Bayesian Spatial Analysis on Global Suicide Rates

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```r
options(repos = c(CRAN = "https://cloud.r-project.org"))
knitr::opts_chunk$set(warning = FALSE, message = FALSE, error = TRUE)
library(tidyverse)
```

```
## -- Attaching core tidyverse packages ------------------------ tidyverse 2.0.0 --
## v dplyr     1.1.4     v readr     2.1.4
## v forcats   1.0.0     v stringr   1.5.1
## v ggplot2   3.4.4     v tibble    3.2.1
## v lubridate 1.9.3     v tidyr     1.3.0
## v purrr     1.0.2
## -- Conflicts ------------------------------------------ tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()
## i Use the conflicted package (<http://conflicted.r-lib.org/>) to force all conflicts to become error
```

```r
theme_set(theme_bw())
require(extraDistr) #need for rdunif
```

```
## Loading required package: extraDistr
##
## Attaching package: 'extraDistr'
##
## The following object is masked from 'package:purrr':
##
##     rdunif
```

```r
library(dplyr)
suppressPackageStartupMessages(require(rstan))

#libraries for spatial data objects
#install these packages if they're not already installed :)
# install.packages(c("sf", "spdep", "rgdal")) #sf for vector datam-> shapefile
# install.packages("terra")   # for raster data
required_packages <- c("sf", "spdep", "terra", "dplyr", "readr", "rnaturalearth", "rnaturalearthdata")
installed_packages <- rownames(installed.packages())

for (pkg in required_packages) {
  if (!(pkg %in% installed_packages)) {
    install.packages(pkg)
  }
}

library(sf)
```

```
## Linking to GEOS 3.10.2, GDAL 3.4.2, PROJ 8.2.1; sf_use_s2() is TRUE
```

```r
library(spdep)
```

```
## Loading required package: spData
## To access larger datasets in this package, install the spDataLarge
## package with: `install.packages('spDataLarge',
## repos='https://nowosad.github.io/drat/', type='source')`
```

```r
library(terra)
```

```
## terra 1.8.42
##
## Attaching package: 'terra'
##
## The following object is masked from 'package:rstan':
##
##     extract
##
## The following object is masked from 'package:tidyr':
##
##     extract
```

```r
library(dplyr)
library(readr)
library(rnaturalearth)
library(rnaturalearthdata)
```

```
##
## Attaching package: 'rnaturalearthdata'
##
## The following object is masked from 'package:rnaturalearth':
##
##     countries110
```

```r
#For fuzzy matching rnaturalearthdata country names to our dataset's country names
install.packages("fuzzyjoin")
```

```
##
## The downloaded binary packages are in
##  /var/folders/h2/0z07kqqn1n99gzcftq2ldtv40000gp/T//RtmpE1j3UM/downloaded_packages
```

```r
library(fuzzyjoin)
```

Introduction

Mental health has become an increasingly important topic and suicide remains a significant public health concern worldwide, with rates varying across regions due to complex social, economic, and cultural factors. Therefore, understanding the geographic distribution of suicide rates may be important for the development of targeted mental health policies and preventative measures. Although place of habitation clearly affect mental and physical lifestyles, there have few studies conducted on the geographical relationship between suicide rates and mental well-being.

In this study, we apply a Bayesian hierarchical model with a conditionally autoregressive (CAR) prior to investigate spatial patterns in suicide rates across countries, **!!TODO: THIS PART IS WRONG NOW: stratified by sex groups. We will be using a Poisson likelihood to obtain a prior distribution.** After which, inferential analysis will be done using Monte Carlo Markov Chain (MCMC) done in Stan.

!! TODO: THIS PART IS WRONG NOW: This research specifically focuses on two recent

years, 2019 and 2021, to investigate any observable changes in suicide patterns potentially influenced by global events like the COVID-19 pandemic. **!! TODO: THIS PART NEEDS TO CHANGE: Our main research question is: Are there identifiable spatial clusters of high suicide rates, and do these rates differ between 2019 and 2021?**

This approach allows us to identify high-risk regions, quantify uncertainty, and better understand how neighboring countries may influence each other's suicide rates. Valuable insights from this analysis may provide the opportunity to create more data-informed mental health interventions.

!! TODO: add link to github repo

Literature Review

TO DO

A similar study on the relationship between location and suicide has been conducted, but the data was limited to regions in London and with the rise of social media and the global pandemic, the information may now be outdated (Congdon, P., 1997). In comparison, our dataset contains over 180 countries and the analysis focuses on 2021, which may show the impact of the COVID-19 pandemic on suicide rates.

"The social restriction practices and policies imposed by different countries secondary to the COVID-19 pandemic might have negatively influenced the fore-said risk factors that has been indirectly led increased rates of suicidal attempts and deaths" (Pathirathna et al., 2022).

Dataset and Data Cleaning

Dataset Name: Crude Suicide Rate (Per 100,000 Population)

Source: https://www.who.int/data/gho/data/themes/mental-health/suicide-rates

Description: The raw dataset has notable features like country, age group, sex, and suicide rate (per 100,000 people) that can be extracted.

Location: Country name

Period: Year (2019, 2021)

Dim1: Sex ("Female", "Both sexes", "Male)

FactValueNumeric: Number of suicide deaths in a year, divided by the population and multiplied by 100 000 (as indicated in the original data source)

FactValueNumericLow: Low estimate

FactValueNumericHigh: High estimate

Note: The FactValueNumeric data are estimates of the number of suicides. "The estimates are derived from the WHO Global Health Estimates (GHE)" [data source]. However, some countries may not have an accurate way of recording the exact number of deaths, potentially leading to inaccurate estimations. Hence there is a high and low in the death rates. "For countries without high-quality death registration data, cause of death estimates are calculated using other data, including household surveys with verbal autopsy, sample or sentinel registration systems, special studies" [data source].

```
data_raw = read.csv("suicide_rate_raw.csv", header = TRUE)
#filter out "both sexes" to avoid duplication
data = as.data.frame(data_raw |> select(Location, Period, Dim1, FactValueNumeric, FactValueNumericLow, F
unique(data$Period)
```

```
## [1] 2021 2019
```

```r
# unique(data$Location)
max(data$FactValueNumeric)
```

```
## [1] 48
```

```r
min(data$FactValueNumeric)
```

```
## [1] 0
```

```r
nrow(data)
```

```
## [1] 740
```

The dataset after filtering consists of 740 observations.

Data Analysis

As we have obtained the cleaned data for suicide rates in 2019 and 2021, we can now declare a prior model from information obtained historically.

Model

Priors:

$\mu \sim \mathcal{N}(9.2, 3)$

$\beta \sim \mathcal{N}(0, 1)$

$\sigma_\phi \sim \text{Exp}(1)$

$\sigma \sim \text{Exp}(1)$

$\phi_{\text{node1}[i]} - \phi_{\text{node2}[i]} \sim \mathcal{N}\left(0, \sigma_\phi^2\right) \quad \text{for } i = 1, \ldots, N_{\text{edges}}$

$\sum_{r=1}^{R} \phi_r^2 \sim \mathcal{N}\left(0, R \cdot \sigma_\phi^2\right)$

Likelihood:

$y_n \sim \mathcal{N}(\mu + \beta \cdot t_n + \phi_{r_n}, \sigma) \quad \text{for } n = 1, \ldots, N$

In 2020, the global average suicide rate was 9.2 people per 100,000 people (World Health Organization, n.d.). Therefore, we've chosen this as the mean for our prior on the estimate of the global suicide rate μ. Additionally, a standard deviation of 3 allows for reasonable uncertainty around the average without being overly tight.

The β parameter represents the effect of time, for which we've chosen a weakly information parameter. **NOTE/QUESTION/TODO: the initial prior we picked was normal(0.1, 0.05) based on research so we did we change this to (0,1)?**

The prior on both the standard deviation of spatial effects σ_ϕ and the observation noise σ is $Exp(1)$, which allows for smaller, more reasonable standard deviations.

Get adjacency pairs

Firstly we need to know which countries are neighbors of each other https://cran.r-project.org/web/packages/rnaturalearth/vignettes/rnaturalearth.html

During the data analysis process we realized that the country names in our dataset did not match with the country names of the rnaturalearthdata dataset that we are using to model the spatial data. This led to the model mistaking the countries as having no neighbors and producing nodes with values 0. To solve this, we renamed the country names in our dataset to match that of rnaturalworld's

```r
# unique(data$Location)
# unique(world_sf$admin)

data_cleaned <- data %>%
  mutate(Location = case_when(
    Location == "Viet Nam" ~ "Vietnam",
    Location == "Türkiye" ~ "Turkey",
    Location == "Iran (Islamic Republic of)" ~ "Iran",
    Location == "Russian Federation" ~ "Russia",
    Location == "Republic of Korea" ~ "South Korea",
    Location == "Syrian Arab Republic" ~ "Syria",
    Location == "Brunei Darussalam" ~ "Brunei",
    Location == "Netherlands (Kingdom of the)" ~ "Netherlands",
    Location == "Republic of Moldova" ~ "Moldova",
    Location == "Lao People's Democratic Republic" ~ "Laos",
    Location == "United Kingdom of Great Britain and Northern Ireland" ~ "United Kingdom",
    Location == "Venezuela (Bolivarian Republic of)" ~ "Venezuela",
    Location == "Bolivia (Plurinational State of)" ~ "Bolivia",
    Location == "Democratic People's Republic of Korea" ~ "North Korea",
    Location == "Micronesia (Federated States of)" ~ "Federated States of Micronesia",
    Location == "Cote d'Ivoire" ~ "Ivory Coast",
    Location == "Eswatini" ~ "eSwatini",
    Location == "Timor-Leste" ~ "East Timor",
    Location == "occupied Palestinian territory, including east Jerusalem" ~ "Palestine",
    Location == "Sao Tome and Principe" ~ "São Tomé and Principe",
    Location == "Bahamas" ~ "The Bahamas",
    Location == "Congo" ~ "Republic of the Congo",
    Location == "Serbia" ~ "Republic of Serbia",
    TRUE ~ Location  # keep all other names unchanged
  ))
#Search for country names in both datasets for debugging and filtering names
#subset(data_cleaned, grepl("Singapore", Location, ignore.case = TRUE))
# unique(subset(world_sf, grepl("Singapore", admin, ignore.case = TRUE)))
```

Now we can join the two datasets so our original dataset will have adjacency parameters from world_sf

```r
#From rnaturalearth dataset
world_sf <- ne_countries(scale = "medium", returnclass = "sf") %>%
  st_make_valid() %>%
  filter(admin %in% data_cleaned$Location) %>%
  arrange(admin)  # ensure a consistent order

world_sf$region_id <- 1:nrow(world_sf)

data_matched <- data_cleaned %>%
  filter(Location %in% world_sf$admin) %>%
  left_join(world_sf %>% st_drop_geometry() %>% select(admin, region_id),
            by = c("Location" = "admin"))

stopifnot(all(!is.na(data_matched$region_id)))

any(world_sf$region_id == 0)
```

```
## [1] FALSE
```

Convert Neighbor List to Adjacency Pairs

```
world_sp <- as(world_sf, "Spatial")

#### making neighbors!!
neighbors <- poly2nb(world_sp, row.names = world_sf$region_id)

num_neighbors <- sapply(neighbors, length)
R <- length(neighbors)

