

Practice Exam # 1

STA 371G, Statistics and Modeling, Spring 2015

- Please answer all problems in the space provided on the exam. The full score is 100.
- Read each question carefully and clearly present your answers.
- You must show all your work and give a complete explanation. No credit will be given for only the answer without an explanation/equation.
- The exam is closed-book. You are allowed one page of notes. You may use a calculator.

$$Z \sim N(0,1)$$

x	P(Z<x)	x	P(Z<x)	x	P(Z<x)	x	P(Z<x)
-3	0.0013	-1.5	0.0668	0	0.5	1.5	0.9332
-2.95	0.0016	-1.45	0.0735	0.05	0.5199	1.55	0.9394
-2.9	0.0019	-1.4	0.0808	0.1	0.5398	1.6	0.9452
-2.85	0.0022	-1.35	0.0885	0.15	0.5596	1.65	0.9505
-2.8	0.0026	-1.3	0.0968	0.2	0.5793	1.7	0.9554
-2.75	0.003	-1.25	0.1056	0.25	0.5987	1.75	0.9599
-2.7	0.0035	-1.2	0.1151	0.3	0.6179	1.8	0.9641
-2.65	0.004	-1.15	0.1251	0.35	0.6368	1.85	0.9678
-2.6	0.0047	-1.1	0.1357	0.4	0.6554	1.9	0.9713
-2.55	0.0054	-1.05	0.1469	0.45	0.6736	1.95	0.9744
-2.5	0.0062	-1	0.1587	0.5	0.6915	2	0.9772
-2.45	0.0071	-0.95	0.1711	0.55	0.7088	2.05	0.9798
-2.4	0.0082	-0.9	0.1841	0.6	0.7257	2.1	0.9821
-2.35	0.0094	-0.85	0.1977	0.65	0.7422	2.15	0.9842
-2.3	0.0107	-0.8	0.2119	0.7	0.758	2.2	0.9861
-2.25	0.0122	-0.75	0.2266	0.75	0.7734	2.25	0.9878
-2.2	0.0139	-0.7	0.242	0.8	0.7881	2.3	0.9893
-2.15	0.0158	-0.65	0.2578	0.85	0.8023	2.35	0.9906
-2.1	0.0179	-0.6	0.2743	0.9	0.8159	2.4	0.9918
-2.05	0.0202	-0.55	0.2912	0.95	0.8289	2.45	0.9929
-2	0.0228	-0.5	0.3085	1	0.8413	2.5	0.9938
-1.95	0.0256	-0.45	0.3264	1.05	0.8531	2.55	0.9946
-1.9	0.0287	-0.4	0.3446	1.1	0.8643	2.6	0.9953
-1.85	0.0322	-0.35	0.3632	1.15	0.8749	2.65	0.996
-1.8	0.0359	-0.3	0.3821	1.2	0.8849	2.7	0.9965
-1.75	0.0401	-0.25	0.4013	1.25	0.8944	2.75	0.997
-1.7	0.0446	-0.2	0.4207	1.3	0.9032	2.8	0.9974
-1.65	0.0495	-0.15	0.4404	1.35	0.9115	2.85	0.9978
-1.6	0.0548	-0.1	0.4602	1.4	0.9192	2.9	0.9981
-1.55	0.0606	-0.05	0.4801	1.45	0.9265	2.95	0.9984
-1.5	0.0668	0	0.5	1.5	0.9332	3	0.9987

Problem 1 (15 points)

A typical customer buys a random number (X) of polo shirts when he shops at a men's clothing store. The distribution of X is given by the following probability distribution: $P(X = 0) = 0.30$, $P(X = 1) = 0.30$, $P(X = 2) = 0.20$, $P(X = 3) = 0.10$, and $P(X = 4) = 0.10$.

(a) **(3 points)** Find the expected number of polo shirts bought by a customer when he visits this clothing store.

(b) **(3 points)** Find the variance of X .

(c) **(3 points)** Assuming that each shirt costs \$50, let Y be the total amount of money (in dollars) spent by a customer when he visits this clothing store. Find the mean of Y .

(d) **(3 points)** Find the variance of Y .

(e) **(3 points)** What's the probability that a customer's expenditure will be more than 1 standard deviation above the mean expenditure level?

Problem 3 (25 points)

Amore Frozen Foods is reviewing the operation of their macaroni and cheese filling machine. Recall that the FDA approved weight control system for the company specifies that a 20 minute batch must be rejected if the average of 5 sample pies taken at the beginning of the batch run is less than 8.0 ounces.

Suppose the weight of the pies has a normal distribution with mean μ , where μ is the target fill rate, and a standard deviation of 0.22.

(a) (5 points) If Amore sets $\mu = 8.5$, what would be the 95% confidence interval of the weight of a randomly selected pie.

(b) (5 points) What should the filling target be set to if Amore wants the probability to be 0.975 that a randomly selected pie will be no less than 8.0 ounces?

(c) (5 points) What should the filling target be set to if Amore wants the probability to be 0.975 that a given batch will pass the FDA approved weight control system (i.e., a probability of 0.975 that the batch is accepted)?

- (d) **(10 points)** Suppose the company decides to set the target fill rate to $\mu = 8.20$. At this fill rate, the cost of ingredients per dozen pies is $\frac{8.20}{8.44} \times \$1.82 = \$1.77$. Also, the cost of packaging is \$0.62 per dozen pies and the cost of labor and overhead is \$0.56 per dozen pies. Therefore, the total cost is $\$1.77 + \$0.62 + \$0.56 = \2.95 per dozen pies.

A batch consists of 1,000 dozen pies (i.e. 12,000 pies). Suppose that if a batch is rejected then all 12,000 pies can be sold at the Thrift Store for \$3.60 per dozen pies. If a batch is not rejected then all 12,000 pies can be sold at wholesale for \$4.50 per dozen pies.

