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Assignment 1

1. Give examples of sets A, B, and C such that A, B, C are all non-empty (i.e., not equal to $\emptyset$ and the following hold or state that no such values of A, B, C are possible to satify:

   - $A \cup B = C$
     Let $A = \{1, 2\}$, $B = \{3\}$, $C = \{1, 2, 3\}$
     Then $A \cup B = \{1, 2, 3\} = C$
   - $A \cup B = A$
     Let $A = \{1, 2\}$, $B = \{1\}$
     Then $A \cup B = \{1, 2\} = A$
   - $A \setminus A = B$
     No such values of A,B,C are possible to satisfy this condition. Since $A \setminus A = \emptyset$.
   - $A \cap B = B \setminus (C \cup A)$
     Let $A = \{1\}$, $B = \{2\}$, $C = \{2\}$
     Then $A \cap B = \{1\} \cap \{2\} = \emptyset$, $B \setminus (C \cup A) = \{2\} \setminus \{\{2\} \cup \{1\}\} = \emptyset$
   - $A \cap B = C$
     Let $A = \{1, 2\}$, $B = \{2, 3\}$, $C = \{2\}$
     Then $A \cap B = \{2\} = C$

2. Write out the following sets:

   - $S_1 = \{x | x \subseteq \{a, b, c\}\}$
     $S_1 = \{\{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}, \emptyset\}$
   - $S_2 = \{x | x \subset \{a, b, c\}\}$
     $S_2 = \{\{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \emptyset\}$
   - $S_3 = \{x | x \subseteq \{a, b, c, d\} \wedge |x| = 2\}$
     $S_3 = \{\{a, b\}, \{b, c\}, \{c, d\}, \{a, c\}, \{a, d\}, \{b, d\}\}$

3. Convert the following descriptions of sets into a set theoretic mathematical representation (i.e., like what is typed out for question 2):

   - S_1: The set of subsets of the set containing a, b, c such that no set has more than one element.
     $S_1 = \{x | x \subseteq \{a, b, c\} \wedge \neg(|x| > 1)\}$
   - S_2: The set containing natural numbers that are divisible by four.
     $S_2 = \{x | x \in N \wedge x \equiv 0 (mod 4)\}$
   - S_3: The set containing all pairs of natural numbers, i.e., 2-tuples like (a,b), such that the second element of the pair is twice the first.
     $S_3 = \{(a, b) | a, b \in N \wedge b = 2a\}$

- Let U be the set of all possible sets. S_4: The set of all sets that contain themelves.
  $S_4 = \{x | x \in U \land x \in x\}$
- Let U be the set of all possible sets. S_5: The set of all sets that don't contain themselves.
  $S_4 = \{x | x \in U \land x \notin x\}$

4. Bonus: In question 3, the set S_5 was the set of all sets that don't contain themselves. Question: is $S_5 \in S_5$? Why or why not?

   Let's look at 2 cases:
   Suppose $S_5 \in S_5$. By definition of S_5: the set of all sets that don't contain themselves, which means $S_5 \notin S_5$. This contradicts what we suppose.
   Suppose $S_5 \notin S_5$. Since R is the set of sets that does not contain themselves, $S_5 \in S_5$.
   Therefore, this is a paradox which contradicts itself.