#regions with at least one neighbor -> we want to leave out countries with no neighbors
valid_indices <- which(num_neighbors > 0)

node1 <- c()
node2 <- c()

for (i in valid_indices) { #only make nodes for countries with neighbors
  for (j in neighbors[[i]]) {
    if (j != 0 && world_sf$region_id[j] != 0) { #i purposely excluded zeros so node2 doesnt have 0 "ind
      node1 <- c(node1, world_sf$region_id[i])
      node2 <- c(node2, world_sf$region_id[j])
    }
  }
}

stopifnot(!any(node2 == 0))
length(node1)
```

```
## [1] 610
```

```
length(node2)
```

```
## [1] 610
```

```
any(node2==0) #a lot of debugging led to this being false
```

```
## [1] FALSE
```

STAN Data List

```
nrow(data_matched)
```

```
## [1] 740
```

```
stan_data <- list(
  N = nrow(data_matched),
  y = data_matched$FactValueNumeric,
  time = as.integer(data_matched$Period == 2021), #remove this
  R = R,
  region = data_matched$region_id,
  N_edges = length(node1),
  node1 = node1,
  node2 = node2,
  num_neighbors = num_neighbors
)
```

Extract posterior data from STAN code file Code reference: https://ubc-stat-ml.github.io/web447/w08_mcmc1/topic06_hands_on.html Why use iter = 2000 and chains = 4:

```r
#setwd("C:/Users/Minh/OneDrive/Documents/Bayesian-Data-Analysis-Project")
model <- stan_model(file = "model.stan")
fit <- sampling(model, data = stan_data, iter = 4000, warmup = 2000, chains = 4, seed = 123)
```

```
##
## SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 0.000373 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 3.73 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: Iteration:    1 / 4000 [  0%]  (Warmup)
## Chain 1: Iteration:  400 / 4000 [ 10%]  (Warmup)
## Chain 1: Iteration:  800 / 4000 [ 20%]  (Warmup)
## Chain 1: Iteration: 1200 / 4000 [ 30%]  (Warmup)
## Chain 1: Iteration: 1600 / 4000 [ 40%]  (Warmup)
## Chain 1: Iteration: 2000 / 4000 [ 50%]  (Warmup)
## Chain 1: Iteration: 2001 / 4000 [ 50%]  (Sampling)
## Chain 1: Iteration: 2400 / 4000 [ 60%]  (Sampling)
## Chain 1: Iteration: 2800 / 4000 [ 70%]  (Sampling)
## Chain 1: Iteration: 3200 / 4000 [ 80%]  (Sampling)
## Chain 1: Iteration: 3600 / 4000 [ 90%]  (Sampling)
## Chain 1: Iteration: 4000 / 4000 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 14.33 seconds (Warm-up)
## Chain 1:                9.45 seconds (Sampling)
## Chain 1:                23.78 seconds (Total)
## Chain 1:
##
## SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 2).
## Chain 2:
## Chain 2: Gradient evaluation took 0.000151 seconds
## Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 1.51 seconds.
## Chain 2: Adjust your expectations accordingly!
## Chain 2:
## Chain 2:
## Chain 2: Iteration:    1 / 4000 [  0%]  (Warmup)
## Chain 2: Iteration:  400 / 4000 [ 10%]  (Warmup)
## Chain 2: Iteration:  800 / 4000 [ 20%]  (Warmup)
## Chain 2: Iteration: 1200 / 4000 [ 30%]  (Warmup)
## Chain 2: Iteration: 1600 / 4000 [ 40%]  (Warmup)
## Chain 2: Iteration: 2000 / 4000 [ 50%]  (Warmup)
## Chain 2: Iteration: 2001 / 4000 [ 50%]  (Sampling)
## Chain 2: Iteration: 2400 / 4000 [ 60%]  (Sampling)
## Chain 2: Iteration: 2800 / 4000 [ 70%]  (Sampling)
## Chain 2: Iteration: 3200 / 4000 [ 80%]  (Sampling)
## Chain 2: Iteration: 3600 / 4000 [ 90%]  (Sampling)
## Chain 2: Iteration: 4000 / 4000 [100%]  (Sampling)
## Chain 2:
## Chain 2:  Elapsed Time: 15.043 seconds (Warm-up)
## Chain 2:                9.021 seconds (Sampling)
```

```
## Chain 2:                24.064 seconds (Total)
## Chain 2:
##
## SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 3).
## Chain 3:
## Chain 3: Gradient evaluation took 0.000145 seconds
## Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 1.45 seconds.
## Chain 3: Adjust your expectations accordingly!
## Chain 3:
## Chain 3:
## Chain 3: Iteration:    1 / 4000 [  0%]  (Warmup)
## Chain 3: Iteration:  400 / 4000 [ 10%]  (Warmup)
## Chain 3: Iteration:  800 / 4000 [ 20%]  (Warmup)
## Chain 3: Iteration: 1200 / 4000 [ 30%]  (Warmup)
## Chain 3: Iteration: 1600 / 4000 [ 40%]  (Warmup)
## Chain 3: Iteration: 2000 / 4000 [ 50%]  (Warmup)
## Chain 3: Iteration: 2001 / 4000 [ 50%]  (Sampling)
## Chain 3: Iteration: 2400 / 4000 [ 60%]  (Sampling)
## Chain 3: Iteration: 2800 / 4000 [ 70%]  (Sampling)
## Chain 3: Iteration: 3200 / 4000 [ 80%]  (Sampling)
## Chain 3: Iteration: 3600 / 4000 [ 90%]  (Sampling)
## Chain 3: Iteration: 4000 / 4000 [100%]  (Sampling)
## Chain 3:
## Chain 3:  Elapsed Time: 15.064 seconds (Warm-up)
## Chain 3:                8.957 seconds (Sampling)
## Chain 3:                24.021 seconds (Total)
## Chain 3:
##
## SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 4).
## Chain 4:
## Chain 4: Gradient evaluation took 0.000148 seconds
## Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 1.48 seconds.
## Chain 4: Adjust your expectations accordingly!
## Chain 4:
## Chain 4:
## Chain 4: Iteration:    1 / 4000 [  0%]  (Warmup)
## Chain 4: Iteration:  400 / 4000 [ 10%]  (Warmup)
## Chain 4: Iteration:  800 / 4000 [ 20%]  (Warmup)
## Chain 4: Iteration: 1200 / 4000 [ 30%]  (Warmup)
## Chain 4: Iteration: 1600 / 4000 [ 40%]  (Warmup)
## Chain 4: Iteration: 2000 / 4000 [ 50%]  (Warmup)
## Chain 4: Iteration: 2001 / 4000 [ 50%]  (Sampling)
## Chain 4: Iteration: 2400 / 4000 [ 60%]  (Sampling)
## Chain 4: Iteration: 2800 / 4000 [ 70%]  (Sampling)
## Chain 4: Iteration: 3200 / 4000 [ 80%]  (Sampling)
## Chain 4: Iteration: 3600 / 4000 [ 90%]  (Sampling)
## Chain 4: Iteration: 4000 / 4000 [100%]  (Sampling)
## Chain 4:
## Chain 4:  Elapsed Time: 16.147 seconds (Warm-up)
## Chain 4:                8.967 seconds (Sampling)
## Chain 4:                25.114 seconds (Total)
## Chain 4:
```

```r
print(fit)
```

```
## Inference for Stan model: anon_model.
## 4 chains, each with iter=4000; warmup=2000; thin=1;
## post-warmup draws per chain=2000, total post-warmup draws=8000.
##
##              mean se_mean   sd   2.5%    25%    50%    75%  97.5%
## mu           9.13    0.32 2.76   3.89   7.33   9.05  10.83  14.93
## beta        -0.28    0.01 0.45  -1.19  -0.58  -0.28   0.02   0.58
## phi[1]      -5.09    0.33 4.21 -13.56  -7.84  -5.09  -2.25   3.11
## phi[2]      -5.37    0.33 4.30 -13.76  -8.25  -5.32  -2.51   3.11
## phi[3]      -6.68    0.32 4.17 -14.92  -9.46  -6.65  -3.88   1.42
## phi[4]      -1.14    0.33 4.28  -9.67  -4.06  -1.08   1.72   7.33
## phi[5]      -8.24    0.32 4.40 -17.00 -11.22  -8.21  -5.23   0.42
## phi[6]      -0.09    0.32 4.20  -8.25  -2.94  -0.09   2.79   8.09
## phi[7]      -6.13    0.32 4.27 -14.54  -8.98  -6.08  -3.20   2.13
## phi[8]       4.57    0.32 4.39  -4.15   1.68   4.69   7.58  12.95
## phi[9]       5.46    0.33 4.22  -2.99   2.65   5.55   8.27  13.52
## phi[10]     -6.63    0.33 4.17 -14.91  -9.42  -6.66  -3.86   1.68
## phi[11]     -4.62    0.33 4.42 -13.35  -7.46  -4.63  -1.58   3.92
## phi[12]     -5.75    0.33 4.31 -14.30  -8.59  -5.72  -2.80   2.48
## phi[13]     -4.69    0.32 4.34 -13.39  -7.61  -4.66  -1.76   3.81
## phi[14]      8.70    0.32 4.24   0.41   5.91   8.74  11.56  17.02
## phi[15]      8.83    0.33 4.26   0.37   6.02   8.85  11.72  17.12
## phi[16]     -4.35    0.34 4.40 -13.27  -7.24  -4.29  -1.41   4.16
## phi[17]     -1.79    0.33 4.30 -10.35  -4.63  -1.73   1.04   6.54
## phi[18]     -3.94    0.32 4.37 -12.82  -6.74  -3.86  -0.98   4.43
## phi[19]     -3.90    0.35 4.27 -12.32  -6.79  -3.87  -1.04   4.52
## phi[20]      1.56    0.33 4.32  -7.00  -1.30   1.59   4.50   9.93
## phi[21]      0.13    0.32 4.25  -8.25  -2.68   0.11   2.98   8.48
## phi[22]     -0.75    0.32 4.11  -8.92  -3.47  -0.66   2.03   7.23
## phi[23]     -6.24    0.33 4.38 -14.92  -9.12  -6.19  -3.36   2.40
## phi[24]      0.27    0.33 4.25  -8.12  -2.54   0.37   3.06   8.55
## phi[25]     -0.79    0.33 4.18  -8.96  -3.61  -0.75   2.06   7.29
## phi[26]     -1.66    0.33 4.27 -10.11  -4.51  -1.65   1.21   6.77
## phi[27]      5.58    0.34 4.34  -2.96   2.66   5.65   8.48  13.93
## phi[28]     -4.19    0.34 4.31 -12.66  -7.08  -4.15  -1.28   4.21
## phi[29]     -0.81    0.33 4.22  -9.39  -3.56  -0.68   2.01   7.31
## phi[30]      2.20    0.33 4.37  -6.38  -0.72   2.19   5.11  10.83
## phi[31]      0.13    0.32 4.22  -8.27  -2.64   0.16   2.90   8.49
## phi[32]     -3.63    0.34 4.24 -12.03  -6.48  -3.58  -0.79   4.65
## phi[33]     -0.48    0.33 4.29  -9.23  -3.33  -0.47   2.37   7.84
## phi[34]     -0.05    0.33 4.05  -7.81  -2.78  -0.03   2.65   7.85
## phi[35]     -3.79    0.34 4.26 -12.16  -6.64  -3.76  -0.92   4.49
## phi[36]     -2.87    0.33 4.37 -11.59  -5.79  -2.86   0.03   5.63
## phi[37]     -1.53    0.34 4.32 -10.20  -4.39  -1.51   1.35   7.08
## phi[38]      6.54    0.34 4.17  -1.84   3.75   6.64   9.34  14.51
## phi[39]      4.86    0.32 4.40  -3.91   1.89   4.93   7.83  13.30
## phi[40]     -5.25    0.32 4.37 -13.80  -8.15  -5.25  -2.34   3.24
## phi[41]      3.89    0.32 4.26  -4.39   1.05   3.88   6.73  12.27
## phi[42]     -0.68    0.33 4.19  -8.95  -3.40  -0.68   2.09   7.45
## phi[43]      1.98    0.32 4.37  -7.02  -0.90   2.09   4.92  10.38
## phi[44]     -0.88    0.33 4.28  -9.15  -3.82  -0.88   2.05   7.50
## phi[45]     -3.91    0.33 4.40 -12.83  -6.78  -3.90  -0.91   4.41
```

