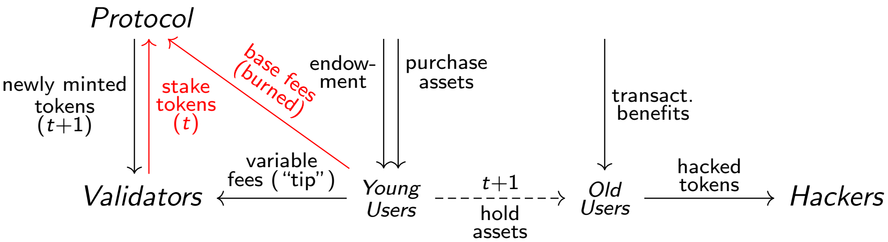
Literature review

- Micro foundations:
  Biais et al. (2019), Budish (2018), Gans and Gandal (2019), Gans and Holden (2022), Huberman et al. (2021), and Nisan et al. (2007), etc.
  → **Block reward and transaction fee as incentive mechanisms**
- Pricing models of cryptocurrencies:
  Bitcoin: *Athey et al. (2016), Biais et al. (2023), Bolt and Van Oordt (2020), Catalini and Gans (2020), Chiu and Koeppl (2017), Garratt and Wallace (2018), Hinzen et al. (2022), and Schilling and Uhlig (2019a, 2019b), etc.*
  Proof-of-stake: *Catalini et al. (2020) and Saleh (2019, 2021)*
  → **Static token supply means all shocks are absorbed by price**
- Monetary policies for stablecoins, platform tokens, and CBDCs:
  Cong, He, and Tang (2022), Cong, Li, and Wang (2022), Cong et al. (2021), d'Avernas et al. (2022), Fernández-Villaverde et al. (2021), Sockin and Xiong (2023a, 2023b), and Zhu and Hendry (2019), etc.
  → **Optimizing policy for defending peg and/or making profit**

Contribution to the literature

- Practical: Most cryptoeconomic systems feature fixed-schedule token supply, creating highly volatile token prices
- Problem: Traditional models for optimal fiscal policy (e.g. Ramsey) and monetary policy (e.g. NK) are not directly transferrable to cryptoeconomies due to transaction fee role in the cryptoeconomy
- Novel hypothesis: Dynamic token supply (via staking and burning policies) can be used to optimize welfare
    - implement in a dynamic equilibrium model framework

A model of proof-of-stake cryptoeconomies



- Inspired by Biais et al. (J Finance 2023) "Equilibrium Bitcoin Pricing"
- Two-period OLG model
- Endogenous token supply (innovation over Biais et al. (2023))
- Assumption: Crypto's fundamental value comes from the stream of future transactional benefits
  - e.g. access to unique goods, not expropriated/taxed/constrained by government, direct internet access

User's problem

- Period t: allocates endowment income e_t^u into three asset classes: risk-free s_t, crypto q_t, and fiat $\hat{q}_t$. To buy crypto they pay a two-part transaction fee φ_t^b (burned) and φ_t^v (goes to validator)
- Period t+1: receive interest r_t, transactional benefits θ_{t+1}, and hacking loss h_{t+1}

$$c_t^u = e_t^u - s_t - (1 + \varphi_t^b + \varphi_t^v) q_t p_t - \hat{q}_t \hat{p}_t \tag{1}$$

$$c_{t+1}^u = s_t(1 + r_t) + (1 - h_{t+1})(1 + \theta_{t+1}) q_t p_{t+1} + \hat{q}_t \hat{p}_{t+1} \tag{2}$$

Validator's problem

- Period t: receive transaction fees φ_t^v, stake L_t
- Period t+1: receive staking yield δ_t

Trade-off:

$$\text{more staking at } t \Rightarrow \begin{cases} \text{more staking yield at } t+1 \\ \text{less circulating supply} \Rightarrow \text{less transact. fees at } t \end{cases}$$

$$c_t^v = e_t^v + \varphi_t^v q_t p_t - L_t p_t \tag{3}$$

$$c_{t+1}^v = (1+\delta_t) L_t p_{t+1} \tag{4}$$

Token supply

$$\begin{aligned} M_{t+1} &= (M_t - L_t) + L_t(1+\delta_t) - \varphi_t^b X_t \\ &= M_t + \delta_t L_t - \varphi_t^b X_t \end{aligned} \tag{5}$$

M_t: total token supply
$X_t \equiv M_t - L_t$: circulating supply
$\Rightarrow$ Token market clearing condition is $X_t = q_t$

* In reality the total reward is $\delta_t L_t^\alpha$, with pre-set $\alpha \in [0,1]$[1]. Here I set $\alpha = 1$ for simplicity.