If the FDA approved weight control system is used, what is the expected profit per batch if the target fill rate is set to $\mu = 8.20$?

Problem 4: Match the Plots (10 points)

Below (Figure 1) are 4 different scatter plots of an outcome variable y versus predictor x followed by 4 four regression output summaries labeled A, B, C and D. Match the outputs with the plots. For example, if you believe Regression D matches Plot 1, you should draw a line to connect them.

Plot 1 (upper-left corner)

Regression A

Plot 2 (upper-right corner)

Regression B

Plot 3 (bottom-left corner)

Regression C

Plot 4 (bottom-right corner)

Regression D

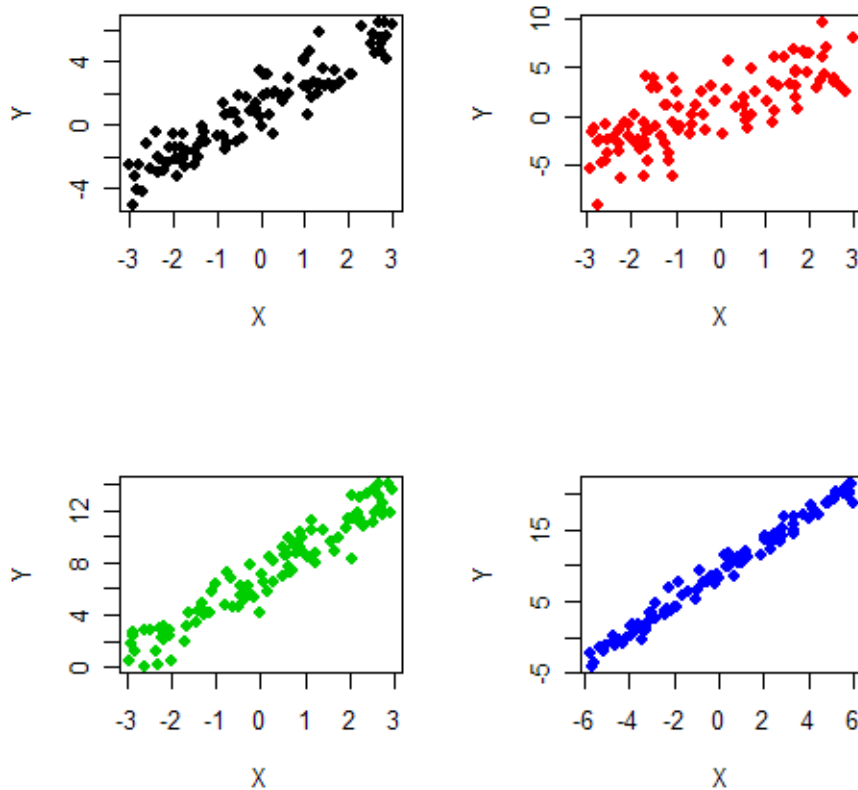


Figure 1: Scatter Plots

Regression A:

Coefficients:

	Estimate	Std. Error
(Intercept)	7.03747	0.12302
(Slope)	2.18658	0.07801

Residual standard error: 1.226

R-Squared: 0.8891

Regression B:

Coefficients:

	Estimate	Std. Error
(Intercept)	1.1491	0.1013
(Slope)	1.4896	0.0583

Residual standard error: 1.012

R-Squared: 0.8695

Regression C:

Coefficients:

	Estimate	Std. Error
(Intercept)	1.2486	0.2053
(Slope)	1.5659	0.1119

Residual standard error: 2.052

R-Squared: 0.6666

Regression D:

Coefficients:

	Estimate	Std. Error
(Intercept)	9.0225	0.0904
(Slope)	2.0718	0.0270

Residual standard error: 0.902

R-Squared: 0.9835

Problem 5 (20 points)

The table below shows the price (in thousands of dollars) for 6 houses (in thousands of square feet) recently sold in City Random. Let X denote Size and Y denote Price.

Size (X)	0.80	0.90	1.00	1.10	1.40	1.40
Price (Y)	140	166	148	186	178	170

- (a) **(2 points)** Calculate the sample means of X and Y .
- (b) **(2 points)** Calculate the sample standard deviations of X and Y .
- (c) **(2 points)** Calculate the sample covariance between X and Y .
- (d) **(2 points)** Calculate the sample correlation between X and Y .
- (e) **(2 points)** Suppose we use simple linear regression to describe how the Price changes as a linear function of the Size. Calculate the least squares estimates of the intercept and slope.
- (f) **(2 points)** What are the units of the intercept and slope.
- (g) **(3 points)** Suppose we change the units of Size from 1000 square feet to 100 square meters (92.9 square meters = 1000 square feet), what would be the intercept and slope?
- (h) **(3 points)** Calculate the coefficient of determination R^2 and explain its meaning.
- (i) **(2 points)** Based on this analysis, briefly describe your understanding of the relationship between Price and Size.

Problem 6 (15 points)

Consider the regression model

$$\text{TI}_t = \beta_0 + \beta_1 \text{SP500}_t + \epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0, \sigma^2),$$

where TI_t and SP500_t represent the returns in month t on Texas Instruments and the S&P 500, respectively. Suppose we know it is true that $\beta_0 = -0.0007$, $\beta_1 = 1.62$ and $\sigma = 0.091$.

- (a) **(5 points)** Suppose the return on the S&P 500 is 3% in a given month, what's the expected return on TI in that month?
- (b) **(5 points)** Suppose the return on the S&P 500 is -5% in a given month, what's the 95% prediction interval for the return on TI in that month?
- (c) **(5 points)** Suppose the return on the S&P 500 is 0% in a given month, what's the 95% prediction interval for the return on TI in that month?