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# Assignment 5

1. Prove that the following language is not regular: $\{a^i b^j c^k : i = j \vee i = k\}$.

   *Proof.* We prove L is not regular by contradiction using pumping lemma.
   *Case 1: If $i = j$, then L is not regular*
   Assume that L is regular. Therefore there exists some pumping length n.
   Let $z = a^i b^i c^k$.
   This word is in L and it is also longer than i: $|z| \geqslant i$.
   Consider $z = uvw$ where $|uv| \leqslant i$ and $|v| > 0$.
   Because our word is $z = a^i b^i c^k$ and $uv = a^i$ that means it can only consist of a. Let $v = a^p$ and $u = a^{i-p}$ where $j \geqslant 1$. Then $w = b^i c^k$.
   Consider the word $z^0 = uv^0 w$. This is equal to $a^{i-p} b^i c^k$. Thus the word is not of the form $z = a^i b^i c^k$ and not an element of L.
   This contradicts what we assume. Therefore the language is not regular.

   *Case 2: If $i = k$, then L is not regular*
   Assume that L is regular. Therefore there exists some pumping length n.
   Let $z = a^i b^j c^i$.
   This word is in L and it is also longer than i: $|z| \geqslant i$.
   Consider $z = uvw$ where $|uv| \leqslant i$ and $|v| > 0$.
   Because our word is $z = a^i b^j c^i$ and $uv = a^i$ that means it can only consist of a. Let $v = a^p$ and $u = a^{i-p}$ where $j \geqslant 1$. Then $w = b^j c^i$.
   Consider the word $z^0 = uv^0 w$. This is equal to $a^{i-p} b^j c^i$. Thus the word is not of the form $z = a^i b^j c^i$ and not an element of L.
   This contradicts what we assume. Therefore the language is not regular.

   Therefore in both cases, the language is not regular language. □

2. Prove that the following language is not regular: $\{0^i 1^j : i \neq j\}$.

   *Proof.* Assume $L = \{0^i 1^j : i \neq j\}$ is a regular language. We apply operations that preserve regularity on $L$.
   We have $\overline{L} \cap 0^\star 1^\star = \{0^i 1^i : i \geqslant 0\}$.
   Since $L$ is regular, then $\overline{L}$ is also regular since it preserves regularity using set complement.
   Since regular languages preserve regularity under intersection (and $0^\star 1^\star$ is regular), we get $\{0^i 1^i : i \geqslant 0\}$ is also regular. This derives contradiction since $\{0^i 1^i : i \geqslant 0\}$ is not a regular language (according to an example in class).
   Therefore, the language $\{0^i 1^i : i \geqslant 0\}$ is not regular. □

3. Consider the alphabet $\Sigma = \{\emptyset, \epsilon, a, b, (, ), +, \star\}$. Provide a context-free grammar that generates all strings in $\Sigma^\star$ that are valid regular expressions. For example, it should be able to generate $aaa\star$ and $a(b + \epsilon)$ but

not () or ))$bc+$.

We have: a context free grammar is $G = (V, \Sigma, P, S)$ where:
$S$ is a starting variable.
$\Sigma = \{\emptyset, \epsilon, a, b, (, ), +, \star\}$.
$V$ is a set of all variables.
$P = \{S \to \emptyset | a | b | SS | S^\star | S + S | (S) | \epsilon\}$