STA 371G: Statistics and Modeling

Multiple Linear Regression

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Multiple Linear Regression

Many problems involve more than one independent variable or factor which affects the dependent or response variable.

- More than size to predict house price!
- Demand for a product given prices of competing brands, advertising, house hold attributes, etc.

In SLR, the conditional mean of Y depends on X. The Multiple Linear Regression (MLR) model extends this idea to include more than one independent variable.

Multiple Linear Regression

Same as always, but with more covariates.

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_p X_p + \epsilon$$

Recall the key assumptions of our linear regression model:

(i) The conditional mean of Y is linear in the X_j variables.

(ii) The error terms (deviations from line)

- are normally distributed
- independent from each other
- identically distributed (i.e., they have constant variance)

$$Y|X_1 \dots X_p \sim N(\beta_0 + \beta_1 X_1 \ldots + \beta_p X_p, \sigma^2)$$

Multiple Linear Regression

Our interpretation of regression coefficients can be extended from the simple single covariate regression case:

$$\beta_j = \frac{\partial E[Y|X_1, \ldots, X_p]}{\partial X_j}$$

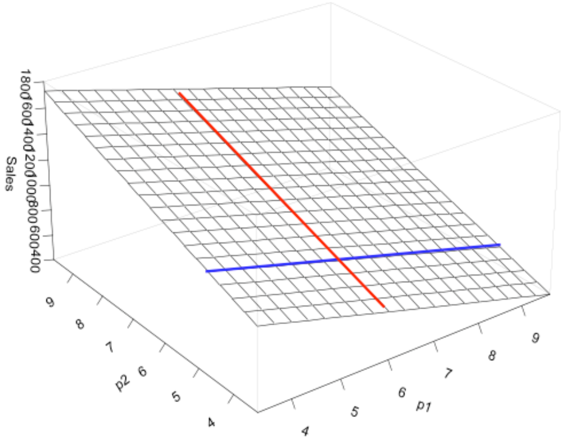
Holding all other variables constant, β_j is the average change in Y per unit change in X_j.

Multiple Linear Regression

If $p = 2$, we can plot the regression surface in 3D.

Consider sales of a product as predicted by price of this product (P1) and the price of a competing product (P2).

$$Sales = \beta_0 + \beta_1 P1 + \beta_2 P2 + \epsilon$$



Least Squares

$$Y = \beta_0 + \beta_1 X_1 \ldots + \beta_p X_p + \varepsilon, \quad \varepsilon \sim N(0, \sigma^2)$$

How do we estimate the MLR model parameters?

The principle of Least Squares is exactly the same as before:

- Define the fitted values
- Find the best fitting plane by minimizing the sum of squared residuals.

Least Squares

The data...

p1	p2	Sales
5.1356702	5.2041860	144.48788
3.4954600	8.0597324	637.24524
7.2753406	11.6759787	620.78693
4.6628156	8.3644209	549.00714
3.5845370	2.1502922	20.42542
5.1679168	10.1530371	713.00665
3.3840914	4.9465690	346.70679
4.2930636	7.7605691	595.77625
4.3690944	7.4288974	457.64694
7.2266002	10.7113247	591.45483
...	...	...

Least Squares

Model: $Sales_i = \beta_0 + \beta_1 P1_i + \beta_2 P2_i + \epsilon_i,\ \epsilon \sim N(0, \sigma^2)$

Regression Statistics	
Multiple R	0.99
R Square	0.99
Adjusted R Square	0.99
Standard Error	28.42
Observations	100.00

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2.00	6004047.24	3002023.62	3717.29	0.00
Residual	97.00	78335.60	807.58		
Total	99.00	6082382.84			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	115.72	8.55	13.54	0.00	98.75	132.68
p1	-97.66	2.67	-36.60	0.00	-102.95	-92.36
p2	108.80	1.41	77.20	0.00	106.00	111.60

$b_0 = \hat{\beta}_0 = 115.72,\ b_1 = \hat{\beta}_1 = -97.66,\ b_2 = \hat{\beta}_2 = 108.80,$

$s = \hat{\sigma} = 28.42$

Plug-in Prediction in MLR

Suppose that by using advanced corporate espionage tactics, I discover that my competitor will charge \$10 the next quarter. After some marketing analysis I decided to charge \$8. How much will I sell?

