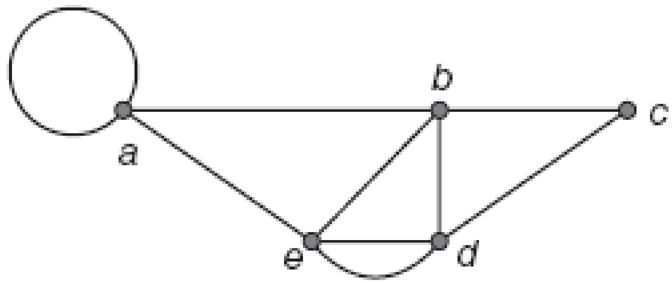
Minh Anh Nguyen
Discrete Mathematics **Assignment-4**

Section 2.1:

1. Consider the following undirected graph.



   (a) How many edges are there in this graph?

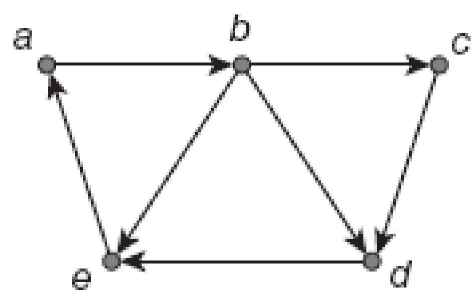
   There are 9 edges in this graph.

   (b) Give the degree of each vertex.
   The dergree of each vertex is:

   a has degree 4
   b has degree 4
   c has degree 2
   d has degree 4
   e has degree 4

   (c) Do these numbers agree with Euler's first observation?
   The sum of the degrees is $4 + 4 + 2 + 4 + 4 = 18$.
   There are 9 edges in the graph.
   The sum of the degrees is doubled the number of edges.
   Hence, these numbers agree with Euler's first observation.

2. Consider the following directed graph.



(a) Give the indegree of each vertex.

The indegree of a is 1.
The indegree of b is 1.
The indegree of c is 1.
The indegree of d is 2.
The indegree of e is 2.

(b) Give the outdegree of each vertex.

The outdegree of a is 1.
The outdegree of b is 3.
The outdegree of c is 1.
The outdegree of d is 1.
The outdegree of e is 1.

(c) Compute the sum of the indegrees and the sum of the outdegrees. What do you notice?
The sum of the indegrees is:

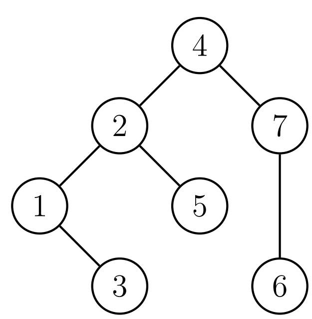
$$1 + 1 + 1 + 2 + 2 = 7$$

The sum of the outdegrees is:

$$1 + 3 + 1 + 1 + 1 = 7$$

I noticed that the sum of the indegrees is equal to the sum of outdegrees and also equal to the numbers of edges.

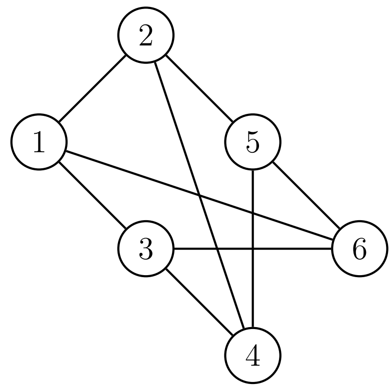
3. A circuit is *simple* if it has no repeated edges. Draw a connected, undirected graph with seven vertices and no simple circuits. How many edges does it have?



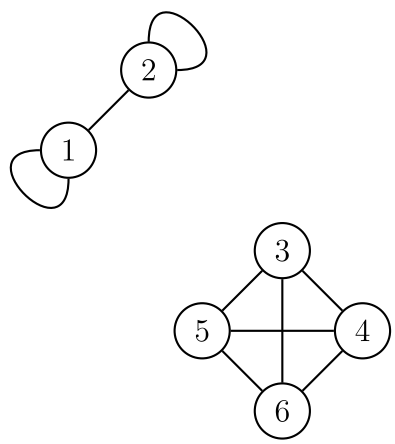
It has 6 edges.

