Phil 377 Assignment 2

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Due: July 26, 2019 at 1 pm

1. For each item below, provide a Fitch-style proof of the claim.

   Proofs from single premises

   (a) $P \to (Q \to R) \vdash (P \to Q) \to (P \to R)$

1	$P \to (Q \to R)$	
2	$\vert\ P \to Q$	
3	$\vert\ \vert\ P$	
4	$\vert\ \vert\ Q \to R$	$\to$E 1, 3
5	$\vert\ \vert\ Q$	$\to$E 2, 3
6	$\vert\ \vert\ R$	$\to$E 4, 5
7	$\vert\ P \to R$	$\to$I 3–6
8	$(P \to Q) \to (P \to R)$	$\to$I 2–7

   (b) $(E \wedge L) \to M \vdash (\neg M \wedge L) \to \neg E$

1	$(E \wedge L) \to M$	
2	$\vert\ \neg M \wedge L$	
3	$\vert\ \neg M$	$\wedge$E 2
4	$\vert\ L$	$\wedge$E 2
5	$\vert\ \vert\ E$	
6	$\vert\ \vert\ E \wedge L$	$\wedge$I 5, 4
7	$\vert\ \vert\ M$	$\to$E 1, 6
8	$\vert\ \vert\ \bot$	$\neg$E 3, 7
9	$\vert\ \neg E$	$\neg$I 5–8
10	$(\neg M \wedge L) \to \neg E$	$\to$I 2–9

   (c) $(\neg D \to \neg B) \wedge (D \to (B \to C)) \vdash \neg B \vee (D \wedge C)$

1	$(\neg D \to \neg B) \land (D \to (B \to C))$	
2	$\neg D \to \neg B$	$\land$E 1
3	$D \to (B \to C)$	$\land$E 1
4	$B \lor \neg B$	LEM
5	$\mid\ \neg B$	
6	$\mid\ \neg B \lor (D \land C)$	$\lor$I 5
7	$\mid\ B$	
8	$\mid\ \mid\ \neg D$	
9	$\mid\ \mid\ \neg B$	$\to$E 2, 8
10	$\mid\ \mid\ \bot$	$\neg$E 7, 9
11	$\mid\ D$	$\neg$I 8–10
12	$\mid\ B \to C$	$\to$E 3, 11
13	$\mid\ C$	$\to$E 12, 7
14	$\mid\ D \land C$	$\land$I 11, 13
15	$\mid\ \neg B \lor (D \land C)$	$\lor$I 14
16	$\neg B \lor (D \land C)$	$\lor$I 4, 5–6, 7–13

Equivalences

(d) $A \land (B \lor C) \dashv\vdash (A \land B) \lor (C \land A)$

1	$A \land (B \lor C)$	
2	A	$\land$E 1
3	$B \lor C$	$\land$E 1
4	$\mid\ B$	
5	$\mid\ A \land B$	$\land$I 2, 4
6	$\mid\ (A \land B) \lor (C \land A)$	$\lor$I 5
7	$\mid\ C$	
8	$\mid\ C \land A$	$\land$I 2, 6
9	$\mid\ (A \land B) \lor (C \land A)$	$\lor$I 8
10	$(A \land B) \lor (C \land A)$	$\lor$E 3, 4–6, 7–9

1	$(A \wedge B) \vee (C \wedge A)$	
2	$\quad A \wedge B$	
3	$\quad A$	$\wedge$E 2
4	$\quad B$	$\wedge$E 2
5	$\quad B \vee C$	$\vee$I 4
6	$\quad A \wedge (B \vee C)$	$\wedge$I 3, 5
7	$\quad C \wedge A$	
8	$\quad C$	$\wedge$E 7
9	$\quad A$	$\wedge$E 8
10	$\quad C \vee B$	$\vee$I 8
11	$\quad A \wedge (B \vee C)$	$\wedge$I 9, 10
12	$A \wedge (B \vee C)$	$\vee$E 1, 2–6, 7–11

(e) $\neg(A \wedge B) \dashv\vdash A \rightarrow \neg B$

1	$\neg(A \wedge B)$	
2	$\quad A$	
3	$\quad\quad B$	
4	$\quad\quad A \wedge B$	$\wedge$I 2, 6
5	$\quad\quad \bot$	$\neg$E 1, 4
6	$\quad \neg B$	$\neg$I 3–5
7	$A \rightarrow \neg B$	$\rightarrow$I 2–6

