Market Statistics and Technical Analysis: The Role of Volume

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ABSTRACT

We investigate the informational role of volume and its applicability for technical analysis. We develop a new equilibrium model in which aggregate supply is fixed and traders receive signals with differing quality. We show that volume provides information on information quality that cannot be deduced from the price statistic. We show how volume, information precision, and price movements relate, and demonstrate how sequences of volume and prices can be informative. We also show that traders who use information contained in market statistics do better than traders who do not. Technical analysis thus arises as a natural component of the agents' learning process.

TECHNICAL ANALYSIS OF MARKET data has long been a pervasive activity in both security and futures markets. Technical analysts believe that price and volume data provide indicators of future price movements, and that by examining these data, information may be extracted on the fundamentals driving returns.[1] If markets are efficient in the sense that the current price impounds all information, then such activity is clearly pointless. But if the process by which prices adjust to information is not immediate, then market statistics may impound information that is not yet incorporated into the current market price. In particular, volume may be informative about the process of security returns.

In this paper we investigate the informational role of volume. That volume may play an important role in markets has long been a subject of empirical research (see, for example, Gallant, Rossi, and Tauchen (1992); Karpoff (1987) provides an excellent review of previous research). This research has documented a remarkably strong relation between volume and the absolute

[1] The classic work on technical analysis is generally regarded as Edwards and Magee (1957). A more recent work that also details the explicit role of volume in technical analysis is Pring (1991). Neftci (1991) and Brock, Lakonishok, and LeBaron (1992) provide empirical testing of some common rules used in technical analysis of price changes. The latter paper suggests that positive returns may accrue to at least some price-based technical strategies.

value of prices changes in both equity markets and futures markets. But why such a pattern exists or even how volume evolves in markets is not clear. Our goal in this research is to determine how the statistical properties of volume relate to the underlying value of the asset and to the behavior of market prices. By establishing these properties, we hope to show what traders could learn from volume and how this could provide one explanation for the use of volume-based technical analysis in markets.

A natural starting place for our research would seem to be the recently developed models looking at the information content of price sequences. In particular, Brown and Jennings (1989) and Grundy and McNichols (1989) consider rational expectations models in which a single price does not reveal the underlying information but a sequence of security prices does. These papers demonstrate that technical analysis of price patterns may be valuable because it facilitates the learning ability of traders. But adapting such models to investigate the role of volume reveals an immediate problem: In standard rational expectations models with aggregate supply uncertainty, volume plays the role of adding noise to the model. Allowing traders to observe volume essentially allows them to know the aggregate supply and this results in a fully revealing single price. In this framework, the informational role of volume is large, but vacuous. With no role to play other than noise, volume in these models can never provide insights into underlying economic fundamentals or give guidance to the process by which information is impounded into the price.

In this paper, therefore, we develop an alternative equilibrium approach for studying the behavior of security markets. Our model is standard in that some fundamental is unknown to all traders and traders receive signals that are informative of the asset fundamental. However, in our model aggregate supply is fixed. The source of noise is the quality of the information; specifically the precision of the signal distribution. Prices alone cannot provide full information on both the magnitude of the signals and their precision. We show that volume provides information about the quality of traders' information that cannot be deduced from the price statistic. We also show how sequences of volume and prices can be informative, and demonstrate that traders who use information contained in the market statistic will do "better" than traders who do not. In our model, technical analysis arises as a natural component of the agents' learning process.

This property of traders using the information contained in volume is a unique and important feature of our model. In other models of volume (see for example Campbell, Grossman, and Wang (1991), Harris and Raviv (1991) and Wang (1991)), volume is interesting for its correlation with other variables, but in itself is unimportant: Traders never learn from volume nor use volume in any decision making. By contrast, in our model volume enters traders' learning problems because they use the specific volume statistic in updating their beliefs. Consequently, volume matters in our model because it affects the behavior of the market, rather than merely describes it.

Our construction of sequences of price and volume also allows us to make predictions about the equilibrium properties of price, price changes, and volume. Our model demonstrates why volume and the absolute value of price changes are positively correlated, and provides interesting comparative static predictions of the effects of information precision and dispersion on the price-volume relationship. Such predictions may be useful to researchers interested in a wide variety of issues in accounting and finance. From a time series perspective, our model provides an intriguing result on the equilibrium behavior of volume. We show that although all traders will learn the asset's value, and prices will thus converge to the full information or strong form efficient price, volume does not converge to zero. In fact, volume has a limit distribution that is nondegenerate. This demonstrates that markets do not shut down as beliefs converge and has the important implication that the "no-trade" equilibrium results so prevalent in the literature may not describe the limit behavior of equilibrium models with learning. Finally, our model also shows why technical analysis of price and volume data can be valuable and provides predictions regarding the type of firms for which it will be particularly useful. Thus, we provide one explanation for the paradoxical existence of technical analysis in seemingly efficient markets.

The paper is organized as follows. In the next section we consider how allowing agents to condition on both prices and volume affects the equilibrium in the standard rational expectations random supply framework. Our purpose in doing so is to demonstrate why such models fail in the presence of volume, and to delineate what must be changed if volume is to be incorporated (and analyzed) in an economically meaningful way. We then investigate in Section II what information is provided by price and volume statistics in a new model in which agents condition on prior market outcomes and there is no aggregate supply uncertainty. Section III examines how the sequence of price and volume statistics reveals information, and provides results on the time series of price and volume. Section IV shows how technical analysis based on price and volume can make agents better off. The paper's final section is a conclusion.

I. Price, Volume, and Technical Analysis

We begin our analysis by examining the role of volume and trade information in the standard rational expectations framework typically employed to investigate how market clearing prices reflect underlying information, and how agents, in turn, learn from prices. Our approach in this section is to investigate simplified versions of models developed by Brown and Jennings and Grundy and McNichols that address the role of price data in technical analysis. These models share a common rational expectations approach, but introduce aggregate supply uncertainty in different ways. As we show, this difference results in major differences in equilibria when volume data are introduced. After investigating the role of volume and trade information in

the Brown and Jennings model and then in the Grundy and McNichols framework, we summarize the problems revealed in the standard approach, and detail how theoretical models must be changed to provide a meaningful analysis of price and volume data.

In the standard approach (see Grossman and Stiglitz (1980)), a collection of agents, indexed by $i = 1, \ldots, I$, trade a risky asset and a riskless asset in a single market. Both Brown and Jennings and Grundy and McNichols consider the limit case where the number of agents I is infinite. In this standard model, trades may occur at time 1 and at time 2. We analyze only time 1 and so do not include time indices. At the end of trading, the riskless asset pays a known dividend of 1 and the risky asset pays a liquidating dividend given by the random variable ψ. Traders begin with identical beliefs about the payoff ψ, which are represented by a normal distribution $N(\psi_0, 1/\rho_0)$.

Traders maximize negative exponential utility functions defined on final wealth (or consumption) of the form

$$U(w_i) = -\exp[-w_i] \tag{1}$$

where w_i is agent i's terminal wealth (we have fixed the coefficient of absolute risk aversion at one). Final period wealth depends on the agents' trading decisions and the assets' payoffs, and so can be written as $w_i = d_i \psi + n_i$, where d_i is agent i's demand for the risky asset and n_i is the number of units of the riskless asset that have a price normalized to one.

Before the start of period 1, each trader receives an endowment of n_0 units of the riskless asset. Each trader also receives a private signal, y_i, on the value of the risky asset which is given by

$$y_i = \psi + e_i \tag{2}$$

where the distribution of each e_i is $N(0, 1/\rho)$. Because the signals' errors are assumed normally distributed with finite variances and are independent across traders, it follows that the average signal, $\bar{y} = \Sigma_{i=1}^I y_i/I$, converges to ψ with probability 1 as the number of traders grows large.

In the Brown and Jennings framework, there is an exogenous supply of the random asset given by the random variable X, with per capita supply X/I, denoted x. As is the case with all random variables in the model, x is normally distributed and is independent of any private signals. Equilibrium requires that

$$x = \sum_{i=1}^I d_i/I \tag{3}$$

or simply that per capita demand equal per capital supply.

In rational expectations models of the form considered here, equilibrium involves a set of price and demand functions that satisfy the following properties. First, given their information sets H^i (to be specified later) agents conjecture the equilibrium price function. Based on these price functions and an observation of the equilibrium price, traders determine their demands for

the risky asset. In an equilibrium, these price conjectures will be correct and per capita demand will equal per capita supply.

To construct such an equilibrium, suppose that each trader conjectures that the price of the risky asset, p, is a linear function of aggregate information ($\bar{y}$) and per capita supply (x):

$$p = \alpha\psi_0 + \beta\bar{y} - \gamma x. \tag{4}$$

Then, the posterior distribution of ψ given $H^i = (y_i, p)$ is normal with mean $E[\psi|H^i]$ and variance $\text{Var}[\psi|H^i]$. Trader i's demand is then

$$d_i = \frac{E[\psi|H^i] - p}{\text{Var}[\psi|H^i]}. \tag{5}$$

Using the equilibrium condition (3), Brown and Jennings then solve for the equilibrium price. They show that it is linear as conjectured and that the coefficient on x is not zero. Thus, prices are not revealing. This allows Brown and Jennings to demonstrate how a sequence of prices could provide information that a single price observation could not, and thus provides a role for technical analysis.

