STA 371G: Statistics and Modeling

Dummy Variables and Interactions

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Example: Detecting Sex Discrimination

Imagine you are a trial lawyer and you want to file a suit against a company for salary discrimination... you gather the following data...

```
 Gender    Salary
1    Male   32.0
2  Female   39.1
3  Female   33.2
4  Female   30.6
5    Male   29.0
... ... ...
208 Female  30.0
```

Detecting Sex Discrimination

You want to relate salary(Y) to gender(X)... how can we do that?

Gender is an example of a categorical variable. The variable gender separates our data into 2 groups or categories. The question we want to answer is: *"how is your salary related to which group you belong to..."*

Could we think about additional examples of categories potentially associated with salary?

- MBA education vs. not
- legal vs. illegal immigrant
- quarterback vs wide receiver

Detecting Sex Discrimination

We can use regression to answer these question but we need to recode the categorical variable into a dummy variable

	Gender	Salary	Sex
1	Male	32.00	1
2	Female	39.10	0
3	Female	33.20	0
4	Female	30.60	0
5	Male	29.00	1
...	...	...	
208	Female	30.00	0

Note: In Excel you can create the dummy variable using the formula:

=IF(Gender="Male",1,0)

Detecting Sex Discrimination

Now you can present the following model in court:

$$Salary_i = \beta_0 + \beta_1 Sex_i + \epsilon_i$$

How do you interpret β_1?

$$\begin{aligned} E[Salary|Sex = 0] &= \beta_0 \\ E[Salary|Sex = 1] &= \beta_0 + \beta_1 \end{aligned}$$

β_1 is the male/female difference

Detecting Sex Discrimination

$$Salary_i = \beta_0 + \beta_1 Sex_i + \epsilon_i$$

Regression Statistics	
Multiple R	0.346541
R Square	0.120091
Adjusted R Square	0.115819
Standard Error	10.58426
Observations	208

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	3149.634	3149.6	28.1151	2.93545E-07
Residual	206	23077.47	112.03		
Total	207	26227.11			

	Coefficients	*tandard Err*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	37.20993	0.894533	41.597	3E-102	35.44631451	38.9735426
Gender	8.295513	1.564493	5.3024	2.9E-07	5.211041089	11.3799841

$\hat{\beta}_1 = b_1 = 8.29...$ on average, a male makes approximately \$8,300 more than a female in this firm.

How should the plaintiff's lawyer use the confidence interval in his presentation?

Detecting Sex Discrimination

How can the defense attorney try to counteract the plaintiff's argument?

Perhaps, the observed difference in salaries is related to other variables in the background and NOT to policy discrimination...

Obviously, there are many other factors which we can legitimately use in determining salaries:

- education
- job productivity
- experience

How can we use regression to incorporate additional information?

Detecting Sex Discrimination

Let's add a measure of experience...

$$Salary_i = \beta_0 + \beta_1 Sex_i + \beta_2 Exp_i + \epsilon_i$$

What does that mean?

$$\begin{aligned} E[Salary|Sex = 0, Exp] &= \beta_0 + \beta_2 Exp \\ E[Salary|Sex = 1, Exp] &= (\beta_0 + \beta_1) + \beta_2 Exp \end{aligned}$$

Detecting Sex Discrimination

The data gives us the “year hired” as a measure of experience...

```
   YrHired Gender Salary  Sex
1       92   Male  32.00   1
2       81 Female  39.10   0
3       83 Female  33.20   0
4       87 Female  30.60   0
5       92   Male  29.00   1
... ... ...
208     62 Female  30.00   0
```

Detecting Sex Discrimination

$$Salary_i = \beta_0 + \beta_1 Sex_i + \beta_2 Exp + \epsilon_i$$

Regression Statistics	
Multiple R	0.700680156
R Square	0.490952681
Adjusted R	0.485986366
Standard E	8.070070757
Observatioı	208

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	12876.27	6438	98.8565	8.7642E-31
Residual	205	13350.84	65.13		
Total	207	26227.11			

