

# Probabilistic Coordination of Heterogeneous Teams From Capability Temporal Logic Specifications

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**Abstract**—This paper explores coordination of heterogeneous teams of agents from high-level specifications. We employ Capability Temporal Logic (CaTL) to express rich, temporal-spatial tasks that require cooperation between many agents with unique capabilities. CaTL specifies combinations of tasks, each with desired locations, duration, and set of capabilities, freeing the user from considering specific agent trajectories and their impact on multi-agent cooperation. CaTL also provides a quantitative robustness metric of satisfaction based on availability of required capabilities for each task. The novelty of this paper focuses on satisfaction of CaTL formulas under probabilistic conditions. Specifically, we consider uncertainties in robot motion (e.g., agents may fail to transition between regions with some probability) and local probabilistic workspace properties (e.g., if there are not enough agents of a required capability to complete a collaborative task). The proposed approach automatically formulates a mixed-integer linear program given agents, their dynamics and capabilities, an abstraction of the workspace, and a CaTL formula. In addition to satisfying the given CaTL formula, the optimization considers the following secondary goals (in decreasing order of priority): 1) minimize the risk of transition failure due to uncertainties; 2) maximize probabilities of regional collaborative satisfaction (if there is an excess of agents); 3) maximize the availability robustness of CaTL for potential agent attrition; 4) minimize the total agent travel time. We evaluate the performance of the proposed framework and demonstrate its scalability via numerical simulations.

**Index Terms**—Formal Methods in Robotics and Automation, Multi-Robot Systems, Planning, Scheduling and Coordination

## I. INTRODUCTION

COORDINATION and control of heterogeneous multi-agent systems has attracted a great deal of attention recently. Many real-world applications require multiple platforms of various capabilities to collaboratively achieve complex tasks. These heterogeneous systems are more capable than large teams of homogeneous platforms, and can cover a larger footprint than a single specialized platform. For example,

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an autonomous vehicle factory consists of a large number of various types of distributed robots, e.g., manipulators, mobile robots, and drones. Consider the following high-level task "at least one mobile robot needs to pick up and deliver auto parts to each manipulator every 30 minutes during the window  $[t_1, t_2]$ , while drones with high resolution cameras and Lidar sensors need to monitor the installation platform every 10 minutes." The complexity of the mission planning increases as the varieties of teams and the number of robots becomes larger. Complicated collaborative task planning across capabilities, times, positions, duration, and asynchronous execution makes heterogeneous robot team planning challenging. This planning problem is more difficult than planning for a large homogeneous team because agents cannot be arbitrarily interchanged in our case. Furthermore, considering high-level, temporal logic tasks that are interleaved in both space and time makes this lack of arbitrary interchangeability even more difficult to handle.

To address these challenges, we employ Capability Temporal Logic (CaTL) to express high-level heterogeneous specifications [1], [2]. CaTL can accommodate various task inter-dependencies in a compact form (e.g., explicit time, position, number of agents with specific capabilities, and duration of execution). In addition, dropout of team members that have responsibilities and capabilities during a mission could result in global specification failure. However, a key feature of CaTL, availability robustness, grants a level of robustness to such attrition. The novelty of this framework centers on accounting for probabilities of collaboratively accomplishing local missions in a given region and the probabilities of successfully navigating between regions of the workspace.

**Literature Review:** Recently, there has been increased interest in synthesizing optimal trajectories for multi-agent systems subject to several high-level temporal logics. Mature tools exist for defining complex tasks in multi-agent systems with Linear Temporal Logic (LTL) [3]–[7]. Abstracting the interactions between robots and environments in [3]–[7] can be computationally expensive, and to improve scalability, [8], [9] have proposed a sampling-based strategy that incrementally builds trees to approximate product models. Since LTL cannot express concrete time spatial-temporal tasks involving multiple agents, [10]–[12] proposed Graph Temporal Logic (GTL) that can be inferred from data, applied to swarm guidance, and adopted to control synthesis of probabilistic manners. As the number of agents increasing, [11], [13], [14] have developed control algorithms for robot swarms subject to complex high-level tasks. However, the aforementioned works do not account for heterogeneous teams.

Prior work within this community has considered

heterogeneous multi-agent teams to some extent. Specifically, decomposition of graph-based automaton for temporal logic formulae using different robot classes have been investigated in [15]–[17]. [18] proposed a Counting Linear Temporal Logic (cLTL+/cLTL) formulation to capture rich heterogeneous specifications, where an Integer Linear Program (ILP) is developed to find feasible paths. Unfortunately, all of these approaches do not consider time bounded execution. [19] formulates integral predicates of Signal Temporal Logic (STL) [20] for heterogeneous systems to include desired time windows with some success. By introducing CaTL, a fragment of STL, in [1], [2], the high-level specifications can be expressed in a more compact form than STL. Given an abstracted interaction between environments and robots, it is normal that some members of the multi-agent system may fail to execute low-level controllers or exceed the maximum allowed time of executions, resulting in uncertainty in transition times. Recent work, such as [17], develops a decentralized and probabilistic controller to address the problem of probabilistic densities over a swarm. In contrast to previous works, we study a more general formulation of heterogeneous probability of agent transition. Moreover, when the number of robots is larger than the minimum required number of agents for task satisfaction, we consider the probabilities of collaboratively accomplishing local missions at different regions, referred to here as regional satisfaction.

**Contributions:** This framework addresses the practical limitations of the original algorithm in [1] by considering the two types of probabilistic analysis described above. We jointly study a multi-objective optimization process that includes (in order of decreasing importance) satisfaction of the CaTL specification, coordination risks, regional success probabilities, robustness, and travel time. The prime innovation is to formulate linear convex constraints for each objective, and design appropriate parameters to control the priorities of each. Consequently, the proposed approach can be converted into a mixed integer linear program (MILP) to efficiently solve the novel problem. We provide a time complexity analysis of adding additional probabilistic constraints and evaluate the developed algorithm via random environments involving various number of heterogeneous agents and environmental configurations. We note that the idea of considering two types of probabilistic settings can be easily adopted with any algorithm abstracting the model as a transitions system (such as cLTL+/cLTL [18]).

## II. PROBLEM FORMULATION

In this section, we introduce a chance-constrained planning problem for heterogeneous multi-robot systems tasked with missions expressed in CaTL.

### A. Environment and Agent Models

Consider a team of robots  $J$  deployed in a common environment  $Env = (Q, E, W, AP, L)$  abstracted as a finite graph with states  $Q$  denoting locations of interests, and transitions  $E \subseteq Q \times Q$  as the possibility of traveling between states. The travel duration between states is captured by

the function  $W : E \rightarrow \mathbb{N}_{\geq 1}$  that assigns positive integer multiples of a common discretization time-step. Waiting at a state  $q$  is captured by self-loops  $(q, q) \in E$  of weight 1, i.e.,  $W((q, q)) = 1$ , for all  $q \in Q$ . States  $q$  are labeled with atomic propositions  $AP$  given by the map  $L : Q \rightarrow AP$ .