Our model is

$$Sales = \beta_0 + \beta_1 P1 + \beta_2 P2 + \epsilon$$

with $\epsilon \sim N(0, \sigma^2)$

Our estimates are $b_0 = 115$, $b_1 = -97$, $b_2 = 109$ and $s = 28$ which leads to

$$Sales = 115 + -97 * P1 + 109 * P2 + \epsilon$$

with $\epsilon \sim N(0, 28^2)$

Plug-in Prediction in MLR

By plugging-in the numbers,

$$\begin{aligned} Sales &= 115 - 97 * 8 + 109 * 10 + \epsilon \\ &= 429 + \epsilon \end{aligned}$$

$$Sales | P1 = 8, P2 = 10 \sim N(429, 28^2)$$

and the 95% Prediction Interval is $(429 \pm 2 * 28)$

$$381 < Sales < 493$$

Least Squares

Just as before, each b_i is our estimate of β_i

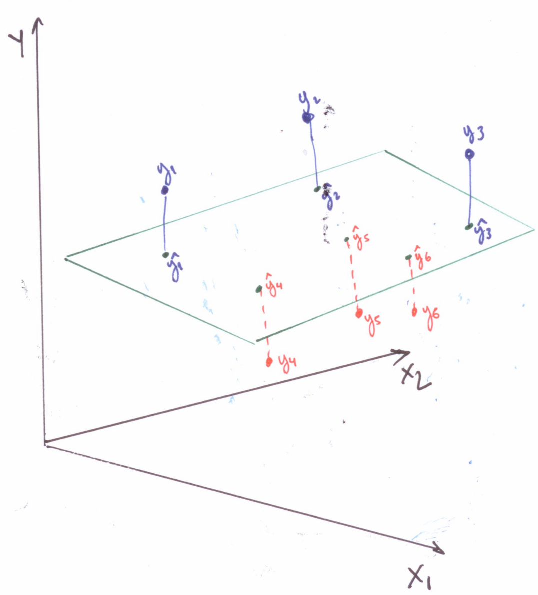
Fitted Values: $\hat{y}_i = b_0 + b_1 x_{1i} + b_2 x_{2i} + \cdots + b_p x_{pi}$.

Residuals: $e_i = y_i - \hat{y}_i$.

Least Squares: Find $b_0, b_1, b_2, \ldots, b_p$ to minimize $\sum_{i=1}^{n} e_i^2$.

In MLR the formulas for the b_i's are too complicated so we won't talk about them...

Least Squares



Residual Standard Error

The calculation for s^2 is exactly the same:

$$s^2 = \frac{\sum_{i=1}^{n} e_i^2}{n-p-1} = \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{n-p-1}$$

- $\hat{y}_i = b_0 + b_1 x_{1i} + \cdots + b_p x_{pi}$
- The residual "standard error" is the estimate for the standard deviation of ϵ,i.e,

$$\hat{\sigma} = s = \sqrt{s^2}.$$

Residuals in MLR

As in the SLR model, the residuals in multiple regression are purged of any linear relationship to the independent variables. Once again, they are on average zero.

Because the fitted values are an exact linear combination of the X's they are not correlated to the residuals.

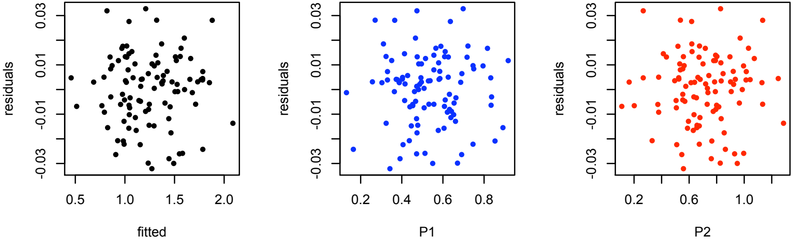
We decompose Y into the part predicted by X and the part due to idiosyncratic error.

$$Y = \hat{Y} + e$$

$$\bar{e} = 0; \quad \mathrm{corr}(X_j, e) = 0; \quad \mathrm{corr}(\hat{Y}, e) = 0$$

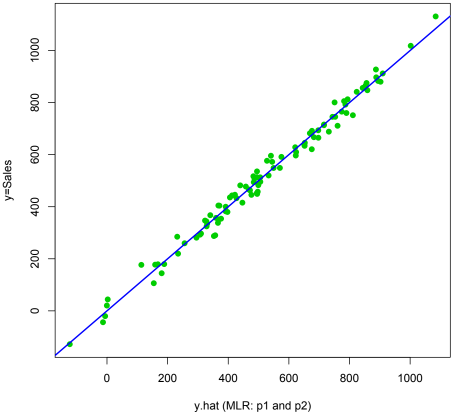
Residuals in MLR

Consider the residuals from the Sales data:



Fitted Values in MLR

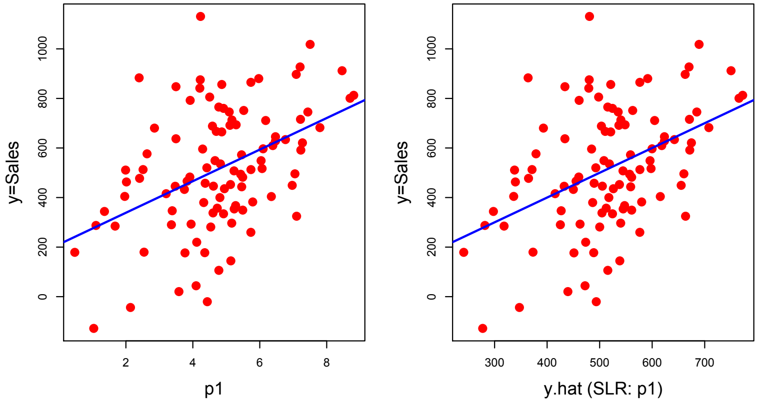
Another great plot for MLR problems is to look at Y (true values) against $\hat{Y}$ (fitted values).



If things are working, these values should form a nice straight line. Can you guess the slope of the blue line?

Fitted Values in MLR

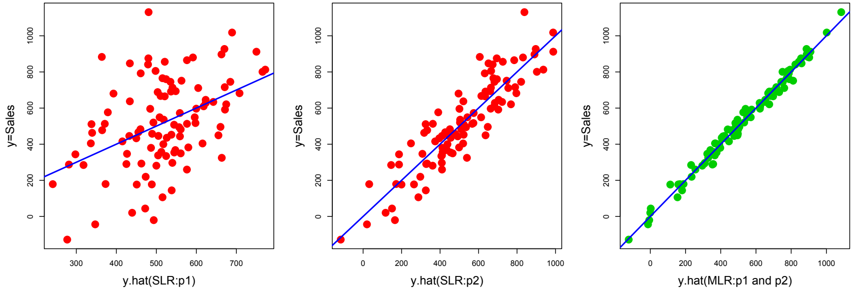
With just $P1$...



- Left plot: *Sales* vs $P1$
- Right plot: Sales vs. $\hat{y}$ (only $P1$ as a regressor)

Fitted Values in MLR

Now, with $P1$ and $P2$...



- First plot: *Sales* regressed on $P1$ alone...
- Second plot: *Sales* regressed on $P2$ alone...
- Third plot: Sales regressed on $P1$ and $P2$

R-squared

- We still have our old variance decomposition identity...

$$SST = SSR + SSE$$

- ... and R^2 is once again defined as

$$R^2 = \frac{SSR}{SST} = 1 - \frac{SSE}{SST}$$

  telling us the percentage of variation in Y explained by the X's.

- In Excel, R^2 is in the same place and "Multiple R" refers to the correlation between $\hat{Y}$ and Y.

Least Squares

$$Sales_i = \beta_0 + \beta_1 P1_i + \beta_2 P2_i + \epsilon_i$$

Regression Statistics	
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$R^2 = 0.99$

Multiple R $= r_{Y,\hat{Y}} = \text{corr}(Y, \hat{Y}) = 0.99$

Note that $R^2 = \text{corr}(Y, \hat{Y})^2$

Back to Baseball

$$R/G = \beta_0 + \beta_1 OBP + \beta_2 SLG + \epsilon$$

Regression Statistics	
Multiple R	0.955698
R Square	0.913359
Adjusted R Square	0.906941
Standard Error	0.148627
Observations	30

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	6.28747	3.143735	142.31576	4.56302E-15
Residual	27	0.596426	0.02209		
Total	29	6.883896			

	Coefficients	*andard Errc*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	-7.014316	0.81991	-8.554984	3.60968E-09	-8.69663241	-5.332
OBP	27.59287	4.003208	6.892689	2.09112E-07	19.37896463	35.80677
SLG	6.031124	2.021542	2.983428	0.005983713	1.883262806	10.17899

$R^2 = 0.913$

Multiple R $= r_{Y,\hat{Y}} = \text{corr}(Y, \hat{Y}) = 0.955$

Note that $R^2 = \text{corr}(Y, \hat{Y})^2$

Intervals for Individual Coefficients

As in SLR, the sampling distribution tells us how close we can expect b_j to be from β_j

The LS estimators are unbiased: $E[b_j] = \beta_j$ for $j = 0, \ldots, d$.

- We denote the sampling distribution of each estimator as

$$b_j \sim N(\beta_j, s_{b_j}^2)$$

Intervals for Individual Coefficients

Intervals and t-statistics are exactly the same as in SLR.

