Spring 2011 Final ENEE 631

Problem 1

(15 points) Write out the steady-state RUKF equations for a 1×1-order $\oplus+$ model and update region

$$\mathcal{U}_{\oplus+} = \{(n_1, n_2), (n_1 - 1, n_2), (n_1 - 2, n_2), \tag{1}$$

$$(n_1 + 2, n_2 - 1), (n_1 + 1, n_2 - 1), (n_1, n_2 - 1), \tag{2}$$

$$(n_1 - 1, n_2 - 1), (n_1 - 2, n_2 - 1)\}. \tag{3}$$

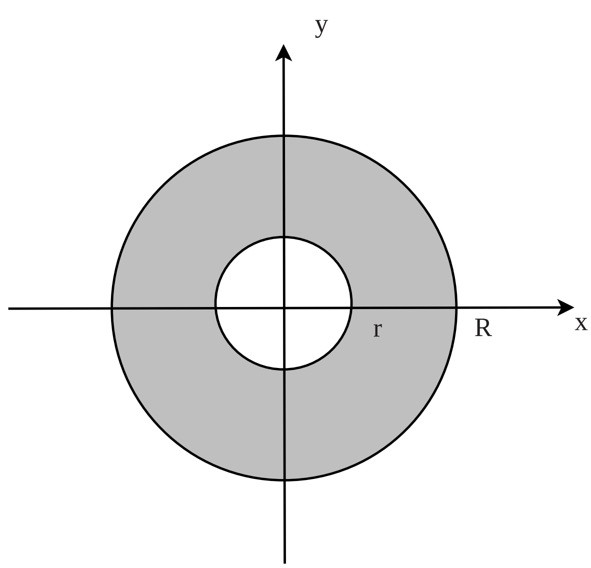
Write these equations in terms of model coefficients $\{c_{10}, c_{01}, c_{11}, c_{-1,1}\}$ and assumed steady-state gain values

$$\{g(0,0), g(1,0), g(2,0), g(-2,1), g(-1,1), g(0,1), g(1,1), g(2,1)\}. \tag{4}$$

Problem 2

(10 points) Calculate the Radon transform of the object defined as

$$f(x,y) = \begin{cases} A, & r^2 \le x^2 + y^2 \le R^2 \\ 0, & \text{otherwise} \end{cases} \tag{5}$$



Problem 3

(10 points) Show that the Markov random field satisfying the noncausal 2-D difference equation (7.6-1 in the textbook), with random input satisfying (7.6-2 in the textbook), has the PSD (7.6-3 in the textbook).

Problem 4

(10 points) Consider using the logarithmic bit assignment rule, for Gaussian variables,

$$B_i = \frac{B}{N} + \frac{1}{2}\log_2 \frac{\sigma_i^2}{\sigma_{gm}^2}, \quad i = 1, ..., N, \tag{6}$$

with $\sigma_{gm}^2 \equiv (\prod_i \sigma_i^2)^{1/N}$ and $N =$ number of channels (coefficients). Apply this rule to the 2×2 DCT output variance set given here:

$$\text{coef. map} = \begin{pmatrix} 00 & 10 \\ 01 & 11 \end{pmatrix} \text{ and corresponding variances} = \begin{pmatrix} 22 & 4 \\ 8 & 2 \end{pmatrix} \tag{7}$$

Assume the total number of bits to assign to these four pixels is $B = 16$. Please resolve any possible negative bit allocations by removing that pixel from the set and reassigning bits to those remaining. Noninteger bit assignments are OK since we plan to use variable-length coding. Please give the bits assigned to each coefficient and the total number of bits assigned to the four pixels.

Problem 5

(15 points) Consider the coding gain in an ideal subband coding system with N channels (the filters used are ideal bandpass filters). Start with the case $N = 2$ before looking at the general case.

(a) Assume that the power spectrum of the input signal $|X(e^{jw})|^2$ is given by

$$|X(e^{jw})|^2 = 1 - \frac{w}{\pi} \quad |w| \leq \pi \tag{8}$$

Give the coding gain as a function of N.

(b) Same as above, but with

$$|X(e^{jw})|^2 = e^{-\alpha w} \quad |w| \leq \pi \tag{9}$$

Give the coding gain as a function of N and α, and compare to (a).