```
## phi[46]      -5.41    0.32  4.39   -14.21    -8.35    -5.38    -2.43     3.12
## phi[47]      -1.16    0.32  4.33    -9.73    -4.04    -1.13     1.78     7.14
## phi[48]      -8.02    0.32  4.25   -16.46   -10.88    -7.95    -5.11     0.27
## phi[49]      -1.25    0.34  4.38    -9.94    -4.08    -1.22     1.72     7.30
## phi[50]      -1.73    0.34  4.36   -10.33    -4.68    -1.67     1.25     6.64
## phi[51]       3.75    0.34  4.33    -5.06     0.97     3.84     6.60    12.16
## phi[52]       7.00    0.32  4.31    -1.40     4.07     7.08     9.94    15.49
## phi[53]      -2.91    0.32  4.22   -11.14    -5.67    -2.90    -0.14     5.49
## phi[54]      11.22    0.33  4.41     2.51     8.27    11.19    14.17    19.85
## phi[55]      -0.12    0.33  4.41    -8.87    -3.12    -0.11     2.86     8.28
## phi[56]       5.68    0.32  4.29    -2.68     2.75     5.70     8.58    14.01
## phi[57]       6.53    0.33  4.26    -2.33     3.83     6.62     9.35    14.65
## phi[58]      -1.08    0.32  4.28    -9.48    -4.00    -1.05     1.84     7.21
## phi[59]      -4.04    0.34  4.37   -12.95    -6.89    -3.97    -1.16     4.43
## phi[60]      -2.38    0.33  4.28   -10.79    -5.21    -2.38     0.52     6.00
## phi[61]       3.92    0.33  4.20    -4.47     1.16     4.00     6.71    12.06
## phi[62]      -3.45    0.33  4.31   -12.17    -6.24    -3.39    -0.58     4.96
## phi[63]      -3.92    0.32  4.26   -12.40    -6.76    -3.93    -1.10     4.44
## phi[64]      -7.86    0.34  4.46   -16.68   -10.79    -7.88    -4.82     0.70
## phi[65]      -3.57    0.31  4.27   -12.08    -6.43    -3.56    -0.69     4.82
## phi[66]      -3.87    0.34  4.19   -12.15    -6.63    -3.87    -1.06     4.37
## phi[67]      -1.50    0.32  4.24    -9.91    -4.35    -1.52     1.35     6.71
## phi[68]      16.65    0.33  4.33     8.05    13.80    16.66    19.62    24.94
## phi[69]      -0.85    0.33  4.32    -9.54    -3.73    -0.72     2.01     7.43
## phi[70]      -5.79    0.33  4.27   -14.22    -8.64    -5.73    -2.95     2.57
## phi[71]       7.71    0.33  4.20    -0.70     4.92     7.77    10.59    15.92
## phi[72]       2.90    0.33  4.39    -5.92    -0.03     2.91     5.85    11.48
## phi[73]       3.34    0.33  4.27    -5.24     0.49     3.35     6.17    11.53
## phi[74]      -7.44    0.32  4.21   -15.92   -10.27    -7.39    -4.64     0.67
## phi[75]      -4.87    0.33  4.22   -13.26    -7.63    -4.83    -2.02     3.35
## phi[76]      -6.03    0.33  4.25   -14.43    -8.88    -6.01    -3.22     2.28
## phi[77]      -0.58    0.32  4.38    -9.27    -3.50    -0.60     2.37     8.11
## phi[78]      -4.70    0.33  4.26   -13.21    -7.49    -4.60    -1.90     3.68
## phi[79]      -1.23    0.33  4.26    -9.62    -4.04    -1.23     1.60     7.24
## phi[80]      -1.69    0.33  4.15    -9.85    -4.44    -1.65     1.03     6.62
## phi[81]      -7.30    0.33  4.42   -15.87   -10.29    -7.34    -4.35     1.62
## phi[82]       8.13    0.32  4.38    -0.61     5.18     8.15    11.12    16.53
## phi[83]      -8.19    0.32  4.26   -16.75   -11.03    -8.18    -5.34     0.07
## phi[84]       7.39    0.33  4.24    -1.18     4.59     7.44    10.29    15.47
## phi[85]      -3.42    0.32  4.21   -11.70    -6.25    -3.36    -0.55     4.68
## phi[86]      10.06    0.33  4.33     1.37     7.19    10.12    12.98    18.45
## phi[87]      -7.15    0.33  4.28   -15.75    -9.93    -7.17    -4.27     1.22
## phi[88]      -1.12    0.33  4.30    -9.80    -4.00    -1.02     1.78     7.31
## phi[89]      -4.10    0.32  4.23   -12.52    -6.93    -4.08    -1.26     4.14
## phi[90]       8.33    0.32  4.32    -0.42     5.44     8.38    11.25    16.62
## phi[91]      -8.23    0.33  4.29   -16.65   -11.08    -8.19    -5.34     0.14
## phi[92]      19.85    0.32  4.31    11.23    16.92    19.88    22.80    28.31
## phi[93]      -3.06    0.33  4.28   -11.63    -5.89    -3.03    -0.17     5.26
## phi[94]      -3.86    0.32  4.19   -12.30    -6.56    -3.85    -1.03     4.32
## phi[95]      15.65    0.34  4.31     7.14    12.77    15.73    18.51    24.12
## phi[96]       1.09    0.33  4.31    -7.45    -1.77     1.15     3.96     9.58
## phi[97]      -2.91    0.33  4.38   -11.70    -5.79    -2.81     0.05     5.64
## phi[98]      -1.48    0.32  4.29   -10.04    -4.39    -1.44     1.52     6.80
## phi[99]      -3.61    0.33  4.31   -12.22    -6.46    -3.63    -0.72     4.73
```

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## phi[100]     -7.29    0.33  4.35  -15.92  -10.21   -7.20   -4.39    1.15
## phi[101]     -4.73    0.33  4.19  -13.00   -7.54   -4.73   -1.89    3.51
## phi[102]     -2.86    0.33  4.41  -11.55   -5.84   -2.82    0.17    5.72
## phi[103]     -6.14    0.31  4.29  -14.47   -9.04   -6.14   -3.33    2.35
## phi[104]      1.13    0.32  4.40   -7.62   -1.75    1.11    4.05    9.89
## phi[105]     -2.28    0.32  4.26  -10.80   -5.09   -2.23    0.60    6.05
## phi[106]      5.26    0.33  4.33   -3.43    2.39    5.30    8.15   13.56
## phi[107]      8.79    0.33  4.40    0.21    5.81    8.91   11.80   17.06
## phi[108]      3.84    0.32  4.30   -4.72    0.98    3.86    6.72   11.98
## phi[109]     -5.92    0.32  4.37  -14.54   -8.88   -5.87   -2.99    2.45
## phi[110]      2.69    0.33  4.26   -5.79   -0.09    2.73    5.47   11.02
## phi[111]     -5.38    0.32  4.20  -13.64   -8.16   -5.40   -2.60    2.89
## phi[112]      0.03    0.33  4.22   -8.36   -2.74    0.07    2.84    8.25
## phi[113]      1.47    0.32  4.34   -6.97   -1.39    1.50    4.32   10.11
## phi[114]      2.45    0.32  4.33   -6.19   -0.43    2.46    5.36   11.11
## phi[115]      2.97    0.33  4.34   -5.56    0.03    3.00    5.85   11.48
## phi[116]     -4.42    0.33  4.26  -13.03   -7.18   -4.34   -1.51    3.77
## phi[117]     -4.37    0.33  4.22  -12.82   -7.16   -4.34   -1.59    4.00
## phi[118]     -4.20    0.32  4.30  -12.82   -7.00   -4.16   -1.28    3.91
## phi[119]      1.36    0.34  4.34   -7.23   -1.54    1.44    4.31    9.72
## phi[120]     -3.25    0.33  4.32  -11.94   -6.07   -3.18   -0.33    5.11
## phi[121]      4.15    0.33  4.25   -4.29    1.36    4.17    6.98   12.42
## phi[122]     -7.66    0.34  4.27  -16.05  -10.52   -7.68   -4.75    0.72
## phi[123]     -3.07    0.32  4.22  -11.51   -5.86   -3.00   -0.23    5.03
## phi[124]     -8.22    0.33  4.28  -16.76  -11.01   -8.22   -5.32   -0.01
## phi[125]     -5.92    0.32  4.30  -14.40   -8.82   -5.89   -2.97    2.51
## phi[126]     -7.09    0.32  4.44  -16.11  -10.03   -7.02   -4.10    1.49
## phi[127]     -1.90    0.33  4.26  -10.26   -4.71   -1.83    0.97    6.26
## phi[128]     -6.57    0.33  4.26  -15.26   -9.33   -6.48   -3.80    1.60
## phi[129]     -5.66    0.33  4.35  -14.21   -8.58   -5.61   -2.75    2.81
## phi[130]      5.52    0.32  4.17   -2.78    2.72    5.50    8.42   13.61
## phi[131]      3.34    0.32  4.32   -5.22    0.45    3.40    6.25   11.66
## phi[132]     -3.22    0.32  4.42  -11.94   -6.24   -3.19   -0.14    5.26
## phi[133]     -4.71    0.34  4.38  -13.48   -7.61   -4.66   -1.75    3.81
## phi[134]      6.17    0.33  4.16   -2.05    3.45    6.16    8.95   14.27
## phi[135]     -1.92    0.32  4.23  -10.42   -4.76   -1.91    0.97    6.22
## phi[136]      1.26    0.33  4.20   -7.11   -1.55    1.38    4.13    9.37
## phi[137]     12.59    0.33  4.03    4.64    9.88   12.62   15.30   20.68
## phi[138]     -0.61    0.35  4.27   -9.22   -3.45   -0.50    2.29    7.68
## phi[139]     -2.64    0.32  4.40  -11.39   -5.55   -2.61    0.33    5.98
## phi[140]     -8.62    0.32  4.31  -17.31  -11.51   -8.59   -5.67   -0.48
## phi[141]      3.40    0.34  4.45   -5.30    0.47    3.36    6.37   12.20
## phi[142]     -7.78    0.32  4.10  -15.91  -10.55   -7.75   -5.00    0.19
## phi[143]     -2.34    0.33  4.23  -10.62   -5.15   -2.36    0.51    5.99
## phi[144]     -3.20    0.32  4.40  -11.88   -6.12   -3.16   -0.25    5.25
## phi[145]     -2.85    0.33  4.32  -11.39   -5.65   -2.81    0.01    5.66
## phi[146]     -0.86    0.33  4.32   -9.41   -3.76   -0.83    2.10    7.46
## phi[147]      1.31    0.33  4.30   -7.10   -1.59    1.31    4.22    9.81
## phi[148]      9.65    0.34  4.29    0.83    6.83    9.71   12.56   17.81
## phi[149]     10.76    0.33  4.38    2.10    7.83   10.81   13.68   19.30
## phi[150]     -1.15    0.33  4.34   -9.80   -4.05   -1.10    1.81    7.30
## phi[151]     12.02    0.33  4.18    3.77    9.22   12.01   14.76   20.29
## phi[152]     18.58    0.32  4.37    9.83   15.73   18.61   21.47   27.14
## phi[153]     -1.50    0.32  4.22   -9.87   -4.27   -1.50    1.33    6.73
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## phi[154]     -0.27    0.33  4.29    -8.73    -3.16    -0.32     2.68     8.18
## phi[155]      6.67    0.32  4.36    -1.96     3.79     6.69     9.58    15.24
## phi[156]     -5.22    0.33  4.23   -13.48    -8.07    -5.27    -2.37     3.08
## phi[157]     14.74    0.33  4.35     5.91    11.84    14.86    17.65    22.97
## phi[158]      5.23    0.34  4.34    -3.26     2.33     5.27     8.11    13.64
## phi[159]      5.05    0.33  4.24    -3.37     2.23     5.11     7.91    13.35
## phi[160]     -8.20    0.34  4.22   -16.44   -11.01    -8.22    -5.40    -0.04
## phi[161]     -8.15    0.34  4.48   -17.02   -11.20    -8.12    -5.10     0.62
## phi[162]     -5.99    0.33  4.31   -14.58    -8.92    -5.94    -3.13     2.44
## phi[163]      5.32    0.33  4.29    -3.09     2.47     5.24     8.26    13.77
## phi[164]     -5.43    0.32  4.40   -14.25    -8.34    -5.43    -2.44     2.97
## phi[165]      0.36    0.33  4.23    -8.28    -2.37     0.42     3.20     8.49
## phi[166]     -4.22    0.33  4.35   -13.05    -7.10    -4.20    -1.29     4.24
## phi[167]      4.62    0.33  4.40    -4.04     1.65     4.63     7.57    13.27
## phi[168]     -6.66    0.33  4.34   -15.30    -9.53    -6.66    -3.78     1.80
## phi[169]     -5.64    0.34  4.26   -13.93    -8.50    -5.59    -2.81     2.79
## phi[170]     -1.88    0.32  4.24   -10.32    -4.71    -1.82     0.95     6.31
## phi[171]     -3.46    0.33  4.29   -11.81    -6.40    -3.48    -0.53     4.86
## phi[172]     13.16    0.32  4.13     4.96    10.37    13.21    15.96    21.14
## phi[173]     -7.25    0.33  4.30   -15.75   -10.15    -7.19    -4.32     0.91
## phi[174]      0.55    0.34  4.39    -8.24    -2.38     0.62     3.44     9.08
## phi[175]     -3.68    0.32  4.17   -11.96    -6.47    -3.71    -0.83     4.50
## phi[176]      6.05    0.33  4.31    -2.66     3.26     6.03     8.90    14.48
## phi[177]     14.83    0.33  4.28     6.33    12.04    14.89    17.73    23.19
## phi[178]      0.04    0.31  4.15    -8.16    -2.75     0.02     2.82     8.32
## phi[179]      6.83    0.32  4.37    -1.78     3.95     6.82     9.75    15.36
## phi[180]     -0.67    0.33  4.26    -9.18    -3.54    -0.64     2.28     7.46
## phi[181]     -1.57    0.33  4.31    -9.93    -4.48    -1.58     1.27     6.92
## phi[182]     -4.87    0.33  4.38   -13.74    -7.73    -4.82    -1.95     3.62
## phi[183]     -2.05    0.34  4.17   -10.22    -4.87    -2.01     0.78     5.93
## phi[184]      7.29    0.33  4.23    -1.07     4.43     7.35    10.19    15.25
## phi[185]     18.58    0.34  4.33    10.07    15.68    18.64    21.51    26.92
## sigma_phi    31.17    0.05  3.52    24.91    28.70    30.89    33.47    38.48
## sigma         6.82    0.00  0.20     6.44     6.68     6.82     6.95     7.23
## lp__      -1841.58    0.23 11.22 -1864.76 -1848.88 -1841.25 -1833.81 -1820.38
##           n_eff Rhat
## mu           72 1.06
## beta       4775 1.00
## phi[1]      167 1.02
## phi[2]      170 1.03
## phi[3]      165 1.03
## phi[4]      164 1.03
## phi[5]      190 1.02
## phi[6]      173 1.02
## phi[7]      179 1.02
## phi[8]      183 1.02
## phi[9]      162 1.03
## phi[10]     159 1.02
## phi[11]     181 1.02
## phi[12]     169 1.02
## phi[13]     184 1.02
## phi[14]     172 1.02
## phi[15]     166 1.03
## phi[16]     169 1.02
```