- A 95% C.I. for β_j is approximately $b_j \pm 2s_{b_j}$
- The t-stat: $t_j = \frac{(b_j - \beta_j^0)}{s_{b_j}}$ is the number of standard errors between the LS estimate and the null value (β_j^0)
- As before, we reject the null when t-stat is greater than 2 in absolute value
- Also as before, a small p-value leads to a rejection of the null
- Rejecting when the p-value is less than 0.05 is equivalent to rejecting when the $|t_j| > 2$

In Excel... Do we know all of these numbers?

Regression Statistics	
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95% C.I. for $\beta_1 \approx b1 \pm 2 \times s_{b_1}$

$$[-97.66 - 2 \times 2.67; -97.66 + 2 \times 2.67] = [-102.95; -92.36]$$

F-tests

- In many situation, we need a testing procedure that can address *simultaneous* hypotheses about more than one coefficient
- Why not the t-test?
- We will look at the Overall Test of Significance... the F-test. It will help us determine whether or not our regression is worth anything!

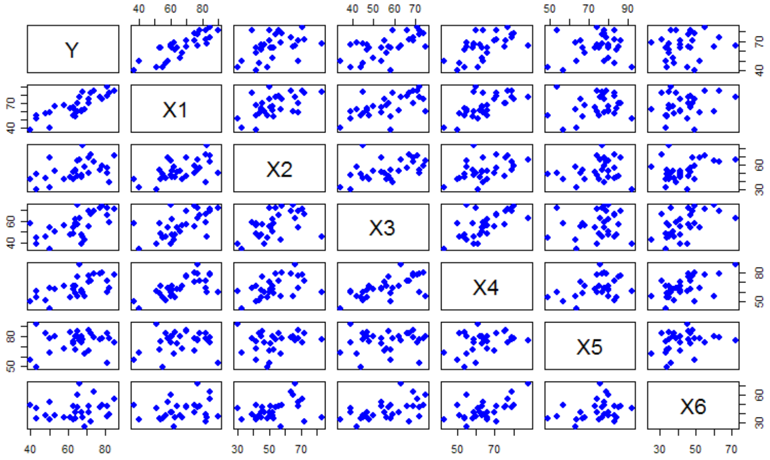
Supervisor Performance Data

Suppose you are interested in the relationship between the overall performance of supervisors to specific activities involving interactions between supervisors and employees (from a psychology management study)

The Data

- Y = Overall rating of supervisor
- X_1 = Handles employee complaints
- X_2 = Does not allow special privileges
- X_3 = Opportunity to learn new things
- X_4 = Raises based on performance
- X_5 = Too critical of poor performance
- X_6 = Rate of advancing to better jobs

Supervisor Performance Data



Supervisor Performance Data

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.855921721
R Square	0.732601993
Adjusted R Square	0.662845991
Standard Error	7.067993765
Observations	30

ANOVA

	df	SS	MS	F	Significance F
Regression	6	3147.966342	524.6611	10.50235	1.24041E-05
Residual	23	1149.000325	49.95654		
Total	29	4296.966667			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%	Lower 99.0%	Upper 99.0%
Intercept	10.78707639	11.58925724	0.930782	0.361634	-13.18712868	34.76128	-21.747859	43.32201173
X1	0.613187608	0.160983115	3.809018	0.000903	0.280168664	0.946207	0.161254	1.06512125
X2	-0.073050143	0.13572469	-0.538223	0.595594	-0.353818055	0.207718	-0.4540749	0.307974622
X3	0.320332116	0.168520319	1.900852	0.069925	-0.028278721	0.668943	-0.152761	0.793425219
X4	0.081732134	0.221477677	0.369031	0.71548	-0.376429347	0.539894	-0.5400301	0.703494319
X5	0.038381447	0.146995442	0.261106	0.796334	-0.265701791	0.342465	-0.3742841	0.451046997
X6	-0.217056682	0.178209471	-1.217986	0.235577	-0.585711058	0.151598	-0.7173505	0.283237125

Is there any relationship here? Are all the coefficients significant?

What about all of them together?

