**MATH 164: Optimization, Summer 2014, Practice Midterm Exam**
Instructor: Ming Yan. Teaching Assistant: Erik Walsberg.

**NAME** ____________________________________________

**STUDENT ID #** ______________________________________

**This is *open to all reference textbooks and class notes*. Calculators are NOT allowed. Please show all your work. Partial credit will be given to partial answers.**
**There are 4 questions of total 20 points.**
**Time: 60 minutes.**

| QUESTION | Maximum | SCORE |
|---|---|---|
| [1] | 5 | |
| [2] | 5 | |
| [3] | 5 | |
| [4] | 5 | |
| TOTAL | 20 | |

**[1]** Foxconn accepts manufacturing orders of iPhone 6 from Apple. The orders from Apple are described in the following table:

| Scheduled Delivery Month | Units Ordered |
|---|---|
| September | 15K |
| October | 20K |
| November | 25K |

However, Foxconn can produce at most 25K, 25K, and 15K units of iPhone 6 in September, October, and November, respectively (Because many Foxconn employees leave for home during the thanksgiving holiday). When Foxconn produce more units than ordered in this month, it has the option to deliver the addition units to a later month, with a storage fee of \$2 per phone per month. For example, if 10 iPhones produced in September are eventually delivered in November, then the storage fee is $10 \times (3 - 1) \times \$2 = \$40$. Assume that

- Foxcoon can not produce more than ordered;
- Foxconn starts producing iPhone 6 in September, i.e., there is no previously produced iPhone 6.

Formulate a linear program for deciding the amount of production in each month so that the total storage fee for Foxconn is minimized. (You do not need to give a standard form or solve this linear program. However, you must clearly explain each decision variable, each constraint and the objective function.)

**[2]** Consider the linear programming

$$
\begin{aligned}
\text{minimize} \quad & z = 10x_1 - 2x_2 + 5x_3 \\
\text{subject to} \quad & x_1 + x_2 - 2x_3 \geq 1 \\
& 2x_1 + x_3 \geq 4 \\
& -x_2 + x_3 \geq 0 \\
& x_1, x_2, x_3 \geq 0
\end{aligned}
$$

(a) Show that $\mathbf{x} = (2, 1, 1)^T$ is a feasible point and label each of the constraints as active or inactive.

(b) Find the set of all feasible directions $\mathbf{p} = (p_1, p_2, p_3)^T$ at $\mathbf{x}$.

(c) Using (b), verify that $\mathbf{p} = (-1, -1, -1)^T$ is a feasible direction at the feasible point $\mathbf{x}$, and determine the maximal step length $\alpha > 0$ such that $\mathbf{x} + \alpha\mathbf{p}$ remains feasible.

**[3]** Consider the linear program.

$$\begin{aligned} \text{minimize} \quad & z = -x_1 - 2x_2 \\ \text{subject to} \quad & -x_1 + x_2 \leq 1 \\ & 2x_1 + x_2 \leq 4 \\ & 0 \leq x_1 \leq 2 \\ & x_2 \geq 0 \end{aligned}$$

(a) Draw the feasible region of this linear problem in a 2D plane.

(b) Give **all** the basic solutions and identify **all** the extreme points of the feasible region.

(c) Is there a degenerate extreme point in the feasible region (yes/no)? If yes, specify it.

(d) Give the standard form of the linear program.

**[4]** Find the optimal solution of the following linear program problem in the standard form using the simplex method.

$$\begin{aligned} \text{minimize} \quad & z = -\frac{4}{3}x_3 + x_4 - 9 \\ \text{subject to} \quad & -2x_1 + x_2 + x_3 = 2 \\ & -x_1 + 2x_2 + x_4 = 7 \\ & x_1 + x_5 = 3 \\ & x_1, x_2, x_3, x_4, x_5 \geq 0 \end{aligned}$$