STA 371G: Statistics and Modeling

Decision Making Under Uncertainty: The Value of Information

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The Value of Information

The decision maker may obtain more sample information before making the terminal decision. Such sample information might be valuable in reducing uncertainty about the state of the world:

- In a medical situation, it may be possible to run further diagnostic tests before making a decision regarding surgery.
- If drilling for oil, the decision maker might run geological tests before deciding whether or not to drill.
- For an investment decision, the investor might be able to obtain more information about the investments under consideration.
- In a market decision, it may be possible to obtain more information about consumers' intentions by conducting a market survey or to gain more information about the future actions of competitors.

The Value of Information

Since there is a cost (money and time) involved in sampling, the decision maker must decide if the additional sample information is expected to be useful enough to justify its cost.

- This type of decision is called a *preposterior decision* because it involves the potential posterior distributions following the proposed *sample*.
- The value of the sample to the decision maker usually depends on the observed result.
- Sequential analysis or sequential decision making: the decision maker could pause after each trail and decide whether to continue sampling or to stop sampling.

Expected Value of Perfect Information (EVPI) from Expected Losses

In the extreme case, if the decision maker is able to get "perfect" information, then the problem of decision making under uncertainty becomes a problem of decision making under certainty.

- If action a^* is optimal under the decision maker's current state of information, then

$$EL(a^*) \leq EL(a) \text{ for all actions } a.$$

- Under perfect information, the decision maker's loss will be zero, so the expected value of perfect information (EVPI) must be equal to

$$\text{EVPI} = EL(a^*) - 0 = EL(a^*).$$

- In words, the EVPI to the decision maker is equal to the expected loss of the action that is optimal under the current state of information.

EVPI Example

- Payoff table

Probability:	0.1	0.4	0.3	0.2	
State:	State I	State II	State III	State IV	ER
Buy Stock A	\$2,000	\$6,000	−\$6,000	\$2,000	\$1,200
Buy Stock B	\$0	\$2,000	−\$2,000	\$2,000	\$600
Buy Stock C	−\$2000	−\$4,000	\$0	\$2,000	−\$1,400

- Loss table

Probability:	0.1	0.4	0.3	0.2	
State:	State I	State II	State III	State IV	EL
Buy Stock A	\$0	\$0	\$6,000	\$0	\$1,800
Buy Stock B	\$2000	\$4,000	\$2,000	\$0	\$2,400
Buy Stock C	\$4000	−\$10,000	\$0	\$0	\$4,400

- *EL* criteria: Buy Stock A
- The optimal action, Buy Stock A, has an expected loss of \$1,800
- Thus the EVPI is \$
- The decision maker should pay up to \$ to obtain perfect information

EVPI Example

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Probability:	0.1	0.4	0.3	0.2	
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- Thus the EVPI is \$1,800
- The decision maker should pay up to \$1,800 to obtain perfect information

EVPI from Expected Payoffs

- The value of perfect information (VPI) under state j is equal to the maximum payoff minus the payoff of the action a^* that is optimal under the current state of information:

$$\mathrm{VPI}(j) = \max_k R(k, j) - R(a^*, j)$$

- The expected value of VPI, or EVPI, can be computed as

$$\mathrm{EVPI} = \sum_j \mathrm{VPI}(j) P(j)$$

- EVPI is also equal to the difference between the expected payoff under perfect information (ERPI) and the expected payoff under the decision maker's current state of uncertainty:

$$\mathrm{EVPI} = \mathrm{ERPI} - ER(a^*), \ \mathrm{ERPI} = \sum_j (\max_k R(k, j)) P(j)$$

- In words,

  EVPI = ER with (free) perfect information − ER without information

EVPI Example

- *ER* criteria: Buy Stock A
- Payoff table

Probability:	0.1	0.4	0.3	0.2	
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Buy Stock B	\$0	\$2,000	−\$2,000	\$2,000	\$600
Buy Stock C	−\$2000	−\$4,000	\$0	\$2,000	−\$1,400
VPI					