Now, we suppose that contemporaneous volume data is publicly available.[2] Volume is typically defined as the number of shares of the risky asset that are traded. Since every trade involves a buyer and a seller, volume could be calculated by simply adding up all buy orders or all sell orders. An equivalent approach in a Walrasian equilibrium is to sum the absolute value of traders' demands and divide by two.[3]

If traders do know volume in the Brown and Jennings framework, then the role for technical analysis dissipates. What causes this to happen is that, if traders use the information conveyed by volume and their own trading behavior, there is a revealing equilibrium. Consequently, with all information revealed to traders, there is no benefit to considering the sequence of prices. To see why this occurs, suppose we let traders condition on per capita volume and the direction of their own trade (i.e., either a buy or a sell).[4] Their

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[2] We conduct a Walrasian analysis and so do not ask how such equilibria could be attained. This is a problem even for rational expectations equilibria without conditioning on volume. Blume and Easley (1990) show that absent some restrictive conditions there does not exist a mechanism that would implement rational expectations equilibria. We view equilibria with conditioning on contemporaneous market statistics as one of several possible approximations to actual market processes. In Section III, we explore an alternative approximation that we find more appealing.

[3] We consider Walrasian equilibria. There is no market maker, so we do not worry about inventory affecting the total quantity traded. If traders have endowments of the risky asset then we need to sum the absolute value of the trader's net demands (see equation (8)).

[4] If traders are not allowed to condition on the direction of their own trade, or if they have differing endowments so that they cannot infer the sign of x from their trade, then the only equilibrium would be nonrevealing. However, the distribution of ψ given their information could not be normal so finding this equilibrium would be a formidable task.

information set is now $H^i = (p, y_i, V, J)$ where J is an indicator variable denoting whether the trader buys or sells, and (per capita) volume is defined by $V = \frac{1}{2I}(\Sigma_{i=1}^{I}|d_i| + |X|)$. Further, let traders conjecture that the equilibrium price function is given by (4) and that price and volume together will be revealing. In this case, each trader's demand function is given by[5]

$$d_i = (\psi_0 \rho_0 + \bar{y}I\rho) - p(\rho_0 + I\rho) \qquad (6)$$

and the price function is given by

$$p = (\psi_0 \rho_0 + \bar{y}I\rho - x)/(\rho_0 + I\rho). \qquad (7)$$

To show that these equations describe an equilibrium, we need to show that the traders' conjectures are correct and that the market clears. First, note that in a revealing equilibrium, every trader will demand the same amount of the risky asset, i.e., $d_i = d_j = d$ for all i and j. So per capita volume will be $\frac{1}{2}(|d| + |x|)$. Now market clearing yields $d = -x$, so $V = |x|$. Thus, each trader infers that $x = -V$ if he is a buyer, i.e., $d_i > 0$, or $x = V$ if he is a seller, i.e., $d_i < 0$.[6] Using this inferred value for x and the market price, p, each trader inverts the price equation (7) to solve for $\bar{y}$. Given knowledge of $\bar{y}$, the optimal demand for any trader is given by (6). It is easy to check that the price given by (7) clears the market when demands are given by (6). Thus, traders have equal demands and their conjectures are correct. Once you know volume, therefore, you can infer the underlying supply uncertainty, prices are revealing, and technical analysis has no role.

Interestingly, the opposite conclusion arises from the Grundy-McNichols approach: volume is devoid of any useful information whatever. The reason for this lies in the uncertainty structure of their model. Unlike the random aggregate supply feature of the Brown and Jennings model, Grundy and McNichols introduce uncertainty by assuming that each of the I traders in the market receives a random endowment of the risky asset. These endowments, x_i, are assumed independently and identically normally distributed with mean μ_x and variance $\sigma_x^2 I$. In this model, some traders receive negative quantities of the risky asset, some receive positive quantities, and trade presumably arises in part to rebalance portfolios.

To ensure that individual traders' endowments carry no information about per capita supply x, Grundy and McNichols consider only the limit economy. In this economy, the variance of x is infinite and the Law of Large Numbers cannot be applied. Note that this assumption of the limit economy (infinite

[5] Those expressions are for any finite economy. The limit economy is degenerate with $p = \psi$. Grundy and McNichols consider the limit to avoid the computational difficulties that arise when individual endowments carry information. That is not a problem here as price and volume are already fully informative. In any case, the limit would be no problem in any economy in which the aggregate signal $\bar{y}$ was not perfect. For example, the common error model used in Brown and Jennings would work.

[6] Obviously, a trader could also infer x from his own demand. We address this possibility later in this section.

traders) is fundamental to their approach. If we consider the finite economy, then there is a finite variance and endowments must provide some information. Each trader also receives a private signal $y_i = \psi + w + \varepsilon_i$ where ψ is the per unit payoff from the risky asset, w is a common error, and ε_i is an idiosyncratic error.

If we examine per capita volume in the limit economy we find the distracting feature that it is infinite. In particular, per capita trading volume is

$$\frac{1}{2} \lim_{I \to \infty} \frac{1}{I} \sum_{i=1}^{I} |d_i - x_i| \tag{8}$$

where d_i is trader i's equilibrium demand and x_i is his endowment. Asymptotically, d_i and x_i are uncorrelated, but x_i has infinite variance. Thus, per capita volume is infinite and it provides no information about the value of the asset.[7]

The fundamental difficulty is the underlying supply structure. Whether supply is introduced by an exogenous random supply or by random endowments, if volume reveals anything it reveals the supply. Consequently, if we allow traders to condition on contemporaneous volume, it is essentially allowing them to remove the "noise" in the pricing equation.[8] With prices then depending only on private signals, the only known equilibrium is one in which price reveals the underlying information.

In this context, volume provides no useful information about any fundamentals relating to the asset but rather is exogenously determined. It seems more reasonable to believe that the volume statistic should capture some endogenous aspect of the trading process not necessarily incorporated in prices. In particular, since volume arises from individual demands, it may be the case that volume reflects aspects of the information structure that traders might wish to know.[9]

But a second difficulty arises in investigating this role. This is the problem created by conditioning on contemporaneous information. Even if volume has some meaningful economic role, when traders use the information conveyed by contemporaneous volume, the only revealing equilibrium is the anomalous one in which volume actually has no information. To see why this is so, consider a Grossman-Stiglitz–style model without the modelling device of

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[7] Our analysis applies to first-period volume, but would apply equally well to volume in any period in which new endowments were distributed as they were in period one. Grundy and McNichols consider volume in period two where no new endowments arrive, and so per capita volume is not infinite. They provide some interesting results on when the "no-trade" equilibrium results break down and volume, per se, is positive. They do not consider the possibility of conditioning on volume, which is the focus of our concern here.

[8] An interesting analysis of the role of noise in noisy rational expectations models is given by Bhattacharya and Spiegel (1991).

[9] In a market microstructure model, Easley and O'Hara (1992) demonstrate that volume may provide information on the existence of new information. In their model, however, trades occur sequentially, so that the information content of volume differs from the role it plays in a call market, rational expectation framework.

random endowments or random supply. Suppose that traders have common preferences and endowments and receive payoff-relevant signals. Now, suppose that there exists a revealing rational expectations equilibrium with conditioning on price and volume. In this equilibrium, traders have common information and they all choose the same trade. But the only such trade that is consistent with market clearing is no trade, and so regardless of the signals, volume is zero—and carries no information.[10] Alternatively, there could be nonrevealing equilibria in which traders condition on price and volume. However, as volume is a sum of absolute values it cannot be normally distributed. So although such an equilibrium might exist there seems to be no hope of constructing it, and hence no hope of using a contemporaneous data approach to study volume.

One way to avoid these difficulties is to allow traders to condition on all information up to but not including the market statistic resulting from their desired trade. This approach, first suggested by Hellwig (1982), avoids the simultaneity problem noted above while retaining the ability to learn from market information. Blume and Easley (1984) use this approach to examine the information content of past market prices. In this paper, we use such conditioning on existing market statistics to investigate the role of price and volume.

This approach, like the approach of conditioning on contemporaneous data, is an abstraction. We offer two justifications for it. First, traders who submit market orders do not know the price at which their order will execute until after the trade occurs. Even traders who use limit orders cannot condition their quantity perfectly on price unless they use incredibly and unrealistically complex orders. But unless traders know the price at which they will trade, and use the information the price contains in selecting their trade, the usual rational expectations approach is not valid. Hence, actual market settings are not consistent with contemporaneous conditioning requirements, but are compatible with the conditioning requirement we consider. Second, asset markets such as the New York Stock Exchange are never in a Walrasian equilibrium: The market is a dynamic process in which continual adjustments occur. The fiction of a Walrasian equilibrium is itself an approximation to workings of the market. Whether this oversimplified description is best constructed with conditioning on past or contemporaneous data depends on how well each model serves its intended purpose.

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[10] The assumption that preferences and endowments are identical is not necessary. Suppose traders have constant absolute risk aversion utility functions of the form $-\exp(-R_i w_i)$ and endowments y_i. If price and volume are revealing, then volume must be

$$\frac{1}{2}\Sigma_i \left| \frac{\Sigma_j y_j}{R_i \Sigma_j (1/R_j)} - y_i \right|.$$

But this does not depend on private information. So unless price alone was revealing, price and volume cannot be revealing.

Conditioning on predetermined rather than contemporaneous information has another advantage. If traders can condition on contemporaneous price information, they can also condition on the information contained in their own net trade.[11] If we include his own net trade in each individual's information set the conventional equilibrium remains as an equilibrium. But there are others as well. Jordan (1983) has shown that with these information sets there always exists a revealing equilibrium.[12] Furthermore, the revealing equilibrium is in a sense more natural as it is robust to the model specification (i.e., exponential utilities and normal distributions) whereas the conventional equilibrium is not robust. Analyses with conditioning on contemporaneous information thus finesse a delicate equilibrium selection problem which does not arise when traders use past information.

In the next section, we investigate how traders learn from market information by developing a Walrasian model in which traders are allowed to use the information conveyed by all past prices and volumes. In our model, price and volume data each convey information about the underlying asset value but the type of information they convey differs. As we demonstrate, this provides a role for technical analysis in which both price and volume data are useful. It should be noted that by technical analysis we do not mean using market statistics from the previous period to infer information from that period. As we do not allow conditioning on current endogenous data, this use of past data occurs by assumption. By technical analysis we mean the use in period t of market statistics from periods $t - 1$, $t - 2, \ldots,$ to make inferences about the future value of assets.

