Homework Assignment 9

STA 371G, Statistics and Modeling

Problem 1: Freemark Abbey Winery (I) (25 points)

This problem is based on the Freemark Abbey Winery case. Please read this case carefully before answering the following questions. You should assume that Freemark Abbey Winery sells the wine in bulk ($1 per bottle) if the storm hits and there is no mold.

(a) Fill the payoff table below using the information given in the case. Find the optimal action with the *maximin* rule, the optimal action with the *maximax* rule.

Table 1: Payoff Table

	0.50*0.40 =0.20 Storm Botrytis	0.50*0.60 =0.30 Storm No Botrytis	0.50*0.40 =0.20 No Storm Sugar 25%	0.50*0.40 =0.20 No Storm Sugre 20%	0.50*0.20 =0.10 No Storm Acidity <0.7%
Harvest Now	2.85*12 =34.2	2.85*12 =34.2	2.85*12 =34.2	2.85*12 =34.2	2.85*12 =34.2
Harvest Later	8*12*0.7 =67.2	2*12/2 =12	3.5*12 =42	3.0*12 =36	2.5*12 =30

maximin: Harvest Now

maximax: Harvest Later

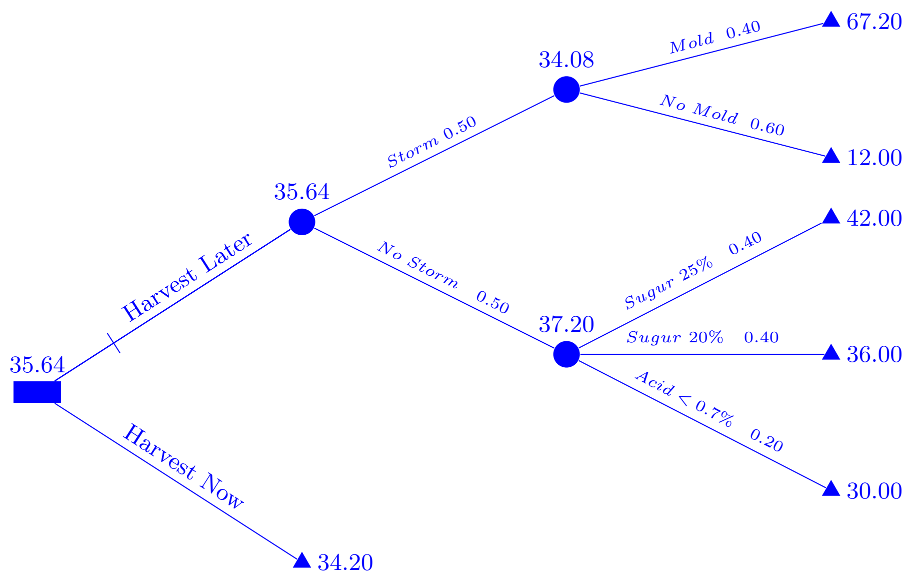
(b) Create a loss table and find the optimal action with the *minimax loss* criterion.

Table 2: Loss Table

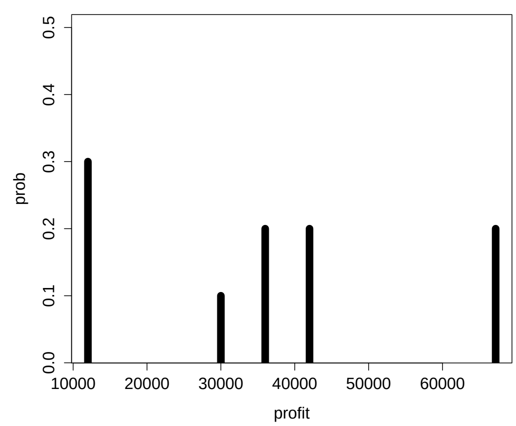
	0.50*0.40 =0.20 Storm Botrytis	0.50*0.60 =0.30 Storm No Botrytis	0.50*0.40 =0.20 No Storm Sugar 25%	0.50*0.40 =0.20 No Storm Sugre 20%	0.50*0.20 =0.10 No Storm Acidity <0.7%
Harvest Now	33	0	7.8	1.8	0
Harvest Later	0	22.2	0	0	4.2

minimax loss: Harvest Later

(c) Construct a decision tree using the information given in the case.



(d) What is the probability distribution that represents the uncertainty regarding the possible outcomes if Jaeger decides to wait to see if the storm hits (rather than harvest immediately)? What is the mean of this distribution?



The mean is \$35.6 K.

(e) What decision would you recommend to Jaeger given the information you have?