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## phi[17]      166 1.02
## phi[18]      182 1.02
## phi[19]      153 1.03
## phi[20]      171 1.02
## phi[21]      178 1.02
## phi[22]      161 1.02
## phi[23]      181 1.02
## phi[24]      168 1.02
## phi[25]      162 1.03
## phi[26]      168 1.03
## phi[27]      167 1.02
## phi[28]      165 1.03
## phi[29]      168 1.02
## phi[30]      172 1.02
## phi[31]      176 1.02
## phi[32]      158 1.03
## phi[33]      166 1.02
## phi[34]      154 1.03
## phi[35]      161 1.03
## phi[36]      176 1.02
## phi[37]      165 1.02
## phi[38]      152 1.03
## phi[39]      186 1.02
## phi[40]      190 1.02
## phi[41]      173 1.03
## phi[42]      158 1.03
## phi[43]      183 1.02
## phi[44]      169 1.02
## phi[45]      178 1.02
## phi[46]      186 1.02
## phi[47]      179 1.02
## phi[48]      173 1.02
## phi[49]      169 1.02
## phi[50]      167 1.02
## phi[51]      162 1.03
## phi[52]      179 1.02
## phi[53]      171 1.02
## phi[54]      184 1.02
## phi[55]      177 1.02
## phi[56]      180 1.02
## phi[57]      164 1.03
## phi[58]      175 1.02
## phi[59]      166 1.03
## phi[60]      168 1.03
## phi[61]      162 1.03
## phi[62]      170 1.02
## phi[63]      174 1.02
## phi[64]      176 1.02
## phi[65]      188 1.02
## phi[66]      153 1.03
## phi[67]      181 1.02
## phi[68]      169 1.03
## phi[69]      169 1.02
## phi[70]      168 1.02
```

```
## phi[71]      163 1.02
## phi[72]      172 1.02
## phi[73]      166 1.02
## phi[74]      174 1.02
## phi[75]      168 1.02
## phi[76]      166 1.03
## phi[77]      187 1.02
## phi[78]      162 1.03
## phi[79]      169 1.02
## phi[80]      162 1.03
## phi[81]      178 1.02
## phi[82]      183 1.02
## phi[83]      177 1.02
## phi[84]      167 1.02
## phi[85]      173 1.02
## phi[86]      176 1.03
## phi[87]      170 1.03
## phi[88]      165 1.02
## phi[89]      175 1.02
## phi[90]      179 1.02
## phi[91]      170 1.02
## phi[92]      185 1.02
## phi[93]      167 1.02
## phi[94]      168 1.02
## phi[95]      158 1.03
## phi[96]      169 1.02
## phi[97]      181 1.02
## phi[98]      177 1.03
## phi[99]      173 1.02
## phi[100]     168 1.02
## phi[101]     164 1.03
## phi[102]     179 1.03
## phi[103]     189 1.02
## phi[104]     188 1.02
## phi[105]     179 1.02
## phi[106]     167 1.03
## phi[107]     183 1.02
## phi[108]     181 1.02
## phi[109]     187 1.02
## phi[110]     165 1.02
## phi[111]     167 1.02
## phi[112]     164 1.02
## phi[113]     184 1.02
## phi[114]     183 1.02
## phi[115]     177 1.02
## phi[116]     171 1.03
## phi[117]     160 1.03
## phi[118]     178 1.02
## phi[119]     165 1.03
## phi[120]     173 1.02
## phi[121]     166 1.03
## phi[122]     162 1.03
## phi[123]     176 1.02
## phi[124]     168 1.03
```

```
## phi[125]     179 1.02
## phi[126]     191 1.02
## phi[127]     163 1.03
## phi[128]     171 1.02
## phi[129]     178 1.02
## phi[130]     166 1.02
## phi[131]     181 1.02
## phi[132]     196 1.02
## phi[133]     163 1.03
## phi[134]     162 1.02
## phi[135]     171 1.02
## phi[136]     166 1.03
## phi[137]     146 1.03
## phi[138]     151 1.03
## phi[139]     188 1.02
## phi[140]     183 1.02
## phi[141]     174 1.02
## phi[142]     160 1.03
## phi[143]     170 1.03
## phi[144]     192 1.02
## phi[145]     176 1.02
## phi[146]     169 1.02
## phi[147]     169 1.03
## phi[148]     160 1.02
## phi[149]     181 1.02
## phi[150]     175 1.02
## phi[151]     161 1.02
## phi[152]     185 1.02
## phi[153]     174 1.03
## phi[154]     165 1.02
## phi[155]     184 1.02
## phi[156]     169 1.03
## phi[157]     175 1.02
## phi[158]     166 1.02
## phi[159]     163 1.03
## phi[160]     151 1.03
## phi[161]     172 1.02
## phi[162]     172 1.02
## phi[163]     170 1.02
## phi[164]     190 1.02
## phi[165]     169 1.02
## phi[166]     178 1.02
## phi[167]     179 1.02
## phi[168]     170 1.02
## phi[169]     161 1.03
## phi[170]     177 1.02
## phi[171]     165 1.02
## phi[172]     164 1.02
## phi[173]     169 1.03
## phi[174]     170 1.02
## phi[175]     166 1.02
## phi[176]     167 1.03
## phi[177]     172 1.02
## phi[178]     178 1.02
```

```
## phi[179]    189 1.02
## phi[180]    162 1.03
## phi[181]    176 1.02
## phi[182]    175 1.02
## phi[183]    150 1.03
## phi[184]    167 1.02
## phi[185]    162 1.03
## sigma_phi  5326 1.00
## sigma      4163 1.00
## lp__       2383 1.00
##
## Samples were drawn using NUTS(diag_e) at Sat Apr 19 16:18:33 2025.
## For each parameter, n_eff is a crude measure of effective sample size,
## and Rhat is the potential scale reduction factor on split chains (at
## convergence, Rhat=1).
```

!!! TODO- evaluation of posterior, e.g. "An appropriate combination of diagnostics, synthetic datasets and other validation strategies."

Model Diagnostics

```r
library(ggplot2)
library(bayesplot)

summary(fit)$summary
```

```
##                    mean     se_mean        sd        2.5%           25%
## mu         9.126953e+00 0.324731903 2.7583882   3.8904780  7.334498e+00
## beta      -2.837201e-01 0.006540313 0.4519360  -1.1899153 -5.810551e-01
## phi[1]    -5.092205e+00 0.325449275 4.2078686 -13.5615908 -7.840893e+00
## phi[2]    -5.367370e+00 0.329762386 4.2995270 -13.7619105 -8.253306e+00
## phi[3]    -6.683763e+00 0.324631103 4.1658533 -14.9247293 -9.463480e+00
## phi[4]    -1.135477e+00 0.334272710 4.2787524  -9.6740174 -4.061437e+00
## phi[5]    -8.237120e+00 0.319162239 4.3962777 -16.9984913 -1.122376e+01
## phi[6]    -9.114193e-02 0.320088608 4.2045219  -8.2500136 -2.941476e+00
## phi[7]    -6.131278e+00 0.318670018 4.2687731 -14.5374540 -8.975650e+00
## phi[8]     4.567687e+00 0.324913098 4.3921442  -4.1505929  1.681541e+00
## phi[9]     5.455532e+00 0.331422483 4.2235146  -2.9925015  2.654372e+00
## phi[10]   -6.631603e+00 0.330819018 4.1742728 -14.9050797 -9.421233e+00
## phi[11]   -4.621690e+00 0.328245738 4.4188624 -13.3475061 -7.460137e+00
## phi[12]   -5.747951e+00 0.331211979 4.3074763 -14.3039579 -8.591440e+00
## phi[13]   -4.690760e+00 0.319952134 4.3438748 -13.3911104 -7.606039e+00
## phi[14]    8.704548e+00 0.323282884 4.2428444   0.4054119  5.908379e+00
## phi[15]    8.825837e+00 0.331086103 4.2638309   0.3730253  6.015868e+00
## phi[16]   -4.351083e+00 0.338803612 4.4025927 -13.2727007 -7.242391e+00
## phi[17]   -1.793152e+00 0.334301687 4.3017488 -10.3544278 -4.632830e+00
## phi[18]   -3.938965e+00 0.324339359 4.3746026 -12.8221434 -6.744689e+00
## phi[19]   -3.901073e+00 0.345400966 4.2690592 -12.3211831 -6.791010e+00
## phi[20]    1.560375e+00 0.330196517 4.3222292  -7.0010354 -1.302585e+00
## phi[21]    1.285830e-01 0.317763054 4.2450216  -8.2488666 -2.679011e+00
## phi[22]   -7.481384e-01 0.323788616 4.1080815  -8.9240069 -3.474617e+00
## phi[23]   -6.235213e+00 0.325449230 4.3778078 -14.9242665 -9.124355e+00
## phi[24]    2.681115e-01 0.328054639 4.2513637  -8.1223096 -2.538878e+00
## phi[25]   -7.891124e-01 0.328333191 4.1791679  -8.9635529 -3.606305e+00
```