II. The Information Content of Volume

We consider a repeated asset market in which agents can trade a risk-free and a risky asset. All trade is between the agents we model; there is no exogenous supply of any asset. Each agent maximizes a negative exponential utility function of the form defined in equation (1). The asset's eventual value is given by the random variable Ψ, where Ψ is normally distributed with mean Ψ_0 and variance $1/\rho_0$. All traders initially have $N(\psi_0, 1/\rho_0)$ as their (common) prior on asset value. We make the usual assumption that all random variables in the model are independent.

Our interest is in the market statistics that arise in a competitive economy with a large number of traders. We develop these statistics by analyzing a market with N traders and providing results as $N \to \infty$. We refer to results obtained by taking the limit as the number of traders grows large as results

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[11] Note that this information includes not only the direction of the trade, but its magnitude as well.

[12] It is easy to calculate this revealing equilibrium in the Grundy-McNichols and Brown-Jennings models. If each trader conjectures that price and his own net trade is revealing then his demand depends only on the mean signal. His demand and price then reveal the per capita supply as well as the mean signal.

for the large economy. Because traders are risk averse, movements in the price elicit portfolio rebalancing trades. In addition, trade may also occur in response to new information on the asset's true value. Each trader in our economy receives an informative signal in every period. We divide the traders into two groups, with $N_I = \mu N$ traders in group 1 and $N_u = (1 - \mu)N$ in group 2. The traders in each group receive signals from a common distribution, but there are different distributions for the two groups. Formally, each informed trader i in group 1, $i = 1, \ldots, N_I$, receives a signal at date t of $y_t^i = \psi + w_t + e_t^i$ where w_t is a common error term distributed $N(0, 1/\rho_w)$. The e_t^i represents an idiosyncratic error which is distributed $N(0, 1/\rho_t^1)$. Similarly, trader i in group 2, $i = N_I + 1, \ldots, N$, receives signal $y_t^i = \psi + w_t + \varepsilon_t^i$ where each $\varepsilon_t^i \sim N(0, 1/\rho^2)$. We keep ρ^2 fixed (and known) to reduce the complexity of our presentation.

The precision of group 1's signals (the ρ_t^1) are random variables.[13] All parameters other than the ρ_t^1's are known to all traders, but each ρ_t^1 is known only to traders in group 1. This randomness in precisions means that the "quality" of signals varies over time. Consequently, the underlying information structure is complex, in that both the level and quality of signals are unknown.

Each trader begins with zero endowment of the risky asset and some exogenous endowment, N_0, of the riskless asset. For simplicity, we set the price of the riskless asset at one. As the utility function is negative exponential and the asset's eventual payoff has a normal distribution, it is well known that a trader's demands for the risky asset will be independent of his wealth. Our interest is in the Walrasian equilibrium price and volume of the risky asset. To calculate these equilibrium statistics, we need only find traders' demands for the risky asset and find the price that clears the market (i.e., makes excess demand zero).

To make it easier to write asset demands, note that for traders in group 1 each signal y_t^i is distributed $N(\psi, 1/\rho_t^{s1})$ where $\rho_t^{s1} = \rho_w \rho_t^1/(\rho_w + \rho_t^1)$.[14] Similarly, for traders in group 2 each y_t^i is distributed $N(\psi, 1/\rho^{s2})$ where $\rho^{s2} = \rho_w \rho^2/(\rho_w + \rho^2)$. Conditional on w_t, each y_t^i is distributed $N(\theta_t, 1/\rho_t^1)$ for traders in group 1 and $N(\theta_t, 1/\rho^2)$ for traders in group 2, where $\theta_t = \psi + w_t$. So by the Strong Law of Large Numbers, the mean signal in each group, $\bar{y}_t^1$ and $\bar{y}_t^2$, converges almost surely to θ_t as $N \to \infty$. In the large economy, the mean signal is almost surely equal to the true value plus the common error.

Initially, we consider a two-period version of the model, and then extend our results to the multiperiod version. Following Brown and Jennings, we

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[13] At this point, we place no assumptions on the ρ_t stochastic process. In the following section some additional structure will be needed.

[14] This is the exogenous distribution of trader i's signal from trader i's point of view. He knows that the signal has unknown mean ψ and known variance $1/\rho_t^s$. Using standard Bayesian updating (see DeGroot (1970)), the expected value of the asset given signal y_t^i is $(\rho_0 \psi_0 + \rho_t^s y_t^i)(\rho_0 + \rho_t^s)^{-1}$.

assume that traders have myopic, or naive, demands so that each trader chooses his demand to maximize expected utility on a period by period basis.[15] Denoting the price of the risky asset by p_1, the first period demand for the risky asset for each trader i in group 1 is given by

$$\rho_0(\psi_0 - p_1) + \rho_1^{s1}(y_1^i - p_1) \tag{9}$$

and by

$$\rho_0(\psi_0 - p_1) + \rho^{s2}(y_1^i - p_1) \tag{10}$$

for each trader i in group 2. The equilibrium first-period price for an economy with N traders is then given by

$$p_1 = \frac{\rho_0\psi_0 + \mu\rho_1^{s1}\bar{y}_1^1 + (1 - \mu)\rho^{s2}\bar{y}_1^2}{\rho_0 + \mu\rho_1^{s1} + (1 - \mu)\rho^{s2}}. \tag{11}$$

By the Strong Law of Large Numbers we know that in the large economy,

$$p_1 = \frac{\rho_0\psi_0 + (\mu\rho_1^{s1} + (1 - \mu)\rho^{s2})\theta_1}{\rho_0 + \mu\rho_1^{s} + (1 - \mu)\rho^{s2}}. \tag{12}$$

An important property of this equilibrium price is that it is not revealing. Because traders in group 2 do not know ρ_1^s they cannot infer the signal θ_1 from the equilibrium price. Hence, while prices reflect the aggregated value of the underlying signals, these traders do not have enough information to discern what this value is. The conditional distribution of θ_1 given price is not normal, so any multiperiod analysis with conditioning on price alone would be quite complex. Traders in group 1, however, do know ρ_1^{s1} and ρ^{s2}, so observing the equilibrium price tells them θ_1, which is everything that can be known about the underlying asset.

Because traders in group 2 cannot recover θ_1 from price alone, there is a reason for them to look at volume. The first period volume can be found by summing the absolute values of demands at price p_1 and dividing by 2. As it

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[15] Ideally, each trader would predict the stochastic process of prices, given his current information, and then solve the intertemporal decision problem taking potential capital gains into account. This problem is tractable if and only if future prices are normally distributed. When precisions are known this occurs and we have solved the resulting dynamic programming problems. In this case, the value function is negative exponential in wealth, so the only change from the analysis in the text is a change of parameters in demands and thus prices. With random precisions we cannot obtain a closed-form solution to the decision problem and we have thus chosen the approximation of myopic demands.

will be easier to consider per capita volume, we define this as

$$V_1 = \frac{1}{2}\frac{1}{N}\left(\sum_{i=1}^{N_I} |\rho_0(\psi_0 - p_1) + \rho_1^{s1}(y_1^i - p_1)| \right.$$

$$\left. + \sum_{i=N_I+1}^{N} |\rho_0(\psi_0 - p_1) + \rho^{s2}(y_1^i - p_1)| \right). \tag{13}$$

Inspection of the volume definition in equation (13) reveals an immediate problem in analyzing the properties of this market statistic. Because volume is based on absolute values of the demands defined in equations (9) and (10) its distribution is complicated. Unfortunately, while demands involve normally distributed random variables, volume per se cannot be normally distributed. Consequently, if we are to understand the market information conveyed by the volume statistic we must find a way to describe its statistical properties. Proposition 1 provides this characterization of the volume statistic.

PROPOSITION 1: *In the large economy, given* θ_1, *per capita volume,* V_1, *is given by*

$$\frac{\mu}{2}\left[2\frac{\rho_1^{s1}}{(\rho_1^1)^{1/2}}\phi\left(\frac{\delta^1(\rho_1^1)^{1/2}}{\rho_1^{s1}} \right) + \delta^1\left(\Phi\left(\frac{\delta^1(\rho_1^1)^{1/2}}{\rho_1^{s1}} \right) - \Phi\left(\frac{-\delta^1(\rho_1^1)^{1/2}}{\rho_1^{s1}} \right) \right) \right]$$

$$+ \frac{(1-\mu)}{2}\left[2\frac{\rho^{s2}}{(\rho^2)^{1/2}}\phi\left(\frac{\delta^2(\rho^2)^{1/2}}{\rho^{s2}} \right) \right.$$

$$\left. + \delta^2\left(\Phi\left(\frac{\delta^2(\rho^2)^{1/2}}{\rho^{s2}} \right) - \Phi\left(\frac{-\delta^2(\rho^2)^{1/2}}{\rho^{s2}.} \right) \right) \right]$$

where ϕ *is the standard normal density,* Φ *is the standard normal cumulative distribution function and* $\delta^i = \rho_0(\psi_0 - p_1) + \rho_1^{si}(\theta_1 - p_1)$, $i = 1, 2$.

Proof: All proofs are given in the Appendix.

From Proposition 1 and equation (12) we know the market statistics for price and volume in period 1. The question of interest is what information do these market statistics provide? We know from our earlier discussion that price alone is not revealing, so traders cannot infer the noisy signal value θ_1 from just the market price. However, if traders observe both the price and the volume, then potentially the volume information can provide sufficient additional information about θ_1.

