* Reliability Engineering

→ Reliability engineering is a multidisciplinary field that focuses on ensuring the dependability and performance of systems, products, and processes over time.

→ The primary goal is to identify and mitigate potential failures and risks to enhance reliability of a system throughout its lifecycle.

⇒ Key Components of Reliability Engineering

① Reliability :-

It is the probability that a product will perform successfully under specified operating conditions for a given period of time.

[Reliability is Quality over Time]

② ~~Fail~~ Failure Rate (λ) :-

A Reliability index that represents the rate at which your product fails.

$$\text{Failure Rate } (\lambda) = \frac{\text{Number of failures}}{\text{Operating time (Cycles)}} = \text{Failures per hour}$$

③ Mean Time To Failure (MTTF) :-

The reliability index for non-repairable units represents the mean time to failure.

→ Mean Time Between Failure (MTBF) :-

The reliabity index for repairable units represents the mean time between failure.

$$MTTF = \theta = \frac{\text{Operating Time (cycles)}}{\text{Number of failures}} = MTBF$$

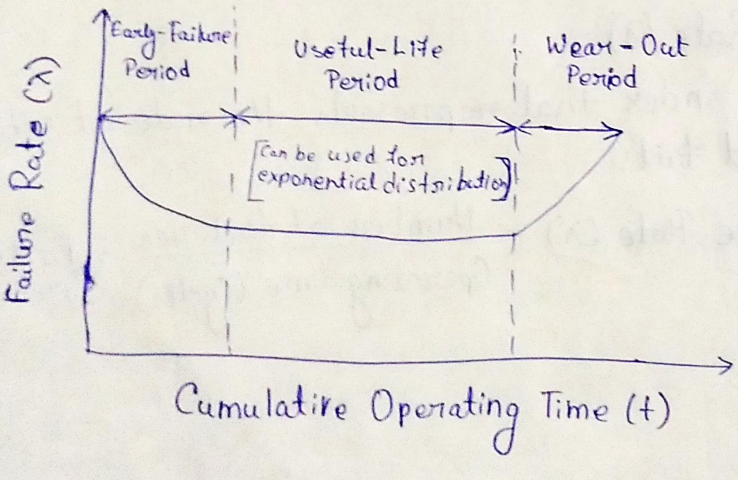
Note

$$\lambda = \frac{1}{\theta}$$

or $\theta = \frac{1}{\lambda}$

⇒ The Bathtub Curve :-

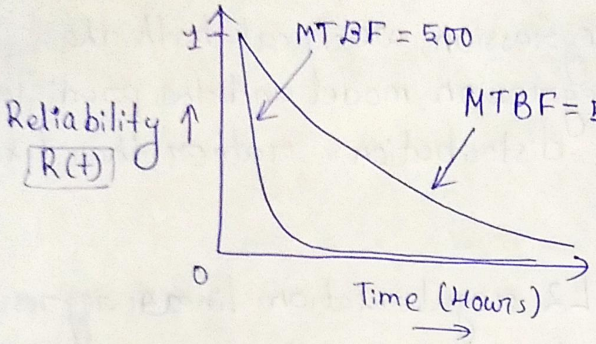
The bathtub curve is a reliability tool that is used to model the reliability of a unit or system over the units entire life.



⇒ The useful-life Period and the Exponential Distribution:-

→ Reliability: $R(t) = e^{-\lambda t}$

$= e^{-t/\theta}$



⇒ The Weibull Distribution :-

⇒ The weibull distribution was discovered by Waloddi Weibull and ~~its~~ is the most versatile distribution in Reliability Engineering because of its ability to model a variety of distributions.

⇒ Reliability: $R(t) = e^{-(t/\theta)^{\beta}}$

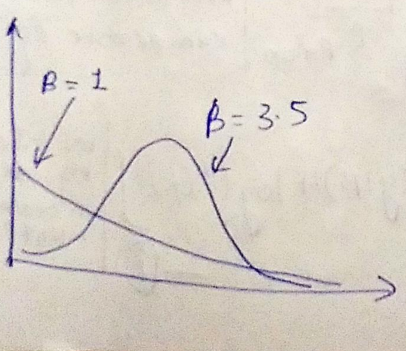
where,

β (Beta) – The Weibull shape parameter

θ (Theta) – The weibull scale parameter

If,

- $\beta < 1$, the Weibull Distribution (WD) represents a system with a decreasing failure rate
- $\beta = 1$, The WD ~~is~~ is approximately equal to the exponential distribution
- $\beta > 1$, The WD represents a system with a increasing failure rate.



[Checkout the video on this By ritvikmath]

* Bayesian Linear Regression.

→ Bayesian linear regression is a statistical method that extends traditional linear regression by incorporating Bayesian principles. ~~In~~

→ In Bayesian linear regression, we treat both the parameters of the regression model and the predicted values as probability distributions rather than fixed values.

