

# Homework Assignment 1

(Due in class on Thursday, 01/23/2014)

STA 371G, Statistics and Modeling, Spring 2014

## Problem 1

A construction company has to complete a project no later than three months from now or there will be significant cost overruns. The manager of the construction company believes that there are four possible values for the random variable  $X$ , the number of months from now it will take to complete the project: 2, 2.5, 3 and 3.5. The manager currently thinks that the probabilities for these four possibilities are in the ratio of 1 to 3 to 4 to 2. That is to say,  $X = 2.5$  is three times more likely than  $X = 2$ .

- (a) Find the probability distribution of  $X$ .
- (b) What is the probability that this project will be completed in no more than three months from now?
- (d) What is the expected completion time of this project from now?
- (e) How much variability exists around the expected completion time? (Hint: calculate the variance/standard deviation)

## Problem 2

A company ships its products from its warehouse in Newark, NJ to Austin, TX by air. If the shipment arrives within 24 hours, it is considered on time. Let  $Y$  be a random variable that  $Y = 1$  if the shipment arrives within 24 hours and  $Y = 0$  if the shipment gets delayed.

Based on the company's historical data, if the weather conditions in both Newark and Austin are normal, then the shipment arrives on time with a 95% probability; if either Newark or Austin has adverse weather conditions, then the shipment arrives on time with a 40% probability; and if both Newark and Austin have adverse weather conditions, then the shipment gets delayed with a 95% probability.

Let  $X$  be a random variable that  $X = 0$  if neither Newark nor Austin has adverse weather conditions,  $X = 1$  if either Newark or Austin has adverse weather conditions, and  $X = 2$  if both Newark and Austin have adverse weather conditions.

- (a) If the company ships its product on a day when both Newark and Austin have adverse weather conditions, then what is the probability of  $Y = 1$ , i.e., the shipment arrives within 24 hours?
  
  
  
  
  
  
  
  
  
  
- (b) Suppose it is forecasted that Newark and Austin have adverse weather conditions with the probabilities of 60% and 10%, respectively, on January 28, 2014. We further assume that the weather conditions of these two cities are independent given the forecast. According to the forecast, what is  $P(X = 2)$ , i.e., the probability that both Newark and Austin have adverse weather conditions on January 28, 2014? How about  $P(X = 0)$ , i.e., the probability that neither Newark nor Austin has adverse weather conditions on January 28, 2014? How about  $P(X = 1)$ ? (Hint: using the joint probability multiplication rule for independent random variables.)
  
  
  
  
  
  
  
  
  
  
- (c) Based on the information provided in (b), if the company ships its products on January 28, 2014, what is  $P(Y = 1)$ , i.e., the probability that the shipment arrives on time? (Hint: first find out all  $P(Y = 1|X = x)$  and all  $P(X = x)$ , then ....)