To determine the value of looking at volume, we need to separate out the information generated by prices from that generated by volume. From the volume equation it is apparent that the volume statistic includes both θ_1 and

ρ_1^1. Using the equilibrium price equation we know that

$$\theta_1 - p_1 = \rho_0(p_1 - \psi_0)/(\mu\rho_1^{s1} + (1 - \mu)\rho^{s2}).\qquad(14)$$

Substituting for θ_1 allows us to write the volume statistic as

$$\frac{\mu}{2}\left[2\frac{\rho^{s1}}{(\rho_1^1)^{1/2}}\phi\left(\frac{\hat{\delta}_1^1(\rho_1^1)^{1/2}}{\rho_1^{s1}}\right) + \hat{\delta}_1^1\left(\Phi\left(\frac{\hat{\delta}_1^1(\rho_1^1)^{1/2}}{\rho_1^{s1}}\right) - \Phi\left(\frac{-\hat{\delta}_1^1(\rho_1^1)^{1/2}}{\rho_1^{s1}}\right)\right)\right]$$

$$+\frac{(1 - \mu)}{2}\left[2\frac{\rho^{s2}}{(\rho^2)^{1/2}}\phi\left(\frac{\hat{\delta}_1^2(\rho^2)^{1/2}}{\rho^{s2}}\right)\right.$$

$$\left.+ \hat{\delta}_1^2\left(\Phi\left(\frac{\hat{\delta}_1^2(\rho^2)^{1/2}}{\rho^{s2}}\right) - \Phi\left(\frac{-\hat{\delta}_1^2(\rho^2)^{1/2}}{\rho^{s2}}\right)\right)\right]\qquad(15)$$

where

$$\hat{\delta}_1^j = \rho_0(p_1 - \psi_0)\left(\frac{\rho_1^{sj}}{\mu\rho_1^{s1} + (1 - \mu)\rho^{s2}} - 1\right),\text{ for }j = 1, 2.$$

Using this expression for volume, we now investigate how volume is related to the underlying parameters in the market. We show that given price, volume conveys information about signal quality, ρ_1^1, which can then be used in the price equation to make an inference about θ_1.

Calculation shows that if $\rho_1^1 \in (\rho^2, \rho_w)$, then volume is increasing the precision of group 1's signal. To explain why this relationship occurs, and how traders use it, we focus on the simple case where $\rho^2 = 0$ and $\rho_1^1 > 0$. As a first property note that given a price p_1, the effect of changing the precision, ρ_1^1, on volume is given by

$$\partial V_1/\partial\rho_1^1 = \frac{\mu}{2}\phi\left(\hat{\delta}_1\frac{(\rho_w + \rho_1^1)}{\rho_w(\rho_1^1)^{1/2}}\right)\left(\frac{\rho_w}{(\rho_1^1)^{1/2}}\right)\frac{(\rho_w - \rho_1^1)}{(\rho_w + \rho_1^1)^{1/2}}.\qquad(16)$$

Thus, for any price, p_1, per capita volume is increasing in the precision, ρ_1^1, of group 1's signal for $\rho_1^1 < \rho_w$ and decreasing in ρ_1^1 for $\rho_1^1 > \rho_w$.

The intuition for this result can be seen by considering what happens when group 1 receives low quality signals, i.e., ρ_1^1 near 0. In this case, traders in group 1 receive very dispersed signals, but place little confidence in them. At the extreme value of $\rho_1^1 = 0$, there are no useful signals and the only possible equilibrium occurs at $p_1 = \psi_0$ and volume of zero. Alternatively, if ρ_1^1 is large, i.e., $\rho_1^1 > \rho_w$, then volume is again low, but for the opposite reason. Now, group 1 receives high-quality signals, but they are also highly correlated. As $\rho_1^1 \to \infty$, group 1 traders all receive the same signal and so do not trade with each other. Now the only trade occurs between group 1 and group 2. This suggests that simple linear predictions of the correlation between information and volume are seriously misspecified; low volume may be as

indicative of new information as is high volume. Volume is related to dispersion of beliefs and the link between dispersion and information is complex.

This relationship between price, volume, and the quality of information can perhaps best be investigated by examining the actual equilibrium outcomes for a specific economy. Because our model provides closed form solutions, it is possible to graph these outcomes for a specific set of parameter values. As a starting point, let the initial prior mean $\psi_0 = 1$, and let the fraction of traders in group 1, μ, be 0.5. For simplicity, let both the precision of the common error in the signal, ρ_w, and the precision of the prior, ρ_0, be 0.5. Finally, let $\rho^2 = 0$, so we know that $\rho_1^1 > \rho^2$. Figure 1 is generated by selecting, for each ρ_1^1 in a grid, various θ's and then calculating the equilibrium price and per capita volume.

Slices of Figure 1 for fixed precision, ρ_1^1, suggest a convex relationship between price and volume. This is in fact true generally, not just for the specific economy graphed in Figure 1. Volume is convex in price with its minimum at price equal to the prior expected value of the asset.
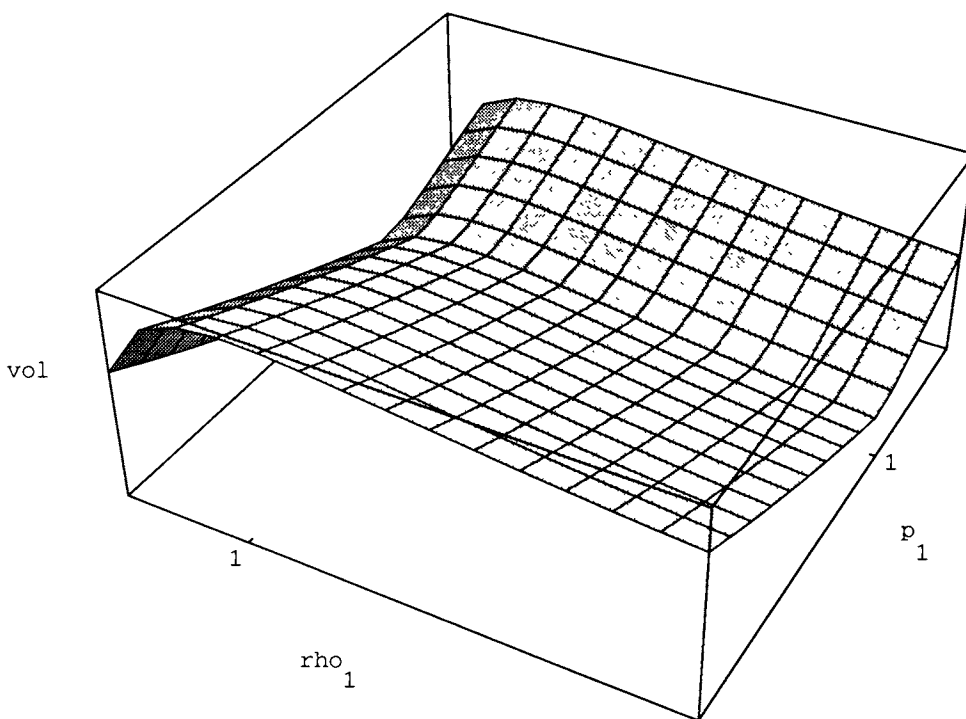

Figure 1. The relation of volume to equilibrium price and information precision. The figure is constructed for an economy with the prior mean $\psi_0 = 1$, $\rho_w = \rho_0 = 1$, $\rho^2 = 0$, and the fraction of traders receiving information $\mu = 0.5$. The graph is generated from equation (15) which shows per capital volume (*vol*) as a function of the precision of group 1's signals (ρ_1) and the equilibrium (p_1).

PROPOSITION 2: *In equilibrium volume, V_1 is strictly convex in price and achieves its minimum at $p_1 = \psi_0$.*

This convexity result in Proposition 2 follows from the fact that the second derivative of volume with respect to price is positive everywhere and that the first derivative is zero at $p = \psi_0$. To see why this occurs, first note that $p = \psi_0$ can happen only if $\theta_1 = p = \psi_0$, so that on average traders posterior means are unchanged from their prior mean. Trade occurs as individuals have differing signals, but it is limited. As the mean signal, θ_1, moves away from ψ_0, and thus price moves away from ψ_0, on average posterior means are changed and the first term in the demand equations (9) and (10) adds to trade. This term reflects portfolio rebalancing in which even a trader with no new information (i.e., $\rho_1^{si} = 0$) will engage. This convex, or V-shaped, relationship can be seen from slices of the curve in Figure 1, but it is even more vividly illustrated in Figure 2 which provides a plot of equilibrium per capita volume against equilibrium price. This figure is drawn for an economy in which $\psi_0 = \psi = 1$, $\rho_0 = \rho_w = 2$, $\rho^2 = 0$, $\mu = 0.5$ and ρ_1^1 is uniformly distributed on $(0, 1)$. The graph was then constructed by drawing 500 pairs of (θ_1, ρ_1^1) from their distributions and calculating the equilibrium for each pair.

