Difference-in-Differences with Multiple Time Periods

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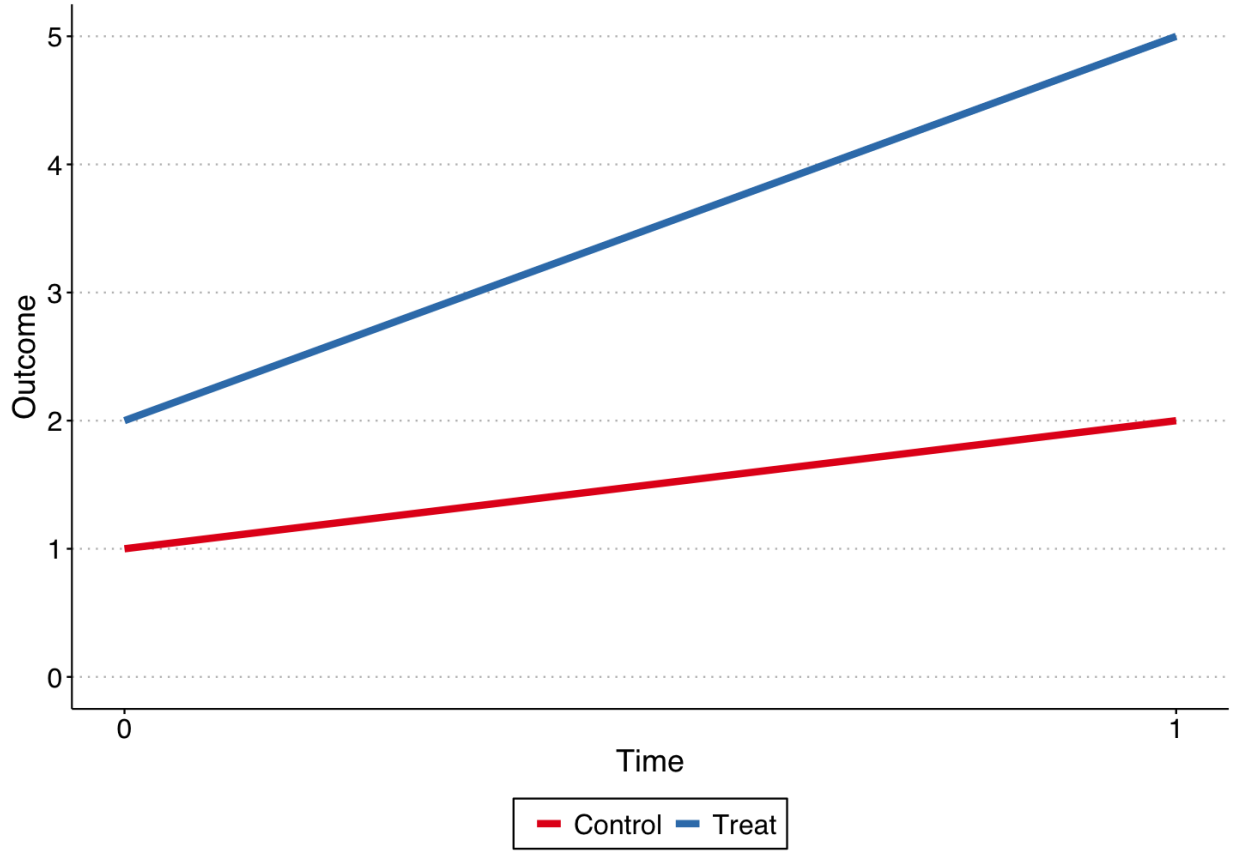
Outline of Lecture

1. Overview of Difference-in-Differences (DiD)
2. Two-Way Fixed Effects (TWFE) Estimator
3. Problems with Using TWFE Estimator for DiD with Multiple Time Periods
4. Simulation Examples
5. Alternative Difference-in-Differences Estimators
6. Conclusion and Recommendations

Difference-in-Differences

- Revisit **Card and Krueger (1993)** minimum wage and employment study comparing NJ and PA.
- 2 units and 2 time periods.
- 1 treated unit (T), which receives treatment in the second period.
- 1 control unit (C), which is never treated.

Difference-in-Differences



Difference-in-Differences

- Building upon **Angrist & Pischke (2008, p. 228)** we can think of the simple 2x2 DiD as a fixed effects estimator.
- Potential Outcomes
  - $Y_{i,t}^0$ = value of dependent variable for unit i in period t without treatment.
  - $Y_{i,t}^1$ = value of dependent variable for unit i in period t with treatment.
- The expected outcome is a *linear function* of unit and time fixed effects:

$$E[Y_{i,t}^0] = \alpha_i + \alpha_t$$

$$E[Y_{i,t}^1] = \alpha_i + \alpha_t + \delta D_{st}$$

- The goal of DiD is to get an *unbiased estimate* of the treatment effect δ.

Difference-in-Differences

- Difference in expectations for the *control* unit between times t = 1 and t = 0:

$$\begin{aligned} E[Y^0_{C,1}] &= \alpha_1 + \alpha_C \\ E[Y^0_{C,0}] &= \alpha_0 + \alpha_C \\ E[Y^0_{C,1}] - E[Y^0_{C,0}] &= \alpha_1 - \alpha_0 \end{aligned}$$