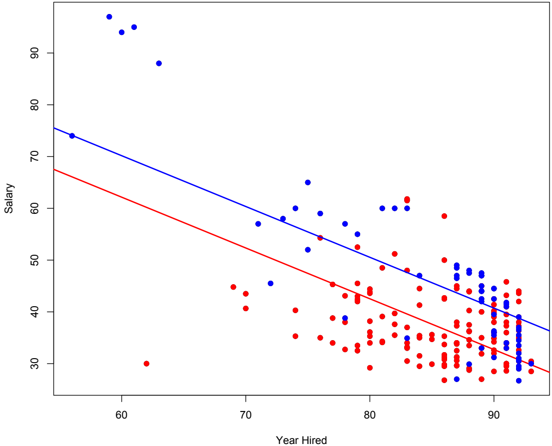
	Coefficients	*tandard Erro*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	121.0212441	6.891851	17.56	9.8E-43	107.433246	134.6092
Gender	8.011885777	1.193089	6.715	1.8E-10	5.65958805	10.36418
YrHired	-0.981150947	0.080285	-12.22	3.7E-26	-1.1394402	-0.822862

$$Salary_i = 121 + 8 Sex_i - 0.98 Exp_i + \epsilon_i$$

Is this good or bad news for the defense?

Detecting Sex Discrimination

$$Salary_i = \begin{cases} 121 - 0.98 Exp_i + \epsilon_i & \text{females} \\ 129 - 0.98 Exp_i + \epsilon_i & \text{males} \end{cases}$$



More than Two Categories

We can use dummy variables in situations in which there are more than two categories. Dummy variables are needed for each category except one, designated as the "base" category.

Why? *Remember that the numerical value of each category has no quantitative meaning!*

Example: House Prices

We want to evaluate the difference in house prices in a couple of different neighborhoods.

	Nbhd	SqFt	Price
1	2	1.79	114.3
2	2	2.03	114.2
3	2	1.74	114.8
4	2	1.98	94.7
5	2	2.13	119.8
6	1	1.78	114.6
7	3	1.83	151.6
8	3	2.16	150.7
...	...	...	...

Example: House Prices

Let's create the dummy variables $dn1$, $dn2$ and $dn3$...

	Nbhd	SqFt	Price	dn1	dn2	dn3
1	2	1.79	114.3	0	1	0
2	2	2.03	114.2	0	1	0
3	2	1.74	114.8	0	1	0
4	2	1.98	94.7	0	1	0
5	2	2.13	119.8	0	1	0
6	1	1.78	114.6	1	0	0
7	3	1.83	151.6	0	0	1
8	3	2.16	150.7	0	0	1
...	...	...				

Example: House Prices

$$Price_i = \beta_0 + \beta_1 dn1_i + \beta_2 dn2_i + \beta_3 Size_i + \epsilon_i$$

$$\begin{aligned}
E[Price|dn1 = 1, Size] &= \beta_0 + \beta_1 + \beta_3 Size \quad \text{(Nbhd 1)} \\
E[Price|dn2 = 1, Size] &= \beta_0 + \beta_2 + \beta_3 Size \quad \text{(Nbhd 2)} \\
E[Price|dn1 = 0, dn2 = 0, Size] &= \beta_0 + \beta_3 Size \qquad\qquad \text{(Nbhd 3)}
\end{aligned}$$

Example: House Prices

$$Price_i = \beta_0 + \beta_1 dn1 + \beta_2 dn2 + \beta_3 Size + \epsilon_i$$

Regression Statistics	
Multiple R	0.828
R Square	0.685
Adjusted R Square	0.677
Standard Error	15.260
Observations	128