```
## phi[26]   -1.664101e+00 0.329175529  4.2719963   -10.1124591 -4.512540e+00
## phi[27]    5.580037e+00 0.336450147  4.3431128    -2.9644390  2.660185e+00
## phi[28]   -4.190581e+00 0.336018133  4.3125452   -12.6567754 -7.084894e+00
## phi[29]   -8.087063e-01 0.325752967  4.2182630    -9.3924747 -3.558701e+00
## phi[30]    2.198389e+00 0.333025159  4.3719513    -6.3842766 -7.182059e-01
## phi[31]    1.324864e-01 0.317434850  4.2153289    -8.2656017 -2.640347e+00
## phi[32]   -3.626271e+00 0.337134017  4.2372378   -12.0316874 -6.478294e+00
## phi[33]   -4.790595e-01 0.333186958  4.2930291    -9.2295561 -3.329100e+00
## phi[34]   -4.923671e-02 0.326450156  4.0463078    -7.8093478 -2.783759e+00
## phi[35]   -3.788927e+00 0.335933283  4.2605730   -12.1569641 -6.642457e+00
## phi[36]   -2.872947e+00 0.328995705  4.3698471   -11.5850409 -5.789055e+00
## phi[37]   -1.531606e+00 0.336510421  4.3234252   -10.1955473 -4.393063e+00
## phi[38]    6.536538e+00 0.337888740  4.1704872    -1.8394729  3.745701e+00
## phi[39]    4.859377e+00 0.322603286  4.4020441    -3.9093595  1.888859e+00
## phi[40]   -5.252168e+00 0.317403685  4.3729575   -13.8017045 -8.153008e+00
## phi[41]    3.894397e+00 0.323668075  4.2606737    -4.3884463  1.050234e+00
## phi[42]   -6.832117e-01 0.333112276  4.1868694    -8.9474912 -3.404029e+00
## phi[43]    1.981022e+00 0.323348136  4.3718381    -7.0215218 -9.038789e-01
## phi[44]   -8.812414e-01 0.329519442  4.2817343    -9.1479985 -3.817268e+00
## phi[45]   -3.913248e+00 0.329557109  4.4017022   -12.8312543 -6.779754e+00
## phi[46]   -5.409911e+00 0.321906424  4.3903850   -14.2144695 -8.347528e+00
## phi[47]   -1.157947e+00 0.324011638  4.3314655    -9.7286541 -4.037638e+00
## phi[48]   -8.019103e+00 0.323103457  4.2495203   -16.4571432 -1.087747e+01
## phi[49]   -1.245405e+00 0.336683863  4.3815685    -9.9435804 -4.081129e+00
## phi[50]   -1.729522e+00 0.336980527  4.3575956   -10.3332583 -4.683205e+00
## phi[51]    3.745826e+00 0.340712605  4.3341877    -5.0560699  9.723147e-01
## phi[52]    7.004786e+00 0.321790535  4.3106472    -1.3951337  4.070629e+00
## phi[53]   -2.910781e+00 0.323052101  4.2234773   -11.1439591 -5.670510e+00
## phi[54]    1.121892e+01 0.325061037  4.4133634     2.5054931  8.267824e+00
## phi[55]   -1.203231e-01 0.331816353  4.4133579    -8.8736190 -3.115699e+00
## phi[56]    5.682238e+00 0.320135648  4.2922010    -2.6841219  2.748580e+00
## phi[57]    6.526381e+00 0.332667687  4.2623097    -2.3285102  3.828325e+00
## phi[58]   -1.079462e+00 0.323420468  4.2773038    -9.4779295 -3.995934e+00
## phi[59]   -4.043471e+00 0.339268057  4.3716119   -12.9452243 -6.889410e+00
## phi[60]   -2.379538e+00 0.329518030  4.2760182   -10.7939048 -5.213551e+00
## phi[61]    3.923293e+00 0.329356314  4.1955230    -4.4731781  1.164638e+00
## phi[62]   -3.454283e+00 0.330587279  4.3054707   -12.1685858 -6.243670e+00
## phi[63]   -3.922339e+00 0.323064574  4.2567531   -12.4048130 -6.762977e+00
## phi[64]   -7.864480e+00 0.336264322  4.4568549   -16.6789381 -1.079283e+01
## phi[65]   -3.571842e+00 0.311600277  4.2684429   -12.0769664 -6.429029e+00
## phi[66]   -3.874491e+00 0.338663207  4.1882991   -12.1472525 -6.631861e+00
## phi[67]   -1.500021e+00 0.315216296  4.2403822    -9.9140442 -4.354698e+00
## phi[68]    1.665069e+01 0.333188265  4.3344456     8.0499443  1.380199e+01
## phi[69]   -8.520414e-01 0.332630718  4.3190559    -9.5441336 -3.731362e+00
## phi[70]   -5.786542e+00 0.329158563  4.2681507   -14.2235969 -8.638594e+00
## phi[71]    7.709212e+00 0.329143333  4.2024261    -0.7033776  4.922249e+00
## phi[72]    2.901825e+00 0.334772420  4.3904646    -5.9163427 -2.985179e-02
## phi[73]    3.336626e+00 0.331336823  4.2660888    -5.2425811  4.860309e-01
## phi[74]   -7.443361e+00 0.319093495  4.2087773   -15.9202365 -1.027127e+01
## phi[75]   -4.868381e+00 0.325140139  4.2152146   -13.2639957 -7.625572e+00
## phi[76]   -6.028899e+00 0.329243734  4.2452323   -14.4310657 -8.876328e+00
## phi[77]   -5.750816e-01 0.319919797  4.3797529    -9.2703679 -3.495428e+00
## phi[78]   -4.699123e+00 0.333938810  4.2568162   -13.2113479 -7.491553e+00
## phi[79]   -1.229889e+00 0.327655634  4.2604194    -9.6157563 -4.043090e+00
```

```
## phi[80]   -1.691652e+00 0.325871122  4.1509242    -9.8520775 -4.440264e+00
## phi[81]   -7.298222e+00 0.331468293  4.4174124   -15.8652286 -1.028616e+01
## phi[82]    8.128895e+00 0.324167715  4.3824005    -0.6105948  5.180300e+00
## phi[83]   -8.193855e+00 0.320118098  4.2572293   -16.7524910 -1.103281e+01
## phi[84]    7.392569e+00 0.328117787  4.2417793    -1.1814147  4.591728e+00
## phi[85]   -3.422009e+00 0.319387346  4.2050824   -11.6973438 -6.252523e+00
## phi[86]    1.005541e+01 0.326007855  4.3298021     1.3710060  7.186461e+00
## phi[87]   -7.147456e+00 0.327647089  4.2770526   -15.7530894 -9.932500e+00
## phi[88]   -1.124717e+00 0.334527386  4.3014903    -9.8021726 -4.004835e+00
## phi[89]   -4.103181e+00 0.319501828  4.2295870   -12.5205540 -6.928089e+00
## phi[90]    8.328338e+00 0.323273987  4.3212379    -0.4164721  5.439871e+00
## phi[91]   -8.226055e+00 0.329337168  4.2902505   -16.6495271 -1.107825e+01
## phi[92]    1.985301e+01 0.317199598  4.3129904    11.2272479  1.692264e+01
## phi[93]   -3.059451e+00 0.330866387  4.2761779   -11.6267205 -5.887304e+00
## phi[94]   -3.863542e+00 0.322767891  4.1867465   -12.2953011 -6.557256e+00
## phi[95]    1.565006e+01 0.342390175  4.3074534     7.1423276  1.277413e+01
## phi[96]    1.088281e+00 0.331141887  4.3098136    -7.4516982 -1.765523e+00
## phi[97]   -2.909981e+00 0.325274115  4.3797402   -11.6953324 -5.789701e+00
## phi[98]   -1.483083e+00 0.322224923  4.2923376   -10.0445217 -4.387064e+00
## phi[99]   -3.614138e+00 0.328009653  4.3142482   -12.2159971 -6.457699e+00
## phi[100]  -7.292734e+00 0.334740456  4.3450414   -15.9240831 -1.020994e+01
## phi[101]  -4.728818e+00 0.327603269  4.1901580   -12.9958062 -7.543204e+00
## phi[102]  -2.863749e+00 0.330141546  4.4128895   -11.5467948 -5.839711e+00
## phi[103]  -6.143337e+00 0.311970306  4.2860543   -14.4705627 -9.035641e+00
## phi[104]   1.134396e+00 0.320862731  4.4044035    -7.6187837 -1.752348e+00
## phi[105]  -2.276993e+00 0.318381284  4.2598369   -10.7995033 -5.088382e+00
## phi[106]   5.261166e+00 0.334820923  4.3326648    -3.4296505  2.390865e+00
## phi[107]   8.794283e+00 0.325324892  4.3971416     0.2111577  5.805877e+00
## phi[108]   3.841509e+00 0.319317269  4.2962490    -4.7176611  9.773254e-01
## phi[109]  -5.924597e+00 0.319082248  4.3672027   -14.5435650 -8.881081e+00
## phi[110]   2.688136e+00 0.331388859  4.2566009    -5.7906237 -8.571086e-02
## phi[111]  -5.384384e+00 0.324989854  4.1994087   -13.6412440 -8.159186e+00
## phi[112]   2.914206e-02 0.330098420  4.2233932    -8.3559334 -2.738718e+00
## phi[113]   1.470992e+00 0.319504218  4.3365946    -6.9693404 -1.385868e+00
## phi[114]   2.452667e+00 0.319620635  4.3295623    -6.1925822 -4.251771e-01
## phi[115]   2.968842e+00 0.326263821  4.3406602    -5.5552402  2.856232e-02
## phi[116]  -4.422093e+00 0.326231799  4.2621778   -13.0265860 -7.183335e+00
## phi[117]  -4.365565e+00 0.333929448  4.2194473   -12.8233407 -7.158697e+00
## phi[118]  -4.204795e+00 0.322518858  4.2999223   -12.8209629 -6.995072e+00
## phi[119]   1.355358e+00 0.337315850  4.3360596    -7.2274636 -1.538540e+00
## phi[120]  -3.250792e+00 0.328915406  4.3209737   -11.9424411 -6.065632e+00
## phi[121]   4.149231e+00 0.329719381  4.2523169    -4.2879704  1.355236e+00
## phi[122]  -7.659792e+00 0.336237322  4.2736755   -16.0531865 -1.051596e+01
## phi[123]  -3.073687e+00 0.318067738  4.2210182   -11.5120890 -5.863181e+00
## phi[124]  -8.217356e+00 0.329986358  4.2763804   -16.7631573 -1.101400e+01
## phi[125]  -5.915401e+00 0.321037547  4.2974943   -14.3963611 -8.821242e+00
## phi[126]  -7.086652e+00 0.320697046  4.4366305   -16.1070181 -1.003064e+01
## phi[127]  -1.897774e+00 0.333388849  4.2608910   -10.2646691 -4.714964e+00
## phi[128]  -6.574451e+00 0.326415854  4.2635108   -15.2639224 -9.328408e+00
## phi[129]  -5.663201e+00 0.325726893  4.3479889   -14.2109007 -8.582346e+00
## phi[130]   5.524335e+00 0.323513155  4.1676074    -2.7752613  2.716596e+00
## phi[131]   3.343291e+00 0.320558238  4.3170858    -5.2239697  4.541790e-01
## phi[132]  -3.220411e+00 0.315933110  4.4234127   -11.9377974 -6.242682e+00
## phi[133]  -4.706125e+00 0.342906934  4.3802878   -13.4826495 -7.608230e+00
```