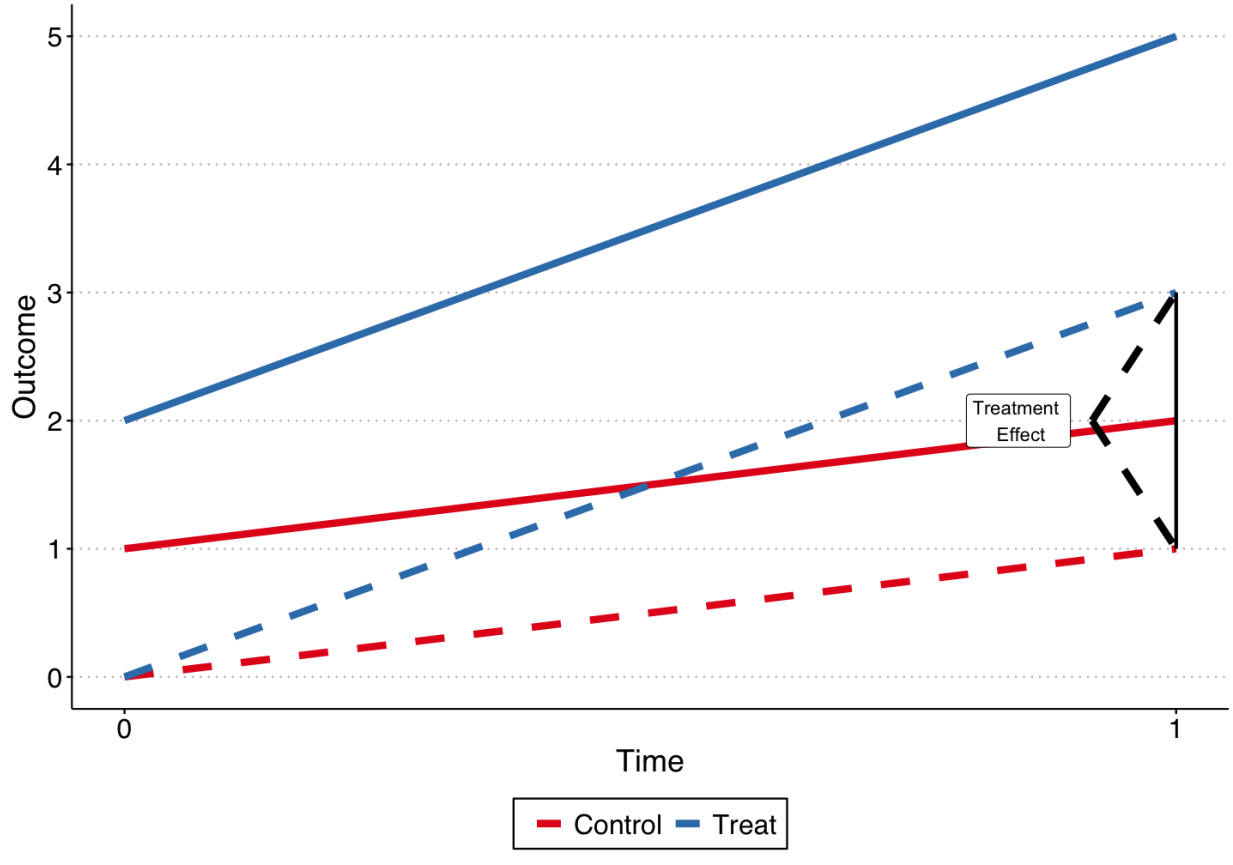
- Now do the same thing for the *treated* unit:

$$\begin{aligned} E[Y^1_{T,1}] &= \alpha_1 + \alpha_T + \delta \\ E[Y^1_{T,0}] &= \alpha_0 + \alpha_T \\ E[Y^1_{T,1}] - E[Y^1_{T,0}] &= \alpha_1 - \alpha_0 + \delta \end{aligned}$$

- If we assume the linear structure of DiD, then unbiased estimate of δ is:

$$\delta = \Big(E[Y^1_{T,1}] - E[Y^1_{T,0}]\Big) - \Big(E[Y^0_{C,1}] - E[Y^0_{C,0}]\Big)$$

Two-Way Differencing



Regression DiD

The DiD can be estimated through linear regression of the form:

$$y_{it} = \alpha + \beta_1 TREAT_i + \beta_2 POST_t + \delta(TREAT_i \cdot POST_t) + \epsilon_{it} \quad (1)$$

The coefficients from the regression estimate in (1) recover the same parameters as the two-way differencing performed above:

$$\alpha = E[y_{it}|i = C, t = 0] = \alpha_0 + \alpha_C$$

$$\begin{aligned}\beta_1 &= E[y_{it}|i = T, t = 0] - E[y_{it}|i = C, t = 0] \\ &= (\alpha_0 + \alpha_T) - (\alpha_0 + \alpha_C) = \alpha_T - \alpha_C\end{aligned}$$

$$\begin{aligned}\beta_2 &= E[y_{it}|i = C, t = 1] - E[y_{it}|i = C, t = 0] \\ &= (\alpha_1 + \alpha_C) - (\alpha_0 + \alpha_C) = \alpha_1 - \alpha_0\end{aligned}$$

$$\begin{aligned}\delta = &(E[y_{it}|i = T, t = 1] - E[y_{it}|i = T, t = 0]) - \\ &(E[y_{it}|i = C, t = 1] - E[y_{it}|i = Ct = 0]) = \delta\end{aligned}$$

TWFE DiD

- Regression DiD provides both estimates of δ and standard errors for the estimates.
- Angrist & Pischke (2008):
  - "It's also easy to add additional (units) or periods to the regression setup... [and] it's easy to add additional covariates."
- Two-way fixed effects estimator :

