STA371G Homework Assignment 2: Solutions

Problem 1

Suppose $Z \sim \mathcal{N}(0, 1)$, i.e., the random variable Z follows a standard normal distribution.

(a) Find $P(Z < -1.25)$.

$P(Z < -1.25) = 0.106$
In Excel: =NORMSDIST(-1.25)
In R: pnorm(-1.25)

(b) Find $P(Z > 1.50)$.

$P(Z > 1.50) = 1 - P(Z \leq 1.50) = 0.067$

(c) Find $P(-0.10 < Z < 1.40)$.

$P(-0.10 < Z < 1.40) = P(Z < 1.40) - P(Z \leq -0.10) = 0.459$

Problem 2

Suppose $X \sim \mathcal{N}(5, 10)$, i.e., X is normally distributed with mean 5 and variance 10. Compute:

(a) $P(X > 5)$

$$P(X > 5) = P\left(Z > \frac{5-5}{\sqrt{10}}\right) = P(Z > 0) = 0.50 \quad \text{(or 50\%)}$$

You can do this without using Excel or R, looking at the normal table or using your calculator... just remember that in the normal distribution, 50% of the probability is above the mean.

(b) $P(X > 5 + 2 \times \sqrt{10})$

$$P(X > 5 + 2 \times \sqrt{10}) \approx 0.025 \quad \text{(or 2.5\%)}$$

Rule of thumb: In the normal distribution, about 95% of the probability is between -2 and 2 standard deviations.

You can also use Excel: = 1-NORMDIST(5+2*sqrt(10),5,sqrt(10),TRUE)
or R: 1-pnorm(5+2*sqrt(10),5,sqrt(10))

(c) $P(X = 8)$

$P(X = 8) = 0$

(d) Express $P(-2 \le X \le 6)$ in terms of Z, the standard normal random variable.

$$P(-2 \le X \le 6) = P\left(\frac{-2-5}{\sqrt{10}} \le Z \le \frac{6-5}{\sqrt{10}}\right) = 0.61$$

Problem 3

Suppose that Z follows a standard normal distribution. The probability is 10% that Z is greater than what number?

We are trying to find $P(Z >?) = 0.10$, i.e., $P(Z \le ?) = 1 - 0.10 = 0.90$.
In Excel: =NORMSINV(0.9)
In R: qnorm(0.90)
The answer is found to be 1.282.

Problem 4

A company can purchase raw material from either Supplier A or Supplier B and is concerned about the amounts of impurity the material contains. A review of the records for each supplier indicates that the percentage impurity levels in consignments of the raw material follow normal distributions with the means and standard deviations given in the table below. The company is particularly anxious that the impurity level in a consignment not exceed 5% and want to purchase from the supplier more likely to meet that specification. Which supplier should be chosen?

	Mean	Standard Deviation
Supplier A	4.4	0.4
Supplier B	4.2	0.6

Let X_A represent the percentage of impurity level in a randomly chosen consignment of raw material from Supplier A. Therefore, $X_A \sim N\left(4.4, 0.4^2\right)$. Similarly, X_B represents the percentage of impurity level in a randomly chosen consignment of raw material from Supplier B, and, $X_B \sim \left(4.2, 0.6^2\right)$.

We need to compute $P(X_A > 5)$ and $P(X_B > 5)$.

$$P(X_A > 5) = P\left(Z > \frac{5-4.4}{0.4}\right) = P(Z > 1.50)$$

and

$$P(X_B > 5) = P\left(Z > \frac{5-4.2}{0.6}\right) = P(Z > 1.33)$$

Since $P(Z > 1.50) < P(Z > 1.33)$, we conclude that Supplier A is better and should be chosen.

Problem 5

The tread life of a particular brand of tires has a normal distribution with mean 40,000 miles and standard deviation 5,000 miles. What proportion of these tires will have tread lives between 35,000 and 45,000 miles? What proportion of these tires will have tread lives between 30,000 and 50,000 miles?

Since the tread life $X \sim \mathcal{N}(40000, 5000^2)$ (X is normal distributed with mean 40000 and variance 5000^2).

$$P(35000 \leq X \leq 45000) = P\left(\frac{35000 - 40000}{5000} \leq Z \leq \frac{45000 - 40000}{5000}\right) = P(-1 \leq Z \leq 1) = 0.68$$

$$P(30000 \leq X \leq 50000) = P\left(\frac{30000 - 40000}{5000} \leq Z \leq \frac{50000 - 40000}{5000}\right) = P(-2 \leq Z \leq 2) = 0.95$$

You can do this without using software. Rule of thumb: In the normal distribution, about 68% of the probability is between -1 and 1 standard deviations, and about 95% of the probability is between -2 and 2 standard deviations