4. Draw an undirected graph with six vertices, each of degree 3, such that the graph is:

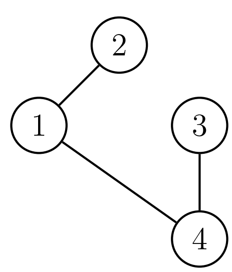
   (a) Connected.



   (b) Not connected.



5. A graph is called simple if it has no multiple edges or loops. Draw five different connected, simple, undirected graphs with four vertices.
   1.



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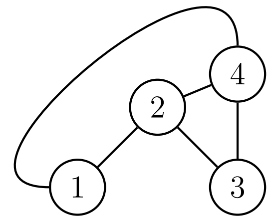

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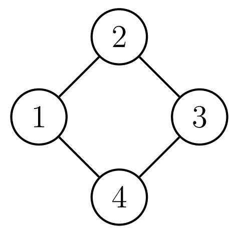

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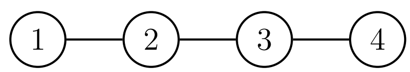

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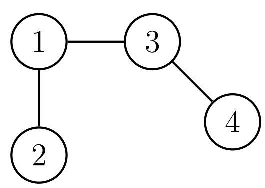
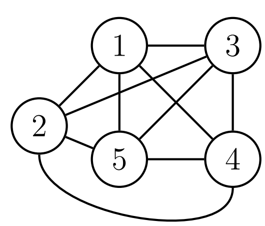


6. An undirected graph is called *complete* if every vertex shares an edge with every other vertex. Draw a complete graph on five vertices. How many edges does it have?

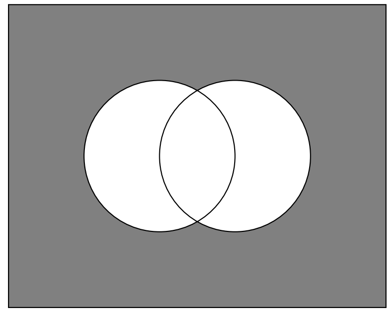


It has 10 edges.

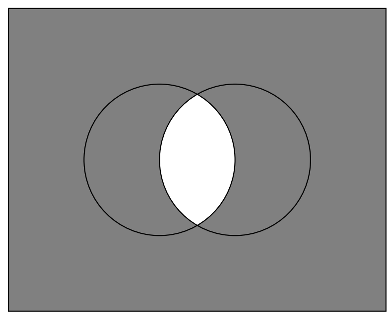
Section 2.2:

1. Draw Venn diagrams to illustrate De Morgan's laws for sets (Theorem 2.1).

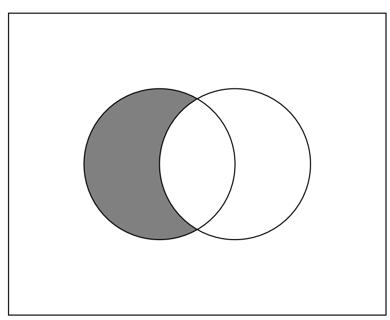
   1. $(A \cup B)' = A' \cap B'$



   2. $(A \cap B)' = A' \cup B'$



2. Draw a Venn diagram to show the region $A \cap B'$. This region is also denoted $A \setminus B$, and is called the set difference.



3. Let $A = \{2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, and suppose the universal set is $U = \{1, 2, \ldots, 9\}$. List all the elements in the following sets.