$$y_{it} = \alpha_i + \alpha_t + \delta^{DD} D_{it} + \epsilon_{it}$$

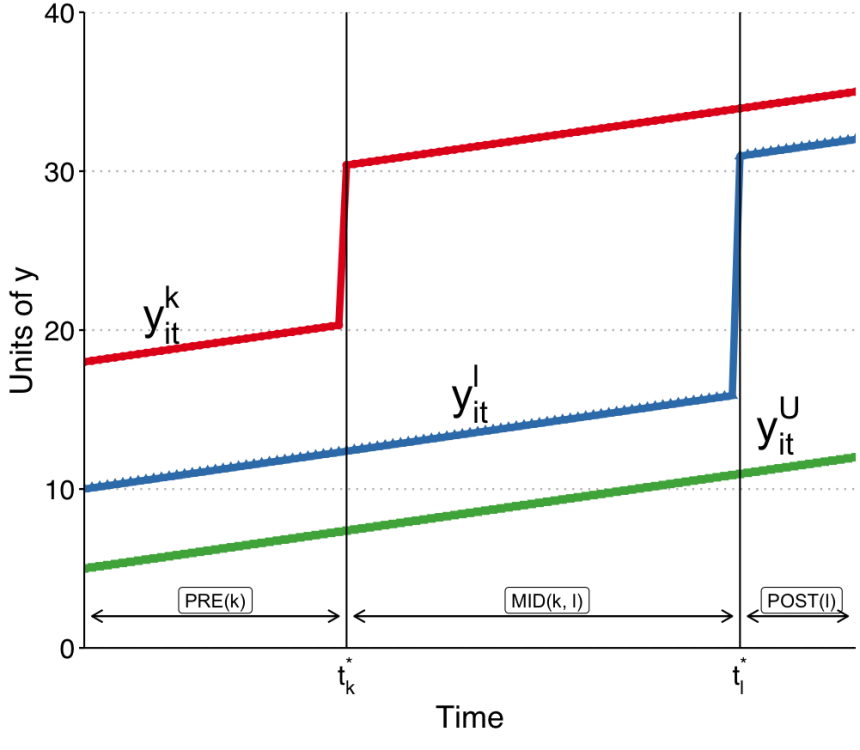
  - α_i and α_t are unit and time fixed effects, D_{it} is the unit-time indicator for treatment.
  - $TREAT_i$ and $POST_t$ now subsumed by the fixed effects.
  - Can be easily modified to include covariate matrix X_{it}, time trends, dynamic treatment effects estimation, etc.

Where TWFE Goes Wrong

- Recent development in econometrics on the issues with TWFE DiD with "staggered treatment timing" (**de Chaisemartin and D'Haultfœuille (2020), Callaway and Sant'Anna (2021), Goodman-Bacon (2021), Sun and Abraham (2021)**)
  - Different units receive treatment at different periods in time.
- Staggered DiD is commonly used. It helps to increase the amount of cross-sectional variation if done correctly.
- What really happened behind the scenes:
  - δ^{DD} with staggered treatment timing is a weighted average of many different treatment effects .
  - The weights are sometimes negative and non-intuitive.
  - Before 2018, people knew little about how TWFE measures when treatment timing varies, how it compares means across groups, or why different specifications change estimates.

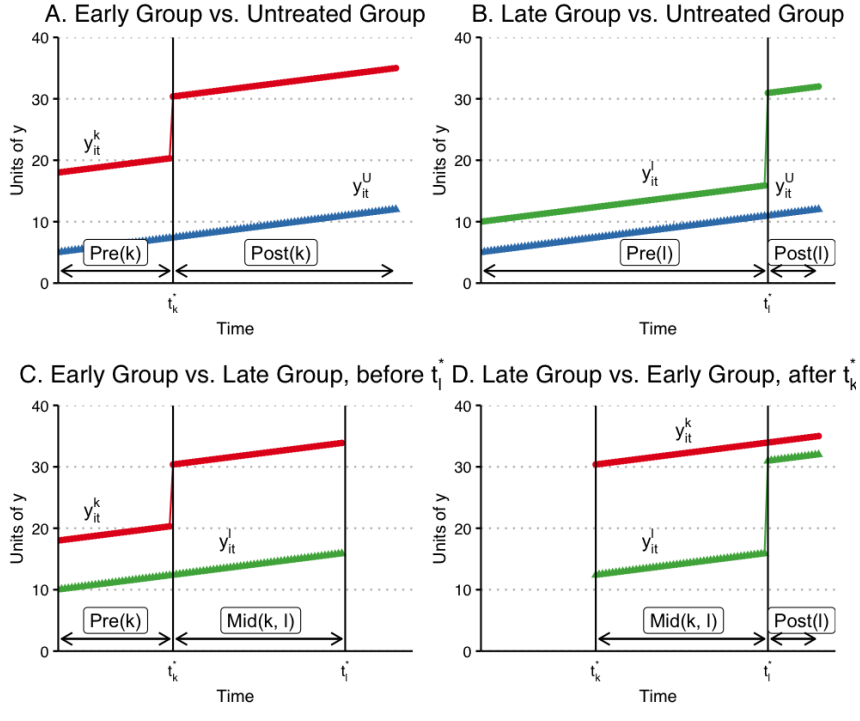
Bias with TWFE - Goodman-Bacon (2021)

- **Goodman-Bacon (2021)** provides a clear graphical intuition for the bias. Assume three treatment groups - never treated units (U), early treated units (k), and later treated units (l).



Bias with TWFE - Goodman-Bacon (2021)

- **Goodman-Bacon (2021)** shows that we can form four different 2x2 groups in this setting, where the effect can be estimated using the simple regression DiD in each group:



Bias with TWFE - Goodman-Bacon (2021)

- Some important Insights
  - δ^{DD} is just the *weighted average* of the four 2x2 treatment effects. The weights are a function of the *size of the sub-sample, relative size of treatment and control units, and the timing of treatment in the sub-sample*.
  - Already-treated units act as controls even though they are treated.
  - Given the weighting function, panel length alone can change the DiD estimates substantially, even when each δ^{DD} does not change.
  - Groups treated closer to middle of panel receive higher weights than those treated earlier or later.

Simulation Exercise

- Can show how easily δ^{DD} goes wrong through a simulation exercise.
- Consider two sets of DiD estimates - one where the treatment occurs in one period, and one where the treatment is staggered.
- The data generating process (DGP) is linear: $y_{it} = \alpha_i + \alpha_t + \delta_{it} + \epsilon_{it}$.
  - $\alpha_i, \alpha_t \sim N(0, 1)$
  - $\epsilon_{i,t} \sim N\left(0, \left(\frac{1}{2}\right)^2\right)$
- We will consider two different treatment assignment set ups for δ_{it}.