- The ERPI is
- Thus the EVPI is
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Buy Stock B	\$0	\$2,000	−\$2,000	\$2,000	\$600
Buy Stock C	−\$2000	−\$4,000	\$0	\$2,000	−\$1,400
VPI	\$0	\$0	\$6000	\$0	

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EVPI Example

- *ER* criteria: Buy Stock A
- Payoff table

Probability: State:	0.1 State I	0.4 State II	0.3 State III	0.2 State IV	ER
Buy Stock A	\$2,000	\$6,000	−\$6,000	\$2,000	\$1,200
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Buy Stock C	−\$2000	−\$4,000	\$0	\$2,000	−\$1,400
VPI	\$0	\$0	\$6000	\$0	

- The ERPI is

$$\$(2000 * 0.1 + 6000 * 0.4 + 0 * 0.3 + 2000 * 0.2) = \$3000$$

- Thus the EVPI is

$$\$6000 * 0.3 = \$3000 - \$1200 = \$1,800$$

- The decision maker should pay up to \$1,800 to obtain perfect information

The Value of Sample Information

In real-world decision-making situations, the decision maker seldom has the opportunity to obtain perfect information, and must take a sample if he/she wants more information.

- Since sampling involves some cost, it would be helpful if the decision maker could determine the *expected value of sample information* (EVSI).
- EVSI will always be less or equal to EVPI.
- The decision maker should pay up to EVSI to obtain sample information.

The Value of Sample Information

Suppose under the prior distribution of the decision maker, action a' maximizes ER, what would be the value to the decision maker of taking a sample and observing the sample result y?

- The decision maker could use this sample to revise her prior distribution and arrive at a posterior distribution.
- Denote the optimal action under the posterior distribution by a'', which means $ER(a''|y) \geq ER(a|y)$ for all actions a.
- The value of sample information y to the decision maker is

$$\text{VSI}(y) = ER(a''|y) - ER(a'|y)$$

- In words, the VSI is the posterior expected payoff of the new optimal act a'' minus the posterior expected payoff of the previous optimal act a'.
- If the sample information y does not change the decision maker's optimal action, then obviously $\text{VSI}(y) = 0$.

VSI Example

Suppose there are two deck of cards. Deck A consists of 26 red cards and 26 green cards; deck B consists of 30 red cards and 13 green cards. A deck is chosen randomly from the two, and you are asked to guess which deck it is, subject to the following payoff table:

	Deck A	*Deck B*
Guess Deck A	4	2
Guess Deck B	1	6

- Your prior probabilities are $P(Deck\ A) = P(Deck\ B) = 0.5$
- maxmin, maximax, minimax loss
- $ER(Guess\ Deck\ A) = 4 * 0.5 + 2 * 0.5 = 3$
  $ER(Guess\ Deck\ B) = 1 * 0.5 + 6 * 0.5 = 3.5$
- $EL(Guess\ Deck\ B) = ?$
- EVPI $= ?$

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  $ER(Guess\ Deck\ B) = 1 * 0.5 + 6 * 0.5 = 3.5$
- $EL(Guess\ Deck\ B) = \quad 1.5$
- $\text{EVPI} = \quad 1.5$

VSI Example

Suppose you can purchase sample information in the form of cards drawn randomly (with replacement) from the deck of cards that has been chosen. Each card observed will cost you \$0.20. Should you choose deck B immediately, or pay \$0.20 to see a single trial?

Sample Outcome	State of the world	Prior Probability	Likelihood	Prior× Likelihood	Posterior Probability
Red Card	Deck A	0.5	0.5	0.250	0.40
	Deck B	0.5	0.75	0.375	0.60
				0.625	
Green Card	Deck A	0.5	0.50	0.250	0.67
	Deck B	0.5	0.25	0.125	0.33
				0.375	

- $ER(Guess\ A \mid red) = 2.8$, $ER(Guess\ B \mid green) = 4.0$
- VSI(Red Card) $= ER(Guess\ B \mid red) - ER(Guess\ B \mid red) = ?$
- $ER(Guess\ A \mid green) = 3.33$, $ER(Guess\ B \mid green) = 2.67$
- VSI(Green Card) $= ER(Guess\ A \mid green) - ER(Guess\ B \mid green) = ?$

VSI Example

Suppose you can purchase sample information in the form of cards drawn randomly (with replacement) from the deck of cards that has been chosen. Each card observed will cost you \$0.20. Should you choose deck B immediately, or pay \$0.20 to see a single trial?