   (a) $(A \cup B)' = \{1, 7, 8, 9\}$

   (b) $(A \cap B) \times A = \{3, 4\} \times A = \{(3, 2), (3, 3)(3, 4), (4, 2), (4, 3), (4, 4)\}$

   (c) $P(B \setminus A) = \{\phi, \{4\}, \{5\}, \{4, 5\}\}$

4. Let the following sets be given.

$$\begin{aligned} \text{C} &= \text{the set of all charitable people.} \\ \text{G} &= \text{the set of all good citizen.} \\ \text{P} &= \text{the set of all polite people.} \end{aligned}$$

   Write the statement, “Everyone who is charitable and polite is a good citizen,” in the language of set theory.

$$C \cap P \subseteq G$$

5. Consider the following sets. The universal set for this problem is **N**. ¡¡¡¡¡¡¡ HEAD

$$\text{A} = \text{The set of all even numbers.}$$

$$\text{B} = \text{The set of all prime numbers.}$$

$$\text{C} =$$

   =======

$$\begin{aligned} \text{A} &= \text{The set of all even numbers.} \\ \text{B} &= \text{The set of all prime numbers.} \\ \text{C} &= \text{The set of all perfect squares.} \\ \text{D} &= \text{The set of all multiples of 10.} \end{aligned}$$

   Using only the symbols 3, A, B, C, D, N, $\in$, $\subseteq$, $=$, $\neq$, $\cap$, $\cup$, x, $'$, ϕ, (, and), write the following statements in set notation.

   (a) None of the perfect squares are prime number.

$$(C \cap B) = \{\phi\}$$

   (b) All multiples of 10 are even numbers.

$$D \subseteq A$$

   (c) The number 3 is a prime number that is not even.

$$3 \in (B \cap A')$$

   (d) If you take all the prime numbers, all the even numbers, all the perfect squares, and all the multiples of 10, you still won’t have all the natural numbers.

$$(B \cup A \cup C \cup D) \neq N$$

6. Consider the following sets. The universal set U for this problem is the set of all residents of India.

$$\begin{aligned} \text{A} &= \text{the set of all English speakers.} \\ \text{B} &= \text{the set of all Hindu speakers.} \\ \text{C} &= \text{the set of all Urdu speakers.} \end{aligned}$$

Express the following sets in the symbols of set theory.

(a) Residents of India who speak English, Hindi, and Urdu.

$$A \cap B \cap C$$

(b) Residents of India who do not speak English, Hindi, or Urdu.

$$(A \cup B \cup C)'$$

(c) Residents of India who speak English, but not Hindi or Urdu.

$$A \cap (B \cup C)'$$

7. Consider the following sets. The universal set for this problem is the set of all quadrilaterals.

A = The set of all parallelograms.
B = The set of all rhombuses.
C = The set of all rectangles.
D = The set of all trapezoids.

Using only the symbols x, A, B, C, D, $\in$, $\subseteq$, $=$, $\neq$, $\cap$, $\cup$, $\times$, $'$, ϕ, (, and), write the following statements in set notation.

(a) The polygon x is a parallelogram, but it isn't a rhombus.

$$x \in (A \cap B')$$

(b) There are other quadrilaterals besides parallelograms and trapezoids.

$$(A \cup D)' \neq \phi$$

(c) Both rectangles and rhombuses are types of parallelograms.

$$(C \cup B) \subseteq A$$

8. Let the following sets be given. The universal set for this problem is the set of all students at some university.

F = the set of all freshmen.
S = the set of all seniors.
M = the set of all math majors.
C = the set of all CS majors.

(a) Using only the symbols F, S, M, C, $||$, $\cap$, $\cup$, $'$, and $>$, translate the following statement into the language of set theory.

There are more freshmen who aren't math majors than there are senior CS majors.

$$(F \cap M') > (S \cap C)$$

(b) Translate the following statement in set theory into everyday English.

$$(F \cap M) \subseteq C$$

All freshmen in Math Major are also CS major.