Agents perform tasks at labeled locations using their capabilities, and  $L^{-1}(\pi) = \{q \mid L(q) = \pi\}$  denotes the set of all regions labeled with  $\pi \in AP$ . Each agent  $j \in J$  is characterized by an initial state  $q_{0,j} \in Q$ , and a set of capabilities  $Cap_j$ . We denote the set of all agents' capabilities by  $Cap$ . The set  $g_j = Cap_j$  determines the agent's class. The set of all agent classes is  $G \subseteq 2^{Cap}$ .

**Definition 1.** The trajectory of an agent  $j \in J$  is denoted by  $s_j : \mathbb{N} \rightarrow Q \cup E$ , which tracks the agent's state or transition at each time  $k \in \mathbb{N}$ . We denote  $u_j : \mathbb{N} \rightarrow E \cup \{\emptyset\}$  as the departure of agent  $j$  at time  $k \in \mathbb{N}$  to traverse transition  $e \in E$ . While traveling along  $e$ , the value of  $u_j(k)$  is  $\emptyset$  for the duration  $W(e)$ . The collection of all control input sequences for all agents is  $u = [u_j]_{j \in J}$ .

**Definition 2.** The team trajectory  $s_J : \mathbb{N} \times G \times Q \rightarrow \mathbb{N}$  tracks the number of agents of each class  $g \in G$  at each state  $q \in Q$  over time  $k \in \mathbb{N}$ . Formally, we have  $s_J(q, g, k) = |\{j \in J \mid s_j = q, Cap_j = g\}|$ , where  $|\cdot|$  is the cardinality of a set. Similarly,  $e_J : \mathbb{N} \times G \times E \rightarrow \mathbb{N}$  tracks the number of agents entering an edge  $e$ , and at each time  $e_J(q, g, k) = |\{j \in J \mid u_j(k) = e, Cap_j = g\}|$ .

**Theorem 1.** [1] *Given a team input signal  $u$ , the induced team trajectory  $s_J$  conforms to Definitions 1-2 if and only if the following conditions hold*

$$\begin{cases} s_J(q, g, k) = \sum_{(q', q) \in E} e_J((q', q), g, k - W(q', q)), \\ \sum_{(q, q') \in E} e_J((q, q'), g, k) = \\ \sum_{(q', q) \in E} e_J((q', q), g, k - W(q', q)), \end{cases} \quad (1)$$

$$\forall q \in Q, g \in G, k = \{0, 1, 2, \dots, N\}.$$

In the following sections, we specify the missions for the heterogeneous teams in CaTL. Then, we extend the task completion semantics to account for probabilistic settings.

### B. Mission Specification

The core unit of CaTL is a *task*, which is a tuple  $T = (d, \pi, \{cp_i\}_{i \in \mathcal{I}_T})$  where  $d \in \mathbb{R}$  is a duration of time,  $\pi \in AP$  is the label for states, each  $cp_i \in Cap \times \mathbb{N}$  is a counting proposition corresponding to how many agents with each capability should be in each region labeled  $\pi$ , and  $\mathcal{I}_T$  is the index set of counting propositions associated with task  $T$ . We denote the label required for  $T$  as  $\pi_T$ .

The syntax of a CaTL formula is defined inductively as

$$\phi := T \mid \phi_1 \wedge \phi_2 \mid \phi_1 \vee \phi_2 \mid \phi_1 \mathcal{U}_{[a,b]} \phi \mid \Diamond_{[a,b]} \phi \mid \Box_{[a,b]} \phi,$$

where  $T$  is a task,  $[a, b]$  is a discrete-time interval for  $a, b \in \mathbb{N}$  and  $a < b$ ,  $\wedge$  and  $\vee$  are the Boolean AND and OR, respectively. The temporal operators  $\mathcal{U}_{[a,b]}$ ,  $\Diamond_{[a,b]}$ , and  $\Box_{[a,b]}$  are the time-bounded *Until*, *Eventually*, and *Always* operators, respectively.

The *qualitative semantics* of CaTL are defined over team trajectories  $s_J$ , the same as STL [17], see [1] for more detail. A team trajectory satisfying a CaTL formula  $\phi$  is denoted by  $s_J \models \phi$ . We denote the set of all tasks  $T$  in  $\phi$  by  $T_\phi$ .

The availability robustness [1] for a given team trajectory  $s_J$  and formula  $\phi$  is computed recursively as:

$$\begin{aligned} \rho(s_J, k, T) &= \min_{c \in \text{cpt}} \min_{k' \in [k, k+d]} \min_{q \in L^{-1}(\pi)} \{ \sum_{g: c \in g} s_J(k', g, q) - \text{cp}(c) \} \\ \rho(s_J, k, \phi_1 \wedge \phi_2) &= \min \{ \rho(s_J, k, \phi_1), \rho(s_J, k, \phi_2) \} \\ \rho(s_J, k, \phi_1 \vee \phi_2) &= \max \{ \rho(s_J, k, \phi_1), \rho(s_J, k, \phi_2) \} \\ \rho(s_J, k, \phi_1 \mathcal{U}_{[a,b]} \phi_2) &= \max_{k' \in [k+a, k+b]} \{ \min \{ \rho(s_J, k', \phi_2), \min_{k'' \in [k, k']} \rho(s_J, k'', \phi_1) \} \} \\ \rho(s_J, k, \Diamond_{[a,b]} \phi) &= \max_{k' \in [k+a, k+b]} \{ \rho(s_J, k', \phi) \} \\ \rho(s_J, k, \Box_{[a,b]} \phi) &= \min_{k' \in [k+a, k+b]} \{ \rho(s_J, k', \phi) \}. \end{aligned} \quad (2)$$

The availability robustness measures the minimum number of agents that can be arbitrarily removed from the trajectory without changing satisfaction of the trajectory with respect to  $\phi$ . We refer readers for more details of CaTL to [1]. We also denote  $\rho(s_J, t, \phi)$  as the robustness of the induced team trajectory  $s_J$  under  $u$  for  $\phi$  at time  $t$ .

