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Design and Analysis of Algorithms CPSC 413 Winter 2020
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PROBLEM SET #[1]

GROUP MEMBERS:

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SOURCES USED:

Lecture Notes 1

Notes from Tutorial

Problem 1. Give a precise definition of the Stable Matching Problem as described on p.3-4 of the textbook.

Answer:

The Stable Matching Problem is the problem of finding a stable matching between two equal-sized sets of elements M and W, given preference list for each man and woman. The result of the problem is a perfect match (everyone is in a pair) and does not have stability, does not exist $(\boldsymbol{m}, \boldsymbol{w}')(\boldsymbol{m}', \boldsymbol{w})$ in the result such that $\boldsymbol{m}$ prefers $\boldsymbol{w}$ and $\boldsymbol{w}$ prefers $\boldsymbol{w}$.

$\longrightarrow$ $\mathcal{A}$nswer

Problem 2. True or false? In every instance of the Stable Matching Problem, there is a stable matching containing a pair (m, w) such that m is ranked first on the preference list of w and w is ranked first on the preference list of m.

False. We have counterexample:
We have $\boldsymbol{M} = \{\boldsymbol{m_1}, \boldsymbol{m_2}\}$, $\boldsymbol{W} = \{\boldsymbol{w_1}, \boldsymbol{w_2}\}$ with preference lists:
$\boldsymbol{m_1} : \boldsymbol{w_1}, \boldsymbol{w_2}$
$\boldsymbol{m_2} : \boldsymbol{w_2}, \boldsymbol{w_1}$
$\boldsymbol{w_1} : \boldsymbol{m_2}, \boldsymbol{m_1}$
$\boldsymbol{w_2} : \boldsymbol{m_1}, \boldsymbol{w_2}$
Consider stable matching cases:
If $\boldsymbol{m_1}$ was paired with the first ranked w on the preference list, then $\boldsymbol{m_1}$ would be pair with $\boldsymbol{w_1}$. However, $\boldsymbol{m_1}$ is not ranked first on $\boldsymbol{w_1}$'s preference list.
If $\boldsymbol{m_2}$ was paired with the first ranked w on the preference list, then $\boldsymbol{m_2}$ would be pair with $\boldsymbol{w_2}$. However, $\boldsymbol{m_2}$ is not ranked first on $\boldsymbol{w_2}$'s preference list.
For this instance, there is no case $(\boldsymbol{m}, \boldsymbol{w})$ such that $\boldsymbol{m}$ is ranked first on the preference list of $\boldsymbol{w}$ and $\boldsymbol{w}$ is ranked first on the preference list of $\boldsymbol{m}$.

Therefore, the statement is false.

$\longrightarrow$ $\mathcal{A}$nswer

Problem 3. True or false? Consider an instance of the Stable Matching Problem in which there exists a man m and a woman w such that m is ranked first on the preference list of w and w is ranked first on the preference list of m. Then in every stable matching S for this instance, the pair (m, w) belongs to S.

True.
We prove by contradiction. Suppose that for the instance that $\boldsymbol{m}$ is ranked first on the preference list of $\boldsymbol{w}$ and $\boldsymbol{w}$ is ranked first on the preference list of $\boldsymbol{m}$, there is a stable matching with the pairs $\boldsymbol{(m, w')}$ and $\boldsymbol{(m', w)}$.
However, $\boldsymbol{w}$ is ranked first on $\boldsymbol{m}$'s preference list, $\boldsymbol{m}$ prefers $\boldsymbol{w}$ than $\boldsymbol{w'}$. And $\boldsymbol{m}$ is ranked first on $\boldsymbol{w}$'s preference list, $\boldsymbol{w}$ prefers $\boldsymbol{m}$ than $\boldsymbol{m'}$.
This is an instability, because $\boldsymbol{m}$ and $\boldsymbol{w}$ prefer each other, they will leave their current partners. Therefore, $\boldsymbol{(m, w)}$ must belong to S.

$\longrightarrow$ $\mathcal{A}$nswer

Problem 4. How many times will the while loop in the Stable Matching algorithm as presented on page 6 of the text be executed? Assume that set M contains n men and W contains n women.

Lower bound - Best case: We have n men and n women, and each man likes different woman. In this case, every man will find a free woman on the first proposal, then the while loop will be executed $\boldsymbol{n}$ times, equal numbers of men/women.

Upper bound - Worst case: We have n men and n women, and every man get matched with his last choice of woman which results in all n men have to propose to n women. In this worst case, the while loop has to iterate through all possible pairs of man and woman. Since we have n men and n women, the total possible pairs is $\boldsymbol{n^2}$. Therefore, the while loop will be executed at most $\boldsymbol{n^2}$ times.

$\longrightarrow$ $\mathcal{A}$nswer

Problem 5. Exercise 3 on pages 22/23

There is not always a stable pairs of schedules for every set of TV shows and ratings. Consider the following counterexample:
Network A has ShowA1 and ShowA2. Network B has ShowB1 and ShowB2.
Timeslot 1: ShowA1 - 20, ShowB2 - 5
Timeslot 2: ShowA2 - 10, ShowB2 - 15
This set cannot have a stable pair of schedules because Network A can switch ShowA1 and ShowA2 to win more slots. That means there exists a schedule such that Network A wins more time slots than the schedule above. Therefore, there is no stable pair of schedules for this set of TV shows and ratings.

$\longrightarrow$ $\mathcal{A}$nswer

Submitted by Minh Hang Chu - 30074056 on October 20, 2020.