A User-Friendly Interface for Relating Timbre and Scale Using Dissonance Curve Analysis

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In the last century, computational techniques for relating the spectrum of a sound—its timbre—and a consonant scale specific to this timbre were developed based on experimental evidence in psychoacoustic research. However, musicians without technical backgrounds are often barred from incorporating these techniques into their music. This paper details the development of a user-friendly interface that relates timbre and scale by executing dissonance curve analysis based on the principle of local consonance. This interface consists of two parts: a Python module for audio and dissonance curve analyses, and an interactive graphical user interface developed for Max/MSP, a visual programming platform commonly used by musicians and multimedia artists. With these tools, this paper then examines a pure sine wave, harmonic timbres, and inharmonic timbres in Javanese gamelan instruments as case studies. This study facilitates musical instrument design and opens new areas for musical exploration by widening the scope of musicians able to apply the principle of local consonance to their music.

Keywords: Timbre-Specific Tuning Systems, Dissonance Curves, Principle of Local Consonance, Javanese Gamelan

I. INTRODUCTION

While common musical instruments are usually performed in the twelve-tone equal-tempered scale (or the chromatic scale), computer synthesizers and sampling tools have opened the doors to new musical sounds with non-harmonic timbres, which are often incompatible with traditional tuning systems. As a result, the last century and decade have seen researchers [1] attempting to answer these two central questions: How does one find an optimal scale for a given timbre? Provided with an arbitrary scale, how does one choose the appropriate timbre?

The selection of tuning systems is often largely based on the level of consonance—the degree to which a sound is pleasant to the listener—of different intervals. In the 16th and 17th centuries, the explanation that the consonance results from the simple frequency ratios between intervals was widely promoted among acoustics researchers. Later, von Helmholtz [2] introduced the alternative explanation that consonance and dissonance result from the absence and presence of beatings when two simple tones are close in frequency. The theory is seconded by Plomp and Levelt [3], whose experiments and statistical analysis of chords in musical compositions led to the conclusion that the difference between consonance and dissonance is related to critical bandwidth. According to [3], the most dissonant intervals correspond with a frequency difference of approximately one quarter of the critical bandwidth, whereas the intervals are perceived as more consonant for frequency differences exceeding the bandwidth.

In the last century, Sethares [1] developed the principle of local consonance with a parametrization of Plomp and Levelt's consonance curves. He related timbre and scale by generating dissonance curves for complex timbres with many component frequencies, expanding on the results of the sine tone studies in [3].

Despite the existence of computational techniques for relating timbre and scale, musicians without technical backgrounds are often barred from incorporating untraditional timbres into their music. Though a description can be found, the "Tuning and Timbre Applet" developed by Forrest [5] is not available online. In addition, its function is limited as it cannot be easily incorporated into future studies given its software-based nature; the user also needs to specify the details of the sound she or he intends to play instead of being able to upload a recorded sample of a timbre.

In this paper, we propose the development of a Python module and a separate interactive graphical user interface for relating timbre and scale based on Sethares' principle of local consonance. Our results facilitate the creation of novel digital musical instruments and opens new areas for musical exploration, allowing for a quantified approach to attaining consonant tunings for a particular timbre.

II. METHODS

In this section, we introduce the methodology used in the development of our interface in three stages: sound analysis, dissonance curve analysis, and creation of the graphical user interface. In Section II.D, the dataset and methodology for our case studies are introduced.

A. Sound Analysis

We analyze the audio input with Python. SciPy is used to read the audio data in WAV format and find prominent frequencies of the sound; Matplotlib generates plots; and NumPy stores data in arrays and performs calculations. After the audio data is read, an algorithm based on fast Fourier transform (FFT) is used to transform the previous time-dependent data into the frequency domain. With the FFT, we find the constituent sinusoidal components of the timbre. Then, to better characterize the timbre, we record a chosen number of the most prominent frequencies. To minimize the effect of potential noise, we eliminate the frequencies with amplitudes below the threshold after obtaining the prominent frequencies.

B. Dissonance Curve Analysis

We find the samples' dissonance curves using Sethares' principle of local consonance. With a parametrized model of Plomp and Levelt's consonance curves, Sethares [1] represented the dissonance function $d(x)$ between sinusoids at frequency f_1 with amplitude v_1 and at frequency f_2 with amplitude v_2 as

$$d(f_1, f_2, v_1, v_2) = v_{12}(e^{-as(f_2 - f_1)} - e^{-bs(f_2 - f_1)}), \qquad (1)$$

where $s = d^*/(s_1 f_1 + s_2)$ and $v_{12} = v_1 v_2$, if the point of maximum dissonance occurs at d^*.