The satisfaction of CaTL specification  $\phi$  is encoded as a set of MILP constraints. For each task  $T \in T_\phi$  s.t.  $T = \{d, \pi, \{c_i, m_i\}_{i \in I_T}\}$  of  $\phi$ , we define binary variables  $z_T(k) \in \{0, 1\}$  s.t.  $z_T(k) = 1$  if the  $T$  of  $\phi$  is accomplished at time  $k + d$ , and binary variables  $z_{\pi, I_T}(k) \in \{0, 1\}$  for each  $T$  s.t.  $z_{\pi, I_T}(k) = 1$  if at least  $m_i$  agents with capability  $c_i$  in each  $q \in L^{-1}(\pi), \forall i \in I_T$  at time  $k$ . Similarly, we define binary variables  $z_{q, \text{cp}_i}(k) \in \{0, 1\}$  s.t.  $z_{q, \text{cp}_i}(k) = 1$  if at least  $m_i$  agents with capability  $c_i$  is in  $q$  at time  $k$ , and binary variables  $z_{\pi, \text{cp}_i}(k) \in \{0, 1\}$  that  $z_{\pi, \text{cp}_i}(k) = 1$  if at least  $m_i$  agents with capability  $c_i$  in each  $q \in L^{-1}(\pi)$  at time  $k$ .

**Theorem 2.** [1] *The satisfaction of a CaTL  $\phi$  can be guaranteed if and only the following MILP constraints hold*

$$\left\{ \begin{array}{l} z_{\pi, \text{cp}_i}(k) \geq \sum_{q \in L^{-1}(\pi)} z_{q, \text{cp}_i}(k) - |L^{-1}(\pi)| + 1, \\ z_{\pi, \text{cp}_i}(k) \leq z_{q, \text{cp}_i}(k), \forall q \in L^{-1}(\pi) \\ z_{\pi, I_T}(k) \geq \sum_{i \in I_T} z_{\pi, \text{cp}_i}(k) - |I_T| + 1, \\ z_{\pi, I_T}(k) \leq z_{\pi, \text{cp}_i}(k), \forall i \in I_T \\ z_T(k) \geq \sum_{l=k}^{k+d} z_{\pi, I_T}(l) - d + 1, \\ z_T(k) \leq z_{\pi, I_T}(l), \forall \{l \in k, k+1, \dots, k+d\} \\ \sum_{\{g | c_i \in g\}} s_J(q, g, k) - m_i + M(1 - z_{\pi, \text{cp}_i}(k)) \geq r_\phi, \\ \sum_{\{g | c_i \in g\}} s_J(q, g, k) - m_i - M(1 - z_{\pi, \text{cp}_i}(k)) \leq r_\phi, \end{array} \right. \quad (3)$$

$$\forall T \in T_\phi \text{ s.t. } T = \{d, \pi, \text{cp}\}$$

where  $r_\phi = 0$ ,  $N$  is the upper time bound of satisfaction, and  $M$  is a sufficiently large constant.

We employ the total travel time of all agents to eliminate spurious agent motion not contributing to mission satisfaction. The total travel time is given as

$$\tau(u) = \sum_{j \in J} \sum_{k=0}^K \widetilde{W}(u_j(k)), \quad (4)$$

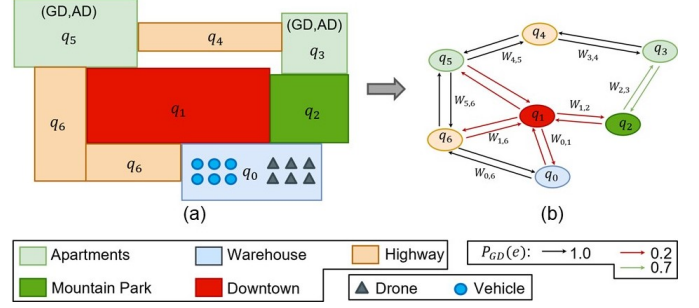


Fig. 1. Environment abstraction for a delivery system. (a) Colors of regions represent different types of labels. Triangles and circles represent two types of agents class i.e. vehicles and drones, which have the set of one capability "ground delivery (GD)" and "air delivery (AD)", respectively. (b) Corresponding abstracted environment. The colored arrows represent different transition uncertainties for the vehicles. And the weight  $W$  of transitions from regions "highway" is larger than others.

where  $\widetilde{W}(u_j(k)) = W(u_j(k))$  if  $u_j(k) \in E \setminus \{(q, q) \mid q \in Q\}$  is the start of a transition between different states, and  $\widetilde{W}(u_j(k)) = 0$  if  $u_j(k)$  is a self-loop that keep the agent stationary or  $u_j(k) = \emptyset$  when the agent is in transition, and  $K$  is the planning horizon computed from the formula [1].

### C. Problem Statement

Only maximizing the availability robustness in [1] omits some practical requirements. Differently, we consider that agents may fail to traverse transitions, and denote the joint probability that all agents in  $J$  successfully traverse their routes induced by  $u$  as  $P_{succ}(u)$ .

In addition to considering agent availability robustness for planning, we also want to impose requirements on distributing agents based on the probability of successfully satisfying tasks using the required capabilities. We denote the probability of completing tasks  $T_\phi$  using agents  $J$  driven by control inputs  $u$  as  $P_{local}(T_\phi, u)$ . The problem can be formulated as

**Problem 1.** Given the team  $J$ , deployed in environment  $Env$ , and CaTL formula  $\phi$ , find a collection of input controls  $u$  for high-level planning to achieve the following multiple objectives with priorities in decreasing order: (i) the team satisfies the mission  $\phi$ ,  $s_J \models \phi$ , (ii) maximizes the probabilities  $P_{succ}(u)$  and  $P_{local}(T_\phi, u)$  of successfully traversing in the environment and performing tasks, (iii) maximizes the availability robustness  $\rho(s_J, 0, \phi)$ , and (iv) minimizes total travel time  $\tau(u)$ .

**Example 1.** As a running example, consider a CaTL formula  $\phi_{\text{case1}} = \Diamond_{[0,9]}(5, \pi_{\text{Apt}}, \{(GD, 2)\}) \wedge \Diamond_{[0,5]}(3, \pi_{\text{Apt}}, \{(AD, 3)\})$  for the environment of a delivery system shown in Fig. 1. In English, the formula states "Within 0 to 9 hours after deployment, each region labeled 'Apartment' requires 2 ground vehicles working there for 5 hours. Within 0 to 5 hours after deployment, there are 3 aerial drones in each region labeled 'Apartment' for 3 hours." We have 6 vehicles and 6 drones.

To save energy, all robots should cross the region "Downtown" and region "Mountain Park" to reach  $q_5$  and  $q_3$ . However, due to the traffic jam and road conditions, there are uncertainties for the ground vehicles crossing these two

regions, detailed in Section III-A. The ideal solution is to avoid the uncertain transitions for ground vehicles, and keep the efficient route for drones. Furthermore, since we can not lose any drones, the maximum availability robustness is 0 for all plans. As a result, we still have two 2 vehicles left that are not properly assigned by optimizing the availability robustness. Since the area  $q_5$  is larger than  $q_3$ , it's reasonable to send extra vehicles to assist the delivery burden in region  $q_5$ . That can be solved via the objective  $P_{local}(T_\phi, u)$ . We note that the two above considerations are not studied in [1].