→ We can derive L1 & L2 regularization linear regression using Bayesian approach.

⇒ Linear Regression formula: $\underset{\text{Response (target value)}}{y} = \underset{\text{Data}}{X}\underset{\text{Parameter (Weights)}}{\beta} + \underset{\text{error}}{\varepsilon}$

⇒ Bayesian Approach:

$$\hat{\beta}_{MAP} = \underset{\beta}{\text{Argmax}}\ \overbrace{P(\beta/y)}^{\text{Posterior}}$$

$$= \underset{\beta}{\text{Argmax}}\ \frac{P(y|\beta)\,P(\beta)}{P(y)}$$

$$= \underset{\beta}{\text{Argmax}}\ \underbrace{P(y|\beta)}_{\text{likelihood}}\ \underbrace{P(\beta)}_{\text{Prior}}$$

∴ We only try to maximize β, we can remove $P(y)$

$$= \underset{\beta}{\text{Argmax}}\left[\log(P(y|\beta)) + \log(P(\beta))\right] \quad \text{—(i)}$$

We take log, and it becomes sum of 2 logs

Note:-
We know that maximizing or minimizing anything is same as maximizing or minimizing it's log().

→ And logs are easier to compute for computers.

→ Now, computing $\log(\mathbb{P}(y/\beta))$

$$y_i \sim N(\beta^T x_i, \sigma^2) \Rightarrow \mathbb{P}(y/\beta) = \prod_{i=1}^{N} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(y_i-\beta^T x_i)^2}{2\sigma^2}}$$

Here y is a vector of observations

Then,

$$\log(\mathbb{P}(y/\beta)) = \sum_{i=1}^{N} \left[\log\frac{1}{\sigma\sqrt{2\pi}} - \frac{(y_i-\beta^T x_i)^2}{2\sigma^2}\right] \quad \text{— (ii)}$$

↳ Since, these parts don't have β in it we can remove it when we put (ii) in (i)

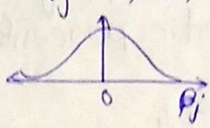
→ Computing Prior :-

(1) If we choose Gaussian distribution for prior :-

$\beta_j \sim N(0, \tau^2)$

$\forall j = 1, \ldots, p$

Assumptions

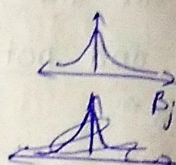


Then our important relevant factor would come out to be

$$-\frac{\sigma^2}{\tau^2}\|\beta\|_2^2 \quad \text{— (iii)}$$

(2) If we choose a Laplacia distribution for prior,

$$\beta_j \sim \frac{1}{2b} e^{-|\beta_j|/b}$$



Then, $\hat{\beta}_{MAP} = \hat{\beta}_{L1}$

$$= \underset{\beta}{\text{Argmin}}\left[\|y - X\beta\|^2 + \lambda\|\beta\|\right]$$

→ Putting (ii) & (iii) in (i), we get

$$\hat{\beta}_{MAP} = \text{Argmax}\left[\sum \|y_i - \beta^T x_i\|^2 - \frac{\sigma^2}{\tau^2}\|\beta\|_2^2\right]$$

$$= \text{Argmin}\left[\|y - X\beta\|^2 + \lambda\|\beta\|^2\right]$$ (In vector form)

$$= \hat{\beta}_{L2} \qquad \left(\lambda = \frac{\sigma^2}{\tau^2}\right)$$

* Stochastic Process

→ A stochastic process is the mathematical model used to describe the evolution of a system or phenomenon over time, where the outcome at any given time is not completely determined by the previous outcomes.

→ Instead, randomness or uncertainty is inherent in the process.

→ A stochastic process is a family of random variables indexed by time (or another parameter), often denoted as $\{X_t : t \in T\}$

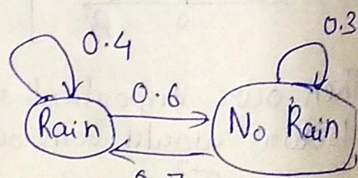
→ X_t represents the state of the system at time t.

→ A stochastic process has the Markove property (Memoryless), if the future behavior depends only on the current state and not on the past.

⇒ This property is important in simplifying the analysis of many stochastic processes.