Harvest Later to maximize the expected payoff.

(f) Would your decision change if the probability changes from 0.4 to 0.2 that the botrytis mold forms given that the storm hits? Why or why not?

Yes, because the expected payoff of Harvest Later changes to \$30.1 K, smaller than \$34.2 K. So the new recommendation that maximizes the expected payoff would be Harvest Now.

(g) Suppose Jaeger's utility function for x thousand dollars is

$$U(x) = 1 - e^{-\frac{x}{100}}.$$

Find the optimal action in terms of expected utility.

The expected utility for Harvest Now is $1 - e^{-34.2/100} = 0.290$.

The expected utility for Harvest Later is $(1 - e^{-67.2/100}) * 0.2 + (1 - e^{-12/100}) * 0.3 + (1 - e^{-42/100}) * 0.2 + (1 - e^{-36/100}) * 0.2 + (1 - e^{-30/100}) * 0.1 = 0.287$.

Thus the optimal action in terms of expected utility is Harvest Now.

Problem 2: Freemark Abbey Winery (II) (15 points)

In this problem, we will study the value of information for Jaeger.

(a) Find the expected value of perfect information (EVPI). This would be the maximum amount of money you would be willing to pay for perfect information.

Under the current state of information, the optimal action is Harvest Later. The expected loss of this action is

$$22.2 * 0.3 + 4.2 * 0.1 = 7.08$$

Thus the EVPI is \$7.08 K and the decision maker shall pay up to \$7.08 K to obtain perfect information.

We may also first find the expected payoff under perfect information (ERPI), which is

$$ERPI = 67.2 * 0.2 + 34.2 * 0.3 + 42 * 0.2 + 36 * 0.2 + 34.2 * 0.1 = 42.72$$

and then calculate EVPI as

$$EVPI = ERPI - 35.64 = 7.08$$

(b) Suppose you could buy perfect information regarding whether or not the storm will hit. What is the most you would be willing to pay for this information?

- Method 1:

  Note that your perfect information about weather won't change the probability for storm to happen or not, it is just used to influence your decisions.

  If you know there is storm, then you will choose Harvest Now since $34.20 > 34.08$, and if you know there is no storm, then you will choose Harvest Later since $34.20 < 37.20$.

  There is a 50% chance that you will be told that there will be storm, thus your expected payoff given perfect information about weather would be

  $$34.20 * 0.5 + 37.20 * 0.5 = 35.70$$

  Thus you would be willing to pay up to

  $$35.70 - 35.64 = 0.06$$

  thousand dollars for perfect weather information.

- Method 2:

  You optimal action under current information is to choose Harvest Later. Given the sample information that there is no storm, your action will still be Harvest Later, so

  $$VSI(No\ Storm) = 0;$$

  given the sample information that there is storm, your action of Harvest Later will have an expected payoff of $ER = 34.08$ and your action of Harvest Now will have an expected payoff of $ER = 34.20$, thus

  $$VSI(Storm) = 34.20 - 34.08 = 0.12.$$

  We have

  $$EVSI = VSI(No\ Storm)*P(No\ Storm)+VSI(Storm)*P(Storm) = 0.12*0.5 = 0.06$$

(c) Suppose you could buy perfect information regarding whether or not the botrytis mold forms if the storm hits. What is the most you would be willing to pay for this information?

- Method 1:

  If you know the botrytis mold would form if the storm hits, then your optimal decision would be Harvest Later, with an expected payoff of

  $$0.5 * 37.2 + 0.5 * 67.2 = 52.2$$

  But if you know the botrytis mold would not form if the storm hits, then your expected payoff of Harvest Later would become

  $$0.5 * 12 + 0.5 * 37.2 = 24.6$$

  and hence you would choose Harvest Now with an expected payoff of 34.2.

  There is a 40% chance that you will be told that the botrytis mold forms, and a 60% chance that you will be be told that the botrytis mold will not form, therefore, your expected payoff under this perfect information is

  $$0.4 * 52.2 + 0.6 * 34.2 = 41.4$$

  You would be willing to pay up to

  $$41.4 - 35.64 = 5.76$$

  thousand dollars for this perfect information about botrytis mold.

- Method 2:

  Since VSI(Mold would form if the storm hits) = 0 and VSI(Mold would not form if the storm hits) = $34.2-(0.5*12+0.5*37.2) = 9.6$, we have EVSI = 9.6*60%= 5.76.