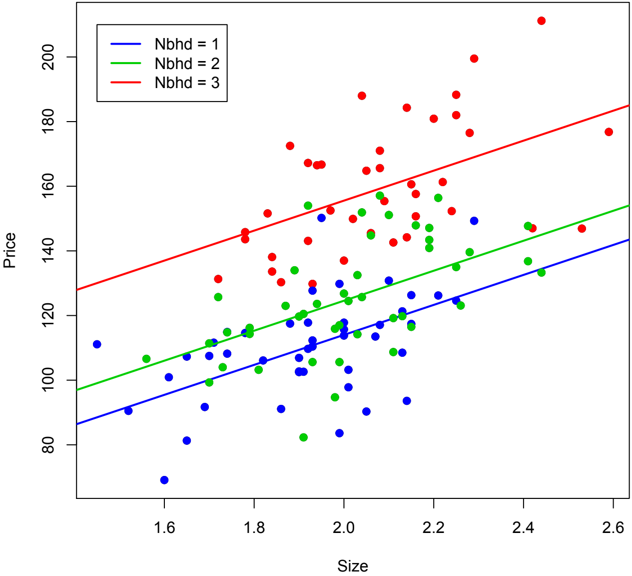
ANOVA

	df	*SS*	*MS*	*F*	*gnificance F*
Regression	3	62809.1504	20936	89.9053	5.8E-31
Residual	124	28876.0639	232.87		
Total	127	91685.2143			

	Coefficients	*Standard Erro*	*t Stat*	*P-value*	*ower 95%*	*pper 95%*
Intercept	62.78	14.25	4.41	0.00	34.58	90.98
dn1	-41.54	3.53	-11.75	0.00	-48.53	-34.54
dn2	-30.97	3.37	-9.19	0.00	-37.63	-24.30
size	46.39	6.75	6.88	0.00	33.03	59.74

$$Price_i = 62.78 - 41.54 dn1 - 30.97 dn2 + 46.39 Size + \epsilon_i$$

Example: House Prices



Example: House Prices

$$Price_i = \beta_0 + \beta_1 Size + \epsilon_i$$

Regression Statistics	
Multiple R	0.553
R Square	0.306
Adjusted R Square	0.300
Standard Error	22.476
Observations	128

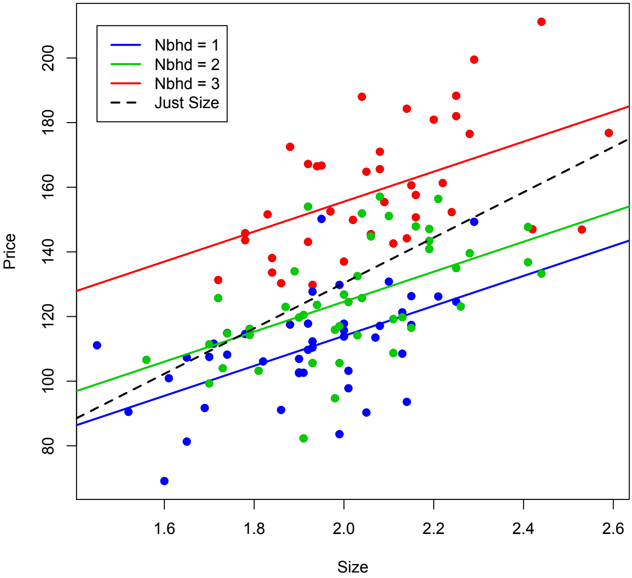
ANOVA

	df	*SS*	*MS*	*F*	*ǥnificance F*
Regression	1	28036.4	28036.36	55.501	1E-11
Residual	126	63648.9	505.1496		
Total	127	91685.2			

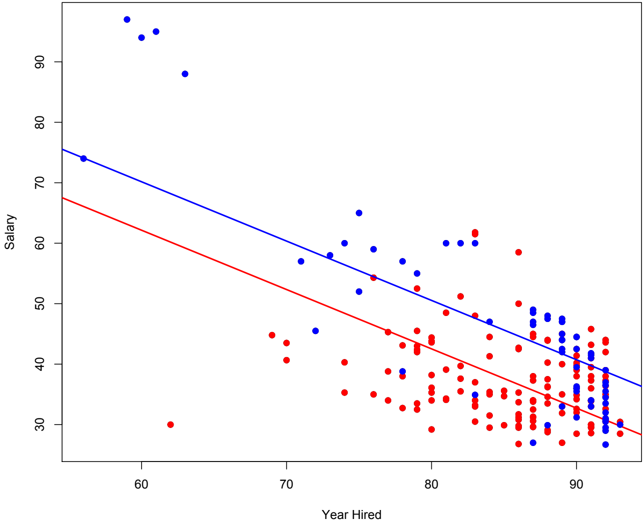
	Coefficients	*andard Err*	*t Stat*	*P-value*	*ower 95%*	*pper 95%*
Intercept	-10.09	18.97	-0.53	0.60	-47.62	27.44
size	70.23	9.43	7.45	0.00	51.57	88.88

$$Price_i = -10.09 + 70.23 Size + \epsilon_i$$

Example: House Prices



Back to the Sex Discrimination Case



Does it look like the effect of experience on salary is the same for males and females?