Interpreting the prior mean ψ_0 as the previous price, this graph also illustrates the obvious positive correlation between volume and the absolute value of price changes. Figure 3 shows this with a plot of volume against the absolute value of price change for the observations in Figure 2. As is apparent, large price changes (either positive or negative) tend to be associated with large volume. Hence, it is the case that absolute price movements and volume are positively related.
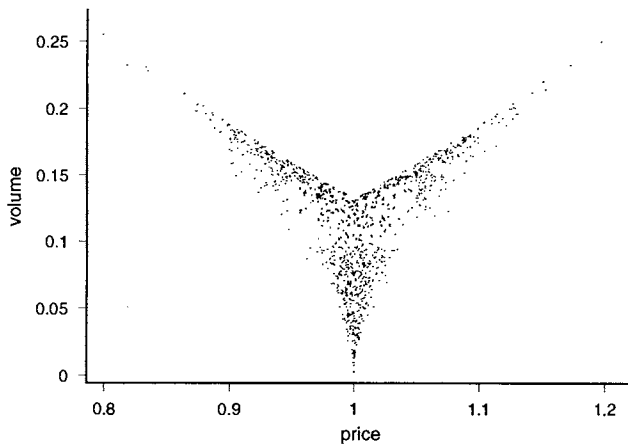

Figure 2. The relation of price and volume. The figure is drawn for parameter values $\psi_0 = \psi = 1$, $\rho_0 = \rho_w = 2$, $\rho^2 = 0$, and $\mu = 0.5$ and $\rho_1^1 \in [0, 1]$. The figure is constructed by drawing 500 pairs of (θ_1, ρ_1^1) from their distributions and calculating the resulting equilibrium. Each point corresponds to the resulting equilibrium price and volume for a single draw.
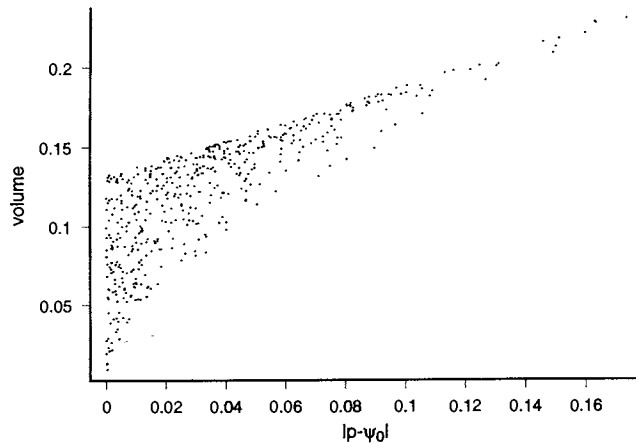
Figure 3. The relation of volume to the absolute value of price changes. The figure is drawn for parameter values $\psi_0 = \psi = 1$, $\rho_0 = \rho_w = 2$, $\rho^2 = 0$, and $\mu = 0.5$ and $\rho_1^1 \in [0, 1]$. The figure is constructed by drawing 500 pairs of (θ_1, ρ_1^1) from their distributions and calculating the resulting equilibrium.

What is particularly intriguing about these results is their remarkable similarity to the findings of empirical researchers. As reported by Karpoff (1987), a V-shape has been found by virtually all empirical investigators of the price-volume relation in equity markets. Moreover, the relation between absolute price changes and volume has been established for both equity and futures markets. In more recent work, Gallant, Rossi, and Tauchen (1991) use time series data to demonstrate the V-shaped pattern between price and volume, and also report that the dispersion of the distribution of the price changes increases uniformly with volume. While the consistency of these results is impressive, what is not well established is why these relations exist. A related question is how, if at all, these phenomena are related to the existence of information.

These issues can be addressed for our sample economy by examining how both the quality (the precision) and the quantity (the dispersion) of information affect the price-volume relation. Figure 4 depicts the resulting price-volume equilibrium outcomes for three different information precisions. What is most striking is that while greater information quality (i.e., larger precision) reduces the dispersion of the points, it does not change the general V-shape of the relation. Indeed, the graph suggests that as the precision approaches its limit, the price-volume relation is characterized by a simple V-shape. These results on information quality suggest a remarkable robustness to the V-shape reported by empirical researchers, but are not consistent with the dispersion results reported by Gallant, Rossi, and Tauchen.

The effect of the quantity of information is perhaps even more interesting. Figure 5 plots the price-volume outcomes for three scenarios corresponding to 10 percent of the traders being in the high-precision group, 50 percent of the
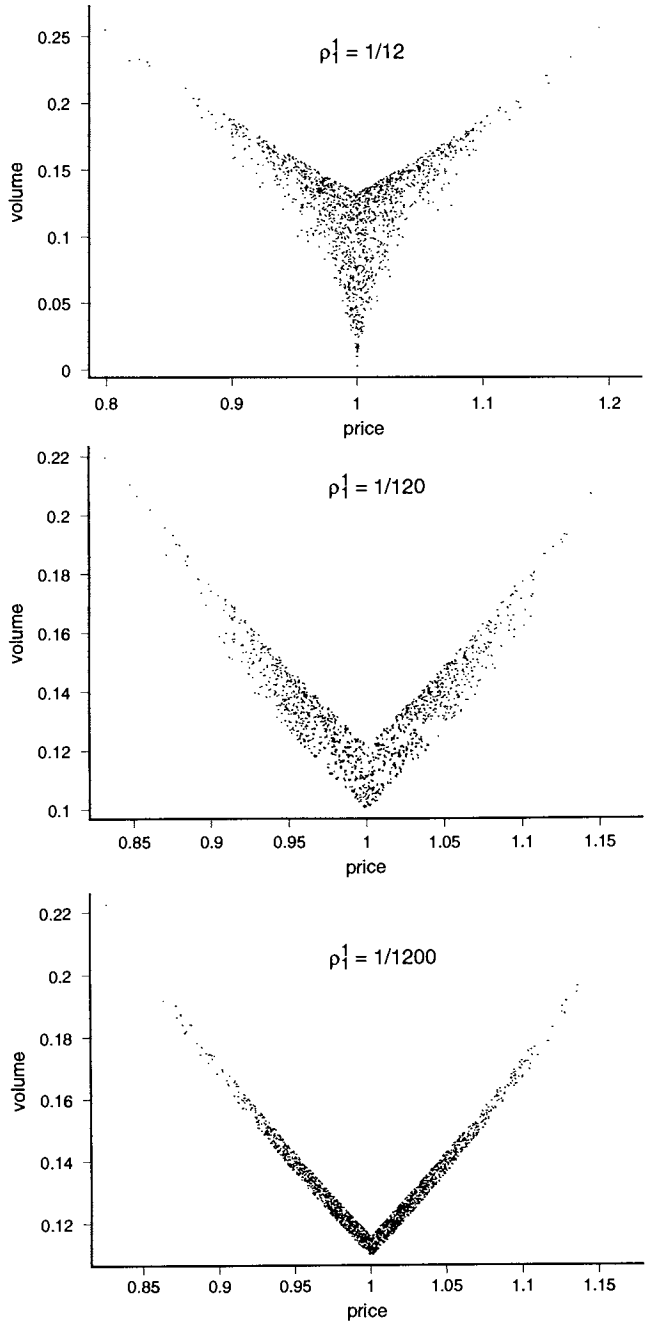
Figure 4. The relation of price and volume for different information quality. The figures are drawn for parameter values $\psi_0 = \psi = 1$, $\mu = 0.5$, $\rho_0 = \rho_w = 2$ and $\rho^2 = 0$. The top panel sets $\rho_1^1 = \frac{1}{12}$, the middle panel sets $\rho_1^1 = \frac{1}{120}$, and the bottom panel sets $\rho_1^1 = \frac{1}{1200}$. Each panel is constructed by drawing 2,000 pairs of (θ_1, ρ_1) from their distributions and calculating the resulting equilibrium. The greater the precision, the better is the information so the bottom panel corresponds to greater information quality.
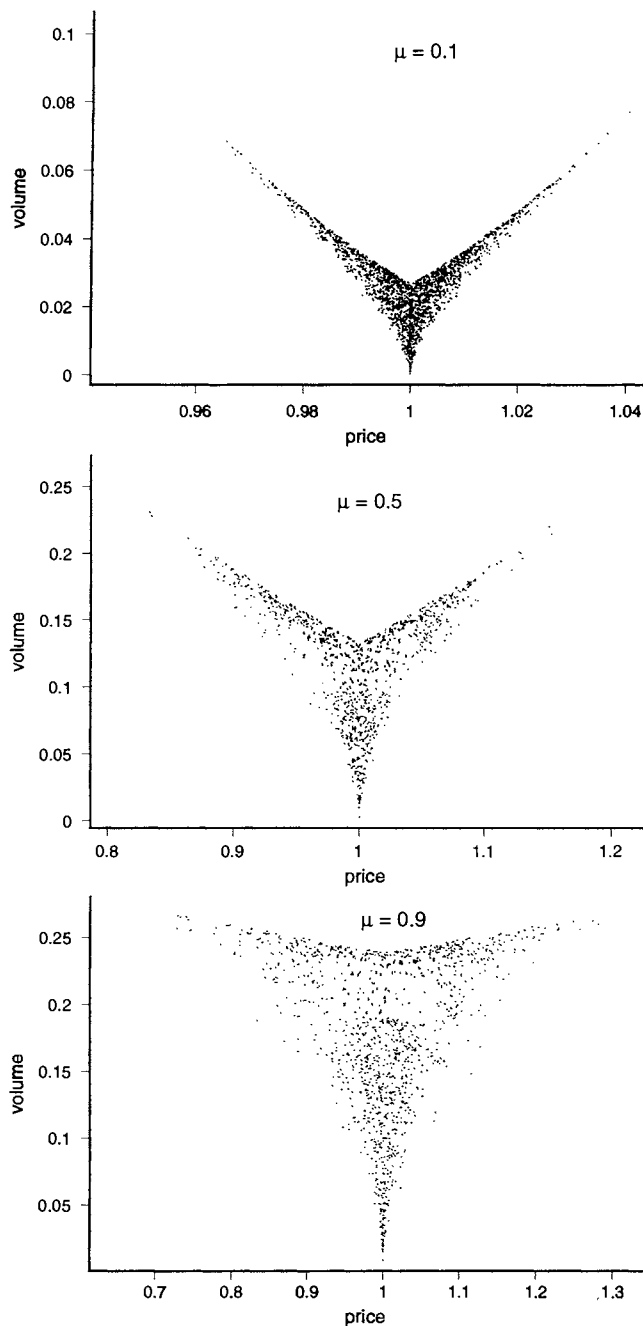
Figure 5. The relation of price and volume for different levels of information dissemination. The figures are drawn for parameter values $\psi_0 = \psi = 1$, $\rho_0 = \rho_w = 2$ and $\rho^2 = 0$, and $\rho_1^1 = \frac{1}{12}$. In the top panel, $\mu = 0.1$, in the middle panel, $\mu = 0.5$ and in the bottom panel, $\mu = 0.9$. Each panel is constructed by drawing 2,000 pairs of (θ_1, ρ_1^1) from their distributions and calculating the resulting equilibrium. The higher is μ, the greater the information dispersion so the bottom panel corresponds to the most information dispersion.

traders being in this group, and 90 percent of the traders being in this group. As information is more widely disseminated, the top of the distribution flattens out, with the V-shape noted earlier virtually disappearing. The dispersion of points, however, increases dramatically, corresponding quite closely to the time series relation depicted in Gallant, Rossi, and Tauchen. In the 90 percent high-precision scenario, it is still the case that the absolute value of prices changes and volume are positively related. What our results suggest, however, is that the sensitivity of this relationship (the slope) is greatly affected by the extent of information dissemination. We intend to explore these information quality and quantity effects further in future work.

