

Large scale optimization for machine learning

Exercise n°2

Exercise 2

Estimation of a quantile response.

Let $x \mapsto F(x)$ be an unknown distribution function such that

$$F(\theta) = \alpha, \quad 0 < \alpha < 1, \quad F'(\theta) > 0.$$

Consider a sequence (z_n) of independent random variables with distribution function $\mathbb{P}(z_n \leq x) = F(x)$, and let the sequence (y_n) to be defined by

$$y_n = 1 \quad \text{if} \quad z_n \leq x_n$$

and $y_n = 0$ otherwise. The sequence (x_n) is the Markov chain studied in the paper of Robbins and Monro defined by

$$x_{n+1} = x_n + a_n \cdot (\alpha - y_n)$$

1. Choose F and α then compute the Markov chain (x_n) for some value of n . Try to solve $F(\theta) = \alpha$ with this Markov chain.
2. Do you observe the convergence of the Markov chain if the assumption are relaxed. For instance :
 - a) if F is a Gaussian distribution (this is not a distribution with compact support).
 - b) if the sequence (a_n) is not of type $1/n$.