For a complex timbre F (that comprises n sine waves with frequencies $f_1 < f_2 < ... < f_n$ and amplitudes v_j for j being integers from 1 to n) with base frequency of f_1, then, Sethares models the dissonance D_F to be

$$D_F = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} d(f_i, f_j, v_i, v_j), \qquad (2)$$

which can also be used to calculate the dissonance between intervals.

In this paper, the dissonance curve analysis is done in Python. A function in the program is dedicated to implementing Sethares' algorithm and plotting the dissonance curve graph. Then, a file containing data from the frequency analysis (Section II.A) and dissonance curve analysis is generated and inputted into the Max interface introduced in the following section.

C. Graphical User Interface on Max/MSP

We create an interactive graphical user interface in Max/MSP, a visual programming platform commonly used by musicians and multimedia artists. Receiving a file generated by the Python program that contains data specific to the timbre, the Max patch iterates through the timbre data encoded in the file and dynamically generates a timbre re-synthesis algorithm and simple user interface.

D. Case Studies

To demonstrate the functionality and test the results of our interface, we complete case studies on a pure sine wave, harmonic timbres, and inharmonic timbres.

The examination of a pure sine wave is important as this pure tone without overtones is the building block of complex tones. We generate a sine tone of 571.5 Hz with Max/MSP for comparison with other timbres. Then, we compare the dissonance curve generated by our interface with the consonance model derived from experimental results by Plomp and Levelt [3].

Next, we conduct case studies on two other basic harmonic waveforms: square and saw waves. A saw wave is composed of a fundamental along with all harmonics of the fundamental. A square wave is composed of the fundamental frequency and odd harmonics, frequencies that are numerical integer multiples of the fundamental. The waves in the case studies are generated on Max with a fundamental frequency of 571.5 Hz. The points of local consonance generated for these waveforms are compared to steps of known scales.

For case studies on inharmonic sounds, we evaluate samples of Javanese gamelan instruments. Gamelan is an ensemble music found in Java and Bali that traditionally uses a tuning system distinct from that of the West. Gamelan orchestras typically play in the pelog (unequal-tempered heptatonic) or slendro (close to equal-tempered pentatonic) scales. Instead of tuning to a single standard reference, gamelan instruments are tuned and timbrally adjusted to the context of different orchestras and are often found to adopt a stretched or compressed octave tuning [6].

The sound library on which our case studies are based consists of, in total, 260 sounds from 11 Javanese gamelan instruments [7]. Among these musical instruments, we focused on 2 tuned instruments: the bonang and the gambang. As case studies, we analyze the audio samples with the Python program mentioned in Section II.A.

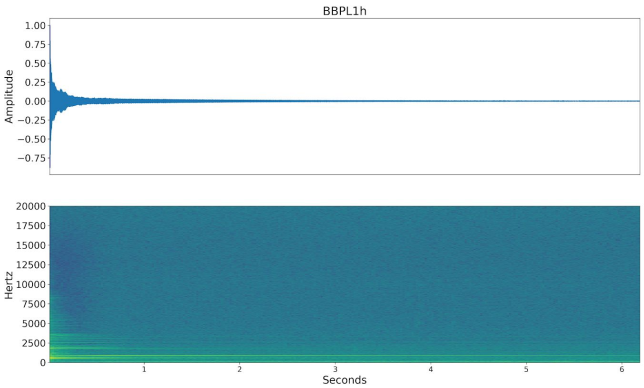

FIG. 1. Time-series plots generated for the bonang sample in Section III.B.3.a. In the spectrogram below, the brighter the color, the larger the amplitude is at the given frequency in the instant.

III. RESULTS AND DISCUSSION

This section offers a description of the Python module and Max interface we developed, followed by case studies on a pure sine wave, harmonic timbres, and inharmonic timbres from gamelan instruments.

A. Python Module and Max Interface

We join audio analysis and dissonance curve analysis in a Python module. The module is available for download on GitHub, where a complete description of the functions in the module and their utilities is provided.[1]

To implement the program, the user first reads in the audio data to obtain its sample rate and impulse response. This step can be completed using the **wavefile.read** function in SciPy. Using **plot_audio**, Two time series plots characterizing the sound can be generated, respectively a graph describing the change of amplitude over time and a spectrogram representing the change of amplitude at each frequency over time (see Fig. 1 for an example).