9. Let E be the set of even numbers, and let P be the set of prime numbers. Use set notation to express the following statement: "2 is the only even prime number."

$$\{2\} = (E \cap P)$$

10. Two sets are called disjoint if they have no elements in common, i.e., if their intersection is the empty set. Prove that finite sets A and B are disjoint if and only if $|A| + |B| = |A \cup B|$. Use the definition of ϕ and the inclusion–exclusion principle (Equation 2.2.3) in your proof.

$$|A \cup B| = |A| + |B| - |A \cap B|$$

Because the intersection of A and B is an empty set.

$$|A \cup B| = |A| + |B| - |\phi|$$

$$|A \cup B| = |A| + |B| - 0$$

$$|A \cup B| = |A| + |B|$$

11. In a class of 40 students, everyone has either a pierced nose or a pierced ear. The professor asks everyone with a pierced nose to raise their hands. Nine hands go up. Then the professor asked everyone with a pierced ear to do likewise. This time there are 34 hands raised. How many students have piercings both on their ears and their noses?

The number of student have piercings both on their ears and noses is:

$$34 + 9 - 40 = 3$$

12. How many integers in the set $\{n \in Z | 1 \leq n \leq 700\}$ are divisible by 2 or 7?

The number of integers in the set are divisible by 2 is:

$$\lfloor \frac{700}{2} \rfloor = 350$$

The number of integers in the set are divisible by 7 is:

$$\lfloor \frac{700}{7} \rfloor = 100$$

The number of integers in the set are divisible by both 2 and 7 is:

$$\lfloor \frac{700}{14} \rfloor = 50$$

Hence, the number of integers in the set divisible by both 2 or 7 is:

$$350 + 100 - 50 = 400$$

13. Let A, B, and C be sets, and let $X = A \cup B$

   (a) Write $|X \cap C|$ in terms of $|A \cap C|$, $|B \cap C|$, and $|A \cap B \cap C|$.

$$|X \cap C| = |A \cap C| + |B \cap C| - |A \cap B \cap C|$$

(b) Write $|A \cup B \cup C|$ in terms of A, B, C, $|A \cap B|$, $|A \cap C|$, $|B \cap C|$, and $|A \cap B \cap C|$. (The result is the inclusion–exclusion principle for three sets.)

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

14. How many integers in the set $\{n \in Z | 1 \leq n \leq 700\}$ are divisible by 2, 5, or 7? The number of integers in the set are divisible by 2 is:

$$\lfloor \frac{700}{2} \rfloor = 350$$

The number of integers in the set are divisible by 5 is:

$$\lfloor \frac{700}{5} \rfloor = 140$$

The number of integers in the set are divisible by 7 is:

$$\lfloor \frac{700}{7} \rfloor = 100$$

The number of integers in the set are divisible by 2 and 5 is:

$$\lfloor \frac{700}{10} \rfloor = 70$$

The number of integers in the set are divisible by 2 and 7 is:

$$\lfloor \frac{700}{14} \rfloor = 50$$

The number of integers in the set are divisible by 5 and 7 is:

$$\lfloor \frac{700}{35} \rfloor = 20$$

The number of integers in the set are divisible by 2, 5 and 7 is:

$$\lfloor \frac{700}{70} \rfloor = 10$$

The number of integers in the set are divisible by 2, 5 or 7 is:

$$350 + 140 + 100 - 70 - 50 - 20 + 10 = 460$$

15. Write down all elements of $(\{1, 2, 3\} \cap \{2, 3, 4, 5\}) \cup \{6, 7\}$.

$$(\{1, 2, 3\} \cap \{2, 3, 4, 5\}) \cup \{6, 7\} = \{2, 3\} \cup \{6, 7\} = \{2, 3, 6, 7\}$$

16. Write down all elements of $\{A, B, C\} \times \{H, K\}$.

$$\{A, B, C\} \times \{H, K\} = \{(A, H), (A, K), (B, H), (B, K), (C, H), (C, K)\}$$

17. Let S = a, b, c. Write down all the elements in the following sets.

(a) $S \times S$

$$S \times S = \{(a, a), (a, b), (a, c), (b, a), (b, b), (b, c), (c, a), (c, b), (c, c)\}$$

(b) $P(S)$

$$P(S) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$$