Given this role for volume, it now becomes apparent why observing price and volume together is more informative than observing price alone. A trader observing only a high price is unable to determine whether the price is high because of a high average signal (the θ_1) or an average signal with a high quality ρ_1^1. In fact, he is left with a curve of (θ_1, ρ_1^1) that are consistent with the price. Volume picks up the quality of the signal in a way different from price because, unlike price, volume is not normally distributed. In our model, as Figure 1 and equation (15) suggest, there are two (θ_1, ρ_1^1) pairs that are consistent with an observation of price and volume. One pair has $\rho_1^1 > \rho_w$ and a low θ_1, the other has $\rho_1^1 < \rho_w$ and a high θ_1. So looking at volume in addition to price reduces the range of possible values of θ_1 to only two points.

The role of volume as a signal of the precision of beliefs means that the volume statistic provides information to the market that is not conveyed by price. Moreover, this information is related to information about the asset value and not to exogenous liquidity or supply shocks. This role for volume is remarkably similar to that claimed by proponents of technical analysis. For example, Pring (1991) explains that "Most indicators [of market movements] are a statistical deviation from price data. Since volume indicators are totally independent of price, they offer a more objective view of the quality of the price trend." In our model, this "independence" of volume is also what allows the quality of information to be inferred from market statistics. It becomes natural to watch volume because it complements the information provided by price. A trader watching only prices cannot learn as much as a trader watching both prices and volume and so faces an unnecessary penalty if he ignores the volume statistic.[16]

In the next section, we extend our model to a multiperiod setting in order to investigate the time series properties of price and volume. To make this analysis tractable, we want the equilibrium in each period to be revealing. (Otherwise, traders will have priors which are mixtures of normals and are thus not normal.) Equation (15) and Figure 1 suggest that, given price, volume will reveal ρ_1^1 as long as $\rho_1^1 > \rho^2$ and ρ_1^1 is known to be above ρ_w or known to be below ρ_w. We assume that $\rho_t^1 \in (\rho^2, \rho_w)$ for all t.

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[16] This complementary role of price and volume is also characteristic of technical analysis techniques. Pring (1991) notes that "It is therefore essential to relate the movement of the volume oscillator (or moving average) to the prevailing movement in price." For a discussion of techniques involving volume see Pring, chapter 18.

PROPOSITION 3: *If $\rho_t^1 \in (\rho^2, \rho_w)$, then in the large economy (ρ_1^1, θ_1) is revealed by (p_1, V_1).*

The proposition demonstrates that under our assumptions market statistics are revealing. Of course, the value of the asset is not known to anyone with certainty because of the common error term, and so it cannot be revealed by these market statistics. Nonetheless, by observing volume in conjunction with price traders can infer all the information available in the market. However, since traders do not know the true asset value (the ψ) it is not the case that the price and the volume reveal complete information. This lack of perfect revelation means that all traders face a learning problem in determining the value of the underlying asset. Since new signals arrive every period, it may be that the sequence of price and volume statistics provides information to all market participants. If this is the case, then technical analysis of past market statistics can be valuable. In the next section we begin our investigation of this role of technical analysis by extending our model to a multiperiod setting.

III. Equilibrium Price-Volume Time Series

To examine the time series of price and volume we extend the asset market model of Section II to multiple periods. Period one price and volume reveal θ_1 so upon entering the second trading period, traders again have a common prior on the asset's eventual value. This prior is, by Bayes's rule, a normal with mean $(\rho_w \theta_1 + \rho_0 \psi_0)/(\rho_w + \rho_0)$ and variance $(\rho_w + \rho_0)^{-1}$. Traders in group 1 then receive signals $y_2^i = \psi + w_2 + e_2^i$ where the precision of the e_2^i distribution is ρ_2^1. Similarly, traders in group 2 receive signals $y_2^i = \psi + w_2 + \varepsilon_2^i$ where the precision of the ε_2^i distribution is ρ^2. The market proceeds as in period one with the only differences being the new common prior, the new randomly drawn precision, the new randomly drawn signals, and the fact that endowments are now equilibrium period one demands. All traders are engaging in technical analysis in the sense that their behavior is influenced by their prior which in turn depends on past market statistics. At this point, however, the use of past data is dictated by our timing convention on when market data is available. It is more interesting to examine the effect of data from more than one previous period.

Our argument for revelation of information through market statistics is constructed inductively. Suppose that market statistics through period $t - 1$ are revealing. Then upon entering trading period t, the traders' common prior is a normal with mean

$$\bar{\theta}_{t-1} = \left[\rho_w \sum_{\tau=1}^{t-1} \theta_\tau + \rho_0 \psi_0 \right] \Big/ [(t-1) \cdot \rho_w + \rho_0] \qquad (17)$$

and variance $(\hat{\rho}_{t-1})^{-1} = [(t-1)\cdot\rho_w + \rho_0]^{-1}$. The traders' gross demands for the risky asset are then given by

$$\hat{\rho}_{t-1}(\bar{\theta}_{t-1} - p_t) + \rho_t^{s1}(y_t^i - p_t) \tag{18}$$

for each trader i in group 1, and by

$$\hat{\rho}_{t-1}(\bar{\theta}_{t-1} - p_t) + \rho^{s2}(y_t^i - p_t) \tag{19}$$

for each trader i in group 2.

An application of the analysis in Section III shows that for the large economy the equilibrium period t price is given by

$$p_t = \frac{\hat{\rho}_{t-1}\bar{\theta}_{t-1} + (\mu\rho_t^{s1} + (1-\mu)\rho^{s2})\theta_t}{\hat{\rho}_{t-1} + \mu\rho_t^{s1} + (1-\mu)\rho^{s2}}. \tag{20}$$

Per capita volume in period t for the large economy is

$$V_t = E\left[\frac{\mu}{2}\Big|\,\hat{\rho}_{t-1}(\bar{\theta}_{t-1} - p_t) + \rho_t^{s1}(y_t^1 - p_t) - \hat{\rho}_{t-2}(\bar{\theta}_{t-2} - p_{t-1})\right.$$

$$\left. - \rho_{t-1}^{s1}(y_{t-1}^1 - p_{t-1})\Big| + \frac{(1-\mu)}{2}\Big|\,\hat{\rho}_{t-1}(\bar{\theta}_{t-1} - p_t) + \rho^{s2}(y_t^2 - p_t)\right.$$

$$\left. - \hat{\rho}_{t-2}(\bar{\theta}_{t-2} - p_{t-1}) - \rho^{s2}(y_{t-1}^2 - p_{t-1})\Big|\right] \tag{21}$$

where the expectation is with respect to the conditionally independent random variables $y_t^1 \sim N(\theta_t, 1/\rho_t^1)$, $y_{t-1}^1 \sim N(\theta_{t-1}, 1/\rho_{t-1}^1)$, $y_t^2 \sim N(\theta_t, 1/\rho^2)$ and $y_{t-1}^2 \sim N(\theta_{t-1}, 1/\rho^2)$.[17]

This volume expression differs from that in period one because traders now have endowments that are their equilibrium period $t-1$ demands. Traders do not know each individual's equilibrium period $t-1$ demands but they do know p_{t-1} and our induction hypothesis is that they have inferred ρ_{t-1}^s, $\hat{\rho}_{t-2}$ and $\bar{\theta}_{t-2}$ from past market statistics. So everything in the per capita volume expression other than y_t^1, y_{t-1}^1, y_t^2, y_{t-1}^2 and ρ_t^s is known. Calculation similar to that in the proof of Proposition 1 shows that per capita volume in period t for the large economy is

$$V_t = \frac{\mu}{2}\Big[2(x_t^1)^{-1}\phi(\delta_t^1 x_t^1) + \delta_t^1\big(\Phi(\delta_t^1 x_t^1) - \Phi(-\delta_t^1 x_t^1)\big)\Big]$$

$$+ \frac{(1-\mu)}{2}\Big[2(x^2)^{-1}\phi(\delta_t^2 x^2) + \delta_t^2\big(\Phi(\delta_t^2 x^2) - \Phi(-\delta_t^2 x^2)\big)\Big], \tag{22}$$

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[17] The expression $\hat{\rho}_{t-2}(\bar{\theta}_{t-2} - p_{t-1}) + \rho_{t-1}^{si}(y_{t-1} - p_{t-1})$ is a random variable representing the period t endowment of group i traders. Traders do not know any individual's endowment, but they do know the distribution of endowments.

where

$$x_t^1 = \left(\frac{(\rho_t^{s1})^2}{\rho_t^1} + \frac{(\rho_{t-1}^{s1})^2}{\rho_{t-1}^1} \right)^{-1/2}, \; x^2 = \left(\frac{2(\rho^{s2})^2}{\rho^2} \right)^{-1/2},$$

$$\delta_t^1 = \hat{\rho}_{t-1}(\bar{\theta}_{t-1} - p_t) + \rho_t^{s1}(\theta_t - p_t) - \hat{\rho}_{t-2}(\bar{\theta}_{t-2} - p_{t-1}) - \rho_{t-1}^{s1}(\theta_{t-1} - p_{t-1}),$$

and

$$\delta_t^2 = \hat{\rho}_{t-1}(\bar{\theta}_{t-1} - p_t) + \rho^{s2}(\theta_t - p_t) - \hat{\rho}_{t-2}(\bar{\theta}_{t-2} - p_{t-1}) - \rho^{s2}(\theta_{t-1} - p_{t-1}).$$

Given our induction hypothesis, the only unknowns in p_t and V_t are ρ_t^1 and θ_t. Now, an argument parallel to that in Proposition 3 shows that (p_t, V_t) reveals (ρ_t^1, θ_t). So, if past market statistics have revealed past information, current market statistics will reveal current information. As a result, the price and per capital volume given above are equilibrium market statistics.