```
## phi[134]    6.169643e+00 0.327209954  4.1618063    -2.0516037  3.452486e+00
## phi[135]   -1.924776e+00 0.323262397  4.2313317   -10.4205749 -4.758341e+00
## phi[136]    1.260861e+00 0.325884958  4.2016837    -7.1084307 -1.548410e+00
## phi[137]    1.259419e+01 0.333676512  4.0335249     4.6392792  9.883071e+00
## phi[138]   -6.071746e-01 0.347247475  4.2659220    -9.2216417 -3.454361e+00
## phi[139]   -2.638426e+00 0.321356860  4.4020344   -11.3937470 -5.548306e+00
## phi[140]   -8.620733e+00 0.318651198  4.3102550   -17.3062793 -1.151083e+01
## phi[141]    3.400984e+00 0.336979120  4.4500126    -5.2994268  4.702537e-01
## phi[142]   -7.782315e+00 0.324376765  4.1008026   -15.9090730 -1.055238e+01
## phi[143]   -2.342477e+00 0.325175168  4.2340662   -10.6242467 -5.150567e+00
## phi[144]   -3.200337e+00 0.317536739  4.4010138   -11.8777461 -6.122057e+00
## phi[145]   -2.847568e+00 0.325648928  4.3168031   -11.3908824 -5.646624e+00
## phi[146]   -8.608897e-01 0.332068582  4.3207307    -9.4078479 -3.760475e+00
## phi[147]    1.312929e+00 0.330839786  4.3014516    -7.1021386 -1.593413e+00
## phi[148]    9.651589e+00 0.338991692  4.2940204     0.8320219  6.832436e+00
## phi[149]    1.075552e+01 0.326026337  4.3836355     2.0968866  7.830893e+00
## phi[150]   -1.145636e+00 0.328523821  4.3426548    -9.7995030 -4.045790e+00
## phi[151]    1.201586e+01 0.329178531  4.1808326     3.7746469  9.216823e+00
## phi[152]    1.857901e+01 0.321325627  4.3695530     9.8271704  1.573193e+01
## phi[153]   -1.496889e+00 0.319507003  4.2185980    -9.8683904 -4.269924e+00
## phi[154]   -2.681474e-01 0.334425603  4.2933794    -8.7264173 -3.155749e+00
## phi[155]    6.669498e+00 0.321737058  4.3623225    -1.9583212  3.785755e+00
## phi[156]   -5.219492e+00 0.325136772  4.2325120   -13.4807105 -8.066084e+00
## phi[157]    1.473602e+01 0.328684332  4.3518957     5.9111042  1.183583e+01
## phi[158]    5.232188e+00 0.336575990  4.3371932    -3.2595723  2.334189e+00
## phi[159]    5.053253e+00 0.332405454  4.2444035    -3.3686780  2.234730e+00
## phi[160]   -8.196900e+00 0.343295539  4.2215525   -16.4369773 -1.100728e+01
## phi[161]   -8.145141e+00 0.341568683  4.4822020   -17.0160185 -1.120154e+01
## phi[162]   -5.993139e+00 0.328283112  4.3079721   -14.5790893 -8.923941e+00
## phi[163]    5.316872e+00 0.329665201  4.2926281    -3.0929788  2.471282e+00
## phi[164]   -5.431599e+00 0.319566526  4.4035795   -14.2494314 -8.337965e+00
## phi[165]    3.590230e-01 0.325411908  4.2334101    -8.2826945 -2.366129e+00
## phi[166]   -4.222935e+00 0.326444201  4.3502353   -13.0538020 -7.104933e+00
## phi[167]    4.621626e+00 0.328699640  4.3960833    -4.0434858  1.653868e+00
## phi[168]   -6.658637e+00 0.332452495  4.3362596   -15.3047637 -9.530363e+00
## phi[169]   -5.644853e+00 0.335166581  4.2558873   -13.9254099 -8.499935e+00
## phi[170]   -1.883086e+00 0.319028633  4.2422530   -10.3222570 -4.710445e+00
## phi[171]   -3.457449e+00 0.334523712  4.2911620   -11.8119535 -6.399850e+00
## phi[172]    1.316024e+01 0.322988776  4.1308815     4.9599767  1.036932e+01
## phi[173]   -7.248515e+00 0.330894811  4.3022417   -15.7511934 -1.015332e+01
## phi[174]    5.510727e-01 0.337305796  4.3932226    -8.2374056 -2.381211e+00
## phi[175]   -3.683194e+00 0.323897622  4.1721464   -11.9585043 -6.467962e+00
## phi[176]    6.045229e+00 0.332923401  4.3086439    -2.6575321  3.262941e+00
## phi[177]    1.483079e+01 0.326984785  4.2847416     6.3331813  1.204049e+01
## phi[178]    4.014086e-02 0.311037283  4.1485861    -8.1578727 -2.754930e+00
## phi[179]    6.830476e+00 0.317346692  4.3674322    -1.7778309  3.954723e+00
## phi[180]   -6.670042e-01 0.334950271  4.2581658    -9.1794295 -3.543671e+00
## phi[181]   -1.571323e+00 0.325048611  4.3068763    -9.9276083 -4.478859e+00
## phi[182]   -4.867189e+00 0.331188048  4.3788426   -13.7385828 -7.734290e+00
## phi[183]   -2.053147e+00 0.340232175  4.1726207   -10.2154849 -4.869358e+00
## phi[184]    7.291297e+00 0.327698703  4.2347191    -1.0678572  4.431282e+00
## phi[185]    1.857862e+01 0.340094778  4.3304459    10.0707339  1.568170e+01
## sigma_phi   3.116984e+01 0.048200509  3.5177687    24.9090213  2.870108e+01
## sigma       6.821583e+00 0.003113815  0.2009159     6.4420667  6.683750e+00
```