Why not look at R^2

- R^2 in MLR is still a measure of goodness of fit.
- However it ALWAYS grows as we increase the number of explanatory variables.
- Even if there is no relationship between the $X's$ and Y, $R^2 > 0$!!
- To see this let's look at some "Garbage" Data

Garbage Data

I made up 6 “garbage” variables that have nothing to do with Y…

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.516876852
R Square	0.26716168
Adjusted R Square	0.075986466
Standard Error	11.70095097
Observations	30

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	6	1147.985	191.3308	1.39747	0.257927747
Residual	23	3148.982	136.9123		
Total	29	4296.967			

	Coefficients	*tandard Err*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 99.0%*	*Upper 99.0%*
Intercept	94.8053024	38.6485	2.453014	0.022169	14.85478564	174.7558	-13.6940154	203.3046202
G1	0.241049359	0.369932	0.651605	0.521115	-0.524213203	1.006312	-0.79747383	1.279572553
G2	-0.739495869	0.341006	-2.168569	0.040705	-1.444921431	-0.03407	-1.69681541	0.217823675
G3	-0.564272368	0.463453	-1.217539	0.235744	-1.522998304	0.394454	-1.86534101	0.736796272
G4	0.156297568	0.291278	0.536592	0.596702	-0.446257444	0.758853	-0.66141832	0.974013455
G5	-0.267328742	0.266723	-1.002269	0.326642	-0.819088173	0.284431	-1.01611092	0.481453434
G6	0.441170035	0.329715	1.338034	0.193965	-0.240897504	1.123238	-0.48445078	1.366790852

Garbage Data

- R^2 is 26% !!
- We need to develop a way to see whether a R^2 of 26% can happen by chance when all the true β's are zero.
- It turns out that if we transform R^2 we can solve this.

Define

$$f = \frac{R^2/p}{(1-R^2)/(n-p-1)}$$

A big f corresponds to a big R^2 but there is a distribution that tells what kind of f we are likely to get when all the coefficients are indeed zero... The f statistic provides a scale that allows us to decide if "big" is "big enough".

The F-test

We are testing:

$$H_0 : \beta_1 = \beta_2 = \dots \beta_p = 0$$

$$H_1 : \text{at least one } \beta_j \neq 0.$$

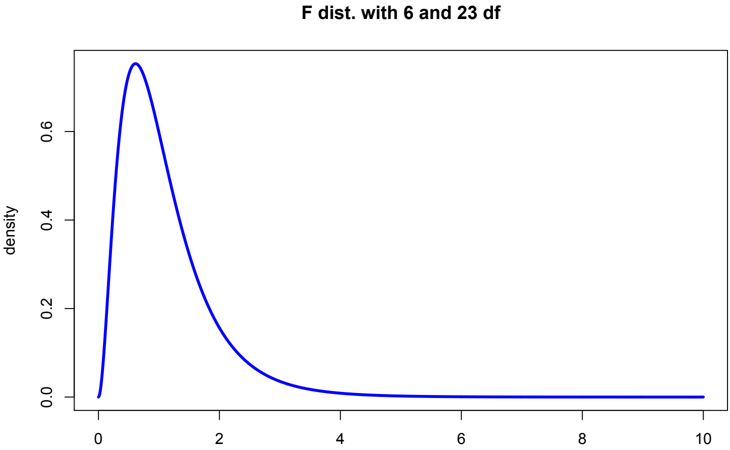
This is the F-test of overall significance. Under the null hypothesis f is distributed:

$$f \sim F_{p,n-p-1}$$

- Generally, $f > 4$ is very significant (reject the null).

The F-test

What kind of distribution is this?



It is a right skewed, positive valued family of distributions indexed by two parameters (the two df values).

The F-test

Let's check this test for the "garbage" data...

ANOVA

	df	SS	MS	F	Significance F
Regression	6	1147.985	191.3308	1.39747	0.257927747
Residual	23	3148.982	136.9123		
Total	29	4296.967			

How about the original analysis (survey variables)...

ANOVA

	df	SS	MS	F	Significance F
Regression	6	3147.966342	524.6611	10.50235	1.24041E-05
Residual	23	1149.000325	49.95654		
Total	29	4296.966667			

F-test

The *p*-value for the F-test is

$$\text{p-value} = Pr(F_{p,n-p-1} > f)$$

- We usually reject the null when the p-value is less than 5%.
- Big f $\rightarrow$ REJECT!
- Small p-value $\rightarrow$ REJECT!

The F-test

In Excel, the p-value is reported under “Significance F”

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	6	1147.985	191.3308	1.39747	0.257927747
Residual	23	3148.982	136.9123		
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ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	6	3147.966342	524.6611	10.50235	1.24041E-05
Residual	23	1149.000325	49.95654		
Total	29	4296.966667			

The F-test

Note that f is also equal to (you can check the math!)

$$f = \frac{SSR/p}{SSE/(n-p-1)}$$

In Excel, the values under MS are SSR/p and $SSE/(n-p-1)$

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	6	1147.985	191.3308	1.39747	0.257927747
Residual	23	3148.982	136.9123		
Total	29	4296.967			

$$f = \frac{191.33}{136.91} = 1.39$$

Understanding Multiple Regression

- There are two, very important things we need to understand about the MLR model:

  1. How dependencies between the X's affect our interpretation of a multiple regression;
  2. How dependencies between the X's inflate standard errors (aka multicolinearity)

- We will look at a few examples to illustrate the ideas...