*Remark 1.* This framework focuses on the high-level planning that can be integrated with low-level navigation controllers to construct a hierarchical structure, and the probabilistic aspects consider the failures of low-level executions and local collaborative satisfaction.

### III. SOLUTION

In this section, we first focus on formulating a mixed integer linear programming (MILP) for transition uncertainties and regional probability of collaborative satisfaction, respectively. Then, we remove the unnecessary constraints to reduce the complexity of the algorithm in section III-C. Finally, we integrate these three aspects to formulate a multi-objective optimization with a specific priority solving Problem 1.

#### A. Transition Uncertainties

In this work, if the agent traversing  $u_j(k)$  fails, we assume agent  $j$  cannot satisfy any tasks after time index  $k$ . As a result, minimizing the probability of failures for all transitions of all agents can reduce attrition, enhancing the availability robustness during executions.

Given an input signal  $u_j(k) = e = (q, q')$ , the probability that an agent  $j$  with capability set  $g_j$  successfully executes  $u_j(k)$  at time index  $k$  is defined as  $P_j(e, g_j, k)$ . For a trajectory  $\mathbf{q}_{N,j} = q_{0,j}q_{1,j} \dots q_{N,j}$  of agent  $j$ , if the agent  $j$  is traversing an edge at time  $k$ , then  $q_{k,j} = \emptyset$ . For a transition  $e = (q, q')$  such that  $q = \emptyset \vee q' = \emptyset$ , then  $P_j(e, g_j, k) = 1$ .

We assume the probability that successfully traversing an edge is independent with respect to time, edge, and agents, s.t.  $P_{succ}(u_j) = \prod_{k=1} P_j(u_j(k), g_j, k)$ ,  $P_j(e, g_j, k) = P_j(e, g_j, k')$  for all  $k, k' \in \mathbb{N}$ , and  $P_{succ}(u) = \prod_{j \in J} P_{succ}(u_j)$ . This assumption is general for a decentralized multi-agent system, where uncertainties only depend on the environment and each robot works independently. Note that we don't assume  $P_j(u_j(k), g_i, k) = P_j(u_j(k'), g_i, k')$  for  $k \neq k'$ .

Given a trajectory  $\mathbf{q}_J = \{\mathbf{q}_{N,1}, \mathbf{q}_{N,2} \dots \mathbf{q}_{N,|J|}\}$  for the group of agents  $J = \{1, 2 \dots |J|\}$  under a team input signal  $u$ . The probability of the group  $J$  successfully traversing the corresponding  $e_J$  can be computed

$$P_{succ}(u) = \prod_{k=0}^{N-1} \prod_{j=1}^{|J|} P_j(e_{j,k}, g_j, k). \quad (5)$$

where  $g_j$  is the capability set of agent  $j$ , and  $e_{j,k}$  is the transition that the agent  $j$  starts to traverse at time  $k$ .

**Assumption 1.** The probabilities of each agent successfully traversing the same edge  $e$  is dependent on capability set i.e.,  $P_j(e, g_j, k) = P_{j'}(e, g_{j'}, k) = p_{e,g}$  for all  $g_j = g_{j'} = g$ .

Assumption 1 indicates the transition uncertainties depend on the type of an agent that is interpreted by its capability set. It's also practical that transition uncertainties depend on different type of heterogeneous agents (e.g. unique agent dynamics) and different edges. We denote the probability of any agents with capability set  $g$  successfully traversing an edge  $e$  at time  $k$  as  $P(e, g, k)$ .

The group of agents is divided into  $|E|$  sub-groups based on all transitions. For instance, given an edge  $e$ ,  $J_e$  denotes a finite index set representing all agents that are traversing edge  $e$ , and  $|J_e|$  represents the number of agents in the sub-group  $J_e \subseteq J$ . Then, (5) can be reformulated as:

$$P_{succ}(u) = \prod_{k=0}^{N-1} P_{succ}(J, k) = \prod_{k=0}^{N-1} \prod_{e \in E} \prod_{j=1}^{|J_e|} P(e, g_j, k). \quad (6)$$

Given a subgroup  $J_e$ , let  $J_{e,g}$  denote a sub-team of  $J_e$  with capability  $g$ . We have  $J_e = \prod_{g \in G} J_{e,g}$ , and (6) can be

$$P_{succ}(u) = \prod_{k=0}^{N-1} \prod_{e \in E} \prod_{g \in G} \prod_{j=1}^{|J_{e,g}|} P(e, g, k). \quad (7)$$

To simplify the analysis, we consider that the probability of each class of robots traversing the same edge is constant i.e.,  $P(e, g, k) = p_{e,g}, \forall k \in \mathbb{N}$ . Such a setting allows us to use the variable  $e_J(e, g, k)$  that represents the number of agents in  $J_{e,g}$  at time  $k$ , and we obtain:

$$P_{succ}(u) = \prod_{k=0}^{N-1} \prod_{e \in E} \prod_{g \in G} p_{e,g}^{e_J(e, g, k)} \quad (8)$$

**Theorem 3.** Given the probabilistic requirement  $P_{succ}(u) \geq \epsilon(u)$  of the routing plan, the probabilistic constraint in the MILP can be of the linear convex form:

$$- \sum_{k=0}^{N-1} \sum_{e \in E} \sum_{g \in G} (e_J(e, g, k) \times \log(p_{e,g})) \leq \log\left(\frac{1}{\epsilon(u)}\right). \quad (9)$$

*Proof.*  $P_{succ}(u) \geq \epsilon \Rightarrow \log(P_{succ}(u)) \geq \log(\epsilon)$ . From (8), we have:

$$\log(P_{succ}(u)) = \sum_{k=0}^{N-1} \sum_{e \in E} \sum_{g \in G} (e_J(e, g, k) \times \log(p_{e,g})). \quad (10)$$

We take  $\log$  for both sides of  $P_{succ}(u) \geq \epsilon(u)$  to obtain the form (9).  $\square$

It's hard to provide a proper value  $\epsilon(u)$  for large heterogeneous teams and a complex CaTL in a large-scale transition system. We treat  $\epsilon(u)$  as one of the multiple objectives in section III-D.

*Remark 2.* Different from [17], we do not model probabilistic transitions as a Markov Decision Process (MDP), because CaTL specifications do not rely on automaton structures. From (2), policies of an MDP that satisfy CaTL are history-dependent.

Thus, the optimization problem becomes computationally expensive.