Problem 3: Freemark Abbey Winery (III) (20 points)

In this problem, we will apply the Bayes' theorem to find out the value of sample information. In problem #2, you computed the expected value of the mold expert's perfect information regarding whether or not the condition of the grapes is such that the botrytis mold will form if the storm hits. Now suppose the information is not perfect. In particular, suppose that if the condition of the grapes is such that the mold will form if the storm hits, the mold expert correctly indicates this 75% of the time; and if the condition of the grapes is such that mold will not form if the storm hits, the mold expert correctly indicates this 85% of the time.

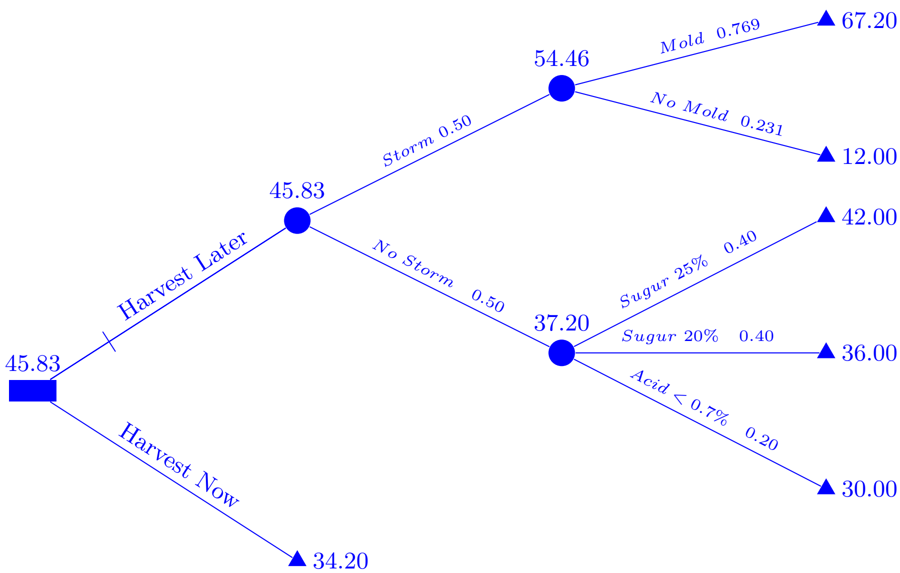
(a) Fill the joint probability table shown below:

	Mold	No Mold	
Expert States Mold			
Expert States No Mold			

	Mold	No Mold	
Expert States Mold	0.40*0.75=0.30	0.60*0.15 =0.09	0.39
Expert States No Mold	0.40*0.25 =0.10	0.60*0.85 = 0.51	0.61
	0.40	0.60	1

(b) If the mold expert states that the mold will form if the storm hits, find the optimal action and the expected payoff under that action.

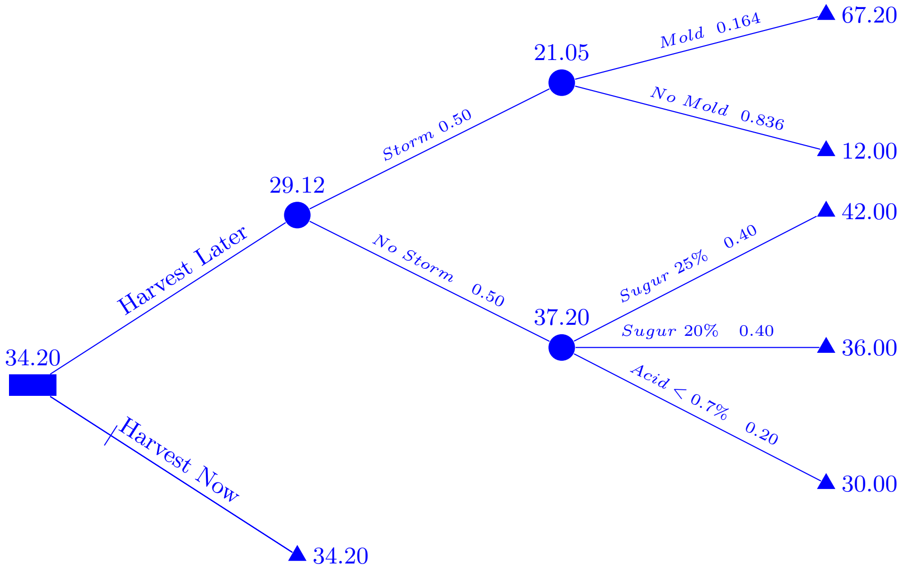
Since $P(\text{Mold} \mid \text{Expert States Mold}) = 0.30/0.39 = 0.769$, we have the following decision tree:



Thus the optimal decision is Harvest Later and the expected payoff is 45.83, if the mold expert states that tho model will form.