1	$A \rightarrow \neg B$	
2	$\quad A \wedge B$	
3	$\quad A$	$\wedge$E 2
4	$\quad B$	$\wedge$E 2
5	$\quad \neg B$	$\rightarrow$E 1, 3
6	$\quad \bot$	$\neg$E 4, 5
7	$\neg(A \wedge B)$	$\neg$I 2–6

Theorems

(f) $\vdash (P \vee P) \rightarrow \neg\neg P$

1	$P \vee P$	
2	$P \vee \neg P$	LEM
3	P	
4	P	R 3
5	P	$\wedge$E 1, 3–4, 3–4
6	$\neg P$	
7	$\bot$	$\neg$E 5, 6
8	$\neg\neg P$	$\neg$I 7–8
9	$(P \vee P) \to \neg\neg P$	$\to$I 1–8

(g) $\vdash (A \to B) \vee (B \to A)$

1	$A \vee \neg A$	
2	A	
3	B	
4	A	R 3
5	$B \to A$	$\to$I 3–4
6	$(B \to A) \vee (A \to B)$	$\vee$I 5
7	$\neg A$	
8	A	
9	$\bot$	$\neg$E 7, 8
10	B	X 9
11	$A \to B$	$\to$I 8–10
12	$(A \to B) \vee (B \to A)$	$\vee$I 11
13	$(A \to B) \vee (B \to A)$	$\vee$E 2, 3–6, 7–12

Proofs from multiple premises

(h) $(L \to M),\ (M \to \neg L) \vdash \neg L$

1	$L \to M$	
2	$M \to \neg L$	
3	$\quad L$	
4	$\quad M$	$\to$E 1, 3
5	$\quad \neg L$	$\to$E 2, 4
6	$\quad \bot$	$\neg$E 3, 5
7	$\neg L$	IP 3–6

(i) $(E \wedge \neg L) \to M,\ \neg(M \vee L) \vdash \neg E$

1	$(E \wedge \neg L) \to M$	
2	$\neg(M \vee L)$	
3	$\quad E$	
4	$\quad\quad L$	
5	$\quad\quad L \vee M$	$\vee$I 4
6	$\quad\quad \bot$	$\neg$E 2, 5
7	$\quad \neg L$	IP 4–6
8	$\quad E \wedge \neg L$	$\wedge$I 3, 7
9	$\quad M$	$\to$E 1, 8
10	$\quad M \vee L$	$\vee$I 9
11	$\quad \bot$	$\neg$E 2, 10
12	$\neg E$	IP 3–11

Inconsistency

(j) $A \vee (\neg B \to C),\ \neg(A \vee C), \neg B \vdash \bot$

1	$A \vee (\neg B \to C)$		
2	$\neg(A \vee C)$		
3	$\neg B$		
4		A	
5		$A \vee C$	$\vee$I 4
6		$\bot$	$\neg$E 2, 5
7		$\neg B \to C$	
8		C	$\to$E 7, 3
9		$C \vee A$	$\vee$I 8
10		$\bot$	$\neg$E 2, 9
11	$\bot$		$\vee$E 1, 4–6, 7–10

2. Consider the propositional language with only $\rightarrow$ and $\perp$ as connectives. Is that language truth functionally complete? Explain your answer. (**2 points**)
The language with only $\rightarrow$ and $\perp$ as connectives is truth functionally complete. A propositional language is Truth Functionally Complete if each truth function can be represented by some formula in the language. We have: $\boldsymbol{A} \rightarrow \perp$ is equivalent to $\neg \boldsymbol{A}$. In semantics, two relations are equivalent when they have the same truth table.

A	$A \quad \rightarrow \quad \perp$
T	F
F	T

A	$\neg A$
T	F
F	T

Since we can represent $\rightarrow$ and $\perp$ as $\rightarrow$ and $\neg$, this language with these connectives are truth functionally complete (according to slide 45, lecture 5).

3. Consider the following truth function ($\mathscr{A} \downarrow \mathscr{B}$) given by the characteristic truth table:

A	B	$A \quad \downarrow \quad B$
T	T	F
T	F	T
F	T	T
F	F	F

Note that the syntax of this connective is that of a binary connective so for example: $\boldsymbol{C} \downarrow (\boldsymbol{B} \downarrow \boldsymbol{A})$ would be a formula.