Eg:-

Present	Next Day
Raining Today	→ 40% chance may Rain Tomorrow
	→ 60% chance may not Rain Tommorow
Non Raining Today	→ 70% chance may Rain Tommorow
	→ 30% chance may not Rain Tommorow



[Markov chain Model]

[Transition Matrix]

$$\begin{array}{c|cc} & \text{Rain} & \text{No Rain} \\ \hline \text{Rain} & 0.4 & 0.6 \\ \text{No Rain} & 0.3 & 0.7 \end{array}$$

Transition Matrix

⇒ Transition Probability Matrix (TPM):

The matrix describing the Markov chain is called transition matrix.

$$P = \begin{array}{c} \\ 1 \\ 2 \\ \vdots \\ n \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & \cdots & n \end{array} \\ \begin{pmatrix} P_{11} & P_{12} & \cdots & P_{1n} \\ P_{21} & P_{22} & \cdots & P_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ P_{n1} & P_{n2} & \cdots & P_{nn} \end{pmatrix} \end{array}\begin{array}{l} \\ \text{— Sum} = 1 \\ \text{— Sum} = 1 \\ \\ \text{— Sum} - 1 \end{array}$$

This matrix is said to be TPM if,

→ It must be square matrix

→ $P_{ij} \geq 0$ for all i and j

→ $\sum P_{ij} = 1$ (row wise)

---

* Statistical Quality Control :-

→ The Statistical Quality Control (SQC) is a set of statistical methods used in the monitoring and control of processes to ensure that products or services meet specified quality standards.

→ SQC is commonly applied in manufacturing but is also used in various other industries where quality is critical.

→ The primary objectives of statistical quality Control are to ~~and~~ identify and ~~maxim~~ minimize variations in the production process, maintain consistency in product quality, and reduce the likelihood of defects.

Variations in quality

- Due to chance
- Due to assignable cause
  - Raw Material
  - difference in machine
  - difference in operators
  - difference of time.

⇒ **Advantages:-**

1. Define the standard
2. Attain the standard
3. Maintain the standard
4. Detect the assignable causes
5. Increase efficiency of worker
6. Confidence / Goodwill
7. Permanent record
8. Efficiency of machine ~~de~~ is determined
9. Continuous ~~chet~~ checking.

⇒ **Types of ~~chart:~~ control charts used in SQC:-**

1. $\bar{X}$ chart (mean chart)
2. R chart (range chart)
3. c chart
4. p chart
5. np chart

① $\bar{x}$ chart :- [Mean chart]

→ Purpose: Monitors the central tendency (mean) of a process.

→ Control Limits: Typically includes central line (CL), upper and lower central lines (UCL & LCL)

[similar for all charts]

→ Calculations:-

$$C.L. = \bar{\bar{x}}$$

$$L.C.L = \bar{\bar{x}} - A_2 \bar{R}$$

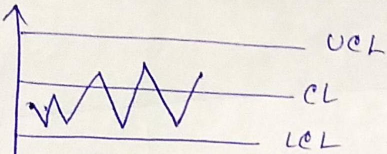
$$U.C.L = \bar{\bar{x}} + A_2 \bar{R}$$

→ A_2 is a constant ~~an~~ for a known no. of samples (n)

→ $\bar{\bar{x}}$ is the mean of all the $\bar{x}$ (means) of the samples.

→ $\bar{R}$ is the mean of all the ranges ($x_{Highest}$ x_{Lowest}) of all the samples.

$\bar{R} = \frac{\Sigma R}{n} = \frac{\Sigma(x_H - x_L)}{n}$



⇒ The graph looks like this (Similar for all the charts)

② $\bar{R}$ chart :- [Range Chart]

→ Purpose : Monitor the variability (range) within each sample of a process.

→ Calculation:-

$$C.L. = \bar{R}$$

$$L.C.L. = D_3 \bar{R}$$

$$U.C.L = D_4 \bar{R}$$

→ D_3 & D_4 are constants for a known no. of samples (n)

③ C-chart:-

→ Purpose : Monitors the number of occurrences (count) of a specific event within a constant sample size.

→ Calculations:-

$$C.L. = \bar{c} = \frac{\sum_{i=1}^{K} c_i}{K}$$

$$U.C.L. = \bar{c} + 3\sqrt{\bar{c}}$$

$$L.C.L. = \bar{c} - 3\sqrt{\bar{c}}$$

c_i = No. of occurences in the i^{th} sample

K = No. of samples

④ NP-chart :-

→ Purpose : Similar to the P chart but used when the sample size is constant.

→ Calculations:-

$$C.L. = \bar{np} = \frac{\sum_{i=1}^{K} \text{No. of defective items in the } i^{th} \text{ sample}}{K}$$

K → (No. of samples)

$$U.C.L. = \bar{np} + 3\sqrt{\bar{np}(1-\bar{p})}$$

$$L.C.L. = \bar{np} - 3\sqrt{\bar{np}(1-\bar{p})}$$

$$\bar{p} = \frac{\bar{np}}{n}$$

n → Inspected items (sample size)

(5) P-chart :-

→ Purpose: Monitors the ~~produ~~ proportion of defective items in a sample.

→ Calculations:-

$$C.L. = \bar{p} = \frac{\sum np}{\sum n}$$

$\sum np$ → Total No. of defectives ~~item~~

$\sum n$ → Total no. of sample

$$U.C.L. = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n \text{ or } \bar{n}}}$$

n → If n is same for all samples

$\bar{n}$ → If n is not same for all samples

$$L.C.L. = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n \text{ or } \bar{n}}}$$