A13073W1

**DEGREE OF MASTER OF SCIENCE IN FINANCIAL ECONOMICS**

# FINANCIAL ECONOMETRICS

**TRINITY TERM 2019**

**Tuesday, 23 April 2019, 14:30 – 17:00**

**Time allowed is TWO HOURS AND THIRTY MINUTES.**

*Candidates should answer **ALL** questions in part A.*
*Candidates should answer **TWO** of four questions in part B.*

*Examiners will place weight 2.5% on each*
*question in Part A and 25% on each question in part B.*

*Use **three** booklets – one for Part A and one for each Part B question.*
*Write the numbers for B questions answered on the cover of the relevant booklet.*

Materials: Calculators.
Calculators must not be removed from the Examination Room.

Do not turn over until told that you may do so.

# Part A: Multiple Choice

**Answer ALL questions in this section.**

*Each question is worth 2.5% of the exam mark.*

### Single Answer Questions

*Select a single answer for each question.*

1. If you roll two fair dice, what is the probability that the product of the two values is greater than 10 if you know one of the die has a value that is 3 or greater?

   (a) $^{15}/_{32}$

   (b) $^{1}/_{2}$

   (c) $^{17}/_{32}$

   (d) $^{3}/_{4}$

2. When evaluating a series of forecasts using the Mincer-Zarnowitz regression $y_{t+h} = \alpha + \beta \hat{y}_{t+h|t} + \gamma x_t + \eta_{t+h}$, what are the values of $\alpha$, $\beta$ and $\gamma$ that should occur when the forecasting model is correctly specified.

   (a) $\alpha = 0, \beta = 0$, no restriction on $\gamma$

   (b) $\alpha = 0, \beta = 0, \gamma = 0$

   (c) $\alpha = 0, \beta = 1, \gamma = 0$

   (d) $\alpha = 1, \beta = 1, \gamma = 1$

3. What are the consequences of using the Cholesky factor when producing impulse response functions?

   (a) Shocks affect all series simultaneously

   (b) The Vector Moving Average coefficients stay the same

   (c) Shocks affect the current series and any series that occurs after the series in the VAR

   (d) Shocks only affect a single series

4. If the 1-month return on a portfolio is $N\left(1.5\%, (3\%)^2\right)$ and the portfolio has assets worth \$5,000,000, what is the 1-month 1% Value-at-Risk?

(a) \$273,952

(b) \$27,395

(c) \$311,374

(d) -\$273,952

5. If you use a Diebold-Mariano to test two models, using $\delta = L_t^A - L_t^B$ and compute a test statistic of -2.5, what do you conclude?

   (a) Model B is correctly specified, and model A is misspecified

   (b) Model B is more accurate than model A

   (c) The models are equally accurate

   (d) Model A is more accurate than model B

6. What transformation is used to model RV as an ARCH-model?

   (a) $\sqrt{RV}_t$

   (b) $\text{sign}(r_t)RV_t^2$

   (c) $\text{sign}(r_t)\sqrt{RV}_t$

   (d) $\text{sign}(r_t)RV_t$

7. What is the 3 step ahead forecast from an ARCH(1) model $\sigma_{t+1}^2 = \omega + \alpha\epsilon_t^2$?

   (a) $\omega + \alpha\epsilon_t^2$

   (b) $\omega + \alpha\omega + \alpha^2\omega + \alpha^3\epsilon_t^2$

   (c) $\frac{\omega}{1-\alpha}$

   (d) $\omega + \alpha\omega + \alpha^2\omega + \alpha\epsilon_t^2$

8. For the OLS estimators $\hat{\beta}_1$ and $\hat{\beta}_2$ in the model $Y_i = \alpha + \beta_1 X_{1i} + \beta_2 X_{2i} + \epsilon_i$ to be consistent, which of the following assumption is necessary?