Then, using **perform_FFT**, the user performs fast Fourier transform on the audio data and characterizes the sound by extracting its prominent frequencies. For dissonance curve analysis, the user calls **diss_measure**, which plots the frequency ratios and corresponding dissonances at each interval and returns information about local minima on the curve. Finally, the user generates a text file containing data to be inputted into the Max interface using the **write_file** function. Alternatively, for a quick solution, the user can input characteristics of the sound into the **write_file_direct** function, which generates the text file directly.

Next, the Max/MSP interface we developed allows the user to audition the timbre transposed to the various scale steps suggested by the previous dissonance curve analysis. The user starts by loading the file containing timbre data into the interface. Then, the scale steps recommended by the dissonance curve analysis appear automatically on the bottom, and clicking on each scale step allows the user to hear the timbre transposed by the given frequency ratio. Two slider widgets on the top right allow the user to change the attack and decay time of the timbre. See Fig. 1 of Appendix A for the appearance of the interface with the timbre data of the bonang sample in Section III.B.3.a as the input.

[1]More information can be found at https://github.com/mingyudeng/2020-RMP-Dissonance-Curve-Analysis.

B. Results from Case Studies

This section presents examples of dissonance curves generated by the Python module of sound samples of a variety of timbres. Then, we compare and relate our observations with other consonance phenomena.

1. Sine Wave

We generate a sine tone of 571.5 Hz. Its spectrum from FFT is shown in Fig. 1 of Appendix B. In its dissonance curve (Fig. 2a), as the interval increases, sensory dissonance increases rapidly until a maximum slightly lower than the minor 2nd (with a ratio of 1.059463:1), after which dissonance continues to decrease, but never as low as that of the 1:1 ratio.

This observation corresponds with the consonance model of simple tones developed by Plomp and Levelt based on experimental data, which indicates that critical bandwidth is the reason behind perception of dissonance, and that the most dissonant intervals are found at frequency differences of around a quarter of this bandwidth [6].

2. Harmonic Timbres

a. Saw Wave

First, we evaluate a saw wave with fundamental frequency of 571.5 Hz and amplitude of $1/n$ at the nth integer multiple of the fundamental frequency for n from 1 to 10 (Appendix B, Fig. 3). In its dissonance curve, it is observed that, although the local minima do not align perfectly with the 12-tet scale steps, all points of local consonance occur at whole number ratios, and the integers in the simple frequency ratios range from 1 to 10 (Fig. 2b). This is in accordance with the justly intoned scales. Table 1 in Appendix B provides a tabulation of this observation.

This observation corresponds with Plomp and Levelt's model of dissonance curve for complex tones under the assumption that the total dissonance of each interval is equal to "the sum of the dissonances of each pair of adjacent partials" [3]. On this curve for a harmonic timbre with 6 partials, points of local consonance occur at simple frequency ratios of the component tones, with the integers