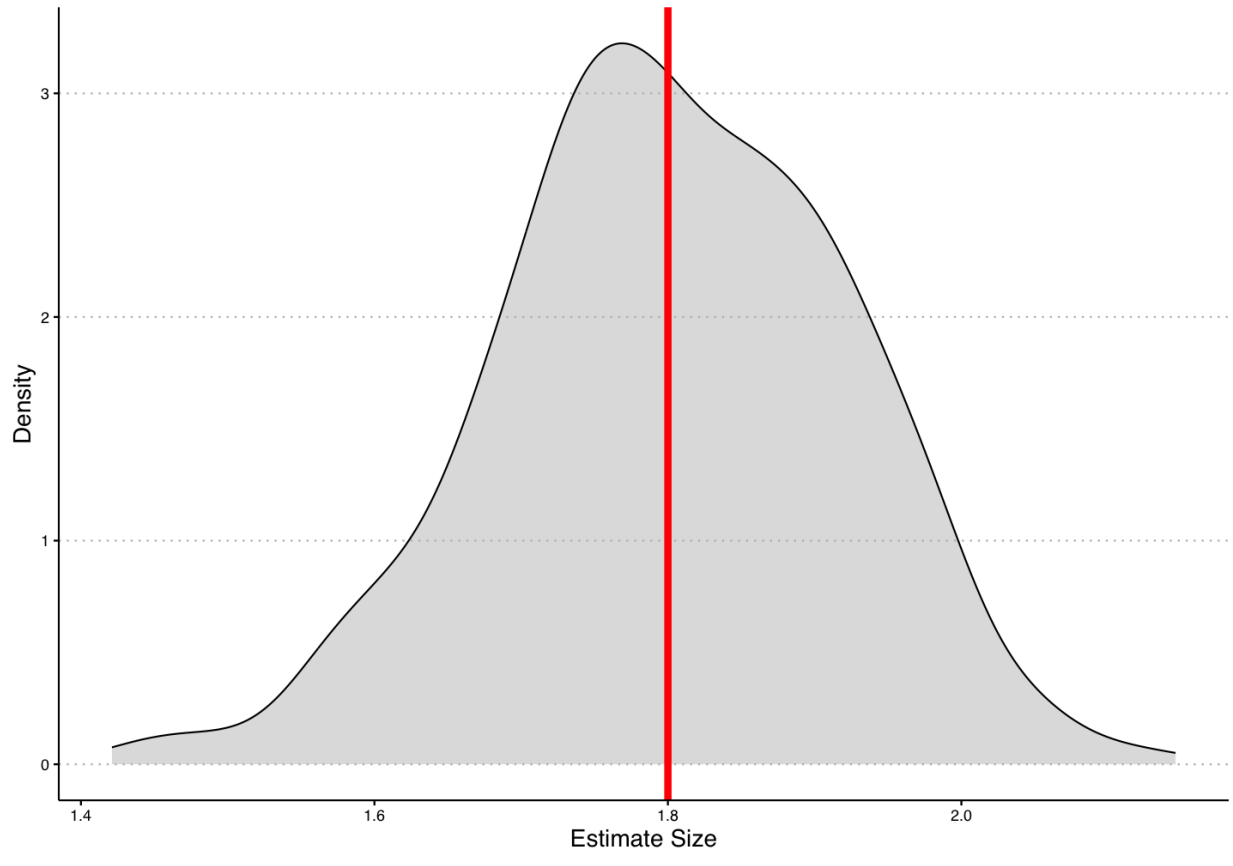
Simulation 1 - 1 Period Treatment

- There are 20 states s, and 200 units i randomly drawn from the 20 states.
- Data covers years 1990 to 2010, and half the states receive "treatment" in 2000.
- For every unit incorporated in a treated state, we pull a unit-specific treatment effect from $\mu_i \sim N(0.3, (1/5)^2)$.
- Treatment effects here are trend breaks rather than unit shifts: the accumulated treatment effect δ_{it} is $\mu_i \times (year - 1995 + 1)$ for years after 2000.
- We then estimate the average treatment effect as $\hat{\delta}$ from:

$$y_{it} = \hat{\alpha}_i + \hat{\alpha}_t + \hat{\delta} D_{it}$$

- Simulate this data 500 times and plot the distribution of estimates $\hat{\delta}$ and the true effect (red line).