### B. Probabilistic Regional Satisfaction

Each task  $T \in T_\phi$  s.t.  $T = \{d, \pi, \{c_i, m_i\}_{i \in I_T}\}$  of CaTL formula  $\phi$  requires every region labeled  $\pi$  to be satisfied using at least  $m_i$  agents with capability  $c_i$  for  $d$  time units. For example, suppose there are 3 regions labeled  $\pi$  and  $m_i = 2$ , the minimum number of agents with capability  $c_i$  for task  $T$  is 6. What if we have 20 agents with capability  $c_i$ ? Obviously, availability robustness is not enough to determine the best allocation of the extra 14 agents, since it is only defined by the minimum number of agents that can be removed.

Consequently, it's worth considering how to coordinate a large number of heterogeneous agents, and properly utilizing additional resources to increase the probability of local satisfaction is important for a large scale heterogeneous multi-agent system. In the following description, we examine how to deal with superfluous agents and optimize local probabilities of CaTL satisfaction. The number of agents with a specific capability  $c_i$  in the region  $q$  at time  $k$  can be obtained:

$$n_{q,c_i}(k) = \sum_{\{g|c_i \in g\}} s_J(q, g, k).$$

Suppose one task in CaTL formula  $\phi$  is to send  $m_i$  number of agents with capability  $c_i$  to regions labeled as  $\pi$  at time  $k$ . For a region  $q \in L^{-1}(\pi)$  and capability  $c_i$ ,  $\underline{n}_{q,c_i} = m_i$  is the minimum requirement of number of agents to satisfy the requirement of  $\phi$ ,  $\bar{n}_{q,c_i}$  represents the number of agents that satisfy the regional ( $q$ ) collaborative sub-task with probability 1, and  $p_{\underline{n}_{q,c_i}}$  is the regional (local) probability of satisfaction for  $\underline{n}_{q,c_i}$  agents.

Based on that, the intuition is to consider a propositional relationship between probabilities of regional satisfaction and number of the heterogeneous agents  $n_{q,c_i}(k)$ . Thus, the regional (local) probability  $\Pr_{q,\pi,c_i}^\phi(n_{q,c_i}(k))$  of satisfaction at time  $k$  is equal to

$$\begin{cases} 0, & \text{if } n_{q,c_i}(k) < \underline{n}_{q,c_i}, \\ 1, & \text{if } n_{q,c_i}(k) > \bar{n}_{q,c_i}, \\ k_{q,\pi,c_i} \cdot (n_{q,c_i}(k) - \underline{n}_{q,c_i}) + p_{\underline{n}_{q,c_i}} & \text{otherwise.} \end{cases} \quad (11)$$

where  $k_{q,\pi,c_i}$  is a constant probabilistic satisfaction rate. From the third statement in (11), we can observe that  $\Pr_{q,\pi,c_i}^\phi(n_{q,c_i}(k))$  is linear with respect to the variable  $n_{q,c_i}(k)$  if  $\bar{n} \geq n_{q,c_i}(k) \geq \underline{n}$ .

**Remark 3.** All parameters  $k_{q,\pi,c_i}$ ,  $\underline{n}_{q,c_i}$ ,  $\bar{n}_{q,c_i}$  of the probabilistic environment are heterogeneous, since they depend on a specific capability  $c_i$  and the region  $q$ .

Given a coordination path  $\mathbf{n}_{q,c_i} = n_{q,c_i}(0), n_{q,c_i}(1), \dots, n_{q,c_i}(N)$  at region  $q$  for capability  $c_i$ , the probability of regional satisfaction can be expressed as

$$\Pr_\phi(q, \pi, c_i) = \sum_{k=0}^N \Pr_{q,\pi,c_i}^\phi(n_{q,c_i}(k)). \quad (12)$$

Based on (12), we can analyze two types of objectives of satisfying label  $\pi$ : 1) maximize the minimum probability of satisfaction and 2) maximize the accumulative probability of satisfaction.

$$\begin{cases} \Pr_{\phi,1}(\pi, c_i) = \min_{q \in L^{-1}(\pi)} \Pr_\pi(q, \pi, c_i), \\ \Pr_{\phi,2}(\pi, c_i) = \sum_{q \in L^{-1}(\pi)} \Pr_\pi(q, \pi, c_i). \end{cases} \quad (13)$$

In summary, the two types of probabilistic environment satisfaction for each capability  $c_i$  at the region  $q$  can be regarded as  $\Pr_{\phi,l}(\pi, c_i)$  for  $l \in \{1, 2\}$ .

**Theorem 4.** Given a regional probabilistic requirement  $\Pr_{\phi,l}(\pi, c_i) \geq \gamma_q$  for  $l \in \{1, 2\}$  of the routing plan, the regional probabilistic constraint in MILP can be of the linear form:

$$\begin{cases} \Pr_{\phi,l}(\pi, c_i) \geq \gamma_q, \\ n \geq \bar{n}_{q,c_i} + 10^{-M} - M(1 - \delta), \\ n \leq \bar{n}_{q,c_i} + M(1 - \delta), \\ \Pr_{q,\pi,c_i}^\phi(n) \leq 1 + M(1 - \delta), \\ \Pr_{q,\pi,c_i}^\phi(n) \geq 1 - M(1 - \delta), \\ \Pr_{q,\pi,c_i}^\phi(n) \leq k_{q,\pi,c_i}(n - \underline{n}_{q,c_i}) + p_{\underline{n}_{q,c_i}} + M\delta, \\ \Pr_{q,\pi,c_i}^\phi(n) \geq k_{q,\pi,c_i}(n - \underline{n}_{q,c_i}) + p_{\underline{n}_{q,c_i}} - M\delta, \\ \Pr_{q,\pi,c_i}^\phi(n) \geq 0, \end{cases} \quad (14)$$

where  $n = n_{q,c_i}(k)$ , and  $\delta \in \{0, 1\}$  is a binary variable.

*Proof.* The conditional statement in (11) can be encoded as a mixed integer program (MIP).  $\delta \in \{0, 1\}$  is used as an indicator. First, we can formulate the equivalent condition  $n_{q,c_i}(k) > \bar{n}_{q,c_i} \iff \delta = 1$  as:

$$\begin{aligned} n_{q,c_i}(k) &\geq \bar{n}_{q,c_i} + 10^{-M} - M(1 - \delta), \\ n_{q,c_i}(k) &\leq \bar{n}_{q,c_i} + M(1 - \delta). \end{aligned} \quad (15)$$

Next we can obtain the following two statements:

$$\begin{aligned} \delta = 1 &\implies \Pr(q, \phi, c_i, k) = 0, \\ \delta = 0 &\implies \Pr(q, \phi, c_i, k) = k_{q,\pi,c_i} \cdot (n_{q,c_i}(k) - \underline{n}_{q,c_i}) + p_{\underline{n}_{q,c_i}}. \end{aligned}$$

