In general the <u>variances</u> of transform coefficients can be computed as follows:

Let $\underline{u}$ be the image vector.

$$\mu_u = E[\underline{u}]$$

$$R_u = E[\underline{u} - \mu_u][\underline{u} - \mu_u]^T$$

Let $\underline{v} = A\underline{u}$

transform

$$\mu_v = E[\underline{v}]$$

$$= A\,E[\underline{u}] = A\mu_u$$

$$R_v = E[(\underline{v} - \mu_v)(\underline{v} - \mu_v)^{*T}]$$

$$= E[(A\underline{u} - A\mu_u)(A\underline{u} - A\mu_u)^{*T}]$$

$$= A R_u A^{*T}$$

The transform coefficient variances are given by the diagonal elements of $R_v$

$$\sigma_v^2(k) = [R_v]_{k,k} = [A R_u A^{*T}]_{k,k}$$

From Parseval's th

$$\sum_{k=0}^{N-1} |v(k)|^2 = \sum_{k=0}^{N-1} | u(n)|^2$$

$$\sum E |V(k)|^2 = \sum E [u(n)]^2$$

~~Let~~ for 2-D separable transform

$$V(k,l) = \sum_{m,n}^{N-1}\sum u(m,n)\, a(k,m)\, a(l,n)$$

$$E[V(k,l)] = \mu_v(k,l) = \sum\sum a(k,m)\, a(l,n)\, \mu_u(m,n)$$

$$\sigma_v^2(k,l) = E\left[\,|V(k,l) - \mu_v(k,l)|^2\right]$$

$$= E\left\{\left[\sum\sum u(m,n)\, a(k,m)\, a(l,n) - \mu_u(m,n)\, a(k,m)\, a(l,n)\right]\left[\quad\right]^*\right\}$$

$$= \sum_{m}\sum_{n}\sum_{m'}\sum_{n'} a(k,m)\, a(l,n)\, r(m,n;m',n')\, a^*(k,m')\, a^*(l,n')$$

Simpler Computation for separable ~~r.f.~~

$$r(m,n;m',n') = r_1(m,m')\, r_2(n,n')$$

$$\sigma_v^2(k,l) = \sigma_1^2(k)\,\sigma_2^2(l)$$

$$= \left[A R_1 A^{*T}\right]_{k,k} \left[A R_2 A^{*T}\right]_{l,l}$$

$R_1 = \{ r_1(m,m') \}$ $\quad R_2 = \{ r_2(n,n') \}$

for stationary r.f

$$\sigma_v^2(k,l) = \sum_m \sum_n \sum_{m'} \sum_{n'} a(k,m)\, a(l,n)$$

$$r(m-m', n-n')\, a^*(k,m')\, a^*(l,n')$$

→ for DFT, DST DCT

Can be Computed in $O(N^2 \log N)$

operations