```
## lp__      -1.841579e+03 0.229789761 11.2177417 -1864.7557116 -1.848876e+03
##                     50%           75%          97.5%      n_eff     Rhat
## mu          9.054159e+00  1.083271e+01  1.492816e+01   72.15407 1.059509
## beta       -2.835411e-01  1.885847e-02  5.819759e-01 4774.81705 1.000662
## phi[1]     -5.086050e+00 -2.254795e+00  3.108304e+00  167.16976 1.022077
## phi[2]     -5.321759e+00 -2.508351e+00  3.105475e+00  169.99617 1.025028
## phi[3]     -6.647753e+00 -3.881874e+00  1.416774e+00  164.67500 1.025358
## phi[4]     -1.077237e+00  1.718292e+00  7.327526e+00  163.84473 1.025808
## phi[5]     -8.208146e+00 -5.227726e+00  4.179700e-01  189.73490 1.021219
## phi[6]     -9.469869e-02  2.793689e+00  8.086060e+00  172.54120 1.020500
## phi[7]     -6.081595e+00 -3.202365e+00  2.128438e+00  179.44185 1.022325
## phi[8]      4.692291e+00  7.582620e+00  1.294890e+01  182.73374 1.021426
## phi[9]      5.551371e+00  8.271620e+00  1.352128e+01  162.39927 1.025622
## phi[10]    -6.657133e+00 -3.859901e+00  1.681063e+00  159.21380 1.024521
## phi[11]    -4.627062e+00 -1.579516e+00  3.920760e+00  181.22694 1.020872
## phi[12]    -5.716978e+00 -2.797638e+00  2.481529e+00  169.13509 1.023273
## phi[13]    -4.662507e+00 -1.758238e+00  3.806999e+00  184.32514 1.019777
## phi[14]     8.738283e+00  1.155912e+01  1.702366e+01  172.24586 1.021258
## phi[15]     8.850739e+00  1.171632e+01  1.712079e+01  165.85098 1.025128
## phi[16]    -4.291569e+00 -1.412447e+00  4.160837e+00  168.85773 1.023761
## phi[17]    -1.726868e+00  1.039957e+00  6.537176e+00  165.58193 1.024581
## phi[18]    -3.856875e+00 -9.834921e-01  4.431315e+00  181.91893 1.021689
## phi[19]    -3.867746e+00 -1.036356e+00  4.516834e+00  152.76267 1.028655
## phi[20]     1.593557e+00  4.499054e+00  9.926020e+00  171.34467 1.024936
## phi[21]     1.096599e-01  2.979820e+00  8.477492e+00  178.46498 1.021768
## phi[22]    -6.611354e-01  2.029191e+00  7.233570e+00  160.97371 1.024426
## phi[23]    -6.191300e+00 -3.356739e+00  2.399746e+00  180.94513 1.024888
## phi[24]     3.655853e-01  3.062945e+00  8.550715e+00  167.94387 1.023989
## phi[25]    -7.494157e-01  2.055783e+00  7.291922e+00  162.01308 1.026076
## phi[26]    -1.646021e+00  1.205415e+00  6.765976e+00  168.42504 1.025499
## phi[27]     5.648054e+00  8.482743e+00  1.393499e+01  166.63291 1.023800
## phi[28]    -4.147638e+00 -1.282874e+00  4.213205e+00  164.71832 1.027301
## phi[29]    -6.777993e-01  2.013971e+00  7.313211e+00  167.68359 1.023907
## phi[30]     2.191745e+00  5.113545e+00  1.082620e+01  172.34415 1.024167
## phi[31]     1.634863e-01  2.901458e+00  8.489384e+00  176.34117 1.024527
## phi[32]    -3.579836e+00 -7.931073e-01  4.647465e+00  157.96487 1.025424
## phi[33]    -4.660630e-01  2.370192e+00  7.844915e+00  166.01666 1.023297
## phi[34]    -2.630548e-02  2.654213e+00  7.848890e+00  153.63285 1.027217
## phi[35]    -3.764545e+00 -9.221155e-01  4.487698e+00  160.85329 1.027093
## phi[36]    -2.864189e+00  2.920712e-02  5.625031e+00  176.42171 1.024198
## phi[37]    -1.507763e+00  1.353505e+00  7.080198e+00  165.06647 1.022631
## phi[38]     6.640150e+00  9.342541e+00  1.450526e+01  152.34428 1.027244
## phi[39]     4.931347e+00  7.827381e+00  1.330023e+01  186.19637 1.020254
## phi[40]    -5.253367e+00 -2.335721e+00  3.239258e+00  189.81327 1.020850
## phi[41]     3.876604e+00  6.729457e+00  1.226630e+01  173.28334 1.025614
## phi[42]    -6.785728e-01  2.085937e+00  7.454184e+00  157.97834 1.025374
## phi[43]     2.087697e+00  4.924602e+00  1.038009e+01  182.80473 1.021685
## phi[44]    -8.759138e-01  2.045738e+00  7.503813e+00  168.84077 1.024296
## phi[45]    -3.895876e+00 -9.071319e-01  4.413224e+00  178.39387 1.022067
## phi[46]    -5.376777e+00 -2.425651e+00  3.115217e+00  186.01413 1.020218
## phi[47]    -1.132347e+00  1.776603e+00  7.139415e+00  178.70986 1.024998
## phi[48]    -7.947530e+00 -5.113165e+00  2.650939e-01  172.98028 1.021347
## phi[49]    -1.224579e+00  1.719957e+00  7.298032e+00  169.36147 1.024291
## phi[50]    -1.669169e+00  1.245961e+00  6.639372e+00  167.21846 1.024155
```

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## phi[51]    3.836235e+00  6.595148e+00  1.216443e+01  161.82255 1.027297
## phi[52]    7.079372e+00  9.941986e+00  1.548518e+01  179.44792 1.020482
## phi[53]   -2.897984e+00 -1.359534e-01  5.492393e+00  170.92091 1.023688
## phi[54]    1.118846e+01  1.416869e+01  1.984537e+01  184.33574 1.019705
## phi[55]   -1.106956e-01  2.855688e+00  8.284326e+00  176.90607 1.023244
## phi[56]    5.699515e+00  8.583720e+00  1.401246e+01  179.75957 1.021123
## phi[57]    6.624378e+00  9.354147e+00  1.465169e+01  164.16054 1.025309
## phi[58]   -1.045870e+00  1.837409e+00  7.206435e+00  174.90620 1.023078
## phi[59]   -3.968809e+00 -1.158562e+00  4.427118e+00  166.03408 1.026650
## phi[60]   -2.380757e+00  5.235783e-01  6.002899e+00  168.39172 1.026154
## phi[61]    3.998283e+00  6.711278e+00  1.205511e+01  162.27074 1.026959
## phi[62]   -3.390524e+00 -5.787136e-01  4.962760e+00  169.61686 1.024792
## phi[63]   -3.930340e+00 -1.100059e+00  4.443971e+00  173.61140 1.022910
## phi[64]   -7.878259e+00 -4.817284e+00  7.042175e-01  175.66911 1.019969
## phi[65]   -3.562291e+00 -6.886339e-01  4.823737e+00  187.64772 1.021300
## phi[66]   -3.874955e+00 -1.062206e+00  4.367838e+00  152.94643 1.029504
## phi[67]   -1.515118e+00  1.348328e+00  6.708766e+00  180.96421 1.023027
## phi[68]    1.665502e+01  1.962163e+01  2.494114e+01  169.23404 1.025673
## phi[69]   -7.185205e-01  2.005919e+00  7.427966e+00  168.59820 1.024319
## phi[70]   -5.728827e+00 -2.948093e+00  2.567630e+00  168.13928 1.023765
## phi[71]    7.770739e+00  1.058969e+01  1.592317e+01  163.01593 1.024596
## phi[72]    2.911900e+00  5.849575e+00  1.148267e+01  171.99729 1.023687
## phi[73]    3.350529e+00  6.173536e+00  1.153117e+01  165.77551 1.023162
## phi[74]   -7.387228e+00 -4.644346e+00  6.658808e-01  173.97065 1.024137
## phi[75]   -4.834439e+00 -2.022203e+00  3.351177e+00  168.07310 1.024355
## phi[76]   -6.006816e+00 -3.221943e+00  2.281820e+00  166.25238 1.026201
## phi[77]   -5.972924e-01  2.368475e+00  8.105806e+00  187.42046 1.021731
## phi[78]   -4.600970e+00 -1.903252e+00  3.683810e+00  162.49351 1.025785
## phi[79]   -1.226291e+00  1.602774e+00  7.244593e+00  169.07112 1.022886
## phi[80]   -1.645889e+00  1.026835e+00  6.616288e+00  162.25491 1.027412
## phi[81]   -7.339108e+00 -4.347671e+00  1.623339e+00  177.60366 1.022666
## phi[82]    8.150894e+00  1.112067e+01  1.652826e+01  182.76145 1.022084
## phi[83]   -8.184455e+00 -5.336051e+00  7.351064e-02  176.86163 1.021557
## phi[84]    7.443872e+00  1.028833e+01  1.547497e+01  167.12314 1.024766
## phi[85]   -3.359613e+00 -5.489373e-01  4.682179e+00  173.34591 1.024463
## phi[86]    1.011969e+01  1.297871e+01  1.844906e+01  176.39244 1.025861
## phi[87]   -7.166274e+00 -4.267097e+00  1.220759e+00  170.40273 1.025820
## phi[88]   -1.019001e+00  1.779725e+00  7.307865e+00  165.33871 1.024842
## phi[89]   -4.078758e+00 -1.260442e+00  4.137398e+00  175.24645 1.022106
## phi[90]    8.383186e+00  1.124899e+01  1.662372e+01  178.67954 1.022640
## phi[91]   -8.186020e+00 -5.344104e+00  1.380564e-01  169.70077 1.024265
## phi[92]    1.988239e+01  2.279600e+01  2.831091e+01  184.88076 1.021000
## phi[93]   -3.033562e+00 -1.685232e-01  5.262503e+00  167.03452 1.024296
## phi[94]   -3.846206e+00 -1.025958e+00  4.317589e+00  168.25682 1.022578
## phi[95]    1.572827e+01  1.851227e+01  2.412043e+01  158.26999 1.025225
## phi[96]    1.145055e+00  3.961076e+00  9.581619e+00  169.39038 1.024901
## phi[97]   -2.814242e+00  5.069888e-02  5.644084e+00  181.29995 1.019119
## phi[98]   -1.438657e+00  1.516842e+00  6.803172e+00  177.44733 1.025014
## phi[99]   -3.630736e+00 -7.153903e-01  4.726527e+00  172.99637 1.022445
## phi[100]  -7.195495e+00 -4.388762e+00  1.147806e+00  168.48895 1.024971
## phi[101]  -4.725094e+00 -1.893004e+00  3.506457e+00  163.59286 1.027303
## phi[102]  -2.815712e+00  1.715063e-01  5.717387e+00  178.66757 1.025499
## phi[103]  -6.140912e+00 -3.332660e+00  2.353187e+00  188.75081 1.019673
## phi[104]   1.113526e+00  4.054434e+00  9.885961e+00  188.42375 1.020144
```

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## phi[105]  -2.227398e+00  5.959471e-01  6.046709e+00  179.01560 1.022802
## phi[106]   5.301155e+00  8.149230e+00  1.356239e+01  167.44995 1.025298
## phi[107]   8.909632e+00  1.179593e+01  1.705616e+01  182.68644 1.022589
## phi[108]   3.858947e+00  6.717559e+00  1.197911e+01  181.02313 1.022047
## phi[109]  -5.870708e+00 -2.990310e+00  2.449983e+00  187.32745 1.022309
## phi[110]   2.727587e+00  5.468249e+00  1.101987e+01  164.98713 1.024420
## phi[111]  -5.398801e+00 -2.596764e+00  2.890874e+00  166.96933 1.024310
## phi[112]   6.564934e-02  2.841325e+00  8.249632e+00  163.69528 1.022143
## phi[113]   1.501882e+00  4.321679e+00  1.011123e+01  184.22326 1.023061
## phi[114]   2.460273e+00  5.359619e+00  1.110580e+01  183.49252 1.022826
## phi[115]   2.997312e+00  5.849286e+00  1.148373e+01  177.00019 1.019993
## phi[116]  -4.344135e+00 -1.514213e+00  3.769078e+00  170.69097 1.027137
## phi[117]  -4.341197e+00 -1.586427e+00  3.999771e+00  159.66205 1.027742
## phi[118]  -4.155062e+00 -1.281825e+00  3.912839e+00  177.75056 1.022500
## phi[119]   1.443518e+00  4.308810e+00  9.724781e+00  165.24069 1.025132
## phi[120]  -3.176638e+00 -3.267935e-01  5.113918e+00  172.58173 1.022276
## phi[121]   4.168158e+00  6.983377e+00  1.241961e+01  166.32682 1.025782
## phi[122]  -7.683197e+00 -4.747400e+00  7.237207e-01  161.55160 1.027171
## phi[123]  -2.997746e+00 -2.303441e-01  5.028607e+00  176.11454 1.022131
## phi[124]  -8.215187e+00 -5.315322e+00 -9.922156e-03  167.94253 1.025776
## phi[125]  -5.885762e+00 -2.966297e+00  2.509634e+00  179.19214 1.021053
## phi[126]  -7.022414e+00 -4.099094e+00  1.488332e+00  191.38884 1.022800
## phi[127]  -1.831469e+00  9.732740e-01  6.257960e+00  163.34231 1.027921
## phi[128]  -6.476974e+00 -3.795486e+00  1.597828e+00  170.60519 1.023666
## phi[129]  -5.606114e+00 -2.754344e+00  2.808599e+00  178.18438 1.024029
## phi[130]   5.497868e+00  8.420921e+00  1.361486e+01  165.95476 1.023614
## phi[131]   3.404839e+00  6.247591e+00  1.165983e+01  181.37084 1.024098
## phi[132]  -3.185607e+00 -1.377332e-01  5.259186e+00  196.03094 1.019269
## phi[133]  -4.661377e+00 -1.745501e+00  3.813617e+00  163.17468 1.027207
## phi[134]   6.156798e+00  8.947361e+00  1.426594e+01  161.77474 1.024192
## phi[135]  -1.909571e+00  9.711563e-01  6.216581e+00  171.33408 1.022051
## phi[136]   1.377179e+00  4.130299e+00  9.369352e+00  166.23332 1.025633
## phi[137]   1.262264e+01  1.529615e+01  2.067629e+01  146.12288 1.027490
## phi[138]  -5.024937e-01  2.290330e+00  7.677130e+00  150.92029 1.027370
## phi[139]  -2.610890e+00  3.296965e-01  5.976879e+00  187.64272 1.020746
## phi[140]  -8.592489e+00 -5.668157e+00 -4.792669e-01  182.96786 1.021841
## phi[141]   3.362781e+00  6.370472e+00  1.219697e+01  174.38796 1.021921
## phi[142]  -7.745655e+00 -4.998297e+00  1.890498e-01  159.82262 1.026278
## phi[143]  -2.359157e+00  5.057461e-01  5.994998e+00  169.54326 1.026581
## phi[144]  -3.164616e+00 -2.525681e-01  5.249184e+00  192.09563 1.019377
## phi[145]  -2.812995e+00  1.165394e-02  5.662714e+00  175.72161 1.023069
## phi[146]  -8.286110e-01  2.097447e+00  7.464351e+00  169.30073 1.022051
## phi[147]   1.308297e+00  4.219448e+00  9.807491e+00  169.04200 1.025299
## phi[148]   9.709316e+00  1.255558e+01  1.780656e+01  160.45383 1.024645
## phi[149]   1.080930e+01  1.367591e+01  1.929509e+01  180.78546 1.023599
## phi[150]  -1.099274e+00  1.809564e+00  7.303822e+00  174.73378 1.023730
## phi[151]   1.201199e+01  1.475894e+01  2.029011e+01  161.31047 1.024963
## phi[152]   1.860541e+01  2.147042e+01  2.713564e+01  184.91975 1.021214
## phi[153]  -1.500382e+00  1.331426e+00  6.729551e+00  174.33136 1.025016
## phi[154]  -3.175039e-01  2.678242e+00  8.181950e+00  164.81605 1.023696
## phi[155]   6.686600e+00  9.579581e+00  1.524012e+01  183.83719 1.020696
## phi[156]  -5.266077e+00 -2.374808e+00  3.077606e+00  169.45884 1.025327
## phi[157]   1.485768e+01  1.764531e+01  2.296500e+01  175.30687 1.021648
## phi[158]   5.268607e+00  8.112629e+00  1.364157e+01  166.05474 1.024559
```

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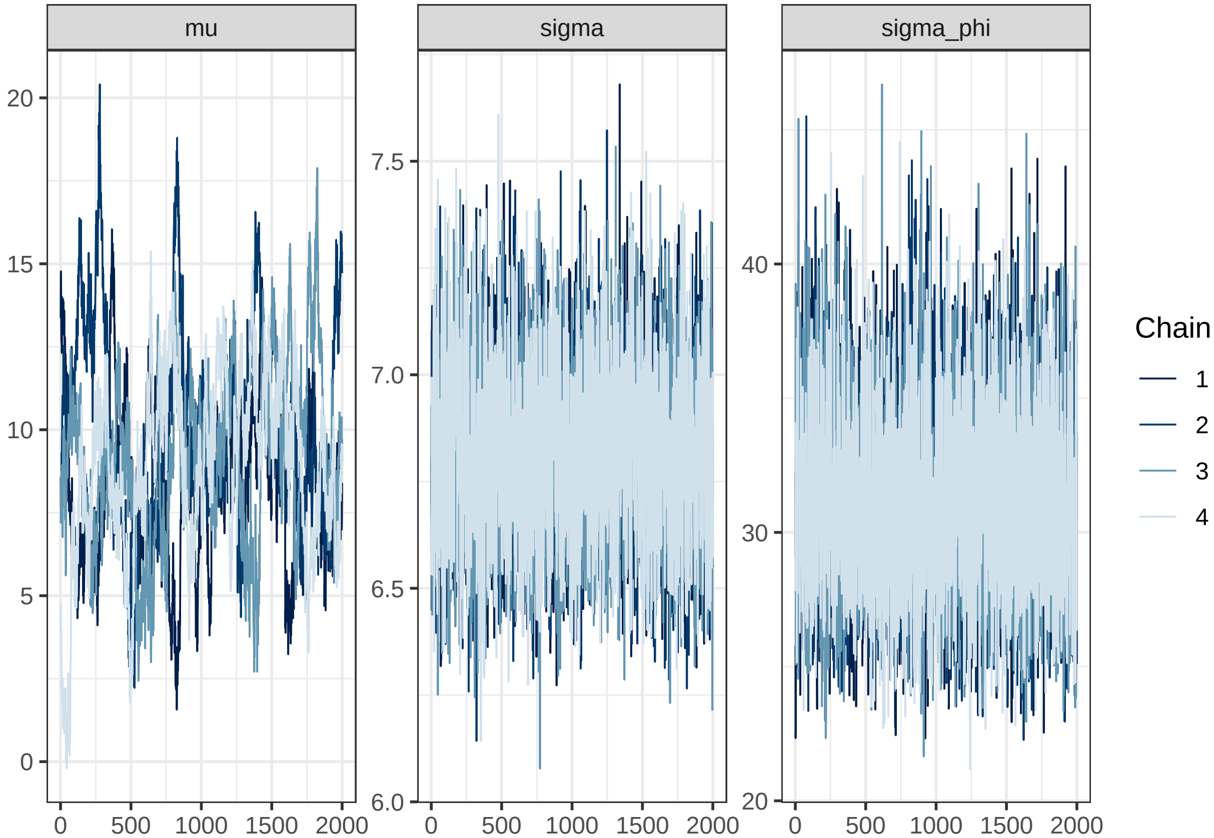
## phi[159]   5.109495e+00  7.908073e+00  1.334753e+01  163.04108 1.025778
## phi[160]  -8.223913e+00 -5.396529e+00 -3.918862e-02  151.21960 1.029035
## phi[161]  -8.120767e+00 -5.095784e+00  6.248917e-01  172.19747 1.023922
## phi[162]  -5.935305e+00 -3.125805e+00  2.435989e+00  172.20616 1.024229
## phi[163]   5.239675e+00  8.261450e+00  1.377262e+01  169.55098 1.023794
## phi[164]  -5.428463e+00 -2.443254e+00  2.971635e+00  189.88432 1.022679
## phi[165]   4.174348e-01  3.202079e+00  8.488196e+00  169.24420 1.023426
## phi[166]  -4.202629e+00 -1.285781e+00  4.237428e+00  177.58553 1.024619
## phi[167]   4.625238e+00  7.569959e+00  1.326575e+01  178.86829 1.024537
## phi[168]  -6.663920e+00 -3.783814e+00  1.799313e+00  170.12626 1.021726
## phi[169]  -5.590786e+00 -2.808834e+00  2.792017e+00  161.23481 1.026096
## phi[170]  -1.819904e+00  9.483820e-01  6.313011e+00  176.82098 1.022351
## phi[171]  -3.479803e+00 -5.346077e-01  4.857349e+00  164.54929 1.024776
## phi[172]   1.320723e+01  1.596281e+01  2.114259e+01  163.57262 1.024132
## phi[173]  -7.185180e+00 -4.322922e+00  9.128960e-01  169.04787 1.025022
## phi[174]   6.174899e-01  3.438137e+00  9.084651e+00  169.63631 1.024154
## phi[175]  -3.709366e+00 -8.270115e-01  4.500790e+00  165.92184 1.022037
## phi[176]   6.027657e+00  8.896624e+00  1.448295e+01  167.49142 1.026296
## phi[177]   1.488697e+01  1.773401e+01  2.319457e+01  171.70945 1.022766
## phi[178]   2.368619e-02  2.815643e+00  8.322278e+00  177.89974 1.022724
## phi[179]   6.817370e+00  9.751150e+00  1.535774e+01  189.40192 1.020831
## phi[180]  -6.352312e-01  2.278642e+00  7.460714e+00  161.61604 1.026479
## phi[181]  -1.580710e+00  1.273287e+00  6.921863e+00  175.56105 1.022313
## phi[182]  -4.823942e+00 -1.949717e+00  3.624320e+00  174.81124 1.024881
## phi[183]  -2.005058e+00  7.752621e-01  5.927687e+00  150.40665 1.027536
## phi[184]   7.352556e+00  1.018503e+01  1.525318e+01  166.99358 1.023599
## phi[185]   1.863946e+01  2.151226e+01  2.691561e+01  162.13072 1.026294
## sigma_phi  3.089310e+01  3.346951e+01  3.847856e+01 5326.36966 1.000148
## sigma      6.820750e+00  6.954506e+00  7.228327e+00 4163.35125 1.000353
## lp__      -1.841250e+03 -1.833807e+03 -1.820383e+03 2383.13983 1.001050
```

```r
mcmc_trace(as.array(fit), pars = c("mu", "sigma", "sigma_phi"))
```




```r
summary(fit, pars = "mu")$summary
```