For (11), the conditional constraint  $\Pr_{q,\pi,c_i}^\phi(n_{q,c_i}(k)) = 1$  for  $n_{q,c_i}(k) > \bar{n}_{q,c_i}$  can be encoded using MIP as

$$\begin{aligned} \Pr_{q,\pi,c_i}^\phi(n_{q,c_i}(k)) &\leq 1 + M(1 - \delta), \\ \Pr_{q,\pi,c_i}^\phi(n_{q,c_i}(k)) &\geq 1 - M(1 - \delta). \end{aligned} \quad (16)$$

For (11), the conditional constraint  $\Pr_{q,\pi,c_i}^\phi(n_{q,c_i}(k)) = k_{q,\pi,c_i} \cdot (n_{q,c_i}(k) - \underline{n}_{q,c_i}) + p_{\underline{n}_{q,c_i}}$  for  $\bar{n} \geq n_{q,c_i}(k) \geq \underline{n}$  can be encoded as:

$$\begin{aligned} \Pr_{q,\pi,c_i}^\phi(n_{q,c_i}(k)) &\leq k_{q,\pi,c_i}(n_{q,c_i}(k) - \underline{n}_{q,c_i}) + p_{\underline{n}_{q,c_i}} + M\delta, \\ \Pr_{q,\pi,c_i}^\phi(n_{q,c_i}(k)) &\geq k_{q,\pi,c_i}(n_{q,c_i}(k) - \underline{n}_{q,c_i}) + p_{\underline{n}_{q,c_i}} - M\delta. \end{aligned} \quad (17)$$

As a result, (15), (16), (17) and  $\Pr_{q,\pi,c_i}^\phi(n_{q,c_i}(k)) \geq 0$  for  $n_{q,c_i}(k) < \underline{n}_{q,c_i}$  can be encoded as the linear constraints to achieve the conditional constraints expressed in (11) into MILP optimization. Based on that, we can apply the variable  $\Pr_{q,\pi,c_i}^\phi(n_{q,c_i}(k))$  into (12) to formulate the objectives in (13), respectively.  $\square$

Theorem 4 allows us to optimize the coordination of extra heterogeneous agents by maximizing the regional probability

of collaborative satisfaction.

**Lemma 1.** *Given a team trajectory  $\bar{q}_J$  under an input  $(u)$ , the optimization objective of probabilistic regional satisfaction can be formulated as a MILP constraint:*

$$P_{local}(T_\phi, u) = \sum_{T \in T_\phi} \sum_{i \in I_T} Pr_{\phi, l}(\pi_T, c_i) \geq \gamma_Q(u), \quad (18)$$

where  $l \in \{1, 2\}$  is the index of two types of objectives in (13), and each  $Pr_{\phi, l}(\pi_T, c_i)$  is formulated based on (14), and  $\gamma_Q(u)$  is desired overall threshold.

*Proof.* The lemma can be directly proved via Theorem 4.  $\square$

In practice, it's also challenging to specify a proper value  $\gamma_Q$  for large heterogeneous teams and a complex CaTL formula. We show how to treat it as one objective and generate an optimized  $\gamma_Q(u)$  in section III-D.

### C. Sparsification of Probabilistic Constraints

As the structure of the transition systems and heterogeneous teams becomes more complex and large, the iterative variables in constraints (9) and (18) will be large. To reduce the complexity, we provide several ways to sparsify the variables for the types of probabilistic constraints.

For (9), we can remove the set of transitions  $E_{safe}$  s.t. any heterogeneous agent can traverse it with probability one. In addition, we can also remove a set of some types of agents  $G_{safe}$  s.t. they can traverse any transition with probability one. For instance, the uncertainties due to rough roads or traffic jam can be ignored by drones. We can remove these cases, and denote the set of uncertain transitions  $E' = E / (E_{safe} \cup (q, q))$ ,  $\forall q \in Q$  and set of capabilities with uncertainties  $G' = G / G_{safe}$ , which are applied to replace  $E$  and  $G$ .

For (18), first, suppose the task of  $\pi$  requires at least  $m_i$  agents with capability  $c_i$ . For each set  $cp_T$  with  $\pi_T$ , since, (18) is to optimally assign the extra agents, we can only consider the group of agents with capability  $c_i$  that has more than  $m_i$  agents, and  $cp_T$  can be refined as  $cp'_T$ . Second, because the variables of (18) are defined from (12), for each set  $L^{-1}(\pi)$ , we can remove the cases where  $\bar{n}_{q, c_i} = \bar{n}_{q, c_i}$  in (11), which means the additional agents with respect to  $m_i$  can't contribute to the regional satisfaction.

In the following sections, we default to applying the compact constraints in (9) and (18) analyzed above.

### D. Multi-objective Function

Given inputs  $u$  for the heterogeneous multi-agent system including a transition system and CaTL specification, the involved objectives from the previous sections can be summarized as: availability robustness  $\rho(u, 0, \phi)$ , total travel time  $\tau(u)$ , transition probability  $\epsilon(u)$ , and probabilistic regional objective  $\gamma_Q(u)$ . The regulation objective function under the same input can be formulated as:

$$J(u) = \beta_1 \cdot \epsilon(u) + \beta_2 \left( \frac{1}{k_{\min}} \cdot \gamma_Q(u) + \frac{1}{|J|+1} (\rho(u, 0, \phi) - \beta_3 \cdot \tau(u)) \right), \quad (19)$$

where  $\beta_1 \gg \beta_2$ ,  $\beta_3 = \frac{\alpha}{|J|N}$  with  $\alpha \in (0, 1)$ , and  $k_{\min} = \min_{q \in L^{-1}(\pi), c_i \in C_\pi, \pi \in \pi_\phi} k_{q, \pi, c_i}$ . We denote  $J(u)$  as the output of objective function under specific input  $u$ .

**Theorem 5.** *By formulating the objective function in (19), one can optimize multiple objectives in the decreasing priority: (i) Maximize the probability of successfully traversing all constraints; (ii) maximize the regional probabilities of collaborative satisfaction for all labeled regions; (iii) maximize the availability robustness; (iv) minimize the total travel time.*

*Proof.* We prove the decreasing order from the lowest priority. Step 1: Let  $J_1(u) = \rho(u, 0, \phi) - \beta_3 \cdot \tau(u)$ . This step shows robustness  $\rho(u, 0, \phi)$  has a higher priority than  $\tau(u)$  e.g.,  $J_1(u_1, z_1) \geq J_1(u_2, z_2)$  if  $r_{a, 0, \phi}(u_1, z_1) > r_{a, 0, \phi}(u_2, z_2)$ . Since  $\rho(u, 0, \phi)$  is an integer value,  $r_{a, 0, \phi}(u_1, z_1) > r_{a, 0, \phi}(u_2, z_2) \implies r_{a, 0, \phi}(u_1, z_1) - r_{a, 0, \phi}(u_2, z_2) \geq 1$ . As for travel time,  $\tau(u)$  can be maximized and minimized respectively as  $\tau_{\max} = |J|N$  and  $\tau_{\min} = 0$ . Therefore,

$$\beta_3 \cdot (\tau(u_1, z_1) - \tau(u_2, z_2)) \leq \beta_3(\tau_{\max} - \tau_{\min}) = \alpha.$$

Thus, one has  $J_1(u_1, z_1) - J_1(u_2, z_2) \geq 1 - \alpha > 0$ .