Back to the Sex Discrimination Case

Could we try to expand our analysis by allowing a different slope for each group?

Yes... Consider the following model:

$$Salary_i = \beta_0 + \beta_1 Exp_i + \beta_2 Sex_i + \beta_3 Exp_i \times Sex_i + \epsilon_i$$

For Females:

$$Salary_i = \beta_0 + \beta_1 Exp_i + \epsilon_i$$

For Males:

$$Salary_i = (\beta_0 + \beta_2) + (\beta_1 + \beta_3) Exp_i + \epsilon_i$$

Both the intercepts and slopes become different.

Sex Discrimination Case

How does the data look like?

	YrHired	Gender	Salary	Sex	SexExp
1	92	Male	32.00	1	92
2	81	Female	39.10	0	0
3	83	Female	33.20	0	0
4	87	Female	30.60	0	0
5	92	Male	29.00	1	92
...	...	...			
208	62	Female	30.00	0	62

Sex Discrimination Case

$$Salary_i = \beta_0 + \beta_1 Sex_i + \beta_2 Exp + \beta_3 Exp * Sex + \epsilon_i$$

Regression Statistics	
Multiple R	0.799130351
R Square	0.638609318
Adjusted R S	0.63329475
Standard Err	6.816298288
Observations	208

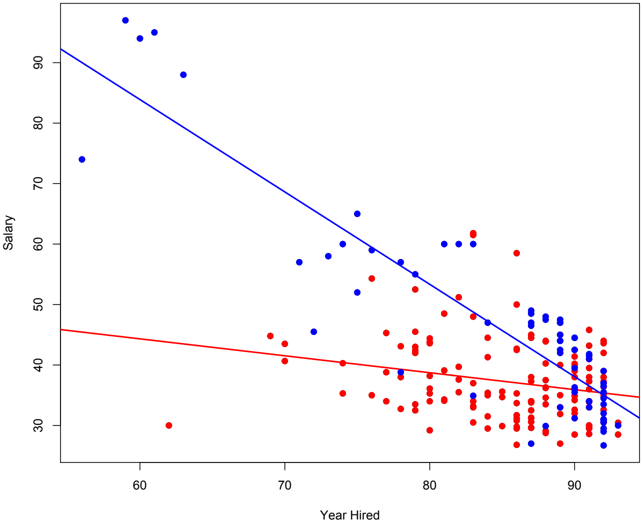
ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	3	16748.88	5582.96	120.16	7.513E-45
Residual	204	9478.232	46.4619		
Total	207	26227.11			

	Coefficients	*Standard Err*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	61.12479795	8.770854	6.96908	4E-11	43.831649	78.41795
Gender	114.4425931	11.7012	9.78041	9E-19	91.371794	137.5134
YrHired	-0.279963351	0.102456	-2.7325	0.0068	-0.4819713	-0.077955
GenderExp	-1.247798369	0.136676	-9.1296	7E-17	-1.5172765	-0.97832

$$Salary_i = 61 + 114 Sex_i + -0.27 Exp + -1.24 Exp * Sex + \epsilon_i$$

Sex Discrimination Case



Is this good or bad news for the plaintiff?

Variable Interaction

So, the effect of experience on salary is different for males and females... in general, when the effect of the variable X_1 onto Y depends on another variable X_2 we say that X_1 and X_2 interact with each other.

We can extend this notion by the inclusion of multiplicative effects through interaction terms.

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 (X_{1i} X_{2i}) + \varepsilon$$

$$\frac{\partial E[Y|X_1, X_2]}{\partial X_1} = \beta_1 + \beta_3 X_2$$

We will pick this up in our next section...