   (a) The error term $\epsilon_i$ is normally distributed for all $i$

   (b) The error term $\epsilon_i$ is homoskedastic for all $i$

   (c) The regressors $X_{1i}$ and $X_{2i}$ are uncorrelated

   (d) The error term $\epsilon_i$ has to be uncorrelated with the regressors $X_{1i}$ and $X_{2i}$

9. Which of the four procedures is not useful for model selection?

   (a) Sequentially add regressors that are statistically significant to the regression model

   (b) Comparing out-of-sample prediction errors for all regression models under consideration

   (c) Find the optimal balance between a higher log-likelihood and an ad-hoc penalty term for adding additional regressors

   (d) Pick the model with the highest $R^2$for all models under consideration

## Multiple Answer Questions

*Select all correct answers. Each question has between 0 and 4 correct answers.*

10. The OLS estimators of $\hat{\boldsymbol{\beta}}$ in the model $y_t = \alpha + \boldsymbol{x_t}\boldsymbol{\beta} + \epsilon_t$:

    (a) Minimizes the sum of squared errors

    (b) Maximizes the model $R^2$

    (c) Are invariant to rescaling the dependent variable, $y_t$

    (d) Are consistent when both $\mathbf{y}$ and $\mathbf{X}$ are $I(1)$ random variables

11. Which of the following are true about the expectations operator, $\mathrm{E}[\cdot]$:

    (a) $\mathrm{E}[XY] = \mathrm{E}[X]\,\mathrm{E}[Y]$ only when the random variables $X$ and $Y$ are independent

    (b) If $g(\cdot)$ is a convex function, then $\mathrm{E}[g(X)] \geq g[\mathrm{E}[X]]$

    (c) $\mathrm{E}[a + bX] = b\mathrm{E}[X]$ where $a$ and $b$ are constants and $X$ is a random variable

    (d) $\mathrm{E}\left[\mathrm{E}\left[X|Y\right]\right] = \mathrm{E}[X]$ only if $X$ and $Y$ are independent

12. If $Y_i \overset{\text{i.i.d.}}{\sim}$ Exponential($\lambda$) and you observe 8 values, $\{5, 7, 5, 3, 9, 4, 6, 1\}$, and the PDF of an exponential random variable is $f_Y(y) = \lambda \exp(-\lambda y)$ for $y \geq 0$, what can you say about $\lambda$

    (a) Its maximum likelihood estimate is 5

    (b) The maximum likelihood estimate of the asymptotic variance of $\hat{\lambda}$ is $1/25$

    (c) A 95% confidence interval includes 0

    (d) The asymptotic distribution of the MLE is $\sqrt{n}\left(\hat{\lambda} - \lambda\right) \xrightarrow{d} N\left(0, \lambda^{-2}\right)$

13. In a hypothesis test, a Type II error

    (a) has probability of $\alpha$ of occurring, where $\alpha$ is the size of the test

    (b) occurs if the null is false

    (c) occurs if the alternative is true

    (d) has a probability of occurring that is equal to 1 minus the power of the test

14. Let $\epsilon_t$ be a white noise process. Identify the covariance stationary processes in the following list:

    (a) $y_t = \phi_0 I_{(t \text{ is even})} + \phi_1 I_{(t \text{ is odd})} + \epsilon_t$, $\phi_0 \neq \phi_1$

    (b) $y_t = y_{t-1} + \epsilon_t$

    (c) $y_t = \delta t + \epsilon_t$

    (d) $y_t = 10 + 0.4 y_{t-1} + 0.4 y_{t-2} + \epsilon_t$

15. Which of the following properties are required of a vector White Noise process?

    (a) Unconditional mean of 0
    (b) Diagonal covariance, so that the shocks are uncorrelated
    (c) Independent across time
    (d) Conditional mean of 0
    (e) Uncorrelated across time

16. What are all of the challenges when computing Model-free Implied Volatility from option price data?

    (a) Out-of-the-money options have poor liquidity
    (b) Options are only available at finitely many strikes
    (c) In-the-money options have poor liquidity
    (d) Option values are from the risk neutral measure

17. Which of the following affect the estimation of realized variance?

    (a) Bid-Ask Bounce
    (b) Errors in data
    (c) Market Closures
    (d) Multiple trades with the same timestamp

