Name: EID: Time of your class:

Midterm Exam #1, STA 371G, Statistics and Modeling, Spring 2017

- You may tear off the blank page at the end as a scratch paper. Please turn in the scratch paper with the exam. The exam is closed-book. You are allowed 1 page (letter/A4 size) of notes. You may use a calculator.
- Please answer all problems in the space provided on the exam. The full score is 100.
- Read each question carefully and clearly present your answers.
- You must show all your work and give a complete explanation. No credit will be given for only the answer without an explanation/equation.

Z ~ N(0,1)

x	P(Z<x)	x	P(Z<x)	x	P(Z<x)	x	P(Z<x)
-3	0.0013	-1.5	0.0668	0	0.5	1.5	0.9332
-2.95	0.0016	-1.45	0.0735	0.05	0.5199	1.55	0.9394
-2.9	0.0019	-1.4	0.0808	0.1	0.5398	1.6	0.9452
-2.85	0.0022	-1.35	0.0885	0.15	0.5596	1.65	0.9505
-2.8	0.0026	-1.3	0.0968	0.2	0.5793	1.7	0.9554
-2.75	0.003	-1.25	0.1056	0.25	0.5987	1.75	0.9599
-2.7	0.0035	-1.2	0.1151	0.3	0.6179	1.8	0.9641
-2.65	0.004	-1.15	0.1251	0.35	0.6368	1.85	0.9678
-2.6	0.0047	-1.1	0.1357	0.4	0.6554	1.9	0.9713
-2.55	0.0054	-1.05	0.1469	0.45	0.6736	1.95	0.9744
-2.5	0.0062	-1	0.1587	0.5	0.6915	2	0.9772
-2.45	0.0071	-0.95	0.1711	0.55	0.7088	2.05	0.9798
-2.4	0.0082	-0.9	0.1841	0.6	0.7257	2.1	0.9821
-2.35	0.0094	-0.85	0.1977	0.65	0.7422	2.15	0.9842
-2.3	0.0107	-0.8	0.2119	0.7	0.758	2.2	0.9861
-2.25	0.0122	-0.75	0.2266	0.75	0.7734	2.25	0.9878
-2.2	0.0139	-0.7	0.242	0.8	0.7881	2.3	0.9893
-2.15	0.0158	-0.65	0.2578	0.85	0.8023	2.35	0.9906
-2.1	0.0179	-0.6	0.2743	0.9	0.8159	2.4	0.9918
-2.05	0.0202	-0.55	0.2912	0.95	0.8289	2.45	0.9929
-2	0.0228	-0.5	0.3085	1	0.8413	2.5	0.9938
-1.95	0.0256	-0.45	0.3264	1.05	0.8531	2.55	0.9946
-1.9	0.0287	-0.4	0.3446	1.1	0.8643	2.6	0.9953
-1.85	0.0322	-0.35	0.3632	1.15	0.8749	2.65	0.996
-1.8	0.0359	-0.3	0.3821	1.2	0.8849	2.7	0.9965
-1.75	0.0401	-0.25	0.4013	1.25	0.8944	2.75	0.997
-1.7	0.0446	-0.2	0.4207	1.3	0.9032	2.8	0.9974
-1.65	0.0495	-0.15	0.4404	1.35	0.9115	2.85	0.9978
-1.6	0.0548	-0.1	0.4602	1.4	0.9192	2.9	0.9981
-1.55	0.0606	-0.05	0.4801	1.45	0.9265	2.95	0.9984
-1.5	0.0668	0	0.5	1.5	0.9332	3	0.9987

Problem 1 (10 points)

Suppose that Claire has a 95% probability to eat breakfast and Kaylee has a 80% probability to eat breakfast. Moreover, assume whether Claire eats breakfast is not related to whether Kaylee eats breakfast.

(a) (3 points) What's the probability that neither Claire nor Kaylee eats breakfast tomorrow morning?

(b) (3 points) If Kaylee had breakfast this morning, what's the probability that Claire also had breakfast this morning?

(c) (4 points) What's the probability that only one of them (*i.e.*, either Claire or Kaylee but not both of them) eats breakfast tomorrow morning?

Problem 2 (10 points)

Suppose $X \sim \mathcal{N}(-10, 100)$, i.e., X is normally distributed with mean -10 and variance 100.

(a) (3 points) Compute $P(X > -10)$

(b) (3 points) Compute $P(X \leq -30)$

(c) (4 points) X is larger than which number with 10.56% probability?

Problem 3 (5 points)

Suppose that X is a continuous random number that is uniformly distributed between -1 and 1, which means that all intervals of the same length between -1 and 1 are equally probable.

(a) (1 points) Compute $P(X = 0)$

(b) (2 points) Plot the probability density function of X (hint: the area under the probability density function is equal to one)

(c) (2 points) Plot the cumulative density function of X (hint: plot $P(X < x)$)

Problem 4 (25 points)

The figure below shows the 2016 McCombs BBA Intern Salary Survey, which is based on 615 voluntary reports of accepted offers.

BBA Intern Salary Survey 2016	Average (Monthly)	Median (Monthly)	Standard Deviation (Monthly)
Internship Overall Salaries	\$3,430	\$3,333	\$1,505

Assume the monthly salary for a BBA intern in 2016 follows a normal distribution, whose mean is equal to the Average Monthly Salary reported in the Survey and whose standard deviation is equal to the Standard Deviation reported in the Survey. Answer Questions (a)-(e).

(a) (5 points) Find the 95% Confidence Interval for the monthly salary of a BBA intern in 2016.

(b) (5 points) Find the probability for a BBA intern in 2016 to have a monthly salary that is between \$4,935 and \$6,440.

