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Calculus 1 Assignment-10
Section: 04
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7. A table of values of an increasing function f is shown. Use the table to find the lower and upper bound estimates of $\int_{10}^{30} f(x)dx$

x	10	14	18	22	26	30
$f(x)$	−12	−6	−2	1	3	8

$$Lower = 4 \times (-12 + -6 + -2 + 1 + 3) = -64$$

$$Upper = 4 \times (-6 + -2 + 1 + 3 + 8) = 16$$

11. Use the Midpoint Rule with the given value of n to approximate the integral. Round the answer to four decimal places.

$$\int_0^8 \sin\sqrt{x}\ dx, n = 4$$

$$\Delta x = \frac{8-0}{4} = 2$$

$$\int_0^8 \sin\sqrt{x} \approx 2 \times (f(\frac{2+0}{2}) + f(\frac{4+2}{2}) + f(\frac{6+4}{2}) + f(\frac{8+6}{2})) \approx 6.1820$$

19. Express the limit as a definite integral on the given interval.

$$\lim_{n\to\infty} \Sigma_{i=1}^n \frac{\sin x_i}{1+x_i}\Delta x, [0,\pi]$$

$$\lim_{n\to\infty} \Sigma_{i=1}^n \frac{\sin x_i}{1+x_i}\Delta x = \int_0^\pi \frac{sinx}{1+x}dx$$

23. Show that the definite integral is equal to $\lim_{n\to\infty} R_n$ and then evaluate the limit.

$$\int_0^4 (x-x^2)dx, R_n = \frac{4}{n}\Sigma_{i=1}^n[\frac{4i}{n} - \frac{16i^2}{n^2}]$$

$$\Delta x = \frac{4-0}{n} = \frac{4}{n}$$

$$x_i = \frac{4i}{n}$$

According to Riemann Sum:

$$\int_0^4 (x - x^2)dx = R_n = \frac{4}{n}\Sigma_{i=1}^n[\frac{4i}{n} - \frac{16i^2}{n}]$$

$$R_n = \frac{4}{n}\Sigma_{i=1}^n[\frac{4i}{n} - \frac{16i^2}{n^2}] = \frac{4}{n}\Sigma_{i=1}^n\frac{4i}{n} - \frac{4}{n}\Sigma_{i=1}^n\frac{16i^2}{n^2}$$

$$= \frac{16}{n^2}\Sigma_{i=1}^n i - \frac{64}{n^3}\Sigma_{i=1}^n i^2 = \frac{16}{n^2}\frac{n(n+1)}{2} - \frac{64}{n^3}\frac{n(n+1)(2n+1)}{6}$$

$$= \frac{16(n+1)}{2n} - \frac{64(n+1)(2n+1)}{6n^2}$$

$$\lim_{n\to\infty} R_n = \lim_{n\to\infty}\frac{16(n+1)}{2n} - \frac{64(n+1)(2n+1)}{6n^2} = \lim_{n\to\infty}\frac{16(n+1)}{2n} - \lim_{n\to\infty}\frac{64(n+1)(2n+1)}{6n^2}$$

$$= \lim_{n\to\infty}\frac{16(1+1/n)}{2} - \lim_{n\to\infty}\frac{64(1+1/n)(2+1/n)}{6} = 8 - \frac{128}{6} = -\frac{40}{3}$$

31. Use the form of the definition of the integral given in Theorem 4 to evaluate the integral.

$$\int_1^5 (3x^2 + 7x)dx$$

$$\Delta x = \frac{5-1}{n} = \frac{4}{n}$$

$$x_i = 1 + \frac{4i}{n}$$

$$\int_1^5 (3x^2+7x)dx = \lim_{n\to\infty}\Sigma_{i=1}^n(3(1+\frac{4i}{n})^2+7(1+\frac{4i}{n}))\frac{4}{n} = \lim_{n\to\infty}\Sigma_{i=1}^n(3+\frac{24i}{n}+\frac{48i^2}{n^2}+7+\frac{28i}{n})\frac{4}{n}$$

$$= \lim_{n\to\infty}\Sigma_{i=1}^n(10 + \frac{52i}{n} + \frac{48i^2}{n^2})\frac{4}{n} = \lim_{n\to\infty}(\Sigma_{i=1}^n 10 + \frac{52}{n}\Sigma_{i=1}^n i + \frac{48}{n^2}\Sigma_{i=1}^n i^2)\frac{4}{n}$$

$$= \lim_{n\to\infty}(10n + \frac{52}{n}\frac{n(n+1)}{2} + \frac{48}{n^2}\frac{n(n+1)(2n+1)}{6})\frac{4}{n}$$

$$= \lim_{n\to\infty}(40 + \frac{208}{n^2}\frac{n(n+1)}{2} + \frac{192}{n^3}\frac{n(n+1)(2n+1)}{6})$$

$$= \lim_{n\to\infty}(40 + 208\frac{(1+1/n)}{2} + 192\frac{(1+1/n)(2+1/n)}{6})$$

$$= 40 + 104 + 64 = 208$$

41. Evaluate the integral by interpreting it in terms of areas.

$$\int_{-2}^5 (10 - 5x)dx$$

Because x = 2 is where the function change from positive to negative.

$$\int_{-2}^5 (10-5x)dx = \int_{-2}^2 (10-5x)dx + \int_2^5 (10-5x)dx = \frac{1}{2}(4\times 20 + 3\times(-15)) = 40 - 45/2 = 17.5$$

52. Given that $\int_0^\pi \sin^4 x dx = \frac{3}{8}\pi$, what is $\int_\pi^0 \sin^4 \theta d\theta$.

$$\int_\pi^0 \sin^4 \theta d\theta = \int_\pi^0 \sin^4 x dx = -\int_0^\pi \sin^4 x dx = -\frac{3}{8}\pi$$

65. Use the properties of integrals to verify the inequality without evaluating the integrals.

$$\int_0^4 (x^2 - 4x + 4) dx \geq 0$$

Because $f(x) = x^2 - 4x + 4 = (x-2)^2 \geq 0$ with all $0 \leq x \leq 4$.
Hence, $\int_0^4 (x^2 - 4x + 4) dx \geq 0$.