18. Which of the following are true about the ARCH-LM test?

    (a) It is a test for autocorrelation in the squared residuals
    (b) It uses a two-step approach where residuals are first computed and then a second-stage regression is estimated
    (c) The asymptotic distribution is a $\chi^2_p$ where $p$ is the number of lags in the regression
    (d) It requires estimating an ARCH($p$) model
    (e) It tests whether the constant is zero in a regression

19. What restrictions on the APARCH model $\sigma_t^\delta = \omega + \alpha \left(|\epsilon_{t-1}| + \gamma\epsilon_{t-1}\right)^\delta + \beta\sigma_{t-1}^\delta$, are correctly paired with the model nested?

    (a) ARCH: $\delta = 2, \gamma = 0, \alpha + \beta = 1$
    (b) GARCH: $\delta = 2, \gamma = 0$
    (c) TARCH: $\delta = 1$
    (d) GJR-GARCH: $\delta = 1$

20. Let $\epsilon_t$ be a white noise process. Identify the ergodic processes in the following list:

(a) $y_t = 2 + \epsilon_t$

(b) $y_t = y_{t-1} + \epsilon_t$

(c) $y_t = 0.4y_{t-1} + 3\epsilon_{t-1} + \epsilon_t$

(d) $y_t = 10 + 0.4y_{t-1} + 0.4y_{t-2} + \epsilon_t$

## Part B: Long Answer

**Answer TWO of the four questions in this section.**

*Each question is worth 25% of the exam mark (i.e., 1/2 of 50%). Within each question points sum to 100% and so will be scaled by 25% when combined in the final exam mark.*

1. Answer the following questions:

   (a) Suppose $y_i = \alpha + \mathbf{x}_i\beta + \epsilon_i$ where $\mathrm{E}\left[\epsilon_i|\mathbf{X}\right] = 0$ and $\mathrm{V}\left[\epsilon_i\right] = \sigma^2$ for all $i$ .

   i. [5%] Define homoskedasticity. What is the asymptotic distribution of $\hat{\beta}$ under homoskedasticity?

   ii. [5%] Define heteroskedasticity. How does the asymptotic distribution of $\hat{\beta}$ change under heteroskedasticity?

   iii. [20%] How do we test for the presence of heteroskedasticity? Discuss all steps of the test procedure.

   iv. [20%] How do we bootstrap the variance estimator for a regression when the data are heteroskedastic. Discuss all steps of the boostrap procedure.

   (b) [15%] How does Filtered Historical Simulation differ from Historical Simulation when forecasting the Value-at-Risk of a portfolio.

   (c) [10%] Describe one other method to forecast Value-at-Risk.

   (d) [15%] How are Value-at-Risk forecasts assessed? Describe two methods that can be sued to detect flawed Value-at-Risk models.

   (e) [10%] What is Expected Shortfall? Why is it preferred to Value-at-Risk?

2. Suppose $\{y_t\}$ is covariance stationary and can be described by the following process:

$$\begin{aligned} y_t &= \phi_0 + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \epsilon_t \\ \epsilon_t &= \sigma_t e_t \\ \sigma_t^2 &= \omega + \alpha \epsilon_{t-1}^2 \\ e_t &\overset{\text{i.i.d.}}{\sim} N(0,1) \end{aligned}$$

(a) What are the values of the following quantities [10%, each]:

i. $\mathrm{E}[y_{t+1}]$

ii. $\mathrm{E}_t[y_{t+1}]$

iii. $\mathrm{E}_t[y_{t+2}]$

iv. $\lim_{h\to\infty} \mathrm{E}_t[y_{t+h}]$

v. $\mathrm{V}_t[\epsilon_{t+1}]$

vi. $\mathrm{V}_t[y_{t+1}]$

vii. $\mathrm{V}_t[y_{t+2}]$

viii. $\mathrm{V}[y_{t+1}]$

(b) [20%] We build a statistical forecasting model for $y_t$ and use it to generate two-step ahead forecasts $\hat{y}_{t+2|t}$. We perform a Mincer-Zarnowitz regression $y_{t+2} = \alpha + \beta \hat{y}_{t+2|t} + \eta_t$. Our forecast evaluation sample has $T = 100$ observations. We get the regression results as presented in the tables below. Perform a Wald hypothesis test and discuss the quality of the forecasts (note that the 5% critical value of $\chi_2^2$ is 5.99).