(c) (5 points) If a BBA student decides to accept an intern offer immediately if he/she receives an offer with a monthly salary that is among the top 16% of all accepted intern offers in 2016, what would be the least amount of monthly salary that will make him/her accept the offer immediately?

(d) (5 points) If you take a random sample of 25 BBA interns in 2016, would the distribution of their average monthly salary be the same as the distribution of the monthly salary of a BBA intern? If Yes, provide your explanations. If No, describe the distribution of their average monthly salary.

(e) (5 points) The Average Monthly Salary of $3,430 reported in the survey provides an estimate of the TRUE average monthly salary, which can only be obtained if all BBA interns in 2016 reported their salaries. Based on the report, find the 95% Confidence Interval of the TRUE average salary of all BBA interns in 2016.

Problem 5 (25 points)

The table below shows the fuel economy of four randomly selected cars. The weight of a car is measured in thousands of kilograms and the fuel economy is measured in KPL (kilometers per liter). Let X denote Weight and Y denote Fuel Economy (Note that four data points are usually far from enough in practice. We choose four points only for illustration purpose).

Weight (X)	2	1.5	1.8	1.1
Fuel Economy (Y)	7	8	5	10

The samples mean of X is $\bar{x} = 1.6$, the samples mean of Y is $\bar{y} = 7.5$, the sample standard deviations of X is $s_x = 0.392$, and the sample standard deviations of Y is $s_y = 2.082$.

(a) (3 points) Calculate the sample covariance between X and Y.

(b) (3 points) Calculate the sample correlation between X and Y.

(c) (3 points) Suppose we use simple linear regression to describe how the Fuel Economy changes as a linear function of the Weight. Calculate the least squares estimates of the intercept and slope.

(d) (3 points) What are the units of the intercept and slope.

(e) (3 points) Suppose we change the units of Weight from 1000 kilograms to 1000 pounds (1000 kilograms = 2204.62 pounds), what would be the new intercept and slope?

(f) (3 points) Suppose we not only change the units of Weight from 1000 kilograms to 1000 pounds, but also change the units of Fuel Economy from KPL (kilometers per litter) to MPG (miles per gallon), where 1 KPL = 2.352 MPG, what would be the new intercept and slope?

(g) (4 points) Calculate the coefficient of determination R^2 and explain its meaning.

(h) (3 points) Based on this analysis, briefly describe your understanding of the relationship between the Weight and Fuel Economy of a car.

Problem 6 (25 points)

The federal Class III milk price, although not the same as, is closely related to the California mailbox price that a milk farmer in California receives for his milk. Based on the monthly milk price data from May 2004 to May 2007, one can run a simple linear regression model to regress the federal Class III milk price on the California mailbox price. The milk price is measured with \$/cwt, where cwt is a unit of measurement that is roughly 100 pound of milk.

The least squares estimation results are presented in the table below:

	A	B	C	D	E	F	G	H
1				**Linear Regression**				
2								
3	**Regression Statistics**							
4	*R*							
5	*R Square*	B5=?						
6	*Adjusted R Square*							
7	*Standard Error*	0.54						
8	*Total number of observations*	37						
9				**Class III = b0 + b1 * Mailbox**				
10								
11	**ANOVA**							
12		*d.f.*	*SS*	*MS*	*F*	*p-level*		
13	*Regression*	1.	138.11		465.63179	0.E+0		
14	*Residual*	35.	C14=?					
15	*Total*	36.	148.49					
16								
17		*Coefficients*	*Standard Error*	*LCL*	*UCL*	*t Stat*	*p-level*	*H0 (5%) rejected?*
18	**Intercept**	-0.93	0.69346	-2.33371	0.48189	-1.3352	0.19043	*No*
19	**Mailbox**	1.14	0.05268	1.02983	1.24372	21.5785	0.E+0	*Yes*
20	*T (5%)*	2.03011						
21	*LCL - Lower value of a reliable interval (LCL)*							
22	*UCL - Upper value of a reliable interval (UCL)*							

Based on the results presented in the table, answers Questions (a)-(d).

(a) (3 points) Suppose the estimated least squares regression line is expressed as

$$\text{Class III Price} = b_0 + b_1 \times \text{Mailbox Price},$$

what's the values of b_0 and b_1?

(b) (3 points) What's the value in cell C14?

(c) (3 points) What's the value in cell B5?

(d) (4 points) We choose b_0 and b_1 to minimize which value in the table?

Consider the regression model

$$\text{ClassIII}_t = \beta_0 + \beta_1 \text{Mailbox}_t + \epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0, \sigma^2),$$

where ClassIII_t represents the milk price in month t for the federal Class III milk and Mailbox_t represents the California mailbox price in month t. Supposing it is true that $\beta_0 = b_0$, $\beta_1 = b_1$ and $\sigma = 0.54$, answers Questions (e)-(g).

(e) (4 points) Suppose the California Mailbox Price is \$12/cwt, what's the 95% Prediction Interval for the price of the federal Class III milk?

(f) (4 points) Suppose the California Mailbox Price is \$12/cwt, what's the probability that the federal Class III milk will be lower than \$10/cwt ?

(g) (4 points) In order to hedge the risk of low milk price in California, in this February, a California milk farmer purchased a put option on the federal Class III milk with a strike price of \$14/cwt. The payoff from the put option is zero, if the strike price is lower than or equal to the Class III milk price, and is equal to the strike price of the put option MINUS the Class III milk price if the strike price is higher than the Class III milk price.

Suppose the California Mailbox Price is \$12/cwt in this August, and the total cost of purchasing and trading the August put option is \$0.79/cwt, what's the probability that this farmer will make a net revenue (mailbox price PLUS payoff from the put option MINUS cost of purchasing and trading the put option) of more than \$13/cwt for his milk in this August?

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Please use this page as your Scratch Paper. This page must be turned in with the exam.