Assignment 4

1. Let $M = (Q, \Sigma, \delta, q_0, A)$ be an ϵ-NFA and let $S \subseteq Q$. Prove that $\epsilon(S) = \epsilon(\epsilon(S))$.

   Proof. We need to prove 2 parts:
   1) $\epsilon(S) \subseteq \epsilon(\epsilon(S))$
   Let $T = \epsilon(S)$. We have $T \subseteq \epsilon(T)$ (According to definition of ϵ closure).
   Therefore, $\epsilon(S) \subseteq \epsilon(\epsilon(S))$ (since $T = \epsilon(S)$).

   2) $\epsilon(\epsilon(S)) \subseteq \epsilon(S)$
   Let $q \in \epsilon(\epsilon(S))$. We have q is reachable from state x such that $x \in \epsilon(S)$ by an ϵ move. (according to definition of ϵ closure)
   If $x \in \epsilon(S)$ then $\delta(x, \epsilon) \subseteq \epsilon(S)$ (according to definition).
   We also know that q is an element of $\delta(x, \epsilon)$ and $\delta(x, \epsilon) \subseteq \epsilon(S)$. Then $q \in \epsilon(S)$.
   Therefore, $\epsilon(\epsilon(S)) \subseteq \epsilon(S)$.

   Since $\epsilon(S) \subseteq \epsilon(\epsilon(S))$ and $\epsilon(\epsilon(S)) \subseteq \epsilon(S)$, then $\epsilon(S) = \epsilon(\epsilon(S))$

   □

2. Let $M = (Q, \Sigma, \delta, q_0, A)$ be an ϵ-NFA. Suppose that there are sets $S \subseteq Q$ and $T \subseteq Q$ such that $S = \epsilon(S)$ and $T = \epsilon(T)$. Prove that $S \cap T = \epsilon(S \cap T)$

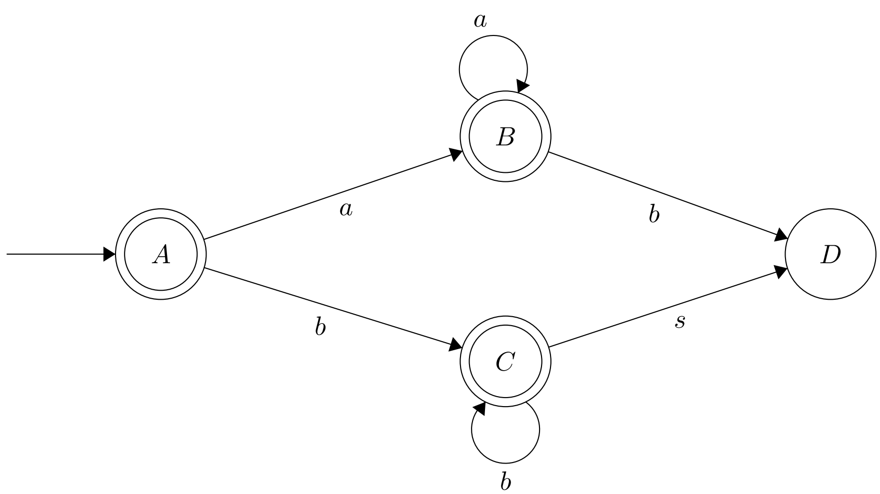
   Proof. We need to prove 2 parts:
   1) $S \cap T \subseteq \epsilon(S \cap T)$
   We have $S \cap T = \epsilon(S) \cap \epsilon(T)$ (since $S = \epsilon(S)$ and $T = \epsilon(T)$).
   Let $q \in Q$ such that $q \in \epsilon(S) \cap \epsilon(T)$. Then $q \in \epsilon(S)$ and $q \in \epsilon(T)$.
   Then $q \in S$ and $q \in T$ (since $S = \epsilon(S)$ and $T = \epsilon(T)$).
   Which means $q \in S \cap T$.
   According to ϵ closure, $S \cap T \subseteq \epsilon(S \cap T)$. Then $q \in \epsilon(S \cap T)$.
   Therefore, $S \cap T \subseteq \epsilon(S \cap T)$.

   2) $\epsilon(S \cap T) \subseteq S \cap T$
   Let $q \in \epsilon(S \cap T)$. Then we have q is reachable from state s such that $s \in S \cap T$ by an ϵ move. (according to ϵ closure). This also means $s \in \epsilon(S) \cap \epsilon(T)$ since $\epsilon(S) = S$ and $\epsilon(T) = T$
   We also know that q is an element of $\delta(s, \epsilon)$ and $\delta(s, \epsilon) \subseteq \epsilon(S) \cap \epsilon(T)$ (according to definition). Then $q \in \epsilon(S) \cap \epsilon(T)$.
   Thus, $q \in S \cap T$.
   Therefore, $\epsilon(S \cap T) \subseteq S \cap T$.

   Since $S \cap T \subseteq \epsilon(S \cap T)$ and $\epsilon(S \cap T) \subseteq S \cap T$, then we have $S \cap T = \epsilon(S \cap T)$. □

3. Let $\Sigma = \{a, b\}$ and let $R = (a\star) + (b\star)$ be a regular expression. Prove that there cannot exist any deterministic finite state machine $M = (Q, \Sigma, \delta, q_0, A)$ such that $|Q| = 3 \wedge L(M) = L(R)$.

   Proof. We will create an DFM with minimized number of states.
   We have DFM for R is:



   We use Table Filling Algorithm to minimize number of states of the machine above.

	A	B	C	D
A	-	×	×	×
B	×	-	×	×
C	×	×	-	×
D	×	×	×	-

   From the table, we can see that all states A, B, C and D are distinguishable and not equivalent. Therefore, the minimum number of states of M for R is 4 states.

   Therefore, there cannot exist any deterministic finite state machine $M = (Q, \Sigma, \delta, q_0, A)$ such that $|Q| = 3 \wedge L(M) = L(R)$.

   $\square$