(a) Represent this function in our language using $(\vee, \wedge, \neg)$ by the standard method, i.e., the method discussed in lecture 5.
The function in our language represents this function is: $(\boldsymbol{A} \wedge \neg \boldsymbol{B}) \vee (\neg \boldsymbol{A} \wedge \boldsymbol{B})$.

(b) Offer a more economical way to represent this function in our propositional language, i.e., you may use any of the connectives $(\neg, \vee, \wedge, \rightarrow, \leftrightarrow, \perp)$, but the goal is to represent the function with fewer symbols than the standard method.
A more economical way to represent is: $\neg(\boldsymbol{A} \leftrightarrow \boldsymbol{B})$.

(c) Consider a propositional language consisting of atoms and $\downarrow$ as the only connective. Would that language be truth functionally complete? Explain your answer.
No. The language consists only atoms and $\downarrow$ as the only connectives are not truth functionally complete. A propositional language is Truth Functionally Complete if each truth function can be represented by some formula in the language. Since $\downarrow$ is represented as $\neg$ and $\leftrightarrow$, the language with these connectives are not truth functionally complete (refer to slide 45, lecture 5)

For the next questions, if you are writing the assignment by hand, please print out these pages and copy your final answers directly on to those pages and either scan them or attach them to your assignment. Highlighting columns: if you are doing it by hand, just use a highlighter, if you are doing it in LaTeX, replace the appropriate column specification in the tabular environment with a 'y'. For example:

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\begin{tabular}{|ccc|c c c c c c c c c c c c|}
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becomes

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\begin{tabular}{|ccc|c c c c c y c c c c y c c|}
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if the relevant columns to highlight were 9 and 14 (starting from the first column).

4. Complete a full truth table showing that $(E \wedge L) \rightarrow M \vDash (\neg M \wedge L) \rightarrow \neg E$. Indicate which column(s) show this by highlighting it(them) in yellow. (**2 points**)

E	L	M	$(E$	$\wedge$	$L)$	$\rightarrow$	M	$\vDash$	$(\neg$	M	$\wedge$	$L)$	$\rightarrow$	$\neg$	E
T	T	T	T	T	T	T	T		F	T	F	T	T	F	T
T	T	F	T	T	T	F	F		T	F	T	T	F	F	T
T	F	T	T	F	F	T	T		F	T	F	F	T	F	T
T	F	F	T	F	F	T	F		T	F	F	F	T	F	T
F	T	T	F	F	T	T	T		F	T	F	T	T	T	F
F	T	F	F	F	T	T	F		T	F	T	T	T	T	F
F	F	T	F	F	F	T	T		F	T	F	F	T	T	F
F	F	F	F	F	F	T	F		T	F	F	F	T	T	F

5. Complete a full truth table showing that $\{A \vee (\neg B \rightarrow C),\ \neg(A \vee C), \neg B\}$ is unsatisfiable. Indicate which column(s) show this by highlighting it(them) in yellow. (**2 points**)

A	B	C	A	$\vee$	$(\neg$	B	$\rightarrow$	$C)$,	$\neg$	$(A$	$\vee$	$C)$,	$\neg$	B
T	T	T	T	T	F	T	T	T	F	T	T	T	F	T
T	T	F	T	T	F	T	T	F	F	T	T	F	F	T
T	F	T	T	T	T	F	T	T	F	T	T	T	T	F
T	F	F	T	T	T	F	F	F	F	T	T	F	T	F
F	T	T	F	T	F	T	T	T	F	F	T	T	F	T
F	T	F	F	T	F	T	T	F	T	F	F	F	F	T
F	F	T	F	T	T	F	T	T	F	F	T	T	T	F
F	F	F	F	F	T	F	F	F	T	F	F	F	T	F

6. Complete a full truth table showing that $(A \rightarrow B) \vee (B \rightarrow C)$ is a tautology. Indicate which column(s) show this by highlighting it(them) in yellow. (**2 points**)

A	B	C	$(A$	$\rightarrow$	$B)$	$\vee$	$(B$	$\rightarrow$	$C)$
T	T	T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T	F	F
T	F	T	T	F	F	T	F	T	T
T	F	F	T	F	F	T	F	T	F
F	T	T	F	T	T	T	T	T	T
F	T	F	F	T	T	T	T	F	F
F	F	T	F	T	F	T	F	T	T
F	F	F	F	T	F	T	F	T	F