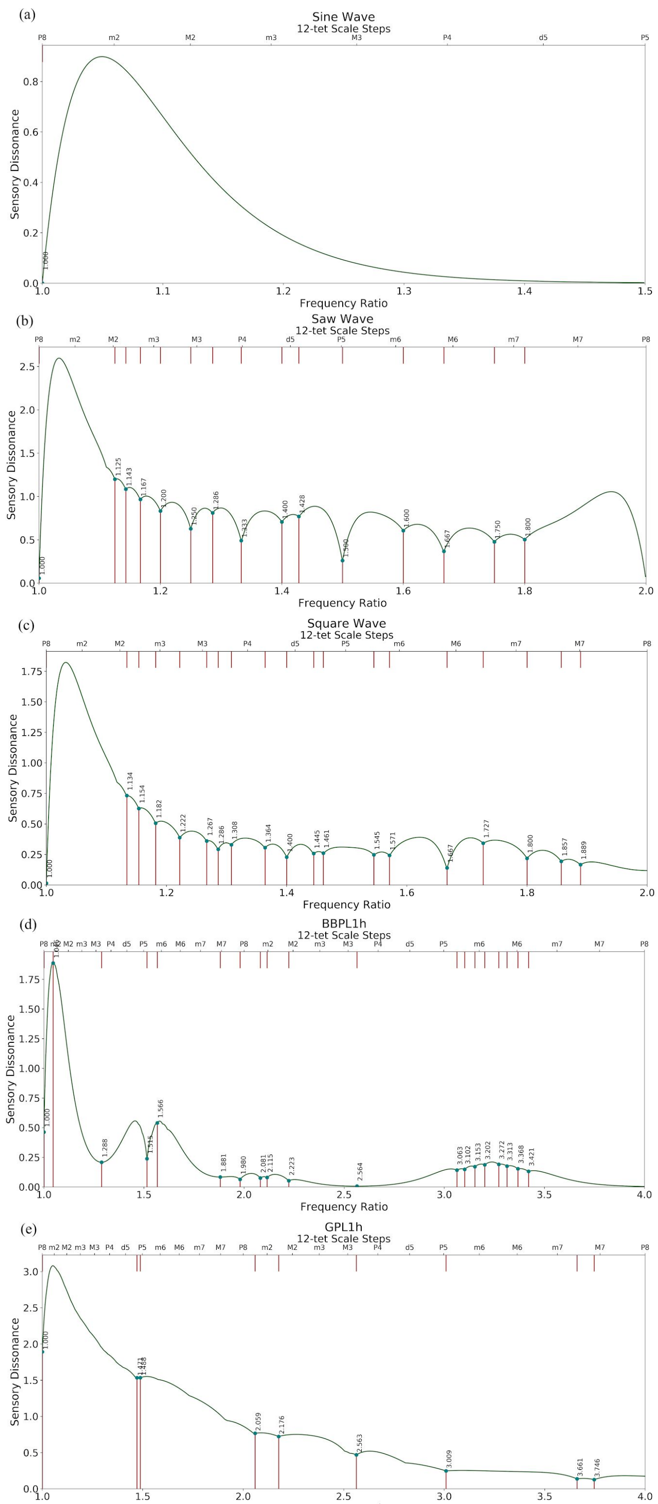

FIG. 2. Dissonance curves of (a) sine wave in Section III.B.1, (b) saw wave in Section III.B.2.a, (c) square wave in Section III.B.2.b, (d) bonang sample in Section III.B.3.a, and (e) gambang sample in Section III.B.3.b.

ratios only containing numbers from 1 to 6. This parallel indicates that the results we obtained conform with expectations from empirical data.

b. Square Wave

Next, we evaluate a square wave with fundamental frequency of 571.5 Hz. Its spectrum from FFT is shown in Fig. 4 of Appendix B. For $n = 1, 2, \ldots, 10$, the amplitude of the nth partial is $1/n$ when n is odd, and 0 when n is even. On its dissonance curve (Fig. 2c), no clear trend similar to that of the saw wave was observed. However, 5 out of the 19 points of local consonance correspond to scale steps on the Bohlen-Pierce scale, which is built from 13 equal divisions of the ratio 1/3.

Interestingly, when square waves with different numbers of partials are tested (ranging from 1 to 10), only specific local minima correspond to Bohlen-Pierce scale steps (such as steps 0, 2, 3, 4, 6, 7) regardless of the total number of points of local consonance. Future study is needed to explain the acoustical basis of this finding. See Table 2 of Appendix B for a tabulation for the comparison between the Bohlen-Pierce scale and the square wave mentioned in the last paragraph.

The relationship between Bohlen-Pierce scale and square waves has been explored by researchers in real life. In particular, the clarinet produces a sound whose partials consist of odd harmonics only. To date, four BP soprano clarinets have been created by Fox [9].

3. Inharmonic Timbres

This section evaluates the inharmonic timbres of two gamelan instruments. While Sethares examines the dissonance curve obtained from an inharmonic gemalen instrument in combination with a harmonic sound, we investigate the dissonance curves of different instruments in the same gamelan [8]. The sample instruments are part of the gamelan at Casa da Música in Oporto, Portugal.

a. The Bonang

The bonang is a collection of gongs (in the shape of kettles) placed in either one or two rows on strings tied to a wooden frame. In our case study, we evaluate a sample of the first note on the second octave of a bonang barung in pelog scale. The sample has a fundamental frequency of 570.63 Hz.

In Fig. 2d, we observe that the points of local consonance are spread out in the first octave, whereas in the second octave, the minima become more clustered (especially in the region between 3.0 and 3.5). A potential explanation is the similarly clustered prominent frequencies around the fundamental frequency (see Fig. 5 of Appendix B for the FFT spectrum). To produce the dissonance curve, two copies of this complex tone are slided against each other to calculate dissonances at each interval. When the two clusters of prominent frequencies slide past each other, the critical bandwidth between each of the partials leads to a region of fluctuating sensory dissonances, thus contributing to the number of local consonances in close proximity.