[1] $\alpha = 0.5$ for Ethereum. See Ethereum Documentation. "Proof-of-Stake Rewards and Penalties"

First order conditions

* using a deterministic version of the model

User's Euler equations:

$$\frac{u'(c_t^u)}{u'(c_{t+1}^u)} = \beta(1+r_t) = \beta\frac{(1+\theta_{t+1})(1-h_{t+1})}{1+\varphi_t^b+\varphi_t^v}\frac{p_{t+1}}{p_t} = \beta\frac{\hat{p}_{t+1}}{\hat{p}_t} \tag{6}$$

Validator's Euler equation:

$$\frac{u'(c_t^v)}{u'(c_{t+1}^v)} = \beta(1+\delta_t)\frac{p_{t+1}}{p_t} \tag{7}$$

Token market equilibrium

Appendix: deterministic math

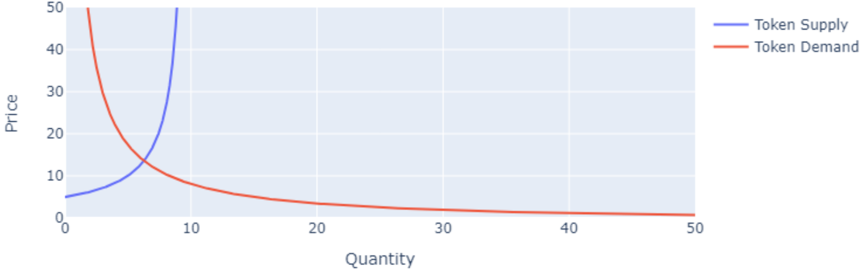


Figure: Illustration of token market equilibrium using artificial parameters. Both supply and demand are convex in q

The Ethereum (ETH) blockchain

Ethereum (created by V. Buterin in 2013) is currently the second largest cryptoeconomy after Bitcoin.

Important historical upgrades to Ethereum's tokenomics:

- London upgrade (Aug 2021): introduced base fee burning via EIP-1559
- "The Merge" (Sep 2022): transition from computing-based (PoW) to staking-based issuance (PoS) i.e. Ethereum 2.0
- Shanghai-Capella upgrade (Apr 2023): unlocked withdrawls of staked ETH [2]

[2] Hypothesis: this changes validators' staking behaviors

Ethereum 2.0 data

Despite having only 6-12 months of history, Ethereum 2.0 data is rich and highly accessible:

- All transactions are publicly broadcasted to the blockchain every 12 seconds (block time), 24/7
- All validators' staking actions and rewards are also recorded
- ETH-USD price data are available from both onchain sources (DEX) and offchain (CEX/"oracle") sources

Features for calibration

- Method: baseline - OLS; advanced - VAR
- Data sources: Ethereum prices from Messari API, txn fees from Dune Analytics, staking data from beaconcha.in
- Proxy for transactional benefits/convenience yields:
  - Throughput measures: average onchain txn processing per second (Cong, He, & Tang, 2022); user operations per second (new measure proposed by industry researchers)
  - Network-based measures: Metcalfe's measure $\log\left(DailyActiveAddresses^2\right)$;
  - Additional controls: index on US crypto regulation (NCSL.org)
- Proxy for hacking: manually index major hacks and wallet losses (problem: frequency is much lower than txn/staking actions)

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Deterministic solutions (cont.)

Inverse demand curve:

$$q_t = \left[\beta \frac{e_t^u}{p_t} - (1+\beta) m_t \frac{\hat{p}_t}{p_t}\right] \cdot \tag{8}$$

$$\left[\beta(1 + \varphi_t^b + \varphi_t^v) + \frac{(1-h_{t+1})(1+\theta_{t+1})}{1+r_t} \frac{p_{t+1}}{p_t}\right]^{-1} \tag{9}$$

Inverse supply curve:

$$q_t = \left(1 + \frac{\beta}{1+\beta}\varphi_t^v\right)^{-1} \left(M_t - \frac{\beta}{1+\beta}\frac{e_t^v}{p_t}\right) \tag{10}$$

Deterministic solutions (cont.)

Token market clearing implies $\frac{1}{p_t^*}$ must satisfy the quadratic equation

$$A_t \frac{1}{p_t^*} = (M_t + B_t \frac{1}{p_t^*})(C_t + D_t \frac{1}{p_t^*}) \tag{11}$$

where

$$A_t := [\beta e_t^u - (1+\beta) m_t \hat{p}_t] \left(1 + \frac{\beta}{1+\beta} \varphi_t^v \right); \quad B_t := -\frac{\beta}{1+\beta} e_t^v; \tag{12}$$

$$C_t := \beta(1 + \varphi_t^b + \varphi_t^v); \quad D_t := \frac{(1 - h_{t+1})(1 + \theta_{t+1})}{1 + r_t} p_{t+1} \tag{13}$$

Deterministic solutions (cont.)

Therefore the closed-form solution for equilibrium token price is given by

$$p_t^* = \frac{2B_tD_t}{A_t - M_tD_t - B_tC_t - \sqrt{(M_tD_t + B_tC_t - A_t)^2 - 4M_tB_tC_tD_t}} \tag{14}$$

under some technical conditions.