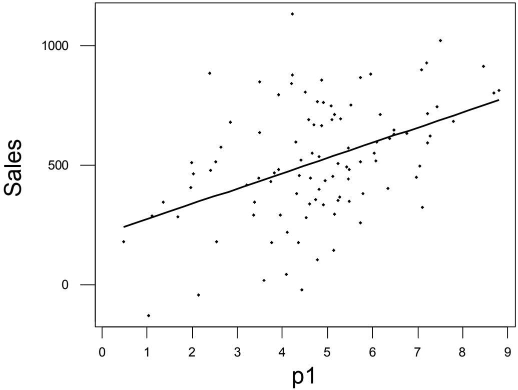
Understanding Multiple Regression

The Sales Data:

- *Sales* : units sold in excess of a baseline
- *P1*: our price in $ (in excess of a baseline price)
- *P2*: competitors price (again, over a baseline)

Understanding Multiple Regression

- If we regress Sales on our own price, we obtain a somewhat surprising conclusion... the higher the price the more we sell!!



- It looks like we should just raise our prices, right? NO, not if you have taken this statistics class!

Understanding Multiple Regression

- The regression equation for Sales on own price (P1) is:

$$Sales = 211 + 63.7P1$$

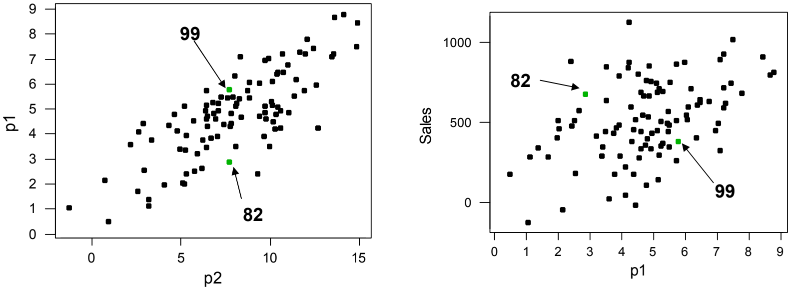
- If now we add the competitors price to the regression we get

$$Sales = 116 - 97.7P1 + 109P2$$

- Does this look better? How did it happen?
- Remember: -97.7 is the affect on sales of a change in $P1$ with $P2$ held fixed!!

Understanding Multiple Regression

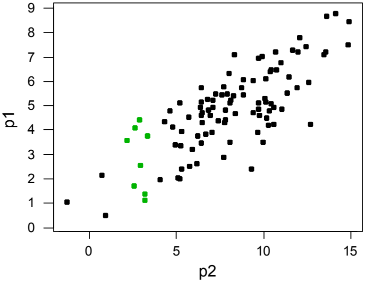
- How can we see what is going on? Let's compare Sales in two different observations: weeks 82 and 99.
- We see that an increase in $P1$, holding $P2$ constant, corresponds to a drop in Sales!

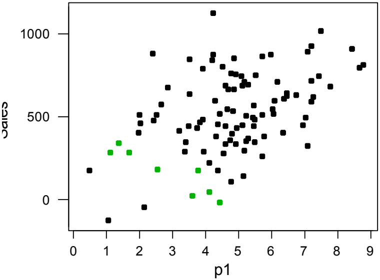


- Note the strong relationship (dependence) between $P1$ and $P2$!!

Understanding Multiple Regression

- Let's look at a subset of points where $P1$ varies and $P2$ is held approximately constant...

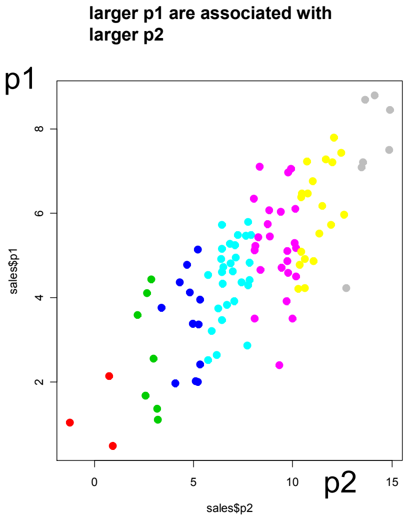


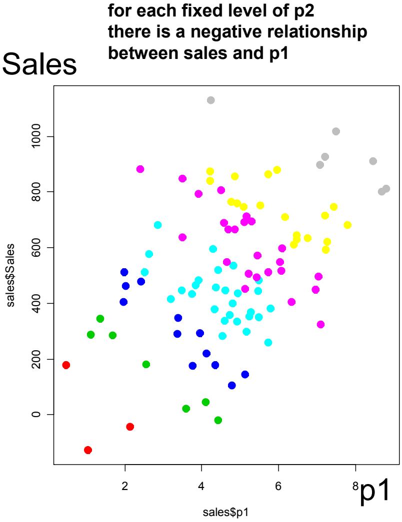


- For a fixed level of $P2$, variation in $P1$ is negatively correlated with Sales!!