$\hat{\Sigma}_{XX}^{-1}\hat{S}\hat{\Sigma}_{XX}^{-1}$

| | $\alpha$ | $\beta$ |
|---|---|---|
| $\alpha$ | 1.156 | -0.169 |
| $\beta$ | -0.169 | 0.828 |

| Parameter | Point estimate | t-statistic |
|---|---|---|
| $\hat{\alpha}$ | 0.18 | 1.65 |
| $\hat{\beta}$ | 1.18 | 12.93 |

3. Answer the following questions:

(a) Suppose $X_i \overset{\text{i.i.d.}}{\sim} \text{Bernoulli}(p)$ where $p$ is an unknown parameter. The likelihood for a single Bernoulli random variables is $f_X(x) = p^x (1-p)^{(1-x)}$.

i. [10%] What is the log-likelihood for $n$ observations from this density?

ii. [10%] What is the maximum likelihood estimator?

iii. [20%] What is the asymptotic distribution of the maximum likelihood estimator?

iv. [15%] Suppose you collected data on coin flips. You observe 56 heads and 44 tails. Using a hypothesis test, is the coin fair? 5% critical values from a $\chi^2_\nu$ are:

| $\nu$ | 1 | 2 | 3 | 98 | 99 | 100 |
|---|---|---|---|---|---|---|
| | 3.84 | 5.99 | 7.81 | 122.11 | 123.23 | 124.34 |

v. [15%] In a sample of 100 coin flips, what is the maximum number of heads that could be seen before the null that the coin is fair can be rejected using a 5% test?

(b) [15%] For which values of the model parameters is the VAR(2),

$$\mathbf{y}_t = \mathbf{\Phi}_0 + \mathbf{\Phi}_1 \mathbf{y}_{t-1} + \mathbf{\Phi}_2 \mathbf{y}_{t-2} + \boldsymbol{\epsilon}_t$$

cointegrated?

(c) [15%] Describe the steps required to implement the Engle-Granger methodology when testing a pair of series for a unit root.

4. Answer the following questions:

   (a) Suppose returns on the equity of a firm depend on the state of the economy. The economy is booming with probability 80%. and in a recession with probability 20%. When in a boom, the annual return is normally distributed $N\left(0.15, 0.3^2\right)$. In a recession, returns on the firm are also normally distributed $N\left(.05, 0.4^2\right)$.

      i. [10%] What is the expected return on this investment?

      ii. [10%] What the variance of the return on this investment?

      iii. [10%] What can you say about the kurtosis of the returns on this firm, when compared to the kurtosis of a normal random variable?

   (b) Consider the AR(2)

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \epsilon_t.$$

      i. [10%] Which of $\phi_1$ and $\phi_2$ produce a covariance time series when $\{\epsilon_t\}$ is a homoskedastic White Noise process ?

      ii. [10%] Suppose the model contains a unit root so that it can be written

$$\Delta y_t = 0.8 \Delta y_{t-1} + \epsilon_t.$$

      What are the values of $\phi_1$ and $\phi_2$?

      iii. Using the process in part ii, what are the values of:

         A. [10%] $\mathrm{V}_t\left[y_{t+1}\right]$

         B. [10%] $\mathrm{V}_t\left[y_{t+2}\right]$

         C. [10%] $\mathrm{V}_t\left[y_{t+1} + y_{t+2}\right]$

         D. [10%] $\lim_{h\to\infty} \mathrm{E}_t\left[\sum_{i=1}^{h} \Delta y_{t+i}\right]$?

         E. [10%] How does the answer to the previous question relate the $\lim_{h\to\infty} \mathrm{E}_t\left[y_{t+h}\right]$?