Math 164: Homework #4, due on Tuesday, July 22
No late homework accepted.

Reading: Chapter 6

[**1**](P178, Exercise 1.3) Find the dual of

$$\begin{aligned} \text{maximize} \quad & z = 6x_1 - 3x_2 - 2x_3 + 5x_4 \\ \text{subject to} \quad & 4x_1 + 3x_2 - 8x_3 + 7x_4 = 11 \\ & 3x_1 + 2x_2 + 7x_3 + 6x_4 \geq 23 \\ & 7x_1 + 4x_2 + 3x_3 + 2x_4 \leq 12 \\ & x_1, x_2 \geq 0, x_3 \leq 0, x_4 \text{ free.} \end{aligned}$$

Verify that the dual of the dual is the primal.

[**2**](P178, Exercise 1.5) Find the dual to the problem

$$\begin{aligned} \text{minimize} \quad & z = \mathbf{c}^T\mathbf{x} \\ \text{subject to} \quad & \mathbf{A}\mathbf{x} = \mathbf{b} \\ & \mathbf{l} \leq \mathbf{x} \leq \mathbf{u}, \end{aligned}$$

where $\mathbf{l}$ and $\mathbf{u}$ are vectors of lower and upper bounds on $\mathbf{x}$.

[**3**](P186, Exercise 2.2) Prove that if both the primal and the dual problems have feasible solutions, then both have optimal solutions, and the optimal objective values of the two problems are equal.

[**4**](P186, Exercise 2.3) Prove that if the primal is unbounded, then the dual is infeasible. If the dual is unbounded, then the primal is infeasible.

[**5**](P186, Exercise 2.4) Prove that if $\mathbf{x}$ is a feasible solution to the primal, $\mathbf{y}$ is a feasible solution to the dual, and $\mathbf{c}^T\mathbf{x} = \mathbf{b}^T\mathbf{y}$, then $\mathbf{x}$ and $\mathbf{y}$ are optimal for their respective problems.

[**6**](P187, Exercise 2.11) Prove that if the problem

$$\begin{aligned} \text{minimize} \quad & z = \mathbf{c}^T\mathbf{x} \\ \text{subject to} \quad & \mathbf{A}\mathbf{x} = \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0}, \end{aligned}$$

has a finite optimal solution, then the new problem

$$\begin{aligned} \text{minimize} \quad & z = \mathbf{c}^T\mathbf{x} \\ \text{subject to} \quad & \mathbf{A}\mathbf{x} = \hat{\mathbf{b}} \\ & \mathbf{x} \geq \mathbf{0}, \end{aligned}$$

cannot be unbounded for any choice of the vector $\hat{\mathbf{b}}$.