PROPOSITION 4: *Suppose $\rho_t^1 \in (\rho^2, \rho_w)$ for all t. In the large economy, the equilibrium price and per capita volume time series are given by* (20) *and* (22).

In this economy, the equilibrium price converges almost surely to ψ. This occurs because all traders are using price-volume data to infer a sequence of θ_t's each of which is normally distributed with mean ψ. So, by the Strong Law of Large Numbers each individual's posterior mean, $\bar{\theta}_t$, is converging to ψ and the posterior variance is converging to zero. In the limit economy, everyone knows ψ, price is ψ and no trade occurs.[18]

One would be tempted to conjecture from this that trade vanishes, i.e., volume goes to zero, as time proceeds. This conjecture is false: Volume in the limit economy is zero, but the limit of volume as time proceeds is not zero. So the limit economy is not a good approximation to an economy after many periods.

The behavior of the price and volume time series for an economy with $\psi_0 = \psi = 1$, $\rho_0 = \rho_w = 2$, $\rho^2 = 0$, $\mu = 0.5$ and ρ_t^1 uniformly and independently distributed on $(0, 1)$ is illustrated in Figure 6. The equilibrium time series was constructed by drawing a sequence of 1,000 pairs of θ_t and ρ_t^1 and constructing the equilibrium at each date given the draws in the sequence up to that date. For this economy, price converges to $\psi = 1$ and volume has a nondegenerate limit distribution.

Trade does not disappear because although traders' beliefs are converging to a common belief their precisions are diverging at the same rate. Intuitively, these effects can be explained by noting that in early periods traders may receive information indicating a wide discrepancy of price from true value, but because they are not very sure of the true value (i.e., their precision is low), they take limited positions. In later periods, when prices are

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[18] Of course any discussion of the limit is straining our story of an asset with an eventual value of ψ a bit too much. But this is intended to be a discussion of the general tendencies of price and volume.
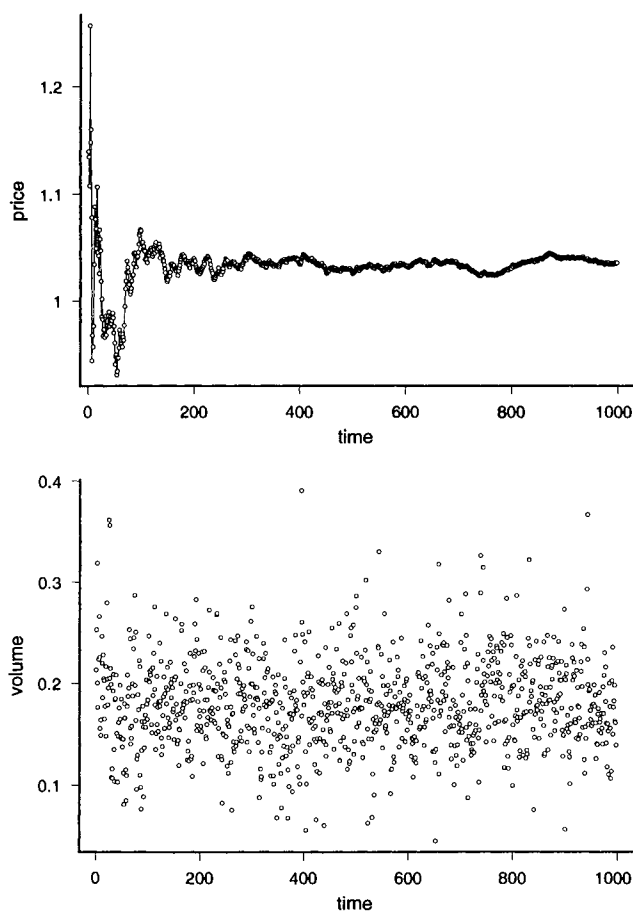
Figure 6. The time series behavior of volume and price. The figures are constructed for an economy with $\psi_0 = \psi = 1$, $\rho_0 = \rho_w = 2$, $\rho^2 = 0$, and $\mu = 0.5$ and ρ_t^1 uniformly and independently distributed on $[0, 1]$. The equilibrium time series was constructed by drawing a sequence of 1,000 pairs of θ and ρ_1^1 and constructing the equilibrium at each date given the draws in the sequence up to that date. Each point depicts the equilibrium actions at that date.

close to true values, traders are more confident (precisions are high) and hence take large positions to exploit even small price discrepancies. Our results on the distribution of volume dictate that these two effects essentially offset. Thus, volume converges to a limit distribution which is very different from volume in the limit economy.

IV. Technical Analysis

Traders in the multiperiod economy developed in Section III are engaging in technical analysis. Their demands depend on past market statistics through their dependence on the sufficient statistic $\bar{\theta}_{t-1}$. Further, knowledge of the

past helps in interpreting current market statistics. A trader who does not engage in technical analysis, and thus does not know $\hat{\rho}_{t-1}$ and $\bar{\theta}_{t-1}$ cannot fully infer current information from the current market statistics. Here traders need to remember the past; they cannot glean everything from current data.

To make the value of technical analysis more precise let $d_i^*(p_t^*, \hat{\psi}, \hat{\rho})$ be uninformed trader i's demand for the risky asset at time t given equilibrium price p_t^*, prior mean $\hat{\psi}$ and prior precision $\hat{\rho}$.[19] Then given a wealth of w_t^i at the beginning of date t, trader i's date $t + 1$ random wealth (assuming that in period $t + 1$ the asset finally pays off) will be

$$w_{t+1}^i\left(p_t^*, \hat{\psi}, \hat{\rho}, \psi\right) = w_t^i + d_t^i\left(p_t^*, \hat{\psi}, \hat{\rho}\right)[\psi - p_t^*], \tag{23}$$

where the predicted distribution of ψ is $N(\hat{\psi}, 1/\hat{\rho})$. Thus trader i's equilibrium expected utility is

$$E_\psi\left[-\exp\left[-w_{t+1}^i\left(p_t^*, \hat{\psi}, \hat{\rho}, \psi\right)\right]\right]. \tag{24}$$

Note that this expected utility depends on the trader's beliefs because of its explicit dependence on $\hat{\psi}$ and $\hat{\rho}$ and because the expectation is taken using the distribution $N(\hat{\psi}, 1/\hat{\rho})$.

A technical analyst believes $\hat{\psi} = \bar{\theta}_{t-1}$ and $\hat{\rho} = \hat{\rho}_{t-1}$. A trader who does not use past data believes $\hat{\psi} = \psi_0$ and $\hat{\rho} = \rho_0$. This divergence in beliefs allows us to calculate the value of technical analysis at time t:

$$E\left[E_\psi\left[-\exp\left[-w_{t+1}^i\left(p_t^*, \bar{\theta}_{t-1}, \hat{\rho}_{t-1}, \psi\right)\right]\Big|\bar{\theta}_{t-1}, \hat{\rho}_{t-1}\right]\right]$$
$$- E_\psi\left[-\exp\left[-w_{t+1}^i(p_t^*, \psi_0, \rho_0, \psi)\right]\right]. \tag{25}$$

The first expected value in equation (25) is the prior expected value of the ability to condition demand on the data revealed by market statistics through period $t - 1$, while the second expected value is simply the expected utility when the trader does not learn from the past. The difference between these expected values is thus the amount that an uninformed trader would be willing to pay at date t to know all past market statistics. (In this calculation we assume that this trader will have no effect on prices as he is one of a countable infinity of traders). This value is clearly nonnegative because the trader can always ignore the data if it is not useful. In our economy, the value will be strictly positive as $\bar{\theta}_{t-1}$ is a consistent estimator of ψ. Proposition 5 demonstrates that the value of technical analysis depends on the quality of information.

PROPOSITION 5: *The value of technical analysis at time t is $1/2(t-1)^{-1}\rho_w[(\psi_0 - p_t^*)^2 + 1/\rho_0] > 0$. This value is increasing in ρ_w and decreasing in ρ_0.*

One aspect of our results that should be stressed is that technical analysis is valuable not because of volume or any other specific statistic but rather

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because there is some underlying uncertainty to be learned in the economy. Technical analysis is valuable for all traders in our model. Where volume matters is in affecting the benefits of technical analysis. Though traders gain from watching prices, they do better still by watching prices and volume.

That technical analysis is more valuable if past market statistics impound higher-quality information (i.e., ρ_w is large), and is less valuable if there is less to be learned from market data (i.e., ρ_0 is large) is an interesting result of our analysis. Because technical analysis helps traders interpret current information, watching the sequence of market statistics allows traders to correctly update beliefs. It seems reasonable that the value of doing so depends on the properties of the information structure. If traders already know a lot about the asset (their prior precision ρ_0 is large) or information in general is not very good (ρ_w small) then watching the market is not very valuable.