Understanding Multiple Regression

- Below, different colors indicate different ranges for $P2$...





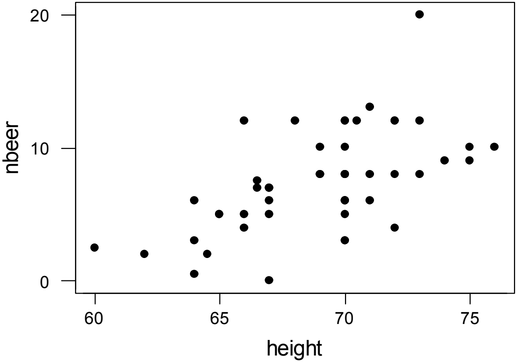
Understanding Multiple Regression

- Summary:
  1. A larger $P1$ is associated with larger $P2$ and the overall effect leads to bigger sales
  2. With $P2$ held fixed, a larger $P1$ leads to lower sales
  3. MLR does the trick and unveils the "correct" economic relationship between Sales and prices!

Understanding Multiple Regression

Beer Data (from an MBA class)

- *nbeer* – number of beers before getting drunk
- *height and weight*



Is number of beers related to height?

Understanding Multiple Regression

$$nbeers = \beta_0 + \beta_1 height + \epsilon$$

Regression Statistics	
Multiple R	0.58
R Square	0.34
Adjusted R Square	0.33
Standard Error	3.11
Observations	50.00

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1.00	237.77	237.77	24.60	0.00
Residual	48.00	463.86	9.66		
Total	49.00	701.63			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	-36.92	8.96	-4.12	0.00	-54.93	-18.91
height	0.64	0.13	4.96	0.00	0.38	0.90

Yes! Beers and height are related...

Understanding Multiple Regression

$$nbeers = \beta_0 + \beta_1 weight + \beta_2 height + \epsilon$$

Regression Statistics	
Multiple R	0.69
R Square	0.48
Adjusted R Square	0.46
Standard Error	2.78
Observations	50.00

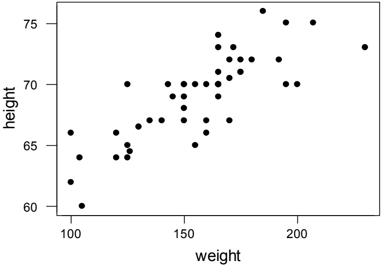
ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2.00	337.24	168.62	21.75	0.00
Residual	47.00	364.38	7.75		
Total	49.00	701.63			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	-11.19	10.77	-1.04	0.30	-32.85	10.48
weight	0.09	0.02	3.58	0.00	0.04	0.13
height	0.08	0.20	0.40	0.69	-0.32	0.47

What about now?? Height is not necessarily a factor...

Understanding Multiple Regression



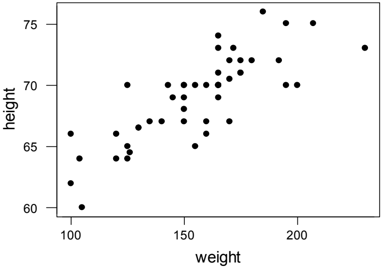
The correlations:

	nbeer	weight
weight	0.692	
height	0.582	0.806

The two x's are highly correlated !!

- If we regress "beers" only on height we see an effect. Bigger heights go with more beers.
- However, when height goes up weight tends to go up as well... in the first regression, height was a proxy for the real *cause* of drinking ability. Bigger people can drink more and weight is a more accurate measure of "bigness".

Understanding Multiple Regression



<u>The correlations:</u>

	nbeer	weight
weight	0.692	
height	0.582	0.806

The two x's are highly correlated !!

- In the multiple regression, when we consider only the variation in height that is not associated with variation in weight, we see no relationship between height and beers.