Next, we compare the local points of consonance with the pelog scale played by the same instrument. However, no notable correlation is observed. This is reasonable, given that specific instruments need to conform to the tuning of the gamelan as a whole, and slight dissonances may be considered necessary in tuning to avoid blandness.

Interestingly, when we compare our dissonance curve to the 5-tet scale, whose steps are considered close to the typical slendro tuning, we find points of correspondence (at 1.515, 1.980, 2.223, and 3.063 on the curve). More experimentation is needed in the future to explain the reasons behind this phenomenon.

b. The Gambang

Similar to a xylophone, the gambang is a percussion instrument composed of wooden bars. It is played by a pair of long thin mallets. In this case study, we evaluate a sample of the first note on the second octave of a gambang in pelog scale. The sample has a fundamental frequency of 576.01 Hz.

As shown in Fig. 2e, the points of local consonance on the dissonance curve generated for the gambang sample are relatively spread out, with only 9 total points in two octaves. This is likely a result of the especially prominent fundamental frequency of the sound. As shown in Fig. 6 of Appendix B, most of the prominent frequencies are clustered in the range of 70 to 250 Hz, yet the fundamental frequency has a much larger amplitude. This discrepancy in amplitudes leads to a dissonance curve with more resemblance to that of the sine wave, thus contributing to the sparse points of local consonance observed on the curve in Fig. 2e.

In fact, the cluster of low frequencies may have been noises in the sample. When we sift out the “noise” by increasing the threshold, the dissonance curve becomes one that is almost identical in shape to that of the sine tone.

In comparison with the pelog scale to which it is tuned, the gambang sample has two points of local consonance that correspond to the pelog scale steps (at 1.488 and 2.059 on the curve). This observation could have resulted from coincidence. Thus, experimentation with more gamelan instruments and orchestras is needed to confirm the validity of these observations.

IV. CONCLUSIONS

In summary, this paper presents a user-friendly interface that joins audio analysis, Sethares’ dissonance curve analysis, and an interactive presentation of the generated scale. The interface is composed of two parts: a Python module and a graphical user interface on Max. With these tools, this paper also examines case studies on a sine wave, harmonic timbres, and inharmonic timbres found in gamelan instruments. This interactive timbre-to-scale interface opens new areas for musical exploration, facilitating the incorporation of inharmonic sounds in music and the creation of new musical instruments.

In the long run, the usefulness and accuracy of the tools we developed are dependent on the validity of Sethares’ principle of local consonance, which, in turn, suffers from the limitations of the model for dissonance calculation. The model is only valid for a “reasonable” scope of frequencies and amplitudes [2]. Directions of future work include developing a set of tools for the reverse process of our research—finding optimal timbres for given scales; completing a comprehensive and systematic evaluation of the tunings of non-western music cultures; and investigating the real-life utility of the principle of local consonance, and thus our results, in the realm of musical instrument design.

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APPENDIX

A. The Max Graphical User Interface

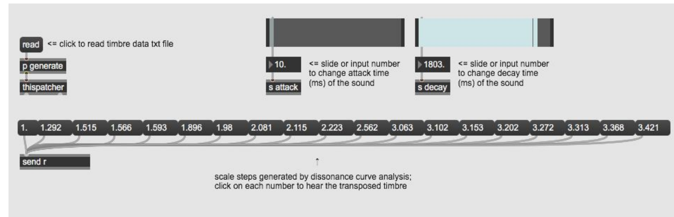

FIG. 1. The appearance of our Max graphical user interface with the bonang sample in Section III.B.3.a as input.

B. Results from Case Studies

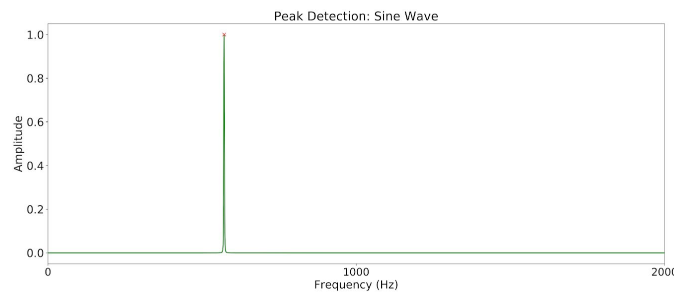

FIG. 2. FFT spectrum of the sine wave in Section III.B.1.