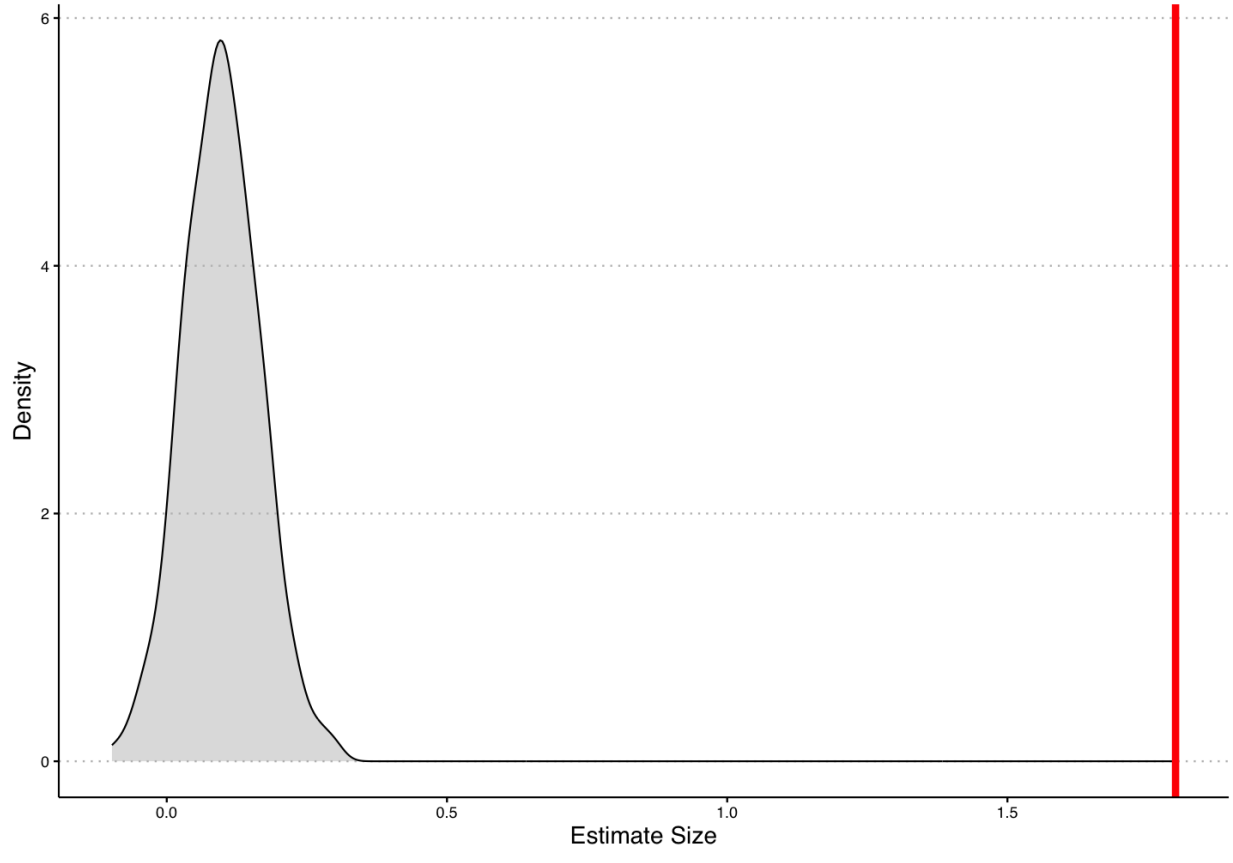
Simulation 1 - 1 Period Treatment



Simulation 2 - Staggered Treatment

- Run similar analysis with staggered treatment.
- The 20 states are randomly assigned into four treatment cohorts of size 50 depending on year of treatment assignment (1994, 1998, 2002, and 2004)
- DGP is identical, except that now δ_{it} is equal to $\mu_i \times (year - \tau_g + 1)$ where τ_g is the treatment assignment year.
- Again, we estimate this data 500 times and plot the distribution of estimates $\hat{\delta}$ and the true effect (red line).

Simulation 2 - Staggered Treatment



Simulation 2 - Staggered Treatment

- Main problem - we use *early treated units* as controls for later treated units, which violates the *parallel trends assumption*!
- When the treatment effect is "dynamic", i.e. takes more than one period to be incorporated into your dependent variable, you are *subtracting* the treatment effects from prior treated units from the estimate of future control units.
- This biases your estimates towards zero when all the treatment effects are the same.

Another Simulation

- Can we actually get estimates for δ that are of the *wrong sign*? Yes, if treatment effects for early treated units are larger (in absolute magnitude) than the treatment effects on later treated units.
- Let's simulate another example in which units are randomly assigned to one of 20 states. The 20 states are randomly assigned into one of 5 treatment groups G_g based on treatment being initiated in 1993, 1997, 2001, 2005, and 2009.
- All treated units incorporated in a state in treatment group G_g receive a treatment effect $\delta_i \sim N(\delta_g, .2^2)$.
- The treatment effect is cumulative or dynamic - $\delta_{it} = \delta_i \times (year - G_g)$.

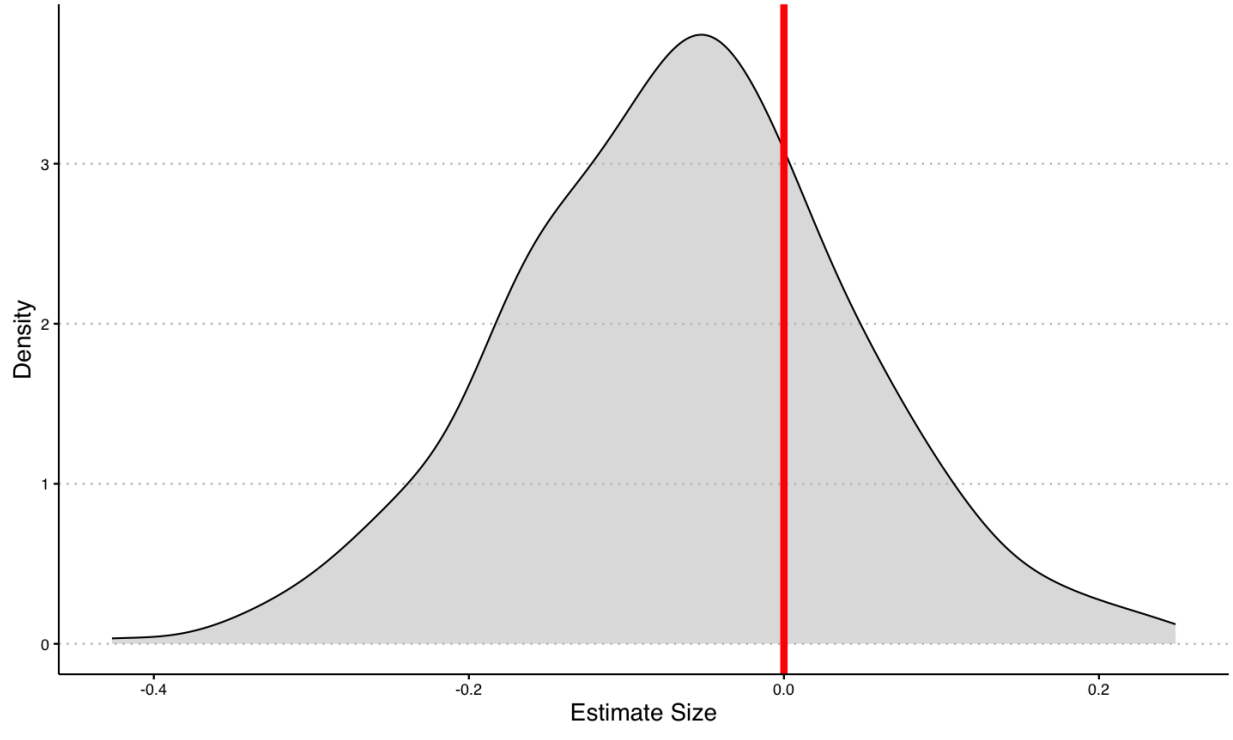
Another Simulation

- The average treatment effect multiple decreases over time:

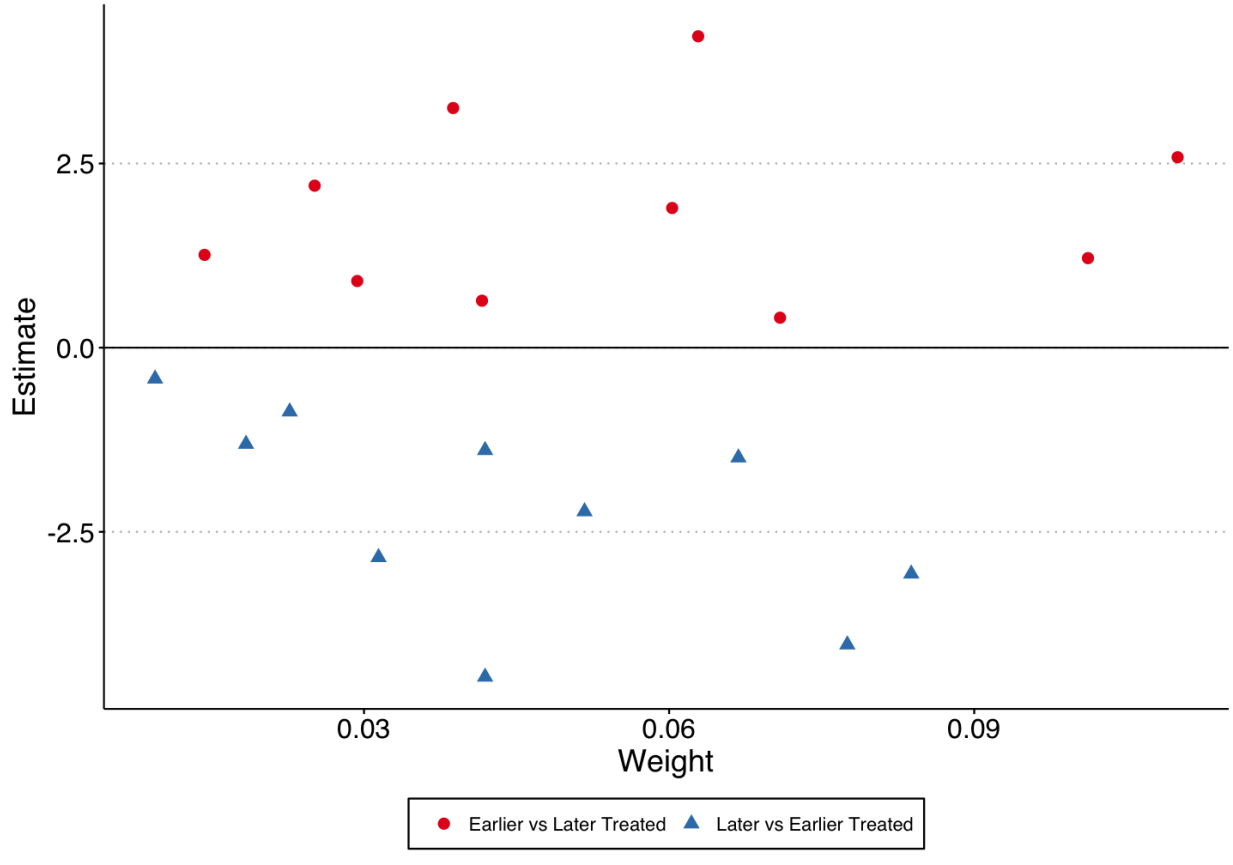
Treatment Effect Averages	
Cohort	Effect
1993	0.5
1997	0.4
2001	0.3
2005	0.2
2009	0.1

Another Simulation

- First let's look at the distribution of δ^{DD} using TWFE estimation with this simulated sample:



Goodman-Bacon Decomposition



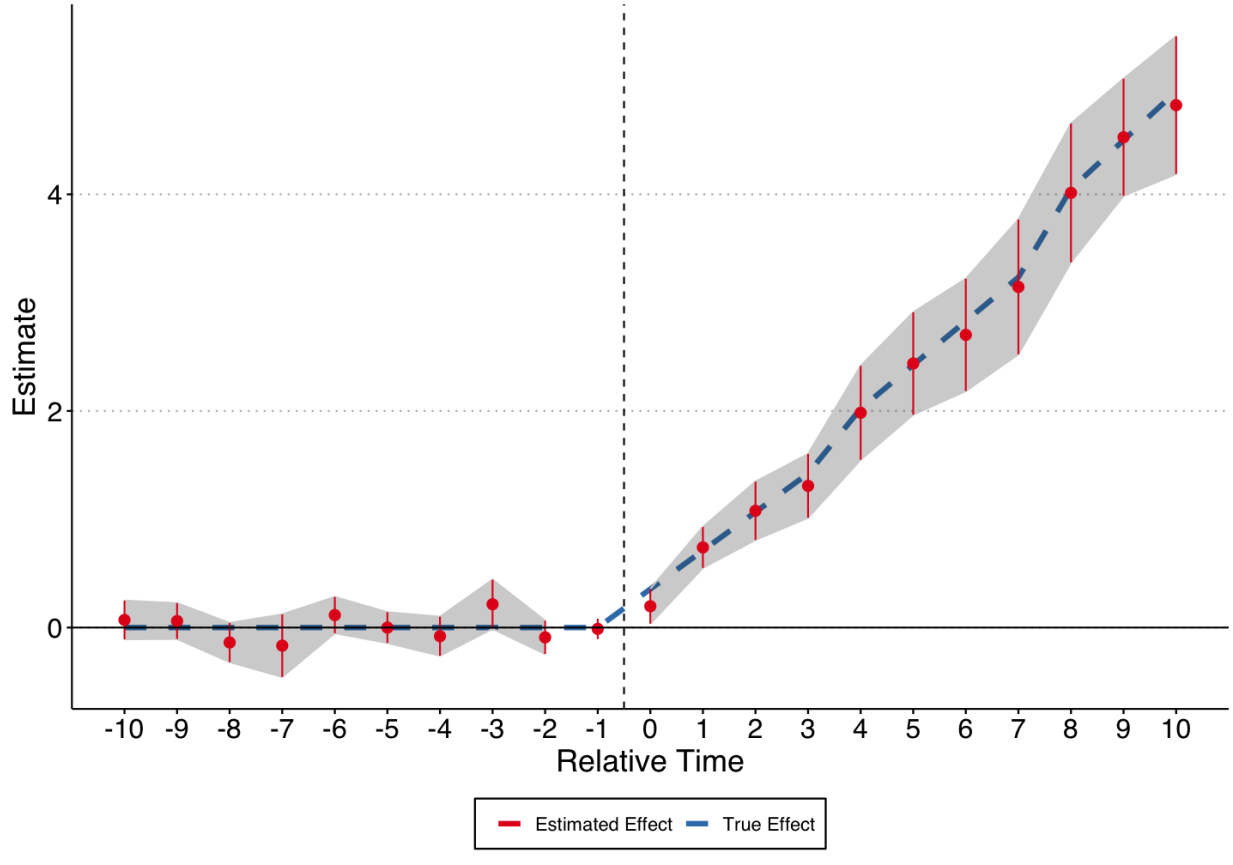
Callaway & Sant'Anna

- Inverse propensity weighted (IPW) long-difference in cohort-specific average treatment effects between treated and untreated units for a given treatment cohort.

$$ATT(g,t) = \mathbb{E}\left[\left(\frac{G_g}{\mathbb{E}[G_g]} - \frac{\frac{p_g(X)C}{1-p_g(X)}}{\mathbb{E}\left[\frac{p_g(X)C}{1-p_g(X)}\right]}\right)\left(Y_t - T_{g-1}\right)\right]$$

- Without covariates, as in the simulated example here, it calculates the simple long difference between all treated units i in relative year k with all potential control units that have not yet been treated by year k.

Callaway & Sant'Anna



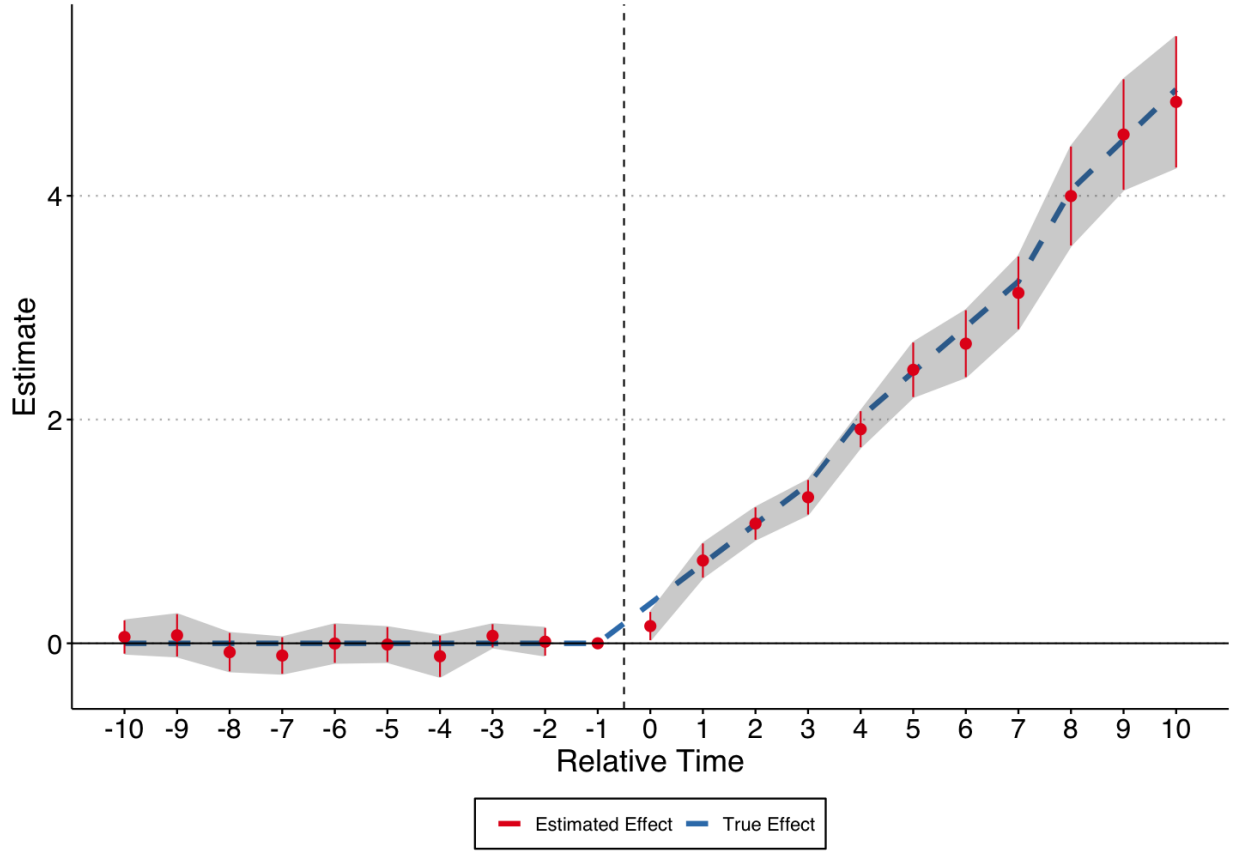
Abraham and Sun

- A relatively straightforward extension of the standard event-study TWFE model:

$$y_{it} = \alpha_i + \alpha_t + \sum_e \sum_{l \neq -1} \delta_{el}(1\{E_i = e\} \cdot D_{it}^l) + \epsilon_{it}$$

- You saturate the relative time indicators (i.e. t = -2, -1, ...) with indicators for the treatment initiation year group, and aggregate to overall aggregate relative time indicators by cohort size.
- In the case of no covariates, this gives you the same estimate as Callaway & Sant'Anna if you *fully saturate* the model with time indicators (leaving only two relative year identifiers missing).
- The authors don't claim that it can be used with covariates, but it seemingly follows if we think it is okay with normal TWFE DiD.

