STA371G Homework Assignment 1: Solutions

Problem 1

A construction company has to complete a project no later than three months from now or there will be significant cost overruns. The manager of the construction company believes that there are four possible values for the random variable X, the number of months from now it will take to complete the project: 2, 2.5, 3 and 3.5. The manager currently thinks that the probabilities for these four possibilities are in the ratio of 1 to 3 to 4 to 2. That is to say, $X = 2.5$ is three times more likely than $X = 2$.

(a) Find the probability distribution of X.

The random variable X has four possible random outcomes: 2, 2.5, 3 and 3.5. Since the summation of the probabilities of all possible outcomes must be equal to one, we have

$$P(X = 2) + P(X = 2.5) + P(X = 3) + P(X = 3.5) = 1.$$

Since $P(X = 2.5) = 3P(X = 2)$, $P(X = 3) = 4P(X = 2)$ and $P(X = 3.5) = 2P(X = 2)$, we have

$$P(X = 2) + 3P(X = 2) + 4P(X = 2) + 2P(X = 2) = 10P(X = 2) = 1.$$

Therefore

$$\begin{aligned} P(X = 2) &= 0.1 \\ P(X = 2.5) &= 0.3 \\ P(X = 3) &= 0.4 \\ P(X = 3.5) &= 0.2 \end{aligned}$$

(b) What is the probability that this project will be completed in no more than three months from now?

$$P(X \leq 3) = P(X = 2) + P(X = 2.5) + P(X = 3) = 0.8$$

(c) What is the expected completion time of this project from now?

The expected completion time of the project is also the mean of the random variable, which can be calculated as

$$\begin{aligned} E(X) &= 2P(X = 2) + 2.5P(X = 2.5) + 3P(X = 3) + 3.5P(X = 3.5) \\ &= 2 \times 0.1 + 2.5 \times 0.3 + 3 \times 0.4 + 3.5 \times 0.2 \\ &= 2.85 \end{aligned}$$

(d) How much variability exists around the expected completion time? (Hint: calculate the variance/standard deviation)

To measure the variability around $E(X)$, we use the variance $Var(X)$ or standard deviation $sd(X)$. The variance is

$$\begin{aligned}Var(X) =&(2 - E(X))^2 P(X = 2) + (2.5 - E(X))^2 P(X = 2.5)\\&+ (3 - E(X))^2 P(X = 3) + (3.5 - E(X))^2 P(X = 3.5)\\=&(2 - 2.85)^2 \times 0.1 + (2.5 - 2.85)^2 \times 0.3\\&+ (3 - 2.85)^2 \times 0.4 + (3.5 - 2.85)^2 \times 0.2\\=&0.2025\end{aligned}$$

The standard deviation is

$$sd(X) = \sqrt{Var(X)} = \sqrt{0.2025} = 0.45$$

Note that we can also calculate the variance as

$$\begin{aligned}Var(X) =&E(X^2) - (E(X))^2\\=&(2)^2 P(X = 2) + (2.5)^2 P(X = 2.5)\\&+ (3)^2 P(X = 3) + (3.5)^2 P(X = 3.5) - 2.85^2\\=&2^2 \times 0.1 + 2.5^2 \times 0.3\\&+ 3^2 \times 0.4 + 3.5^2 \times 0.2 - 2.85^2\\=&0.2025\end{aligned}$$

(e) The project requires a special equipment that the company has already paid 3550 dollars to rent. To continue renting this equipment until the end of the project, for each additional half month, the company will have to pay a rental fee of 500 dollars. What is the expected total rental cost for this equipment? How much variability exists around the expected total rental cost?

Denote Y as the total rental cost. We have

$$Y = 3550 + 500 \times 2 \times X = 3550 + 1000X.$$

The expected total rental cost is

$$E(Y) = 3550 + 1000 \times E(X) = 3550 + 1000 \times 2.85 = 6400 \text{ dollars.}$$

The variance of Y is

$$Var(Y) = 1000^2 \times Var(X) = 1000^2 \times 0.2025 = 2.025 \times 10^5.$$

The standard deviation of Y is

$$sd(Y) = \sqrt{Var(Y)} = 450.$$

Problem 2

Suppose that Jennifer has a 90% probability to eat breakfast and David has a 30% probability to eat breakfast. Moreover, assume whether Jennifer eats breakfast is not related to whether David eats breakfast.

(a) What's the probability that both Jennifer and David eat breakfast tomorrow morning?

$$90\% \times 30\% = 27\%$$

(b) What's the probability that Jennifer eats breakfast or David eats breakfast tomorrow morning?

$$90\% + 30\% - 27\% = 93\%$$

(c) What's the probability that either Jennifer or David (not both of them) eats breakfast tomorrow morning?

$$90\% \times (1 - 30\%) + (1 - 90\%) \times 30\% = 66\%$$