Homework Assignment 8

STA 371G, Statistics and Modeling

Problem 1

Suppose that 60% of the students at UT are football fans, 10% of the students at UT are from McCombs Business School, and 80% of the McCombs students are football fans.

(a) (3 points) If we choose a student at random from UT, what is the probability that this student is a football fan from McCombs Business School?

Let $F = 1$ if the student is a football fan and $F = 0$ otherwise, and $M = 1$ if the student is from McCombs and $M = 0$ otherwise. Since $P(F = 1) = 0.60$, $P(M = 1) = 0.10$ and $P(F = 1|M = 1) = 0.80$, we have

$$P(F = 1, M = 1) = P(M = 1)P(F = 1|M = 1) = 0.10 \times 0.80 = 8\%$$

(b) (3 points) If we choose a student at random from UT, what is the probability that this student is from McCombs Business School but *not* a football fan?

$$P(F = 0, M = 1) = P(M = 1) - P(F = 1, M = 1) = 0.10 - 0.08 = 2\%$$

(c) (3 points) If we choose a student at random from UT, what is the probability that this student is neither a football fan nor from McCombs Business School?

$$P(F = 0, M = 0) = P(F = 0) - P(F = 0, M = 1) = (1 - 0.6) - 0.02 = 38\%$$

(d) (3 points) If we choose a football fan student at random from UT, what is the probability that this football fan student is from McCombs Business School?

$$P(M = 1|F = 1) = \frac{P(F = 1, M = 1)}{P(F = 1)} = \frac{0.08}{0.60} = 13.3\%$$

(e) (3 points) If we choose a student who is *not* from McCombs Business School at random, what is the probability for this non-McCombs student to be a football fan?

$$P(F = 1|M = 0) = \frac{P(F = 1, M = 0)}{P(M = 0)} = \frac{P(F = 1) - P(F = 1, M = 1)}{1 - P(M = 1)} = \frac{0.6 - 0.08}{1 - 0.1} = 57.8\%$$

Comment: one easy way to do this kind of problem is using a table shown below.

	McCombs (M=1)	Not McCombs (M=0)	
Football Fan (F=1)	0.08	0.52	0.60
Not Football Fan (F=0)	0.02	0.38	0.40
	0.10	0.90	1

Problem 2: Freemark Abbey Winery (I)

This problem is based on the Freemark Abbey Winery case. Please read this case carefully before answering the following questions. You should assume that Freemark Abbey Winery sells the wine in bulk ($1 per bottle) if the storm hits and there is no mold.

(a) Fill the payoff table below using the information given in the case. Find the optimal action with the *maximin* rule, the optimal action with the *maximax* rule.

Table 1: Payoff Table

	0.50*0.40 =0.20 Storm Botrytis	0.50*0.60 =0.30 Storm No Botrytis	0.50*0.40 =0.20 No Storm Sugar 25%	0.50*0.40 =0.20 No Storm Sugre 20%	0.50*0.20 =0.10 No Storm Acidity <0.7%
Harvest Now	2.85*12 =34.2	2.85*12 =34.2	2.85*12 =34.2	2.85*12 =34.2	2.85*12 =34.2
Harvest Later	8*12*0.7 =67.2	2*12/2 =12	3.5*12 =42	3.0*12 =36	2.5*12 =30

maximin: Harvest Now

maximax: Harvest Later

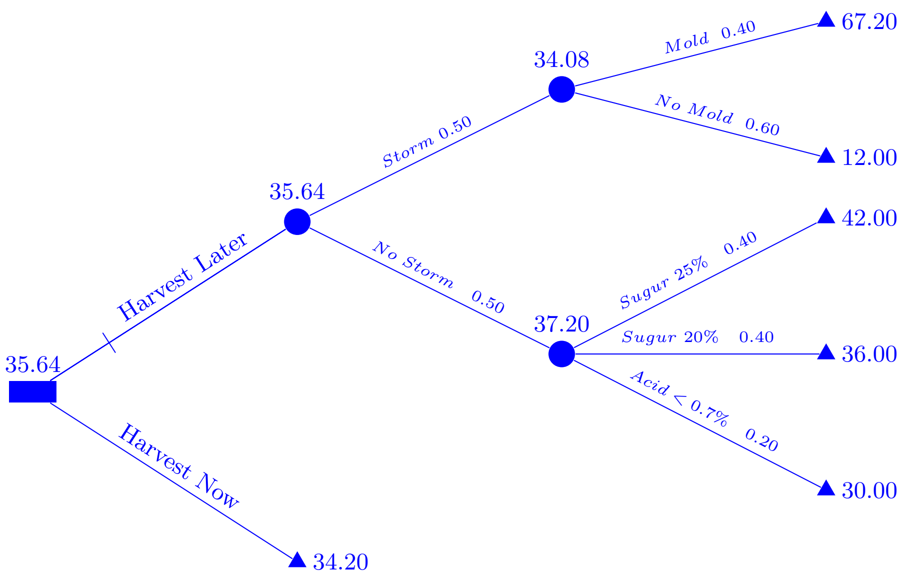
(b) Create a loss table and find the optimal action with the *minimax loss* criterion.

Table 2: Loss Table

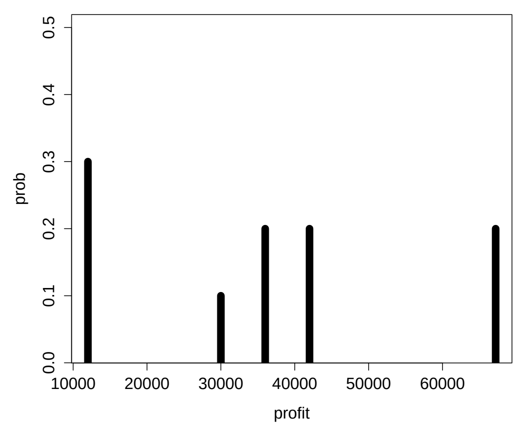
	0.50*0.40 =0.20 Storm Botrytis	0.50*0.60 =0.30 Storm No Botrytis	0.50*0.40 =0.20 No Storm Sugar 25%	0.50*0.40 =0.20 No Storm Sugre 20%	0.50*0.20 =0.10 No Storm Acidity <0.7%
Harvest Now	33	0	7.8	1.8	0
Harvest Later	0	22.2	0	0	4.2

minimax loss: Harvest Later

(c) Construct a decision tree using the information given in the case.



(d) What is the probability distribution that represents the uncertainty regarding the possible outcomes if Jaeger decides to wait to see if the storm hits (rather than harvest immediately)? What is the mean of this distribution?



The mean is \$35.6 K.

(e) What decision would you recommend to Jaeger given the information you have?

Harvest Later to maximize the expected payoff.

(f) Would your decision change if the probability changes from 0.4 to 0.2 that the botrytis mold forms given that the storm hits? Why or why not?

Yes, because the expected payoff of Harvest Later changes to \$30.1 K, smaller than \$34.2 K. So the new recommendation that maximizes the expected payoff would be Harvest Now.

(g) Suppose Jaeger's utility function for x thousand dollars is

$$U(x) = 1 - e^{-\frac{x}{100}}.$$

Find the optimal action in terms of expected utility.

The expected utility for Harvest Now is $1 - e^{-34.2/100} = 0.290$.

The expected utility for Harvest Later is $(1 - e^{-67.2/100}) * 0.2 + (1 - e^{-12/100}) * 0.3 + (1 - e^{-42/100}) * 0.2 + (1 - e^{-36/100}) * 0.2 + (1 - e^{-30/100}) * 0.1 = 0.287$.

Thus the optimal action in terms of expected utility is Harvest Now.