Sample Outcome	State of the world	Prior Probability	Likelihood	Prior× Likelihood	Posterior Probability
Red Card	Deck A	0.5	0.5	0.250	0.40
	Deck B	0.5	0.75	0.375	0.60
				0.625	
Green Card	Deck A	0.5	0.50	0.250	0.67
	Deck B	0.5	0.25	0.125	0.33
				0.375	

- $ER(Guess\ A \mid red) = 2.8$, $ER(Guess\ B \mid green) = 4.0$
- VSI(Red Card) $= ER(Guess\ B \mid red) - ER(Guess\ B \mid red) = \quad 0$
- $ER(Guess\ A \mid green) = 3.33$, $ER(Guess\ B \mid green) = 2.67$
- VSI(Green Card) $= ER(Guess\ A \mid green) - ER(Guess\ B \mid green) = \quad 0.66$

Expected Value of Sample Information (EVSI)

The value of sample information (VSI) depends on the specific sample result y. You must decide whether or not to purchase sample information in advance of the actual sample, so you need to find out the *expected value of sample information* (EVSI) as

$$\text{EVSI} = \sum_y \text{VSI}(y) P(y)$$

- VSI(Red Card) $= 0$, VSI(Green Card) $= 0.66$
- $P(\text{Red Card}) = 0.5 * 0.5 + 0.75 * 0.5 = 0.625$
  $P(\text{Green Card}) = 0.5 * 0.5 + 0.25 * 0.5 = 0.375$
- $$\begin{aligned} \text{EVSI} &= \text{VSI(Red Card)} * P(\text{Red Card}) \\ &\quad + \text{VSI(Green Card)} * P(\text{Green Card}) \\ &= \end{aligned}$$
- You should pay up to \$ to see a single trial

Expected Value of Sample Information (EVSI)

The value of sample information (VSI) depends on the specific sample result y. You must decide whether or not to purchase sample information in advance of the actual sample, so you need to find out the *expected value of sample information* (EVSI) as

$$\text{EVSI} = \sum_y \text{VSI}(y)P(y)$$

- $\text{VSI}(\text{Red Card}) = 0$, $\text{VSI}(\text{Green Card}) = 0.66$
- $P(\text{Red Card}) = 0.5 * 0.5 + 0.75 * 0.5 = 0.625$
  $P(\text{Green Card}) = 0.5 * 0.5 + 0.25 * 0.5 = 0.375$
- $$\begin{aligned} \text{EVSI} &= \text{VSI}(\text{Red Card}) * P(\text{Red Card}) \\ &\quad + \text{VSI}(\text{Green Card}) * P(\text{Green Card}) \\ &= 0.25 \end{aligned}$$
- You should pay up to $0.25 to see a single trial

Expected Value of Sample Information (EVSI)

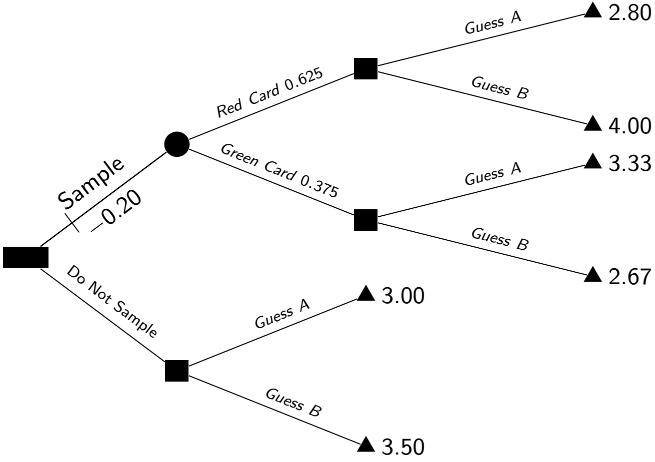
EVSI can also be calculated as

$$\begin{aligned} \text{EVSI} &= E_y[ER(a''|y)] - E_y[ER(a'|y)] = E_y[ER(a''|y)] - ER(a') \\ &= 3.75 - 3.50 = 0.25 \end{aligned}$$

