Math 164: Homework #1, due on Tuesday, July 1
No late homework accepted.

Reading: Chapter 1 and Chapter 3.2

[**1**](P12, Exercise 4.1) Consider the production scheduling problem of the perfume Polly named after a famous celebrity. The manufacturer of the perfume must plan production for the first four months of the year and anticipates a demand of 4000, 5000, 6000, and 4500 gallons in January, February, March, and April, respectively. At the beginning of the year the company has an inventory of 2000 gallons. The company is planning on issuing a new and improved perfume called Pollygone in May, so that all Polly produced must be sold by the end of April. Assume that the production cost for January and February is \$5 per gallon and this will rise to \$5.5 per gallon in March and April. The company can hold any amount produced in a certain month over to the next month at an inventory cost of \$1 per unit. Formulate a linear optimization model that will minimize the costs incurred in meeting the demand for Polly in the period January through April. Assume for simplicity that any amount produced in a given month may be used to fulfill demand for that month.

[**2**](P85, Exercises 2.1(iv))Compute a basis matrix for the null space of the matrix $\mathbf{A}$ and express the points $\mathbf{x}_i$ as $\mathbf{x}_i = \mathbf{p}_i + \mathbf{q}_i$ where $\mathbf{p}_i$ is in the null space of $\mathbf{A}$ and $\mathbf{q}_i$ is in the range space of $\mathbf{A}^T$.

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 0 & 0 & 2 \\ 1 & -1 & -1 & 1 \end{pmatrix} \quad \mathbf{x}_1 = \begin{pmatrix} 3 \\ 1 \\ 1 \\ 2 \end{pmatrix} \quad \mathbf{x}_2 = \begin{pmatrix} 8 \\ 9 \\ -2 \\ -4 \end{pmatrix}$$

[**3**](P85, Exercise 2.5) Let $\mathbf{A}$ be an $m \times n$ matrix with $m < n$. Prove that any n-dimensional vector $\mathbf{x}$ can be written uniquely as the sum of a null-space and a range-space component:

$$\mathbf{x} = \mathbf{p} + \mathbf{q},$$

where $\mathbf{p} \in \mathcal{N}(\mathbf{A})$ and $\mathbf{q} \in \mathcal{R}(\mathbf{A}^T)$.

[**4**] Let $\mathbf{A} \in \mathbf{R}^{m\times n}$ and $\mathbf{B} \in \mathbf{R}^{n\times p}$. Show that

$$\|\mathbf{AB}\| \leq \|\mathbf{A}\|\|\mathbf{B}\|,$$

where $\|\cdot\|$ is the induced norm for matrix.

[**5**] Four buildings are to be connected by electrical wires. The positions of the buildings are as follows: the first building's shape is an ellipse with center $(0,0)^T$ and horizontal and vertical axes 0.5 and 0.6. The second building is a disk with center $(4,0)^T$ and radius 1. The other two buildings are squares centered at $(0,4)^T$ and at $(4,4)^T$, with sides parallel with the axes and of length 2. The electrical wires will be joined at some central point $(x_0, y_0)^T$, and will connect to building i at position $(x_i, y_i)^T$.

(a) Plot the positions of the four buildings in the plane.
(b) Formulate the non-linear optimization problem that minimizes the amount of wire used.

[**6**] Assume that m points (t_i, b_i) are given in the plane, $i = 1, 2, \cdots, m$. Formulate a least-squares minimization problem in the unknown $\mathbf{x} = (x_1, x_2, x_3, x_4, x_5)^T$, for fitting a curve defined by $b(t) = x_1 + x_2 e^{x_3 t} + x_4 e^{x_5 t}$ through the points in an optimal way.