

7. A table of values of an increasing function f is shown. Use the table to find the lower and upper bound estimates of $\int_{10}^{30} f(x)dx$

x	10	14	18	22	26	30
$f(x)$	-12	-6	-2	1	3	8

$$Lower = 4 \times (-12 + -6 + -2 + 1 + 3) = -64$$

$$Upper = 4 \times (-6 + -2 + 1 + 3 + 8) = 16$$

11. Use the Midpoint Rule with the given value of n to approximate the integral. Round the answer to four decimal places.

$$\int_0^8 \sin \sqrt{x} \, dx, n = 4$$

$$\Delta x = \frac{8 - 0}{4} = 2$$

$$\int_0^8 \sin \sqrt{x} \approx 2 \times (f(\frac{2+0}{2}) + f(\frac{4+2}{2}) + f(\frac{6+4}{2}) + f(\frac{8+6}{2})) \approx 6.1820$$

19. Express the limit as a definite integral on the given interval.

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{\sin x_i}{1 + x_i} \Delta x, [0, \pi]$$

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{\sin x_i}{1 + x_i} \Delta x = \int_0^\pi \frac{\sin x}{1 + x} dx$$

23. Show that the definite integral is equal to $\lim_{n \rightarrow \infty} R_n$ and then evaluate the limit.

$$\int_0^4 (x - x^2) dx, R_n = \frac{4}{n} \sum_{i=1}^n [\frac{4i}{n} - \frac{16i^2}{n^2}]$$

$$\Delta x = \frac{4 - 0}{n} = \frac{4}{n}$$

$$x_i = \frac{4i}{n}$$

According to Riemann Sum:

$$\begin{aligned}\int_0^4 (x - x^2) dx &= R_n = \frac{4}{n} \sum_{i=1}^n \left[\frac{4i}{n} - \frac{16i^2}{n} \right] \\ R_n &= \frac{4}{n} \sum_{i=1}^n \left[\frac{4i}{n} - \frac{16i^2}{n} \right] = \frac{4}{n} \sum_{i=1}^n \frac{4i}{n} - \frac{4}{n} \sum_{i=1}^n \frac{16i^2}{n} \\ &\quad \frac{4}{n} \sum_{i=1}^n \frac{4i}{n} - \frac{4}{n} \sum_{i=1}^n \frac{16i^2}{n}\end{aligned}$$