Math 164: Homework #3, due on Tuesday, July 15
No late homework accepted.

Reading: Chapter 5

[**1**](P142, Exercise 2.2(ii,iii)) Solve the following linear programs using the simplex method. If the problem is two dimensional, graph the feasible region, and outline the progress of the algorithm.
(ii)

$$\begin{aligned} \text{maximize} \quad & z = 5x_1 + 3x_2 + 2x_3 \\ \text{subject to} \quad & 4x_1 + 5x_2 + 2x_3 + x_4 \le 20 \\ & 3x_1 + 4x_2 - x_3 + x_4 \le 30 \\ & x_1, x_2, x_3, x_4 \ge 0 \end{aligned}$$

(iii)

$$\begin{aligned} \text{minimize} \quad & z = 3x_1 + 9x_2 \\ \text{subject to} \quad & -5x_1 + 2x_2 \le 30 \\ & -3x_1 + x_2 \le 12 \\ & x_1, x_2 \ge 0 \end{aligned}$$

[**2**](P142, Exercise 2.4) Find all the values of the parameter a such that the following linear program has a finite optimal solution:

$$\begin{aligned} \text{minimize} \quad & z = -ax_1 + 4x_2 + 5x_3 - 3x_4 \\ \text{subject to} \quad & 2x_1 + x_2 - 7x_3 - x_4 = 2 \\ & x_1, x_2, x_3, x_4 \ge 0. \end{aligned}$$

[**3**](P143, Exercise 2.8) Prove that in the simplex method a variable which has just left the basis cannot re-enter the basis in the following iteration.

[**4**](P148, Exercise 3.4) Suppose that the optimal solution to a linear program has been found, and a reduced cost associated with a nonbasic variable is zero. Must the linear program have multiple solutions? Explain your answer.

[**5**](P169, Exercise 4.4)Solve the following problem using the two-phase or big-M method:

$$\begin{aligned} \text{minimize} \quad & z = 2x_1 - 2x_2 - x_3 - 2x_4 + 3x_5 \\ \text{subject to} \quad & -2x_1 + x_2 - x_3 - x_4 = 1 \\ & x_1 - x_2 + 2x_3 + x_4 + x_5 = 4 \\ & -x_1 + x_2 - x_5 = 4 \\ & x_1, x_2, x_3, x_4, x_5 \ge 0. \end{aligned}$$