The properties of technical analysis derived above suggest that it may be particularly appropriate for small, less widely followed stocks. Such firms tend to have greater uncertainty about their future prospects and hence have a low prior precision. Moreover, such stocks may be more affected by private rather than public information, meaning that the effect of private signals may be higher. For such stocks, technical analysis of market statistics may play an important role in providing information to the market and, consequently, efficiency to the price.

V. Conclusions

We have investigated how technical analysis can be valuable to traders in an economy in which the only uncertainty arises from the underlying information structure. In our model, technical analysis is valuable because current market statistics may be sufficient to reveal some information, but not all. Because the underlying uncertainty in the economy is not resolved in one period, sequences of market statistics can provide information that is not impounded in a single market price.

Where we believe our results are most interesting is in delineating the important role played by volume. In our model, volume provides information in a way distinct from that provided by price. As is true in most rational expectations models, price impounds information about the average level of trader's private information. But unique to our model is the feature that volume captures the important information contained in the quality of traders' information signals. Because the volume statistic is not normally distributed, if traders condition on volume they can sort out the information implicit in volume from that implicit in price. We have shown that volume plays a role beyond simply being a descriptive parameter of the trading process.

Our focus on the quality, or precision, of information suggests that the value of particular market statistics may vary depending upon characteristics of the information structure. While we have discussed the potential applications of technical analysis for small, thinly followed stocks, it seems likely

that even (or perhaps, especially) in active markets volume may play an important role. The existence of the Dow-Jones "Rumor Wire" suggests that even "bad" (in the sense of imprecise) information can affect stock prices, so that using the information conveyed by volume may be particularly useful to traders operating in such volatile markets. Indeed, one criticism of program trading voiced by professional traders is that it distorts the information typically provided by trading volume. As our analysis here suggests, introducing trading volume unrelated to the underlying information structure would surely weaken the ability of uninformed traders to interpret market information accurately.

Appendix

Proof of Proposition 1: We first provide a useful lemma.

LEMMA 1: *Let* $y \sim N(\theta, 1/\rho)$, *then*

$$E[|\gamma y + a|] = 2 \frac{\gamma}{\sqrt{2\pi\rho}} \exp\left[-\frac{1}{2} \left(\frac{\delta\rho^{1/2}}{\gamma} \right)^2 \right] + \delta \left[\Phi\left(\frac{\delta\rho^{1/2}}{\gamma} \right) - \Phi\left(\frac{-\delta\rho^{1/2}}{\gamma} \right) \right],$$

for $\delta = a + \gamma\theta$, *and* Φ *the cumulative normal.*

Proof of Lemma 1: Note that $\Pr(|\gamma y + a| \le \alpha) = \Pr(-\alpha \le \gamma y + a \le \alpha) =$

$$\Pr\left(\frac{\rho^{1/2}}{\gamma}(-\alpha - \gamma\theta - a) \le z \le \frac{\rho^{1/2}}{\gamma}(\alpha - \gamma\theta - a) \right)$$

$$= \Phi\left(\frac{\rho^{1/2}}{\gamma}(\alpha - a - \gamma\theta) \right) - \Phi\left(\frac{\rho^{1/2}}{\gamma}(-\alpha - a - \gamma\theta) \right),$$

for z a standard normal. Thus, the density of the random variable $|\gamma y + a|$ is $f(\alpha) = \rho^{1/2}/\gamma[\phi((\rho^{1/2}/\gamma)(\alpha - a - \gamma\theta)) + \phi((\rho^{1/2}/\gamma)(-\alpha - a - \gamma\theta))]$, for ϕ the standard normal density. The claim in the lemma now follows by computing the integral $\int_0^\infty \alpha f(\alpha) d\alpha$.   Q.E.D.

Rewriting equation (13) we have:

$$V_1 = \frac{\mu}{2} \left[\frac{1}{N_I} \sum_{i=1}^{N_I} \left| \rho_0(\psi_0 - p_1) + \rho_1^{s1}(y_1^i - p_1) \right| \right]$$

$$+ \frac{(1-\mu)}{2} \left[\frac{1}{N_u} \sum_{i=N_I+1}^{N} \left| \rho_0(\psi_0 - p_1) + \rho^{s2}(y_1^i - p_1) \right| \right].$$

By the strong law of large numbers as $N \to \infty$ this sequence of random variables converges almost surely to

$$\frac{\mu}{2}E\left[\left|\rho_0(\psi_0 - p_1) + \rho_1^{s1}(y^1 - p_1)\right|\right]$$

$$+ \frac{(1 - \mu)}{2}E\left[\left|\rho_0(\psi_0 - p_1) + \rho^{s2}(y^2 - p_1)\right|\right]$$

where

$$y^1 \sim N(\theta_1, 1/\rho_1^1) \text{ and } y^2 \sim N(\theta_1, 1/\rho^2).$$

Using Lemma 1 we have limit per capita volume almost surely equal to the expression given in Proposition 1.

Proof of Proposition 2: Differentiating volume, equation (15) in the text, with respect to p_1 yields

$$\frac{\partial V_1}{\partial p_1} = \frac{\mu}{2}\frac{\partial \hat{\delta}_1^1}{\partial p_1}\left[\Phi\left(\frac{\hat{\delta}_1^1(\rho_1^1)^{1/2}}{\rho_1^{s1}}\right) - \Phi\left(-\frac{\hat{\delta}_1^1(\rho_1^1)^{1/2}}{\rho_1^{s1}}\right)\right]$$

$$+ \frac{(1 - \mu)}{2}\frac{\partial \hat{\delta}_1^2}{\partial p_1}\left[\Phi\left(\frac{\hat{\delta}_1^2(\rho^2)^{1/2}}{\rho^{s2}}\right) - \Phi\left(-\frac{\hat{\delta}_1^2(\rho^2)^{1/2}}{\rho^{s2}}\right)\right].$$

At $p_1 = \psi_0$ we have $\hat{\delta}_1^1 = \hat{\delta}_1^2 = 0$. Thus, $\partial V_1/\partial p_1 = 0$ at $p_1 = \psi_0$.

The second derivative of volume with respect to price is

$$\frac{\partial^2 V_1}{\partial p_1^2} = \mu\left(\frac{\partial \hat{\delta}_1^1}{\partial p_1}\right)^2\frac{(\rho_1^1)^{1/2}}{\rho_1^{s1}}\phi\left(\frac{\hat{\delta}_1^1(\rho_1^1)^{1/2}}{\rho_1^{s1}}\right)$$

$$+ (1 - \mu)\left(\frac{\partial \hat{\delta}_1^2}{\partial p_1}\right)^2\frac{(\rho^2)^{1/2}}{\rho^{s2}}\phi\left(\frac{\hat{\delta}_1^2(\rho^2)^{1/2}}{\rho^{s2}}\right) > 0.$$

Thus, volume is convex in p_1 and achieves its minimum at $p_1 = \psi_0$.

Proof of Proposition 3: Differentiating volume, equation (15) in the text, with respect to ρ_1^1 yields

$$\frac{\partial V_1}{\partial \rho_1^1} = \frac{\mu}{2}\left[\left(\frac{\rho_w(\rho_w - \rho_1^1)}{\rho_1^1(\rho_w + \rho_1^1)^2}\right)\phi\left(\frac{\hat{\delta}_1^1(\rho_w + \rho_1^1)}{\rho_w(\rho_1^1)^{1/2}}\right) + \left(\frac{\partial \hat{\delta}_1^1}{\partial \rho_1^1}\right)\left\{\Phi\left(\frac{\hat{\delta}_1^1(\rho_w + \rho_1^1)}{\rho_w(\rho_1^1)^{1/2}}\right)\right.$$

$$\left.- \Phi\left(-\frac{\hat{\delta}_1^1(\rho_w + \rho_1^1)}{\rho_w(\rho_1^1)^{1/2}}\right)\right\}\right] + \frac{(1 - \mu)}{2}\left(\frac{\partial \hat{\delta}_1^2}{\partial \rho_1^1}\right)\left[\Phi\left(\frac{\hat{\delta}_1^2(\rho_w + \rho^2)}{\rho_w(\rho^2)^{1/2}}\right)\right.$$

$$\left.- \Phi\left(-\frac{\hat{\delta}_1^2(\rho_w + \rho^2)}{\rho_w(\rho^2)^{1/2}}\right)\right].$$

The term involving the normal density, ϕ, is positive if $\rho_w \geq \rho_1^1$. The terms involving the differences of cumulative normals are also positive as $\partial \hat{\delta}_1^i / \partial \rho_1^1$ the difference of the cumulative normals always have the same sign as long as $\rho_1^1 > \rho^2$. Thus, $\partial V_1 / \partial \rho_1^1 > 0$ for $\rho_1^1 \in (\rho^2, \rho_w)$.

So, given the price p_1 and the equilibrium pricing equation, the only unknown in volume is ρ_1^1 and volume is strictly increasing in ρ_1^1. Thus, given p_1 volume reveals ρ_1^1. Then θ_1 is revealed by the pricing equation (12). Q.E.D.

Proof of Proposition 4: Given the induction hypothesis the proof that (ρ_1^1, θ_t) is revealed by (p_t, V_t) follows directly from the proof of Proposition 3. We know by Proposition 3 that (ρ_1^1, θ_1) is revealed by (p_1, V_1). This completes the induction argument. Now the fact that (20) and (22) describe period t equilibrium follows from the calculations in the text.

Proof of Proposition 5: Calculation shows that the value of technical analysis given by equation (25) is

$$\tfrac{1}{2}(t-1)^{-1}\rho_w\Big[(\psi_0 - p_t^*)^2 + 1/\rho_0\Big].$$

This value is strictly positive, increasing in ρ_w and decreasing in ρ_0.

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