Step 2: Similarly, let  $J_2(u) = \frac{1}{k_{\min}} \cdot \gamma_Q(u) + \frac{1}{|J|+1} J_1(u)$ . We show that  $\gamma_Q(u)$  has a higher priority than  $J_1(u)$  e.g.,  $J_2(u_1, z_1) \geq J_2(u_2, z_2)$  if  $\gamma_Q(u_1, z_1) > \gamma_Q(u_2, z_2)$ . One has

$$0 \leq \frac{1}{|J|+1} (J_1(u_1, z_1) - J_1(u_2, z_2)) \leq 1.$$

From (11),  $k_{\min}$  is the minimum increasing rate of the probabilities of regional satisfaction, if  $\gamma_Q(u_1, z_1) > \gamma_Q(u_2, z_2)$ , the following condition holds

$$\frac{1}{k_{\min}} (\gamma_Q(u_1, z_1) - \gamma_Q(u_2, z_2)) \geq 1.$$

Therefore, one has  $J_2(u_1, z_1) - J_2(u_2, z_2) > 0$ .

Step 3: By selecting  $\beta_1 \gg \beta_2$ , we can ensure  $\epsilon(u)$  has higher higher priority than  $J_2(u)$  e.g.,  $J(u_1, z_1) \geq J(u_2, z_2)$  if  $\epsilon(u_1, z_1) > \epsilon(u_2, z_2)$ .  $\square$

**Lemma 2.** *Given a CaTL formula  $\phi$ , Problem 1 is equivalent to solving the MILP*

$$\begin{aligned} & \max_{\{u_{e, q, k}\}} J(u) \\ & \text{subject to} \\ & (1), (3), (9), (18). \end{aligned} \quad (20)$$

*Proof.* We can directly verify the lemma from Theorem 3, Theorem 4, Lemma 1, and Theorem 5.  $\square$

## IV. COMPUTATIONAL EXPERIMENTS

In the section, we assume the high-level plan can be automatically executed by the low-level controllers, and focus on demonstrating how our algorithm generates the pre-plans to avoid the transitions that have higher probabilities of failures during executions. We show the performance and computational analysis of the route planning algorithm. The MILP optimization problems are solved via Gurobi [21]. We compare the results of our framework denoted as "Probabilistic" and the method in [1] denoted as "Robust" that only consider objectives of availability robustness.



### A. Demonstration

In this subsection, we analyze results of two concrete case studies to demonstrate the proposed framework.

**Case 1:** Continuing with example 1, we set the time weights of transitions connecting to the regions labeled as "Highway" as 2 i.e.,  $W(q_5, q_6) = 2$ ,  $W(q_3, q_4) = 2$  etc, and the weights of other transitions are equal to 1. We only consider one regional probability at  $q_5$  s.t.  $k_{q_5, \phi_{\text{case1}}, \text{GD}} = 0.25$ ,  $\underline{n}_{q_5, \text{GD}} = 2$ ,  $\bar{n}_{q_5, \text{GD}} = 4$  and  $p_{\underline{n}_{q_5, \text{GD}}} = 0.5$ . The results are shown in Fig. 2 (a) and (b), where the dashed lines represent the trajectories of two types of robots and the final decomposition of task satisfaction are shown based on the number of heterogeneous agents at region  $q_5$  and  $q_3$ . Fig. 2 (a) shows that our algorithm drives the ground vehicles to avoid the transitions with uncertainties, and coordinate more robots to the larger area  $q_5$  for regional satisfaction. In contrast, the method "Robust" generates plans with optimal path only respect to availability robustness and travel time.

Then, we add 2 additional drones with capability "AD" at initial state  $q_0$ , and we denote this case as "Extended Example 1". By applying these two methods shown in Fig. 2 (a) and (b), respectively, the additional drones follow the same trajectories, and availability robustness of optimal plans are both equal to 1 such that  $q_3$  and  $q_5$  have 4 drones. This means the CaTL task can still be satisfied if any one of the heterogeneous robots is removed during low-level execution.

**Case 2:** As shown in Fig. 3, we consider each type of agent as having several capabilities, and the set of labels is  $AP = \{\pi_{\text{blue}}, \pi_{\text{orange}}, \pi_{\text{yellow}}, \pi_{\text{green}}, \pi_{\text{cyan}}\}$  based on the colored map. The CaTL formula is  $\phi_{\text{case}} = \Diamond_{[0,20]} T_1 \wedge \Box_{[20,40]} \Diamond_{[0,10]} T_2 \wedge \Diamond_{[5,25]} T_3 \wedge \Diamond_{[3,18]} T_4 \wedge \Diamond_{[20,30]} T_5$  where

$$\begin{cases} T_1 = (1, \pi_{\text{green}} \{(\text{IR}, 2), (\text{Vis}, 2)\}), \\ T_2 = (1, \pi_{\text{blue}} \{(\text{UV}, 1), (\text{Mo}, 2)\}), \\ T_3 = (2, \pi_{\text{yellow}} \{(\text{UV}, 2), (\text{Vis}, 2)\}), \\ T_4 = T_5 = (2, \pi_{\text{orange}} \{(\text{Vis}, 2)\}). \end{cases}$$

There are transition uncertainties of 0.8 for the edges connecting to regions labeled as  $\pi_{\text{red}}$ . In addition, we select the region  $q_{10}$  labeled with  $\pi_{\text{orange}}$  for capability Vis to add the regional satisfaction s.t.  $k_{q_i, \phi_{\text{case}}, \pi_{\text{orange}}} = 0.2$ ,  $\underline{n}_{q_{10}, \pi_{\text{orange}}} = 2$ ,  $\bar{n}_{q_{10}, \text{Vis}} = 4$ , and  $p_{\underline{n}_{q_{10}, \text{Vis}}} = 0.6$ . The optimal trajectories for different colored agents are shown in Fig. 3 (b).

**Summary:** The above comparisons show that the previous method [1] has no benefits for cases where maximum availability robustness is 0, and it can not coordinate the extra agents. The results of 50 runs for each case are shown in Table I, which illustrates that our algorithm is capable of satisfying more practical requirements for cases where the availability robustness can be optimized in the same way as the method in [1].