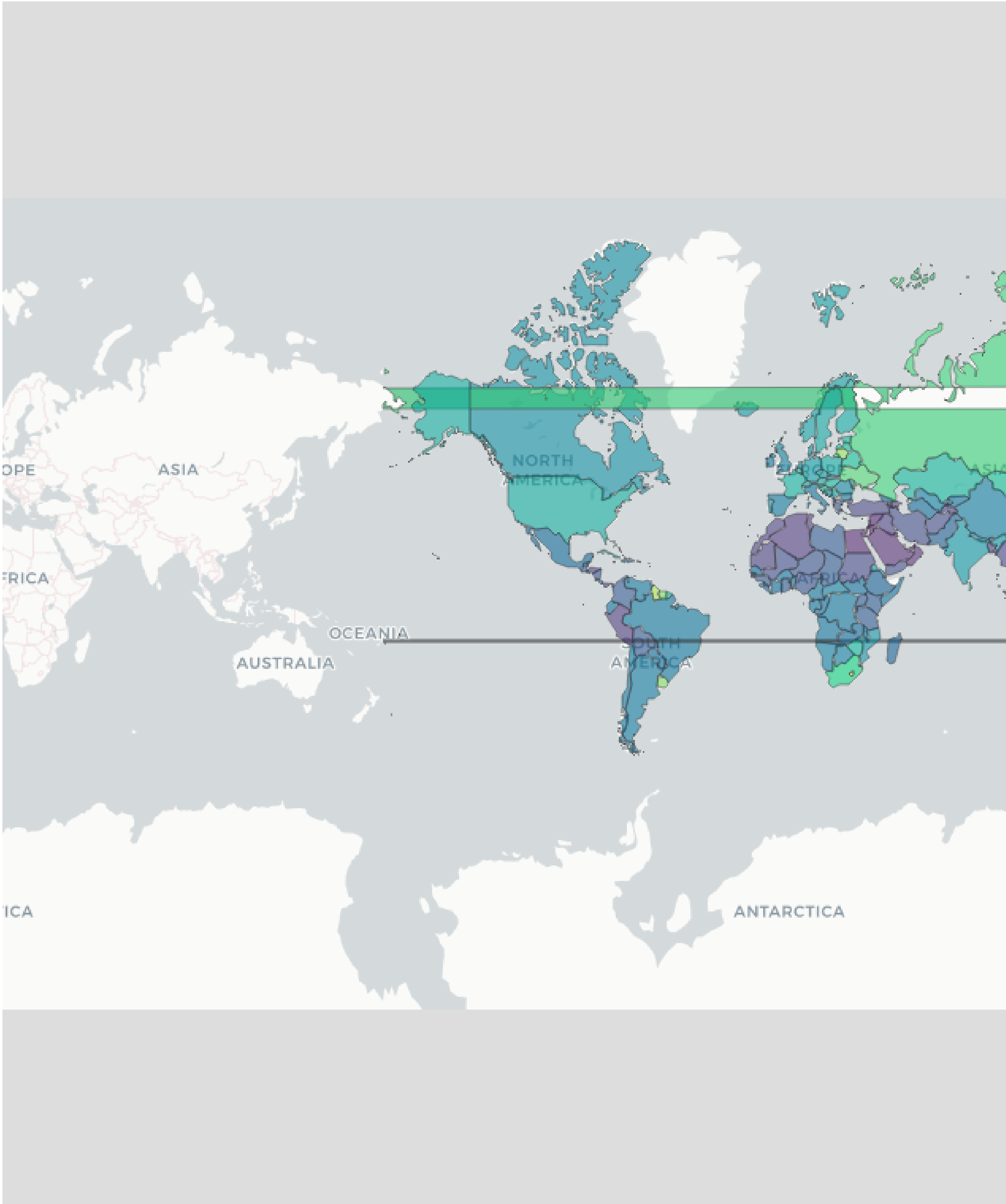
```
##         mean   se_mean       sd     2.5%      25%      50%      75%    97.5%
## mu 9.126953 0.3247319 2.758388 3.890478 7.334498 9.054159 10.83271 14.92816
##       n_eff     Rhat
## mu 72.15407 1.059509
```

```r
install.packages("webshot")
```

```
##
## The downloaded binary packages are in
##  /var/folders/h2/0z07kqqn1n99gzcftq21dtv40000gp/T//RtmpE1j3UM/downloaded_packages
```

```r
webshot::install_phantomjs()

# Extract posterior samples for 'phi' (spatial random effects)
phi_samples <- rstan::extract(fit)$phi

# Calculate posterior mean of phi for each region
phi_mean <- apply(phi_samples, 2, mean)

# Add phi_mean to the world_sf dataset
world_sf$phi_mean <- phi_mean

# Visualize the posterior mean of phi on the map
library(mapview)
library(webshot)
library(htmlwidgets)
```

```r
m <- mapview(world_sf, zcol = "phi_mean")
mapshot(m, file = "phi_map.png")

knitr::include_graphics("phi_map.png")  # include in PDF output
```



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AMERICA
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AFRICA
OCEANIA
AUSTRALIA
SOUTH
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ANTARCTICA

Discussion

Results

TODO!!

Limitations

The spatial prior assumes that nearby countries have similar suicide rates, so if there are sharp differences between neighboring regions, the model may over-smooth and underrepresent the true variation.

Additionally, the focus of this analysis is whether or not there is a spatial relationship between location and suicide rate, but it does not consider underlying factors, such as culture, mental health resources, and economic state. For example, the topic of mental health is considered to be taboo in many countries, resulting in limited access to mental health resources.

The next step would be to look further into these underlying factors and determine whether or not there is a relationship between the factor and suicide rate (e.g. is there a relationship between suicide rate and lower income households in the United States and Canada?). This can be combined with information about distribution of suicide rates across sex and age to give more insight into which subset of groups should be targeted for suicide prevention methods in certain countries.

Member Contributions

!!!TODO - small paragraph discussing what each member did

Appendix

TODO!! - move all code here at the end

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