Abraham and Sun



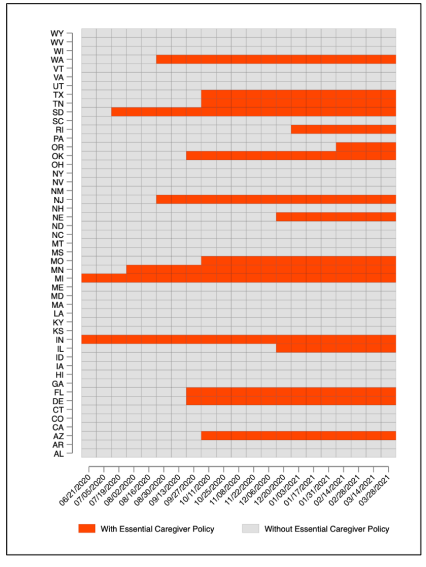
Model Comparison

- Callaway & Sant'Anna
  - Can be *very* flexible in determining which control units to consider, including never-treated and not-yet-treated.
  - Has a more flexible functional form as well (easier to adjust for pre-treatment covariates).
  - *Doubly robust* estimation: combine propensity score and linear regression so that we will get unbiased estimate if either model specification is correct.
- Abraham & Sun
  - Very similar to regular TWFE OLS and hence easy to explain.
  - Control units are all units not treated within the data sample. If most of your units are treated by the end (or all), this can make control units very non-representative and restricted.
- Without covariates, Callaway & Sant'Anna and Abraham & Sun yield same results.

What Are The Best Practices?

- It is still safe to use TWFE DiD when:
  - there is only a **single treatment period**;
  - the treatment effects are **homogeneous**.
- When there are multiple time periods, we should consider to:
  - plot the **treatment timing across cohorts**;
  - decompose the TWFE estimator with **Bacon-decomposition**.
- When using event-study specification, we should **avoid binning relative-time periods** unless we have reasons to believe homogeneous effects apply in the relative-time periods within a bin.

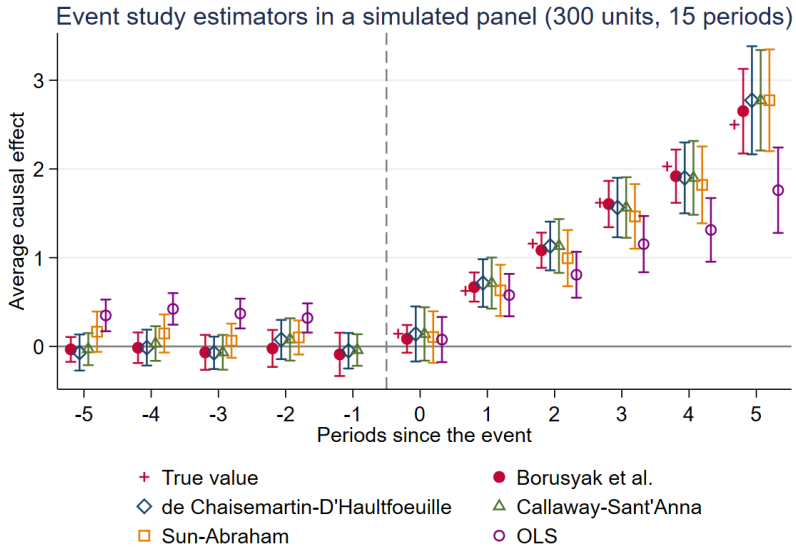
Plot Treatment Timing Across Cohort



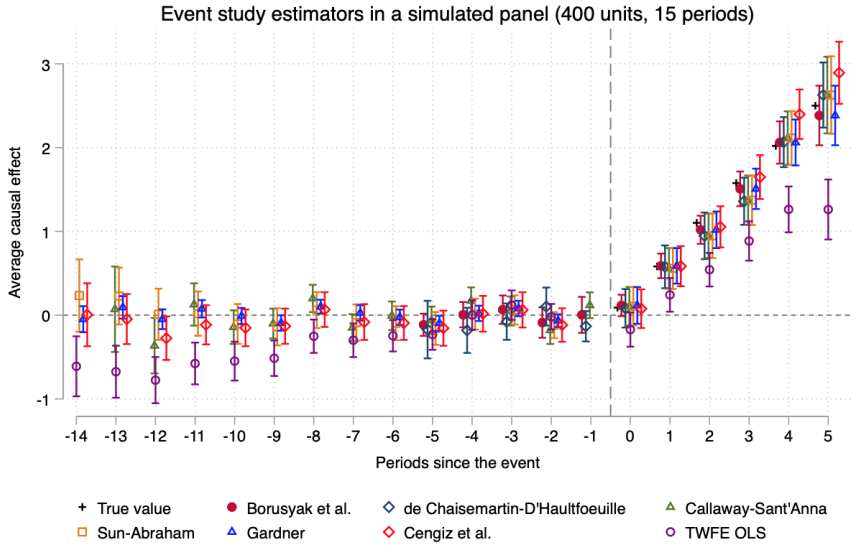
What Are The Best Practices?

- If there is justifiable concern for bias, we should apply at least one of the **alternative estimators**.
    - *Callaway & Sant'Anna, Abraham and Sun, de Chaisemartin-D'Haultfoeuille, etc.*
- When using the alternative estimators, we should justify their choice of **clean comparison groups** (not-yet treated, last treated, or never treated) and articulate why the parallel-trends assumption is likely to apply.
- Regardless of the estimators used, static DiD estimates should be accompanied by **event-study estimates**.
- It is good practice to use more than one alternative estimators and compare their estimates. However, it is really uncessary to include all the available estimators...

This Is Fine


Event study estimators in a simulated panel (300 units, 15 periods)
Average causal effect
0
1
2
3
-5
-4
-3
-2
-1
0
1
2
3
4
5
Periods since the event
True value
Borusyak et al.
de Chaisemartin-D'Haultfoeuille
Callaway-Sant'Anna
Sun-Abraham
OLS

This Is Too Much



Takeaways

- TWFE estimator is a power tool and we should still use it for DiD when appropriate.
- However, we should make sure we understand what we're doing. DiD is a **comparison of means** and at a minimum we should know which means we're comparing.
- Multiple new methods have been proposed, all of which ensure that you **aren't using prior treated units as controls**.
- You should probably tailor your selection of method to your **data structure**: they use and discard different amount of control units and depending on your setting this might matter.
- You don't have to use all the new methods in your project!

References

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