### B. Random Environments

To test scalability, we maintain a fixed team size at 30 and vary the number of states in the experiment. The environments are all grid-worlds with transitions between adjacent states. Edge weights are chosen uniformly from  $\{1, 3\}$ . We consider all agents from four classes, each of which is associated with

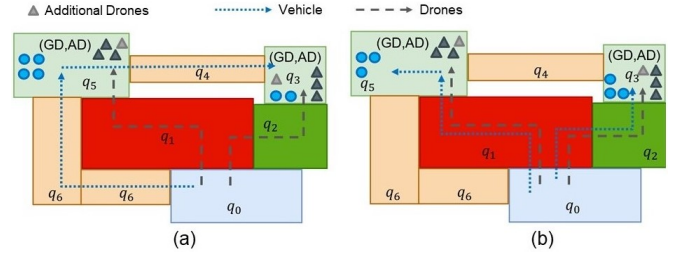


Fig. 2. Optimal motion plans and final coordination for example 1 from two methods. (a) "Probabilistic". (b) "Robust".

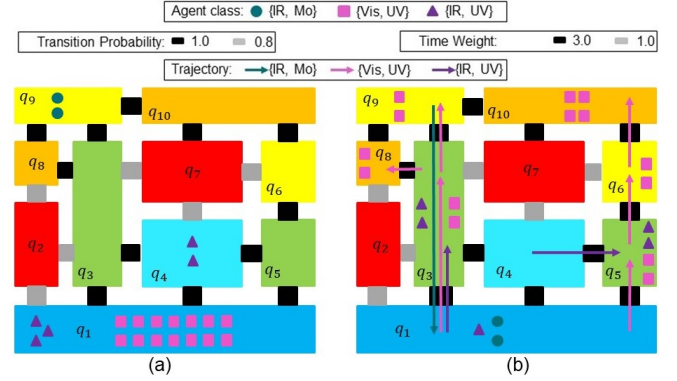


Fig. 3. Demonstration of agents with a set of capability. The probabilities and weights of each edge, and colors of agent classes are shown in the top. (a) Initial location and number of the heterogeneous team. (b) Optimal route plan and final coordination using method "Probabilistic".

two capabilities from  $\{\text{Vis}, \text{UV}, \text{IR}, \text{Mo}\}$ , and we also ensure all individual capabilities are covered in the classes. The initial states of all agents are selected uniformly from all states. The CaTL formula is still  $\phi_{\text{case}}$ .

The probability of a region being labeled was 0.3, and the label of each labeled region in the graph is drawn uniformly from  $AP = \{\pi_{\text{blue}}, \pi_{\text{orange}}, \pi_{\text{yellow}}, \pi_{\text{green}}\}$ . We also make sure that all the labels are selected. In addition, we select regions labeled with  $\pi_{\text{orange}}$  for capability Vis to add the regional satisfaction s.t.  $k_{q_i, \phi_{\text{case2}}, \pi_{\text{orange}}} = 0.3$ ,  $\underline{n}_{q_i, \pi_{\text{orange}}} = 2$  and  $\bar{n}_{q_i, \text{Vis}} = 4$ . For each edge, there's a probability 0.3 to be assigned an uncertainty generated uniformly from  $\{0.80, 1.0\}$ .

We compared our framework with the method [1] as shown in Fig. 4. Fig. 4 (a) illustrates the computational complexity of adding the two types of probabilistic constraints, and also shows the comparison of the time of finding the first feasible solution with these methods. Fig. 4 (b) (c) (d) compare the transition probability, regional satisfaction and availability robustness using these two methods. We can conclude that this work provides a more flexible and comprehensive MILP approach for heterogeneous teams with CaTL.

### V. CONCLUSION

In this article, we have presented a framework for coordination of heterogeneous multi-agents subject to CaTL formulas that is compact and efficient for specifying desired time-bounded behaviors with probabilistic characteristics. We also develop linear convex constraints of probabilistic

TABLE I  
RESULTS SUMMARY

Case Study	Method	Computation Time (mean / max) (s)	Variables	Transition $\epsilon$	Regional $\gamma$	Robust $\rho$
Example 1	Probabilistic	0.04/0.04	1339	1.0	14.0	0
	Robust	0.03/0.03	1276	$1.18e-9$	8.75	0
Extended Example 1	Probabilistic	0.08/0.08	1339	1.0	14.0	1
	Robust	0.05/0.05	1276	$9.73e-23$	8.75	1
Case Study	Probabilistic	34.93/50.80	12556	1.0	31.0	0
	Robust	183.25/212.41	12281	$9.23e-18$	18.6	0

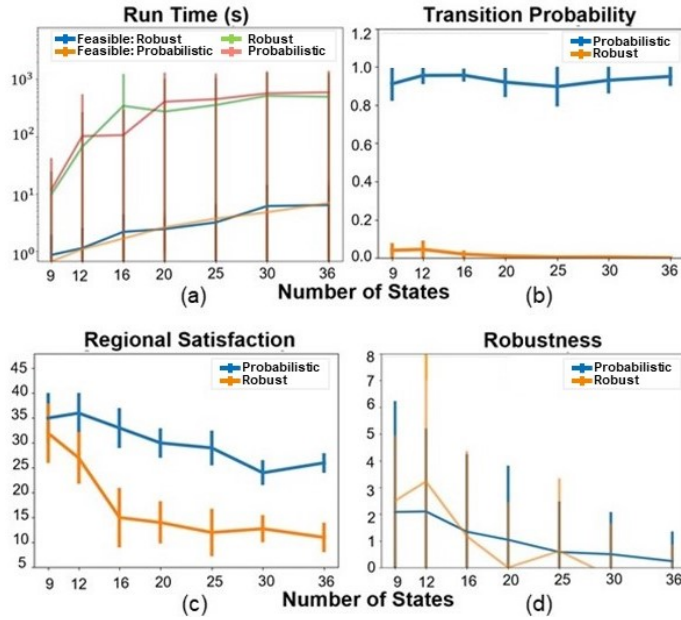


Fig. 4. Results for random environments with varying the number of states with methods "Probabilistic" and "Robust" respectively. (a) Overall computational time to find the optimal and first feasible solutions. (b) Transition probability generated by optimal solutions. (c) Regional satisfaction from optimal solutions. (d) Availability robustness for optimal solutions.

conditions, allowing us to synthesize MILP-based optimal trajectories. The algorithm not only minimizes the risk of motion uncertainties, but also is robust to agent attrition. The two probabilistic constraints considered can be easily extended with cLTL+LTL that define high-level specifications over an infinite horizon. We demonstrate our approach by comparing it to previous work [1] with a series of randomized computational experiments.

To further reduce the complexity, we will explore distributed optimization methods. Another future direction is to consider failure recovery. As the number of agents increases, other future work might consider probabilistic analysis of heterogeneous tasks over swarm-level tasks.

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