(c) If the mold expert states that the mold will not form if the storm hits, find the optimal action and the expected payoff under that action.

Since $P(\text{Mold} \mid \text{Expert States No Mold}) = 0.10/0.61 = 0.164$, we have the following decision tree:



Thus the optimal decision is Harvest Now and the expected payoff is 34.20, if the mold expert states that tho model will not form.

(d) How much are you willing to pay for the mold expert's imperfect information?

- Method 1:
  Since $P(\text{Expert States Mold}) = 0.39$ and $P(\text{Expert States No Mold}) = 0.61$, the expected payoff with the expert's information would be

$$0.39 * 45.83 + 0.61 * 34.20 = 38.74$$

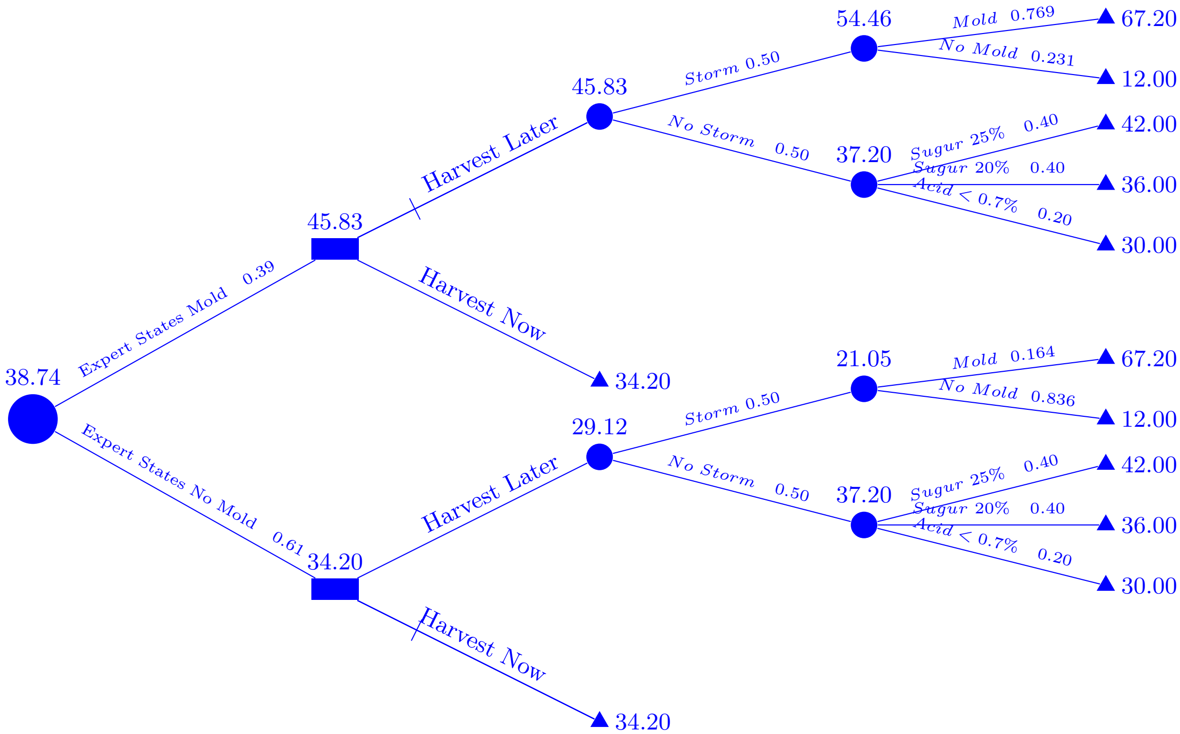
  Thus we should pay up to

$$38.74 - 35.64 = 3.10$$

  thousand dollars for the expert's information.

- Method 2:
  If the Expert States Mold, the decision would not change and VSI(Expert States Mold) = 0; but if the Expert States No Mold, the decision would change and VSI(Expert States No Mold) = 34.20 − 29.12 = 5.08. The EVSI for the expert's opinion would be 5.08*0.61 = 3.10.

The whole process can be described by the following decision tree:


38.74
Expert States Mold 0.39
Expert States No Mold 0.61
45.83
34.20
Harvest Later
Harvest Now
45.83
29.12
34.20
34.20
Storm 0.50
No Storm 0.50
54.46
37.20
21.05
37.20
Mold 0.769
No Mold 0.231
Mold 0.164
No Mold 0.836
Sugur 25% 0.40
Sugur 20% 0.40
Acid < 0.7% 0.20
67.20
12.00
42.00
36.00
30.00