1 Homework week 3 : Linear Regression

Exercise 1: Linear Regression

$$t = y(x, w) + noise = N(y(x, w), \beta^{-1})$$
$$\Rightarrow p(t|x, w, \beta) = N(t|y(x, w), \beta^{-1})$$

The likelihood function:

$$p(t|x, w, \beta) = \prod_{n=1}^{N} N(t_n|y(x_n, w), \beta^{-1})$$

...
Minimize Loss function

$$L = \frac{1}{2}\sum_{i=1}^{N}(y(x_n, w) - t_n)^2$$

with $y(x_n, w) = XW$, $x = \begin{bmatrix} x_1 \\ ... \\ x_n \end{bmatrix}$, $t = \begin{bmatrix} t_1 \\ ... \\ t_n \end{bmatrix}$, $W = \begin{bmatrix} w_0 \\ w_1 \end{bmatrix}$

$$\Rightarrow L = \sum_{i=1}^{n}(x_i.TW - t_i)^2$$

$$= (XW - t)^T(XW - t)$$
$$= (W^T X^T XW - W^T X^T t - t^T XW + t^T t)$$

To minimize L, take derivative and set to zero
$\frac{\delta L}{\delta W} = \frac{1}{2}(X^T XW + X^T XW - 2X^T t) = X^T XW - X^T t = 0$

$$\Rightarrow X^T XW = X^T t$$

$$\Rightarrow W = (X^T X)^{-}1X^T t$$