In words,

EVSI = ER with (free) sample information − ER without information

We can use a decision tree to repeat the whole process:



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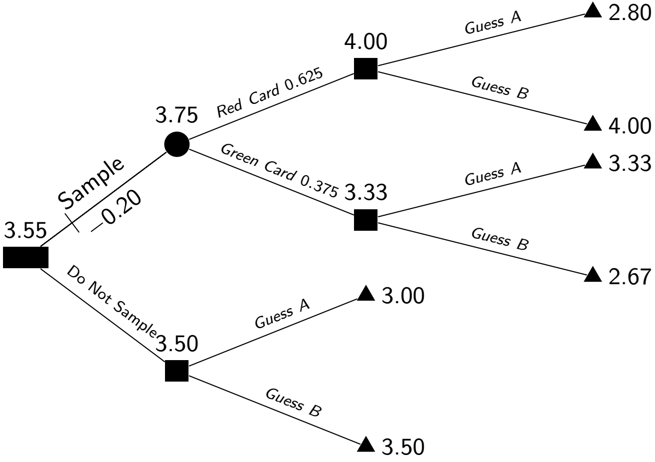

Case Study: Freemark Abbey Winery

In September 1996, William Jaeger had to make a decision: should he harvest the Riesling grapes immediately, or leave them on the vines despite the approaching storm? Shown below is his payoff table.

Table : Payoff Table (in thousands of dollars)

	0.50*0.40 =0.20 Storm Botrytis	0.50*0.60 =0.30 Storm No Botrytis	0.50*0.40 =0.20 No Storm Sugar 25%	0.50*0.40 =0.20 No Storm Sugre 20%	0.50*0.20 =0.10 No Storm Acidity $<0.7\%$
Harvest Now	2.85*12 =34.2	2.85*12 =34.2	2.85*12 =34.2	2.85*12 =34.2	2.85*12 =34.2
Harvest Later	8*12*0.7 =67.2	2*12/2 =12	3.5*12 =42	3.0*12 =36	2.5*12 =30

- maximin, minimax, and maxmin loss
- Construct a decision tree
- Risk profile
- Sensitivity analysis
- Find the optimal action if Jaeger's utility function is $U(x) = 1 - e^{-\frac{x}{100}}$

Case Study: Freemark Abbey Winery

The value of perfect information:

- Find EVPI
- Find the value of perfect information on weather
- Find the value of perfect information regarding whether or not the botrytis mold forms if the storm hits

Case Study: Freemark Abbey Winery

Suppose that if the condition of the grapes is such that the mold will form if the storm hits, the mold expert correctly indicates this 75% of the time; and if the condition of the grapes is such that mold will not form if the storm hits, the mold expert correctly indicates this 85% of the time. If storm hits, then

- $P(\text{Mold}) = 40\%$
- Joint probability table

	Mold	No Mold	
Expert States Mold			
Expert States No Mold			

- Find $P(\text{Mold} \mid \text{Expert States Mold})$
- Find $P(\text{No Mold} \mid \text{Expert States Mold})$
- Find $P(\text{Mold} \mid \text{Expert States No Mold})$
- Find $P(\text{No Mold} \mid \text{Expert States No Mold})$

Case Study: Freemark Abbey Winery

Using these conditional probabilities, which can be calculated by Bayes' theorem, to

- Find the VSI if the mold expert states that the mold will form if the storm hits.
- Find the VSI if the mold expert states that the mold will not form if the storm hits.
- Find the EVSI, which is the maximum Jaeger should pay for the mold expert's opinion.

The whole process can be described by the following decision tree: