ENEE631 Spring 2011 Homework 3

Problem 1

Consider a wide-sense stationary process $\{x[n]\}$ and its filtered version $y[n] = \sum_k h[k]x[n-k]$, where $h[k]$ is a stable and causal filter.

(a) Show that the mean of $\{y[n]\}$ is independent of n. Show that the covariance function of $\{y[n]\}$, $K_Y[n,m] = cov(y[n] \cdot y[m])$ is a function of $k \equiv (n-m)$ only, and given by

$$K_Y[k] = \sum_{\tilde{n}=0}^{\infty} \sum_{\tilde{m}=0}^{\infty} h[\tilde{n}]h[\tilde{m}]K_X[k - (\tilde{n} - \tilde{m})] \tag{1}$$

(b) Assuming zero-mean input, show that

$$K_{XY}[m] = \sum_{k=0}^{\infty} h[k]K_X[m-k] \tag{2}$$

(c) Consider now one-sided wide-sense stationary processes, which can be thought of as wide-sense stationary processes that are ”turned on” at time 0. Consider filtering of such processes by causal FIR and IIR filters, respectively. What can be said about $E(Y[n]), n \geq 0$ in these cases?

Problem 2

The *lifted* SWT is defined starting from the *Lazy* SWT, which just separates the input sequence x into even and odd terms. In the lifted SWT, this is followed by prediction and update steps as specified by the operators P and U, respectively, as shown in Figure 5.33 in the textbook. The output multipliers α_0 and α_1 may be needed for proper scaling.

(a) Find the corresponding prediction and update operators for the SWT that uses the Haar filter set

$$h_0(n) = \delta(n) + \delta(n-1),$$
$$h_1(n) = \delta(n) - \delta(n-1).$$

(b) A Key property of lifting is that the operators P and U are not constrained at all. Show the ISWT for the lifted transform in Figure 5.33 in the textbook by starting from the right-hand side of the figure and first undoing the update step and then undoing the prediction step. Note that this is possible even for nonlinear operators P and U.

(c) Can you do the same for the LeGall-Tabatabai (LGT) $5/3$ analysis filter set?

$$h_0(n) = \left\{ -\frac{1}{8}, \frac{1}{4}, \frac{3}{4}, \frac{1}{4}, -\frac{1}{8} \right\},$$
$$h_1(n) = \left\{ -\frac{1}{2}, 1, \frac{1}{2} \right\}.$$

Reference to
A. cohen, I. Daubechies, and J.-C. Feauveau, ”Biorthogonal Bases of Compactly Supported Wavelets,” Commun. Pure Appl. Math. XLV, 485-560, 1992.
may be necessary.

Problem 3

Show that the inverse filter of a spatially invariant system will also be spatially invariant.

Problem 4

Prove that the spectral density function is a real, non-negative function.