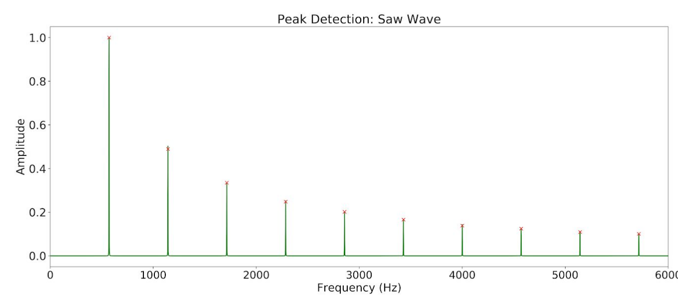

FIG. 3. FFT spectrum of the saw wave in Section III.B.2.a

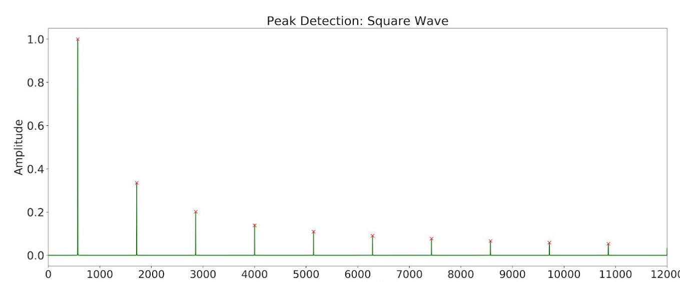

FIG. 4. FFT spectrum of the square wave in Section III.B.2.b.

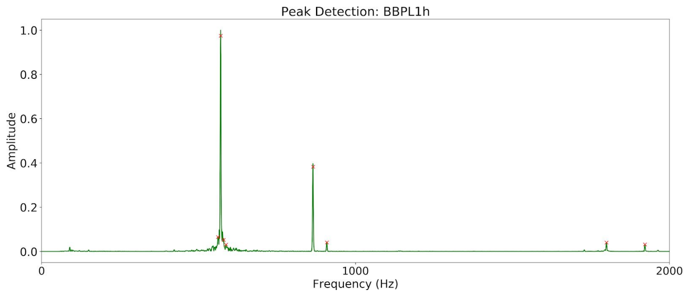

FIG. 5. FFT spectrum of the bonang sample in Section III.B.3.a.

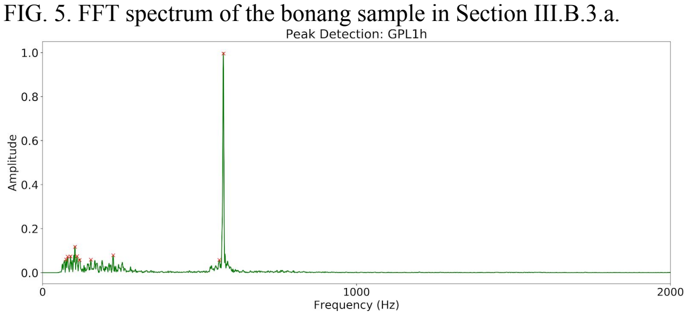

FIG. 6. FFT spectrum of the gambang sample in Section III.B.3.b.

Location of minima	Nearest whole number ratio
1.000	1:1
1.125	9:8
1.143	8:7
1.167	7:6
1.200	6:5
1.250	5:4
1.286	9:7
1.333	4:3
1.400	7:5
1.428	10:7
1.500	3:2
1.600	8:5
1.667	5:3
1.750	7:4
1.800	9:5

TABLE 1. Locations of minima on the dissonance curve of the saw wave in Section III.B.2.a and the nearest whole number ratios.

Location of minima	Bohlen-Pierce Scale Steps	Equal-tempered interval
1.000	0	$3^{0/13}$
1.286	3	$3^{3/13}$
1.400	4	$3^{4/13}$
1.667	6	$3^{6/13}$
1.800	7	$3^{7/13}$
2.140	9	$3^{9/13}$
2.330	10	$3^{10/13}$
3.000	13	$3^{13/13}$

TABLE 2. Locations of minima on the dissonance curve of the square wave in Section III.B.2.b compared with the Bohlen Pierce scale steps. Only locations of minima that correspond with the Bohlen-Pierce scale steps are shown.