Understanding Multiple Regression

$$nbeers = \beta_0 + \beta_1 weight + \epsilon$$

Regression Statistics	
Multiple R	0.69
R Square	0.48
Adjusted R	0.47
Standard E	2.76
Observatio	50

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regressior	1	336.0317807	336.0318	44.11878	2.60227E-08
Residual	48	365.5932193	7.616525		
Total	49	701.625			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	-7.021	2.213	-3.172	0.003	-11.471	-2.571
weight	0.093	0.014	6.642	0.000	0.065	0.121

Why is this a better model than the one with weight and height??

Understanding Multiple Regression

In general, when we see a relationship between y and x (or x's), that relationship may be driven by variables "lurking" in the background which are related to your current x's.

This makes it hard to reliably find "causal" relationships. Any correlation (association) you find could be caused by other variables in the background... correlation is NOT causation

Any time a report says two variables are related and there's a suggestion of a "causal" relationship, ask yourself whether or not other variables might be the real reason for the effect. Multiple regression allows us to control for all important variables by including them into the regression. "Once we control for weight, height and beers are NOT related"!!

Understanding Multiple Regression

- With the above examples we saw how the relationship amongst the X's can affect our interpretation of a multiple regression... we will now look at how these dependencies will inflate the standard errors for the regression coefficients, and hence our uncertainty about them.
- Remember that in simple linear regression our uncertainty about b_1 is measured by

$$s_{b_1}^2 = \frac{s^2}{(n-1)s_x^2} = \frac{s^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

- The more variation in X (the larger s_x^2) the more "we know" about β_1... ie, $(b_1 - \beta_1)$ is smaller.

Understanding Multiple Regression

- In Multiple Regression we seek to relate the variation in Y to the variation in an X holding the other X's fixed. So, we need to see how much each X varies on its own.
- in MLR, the standard errors are defined by the following formula:

$$s_{b_j}^2 = \frac{s^2}{\text{variation in } X_j \text{ not associated with other } X\text{'s}}$$

- How do we measure the bottom part of the equation? We regress X_j on all the other X's and compute the residual sum of squares (call it SSE_j) so that

$$s_{b_j}^2 = \frac{s^2}{SSE_j}$$

Understanding Multiple Regression

- What happens if we are regressing Y on X's that are highly correlated. SSE_j goes down and the standard error s_{b_j} goes up!
- What is the effect on the confidence intervals $(b_j \pm 2 \times s_{b_j})$? They get wider!
- This situation is called Multicolinearity
- If a variable X does nothing "on its own" we can't estimate its effect on Y.

Back to Baseball – Let's try to add AVG on top of OBP

Regression Statistics	
Multiple R	0.948136
R Square	0.898961
Adjusted R Square	0.891477
Standard Error	0.160502
Observations	30

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	6.188355	3.094177	120.1119098	3.63577E-14
Residual	27	0.695541	0.025761		
Total	29	6.883896			

	Coefficients	*andard Errc*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	-7.933633	0.844353	-9.396107	5.30996E-10	-9.666102081	-6.201163
AVG	7.810397	4.014609	1.945494	0.062195793	-0.426899658	16.04769
OBP	31.77892	3.802577	8.357205	5.74232E-09	23.9766719	39.58116

$$R/G = \beta_0 + \beta_1 AVG + \beta_2 OBP + \epsilon$$

Is AVG any good?

Back to Baseball - Now let's add SLG

Regression Statistics	
Multiple R	0.955698
R Square	0.913359
Adjusted R Square	0.906941
Standard Error	0.148627
Observations	30

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	6.28747	3.143735	142.31576	4.56302E-15
Residual	27	0.596426	0.02209		
Total	29	6.883896			

	Coefficients	*andard Errc*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	-7.014316	0.81991	-8.554984	3.60968E-09	-8.69663241	-5.332
OBP	27.59287	4.003208	6.892689	2.09112E-07	19.37896463	35.80677
SLG	6.031124	2.021542	2.983428	0.005983713	1.883262806	10.17899

$$R/G = \beta_0 + \beta_1 OBP + \beta_2 SLG + \epsilon$$

What about now? Is SLG any good

Back to Baseball

Correlations

AVG	1		
OBP	0.77	1	
SLG	0.75	0.83	1

- When AVG is added to the model with OBP, no additional information is conveyed. AVG does nothing "on its own" to help predict Runs per Game...
- SLG however, measures something that OBP doesn't (power!) and by doing something "on its own" it is relevant to help predict Runs per Game. (Okay, but not much...)

Things to remember:

- Intervals are your friend! Understanding uncertainty is a key element for sound business decisions.
- Correlation is NOT causation!
- When presented with an analysis from a regression model or any analysis that implies a causal relationship, skepticism is always a good first response! Ask question... "is there an alternative explanation for this result"?
- Simple models are often better than very complex alternatives... remember the trade-off between complexity and generalization (more on this later)