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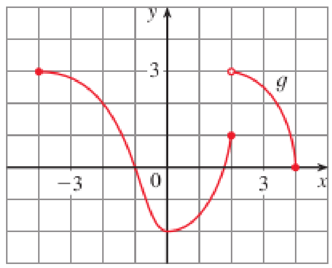
Calculus 1 **Written HW - 1**

Section 1.1:

3. The graph of a function g is given.



(a) State the values of $g(-2)$, $g(0)$, $g(2)$, and $g(3)$.

Answer: $\boxed{\textbf{g(-2) = 2,g(0) = -2,g(2) = 1,g(3) = 2.5}}$

(b) For what value(s) of x is $g(x) = 3$?

Answer: $\boxed{\textbf{x = -4}}$

(c) For what value(s) of x is $g(x) \leq 3$

Answer: $\boxed{\textbf{[-4, 4]}}$

(d) State the domain and range of g.

Domain: $\boxed{\textbf{[-4, 4]}}$

Range: $\boxed{\textbf{[-2, 3]}}$

(e) On what interval(s) is g increasing?

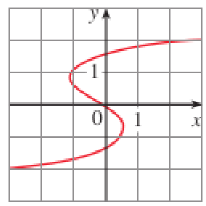
Answer: $\boxed{\textbf{[0, 2]}}$

7. Determine whether the equation $3x - 5y = 7$ defines y as a function of x.
Answer:

$$3x - 5y = 7$$
$$5y = 3x - 7$$
$$\boxed{y = \frac{3}{5}x - \frac{7}{5}}$$

So I can conclude that y is a function of x.

15. Determine whether the curve is the graph of a function of x. If it is, state the domain and domain of the function.



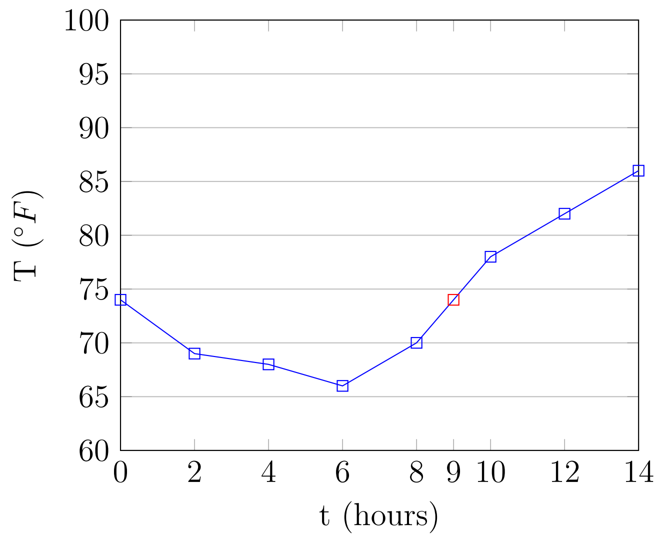
Answer:

If a vertical line is drawn at x = 0 , it intersects the graph at three points, indicating that the graph does not represent a function of x.

31. Temperature readings T (in $°F$) were recorded every two hours from midnight to 2:00PM in Atlanta on a day in June. The time t was measured in hours from midnight.

t	0	2	4	6	8	10	12	14
T	74	69	68	66	70	78	82	86

(a) Use the reading to sketch a rough graph of T as a function of t.
Answer:



(b) Use your graph to estimate the temperature at 9:00AM.

Answer:

It is known that 9:00AM means t = 9. Therefore, the temperature is around 74°$\boldsymbol{F}$(The red square).

37. Evaluate the difference quotient for the given function. Simplify your answer.

$$f(x) = \tfrac{1}{x},\ \tfrac{f(x)-f(a)}{x-a}$$

Answer:

$$\tfrac{f(x)-f(a)}{x-a}$$

$$(\tfrac{1}{x} - \tfrac{1}{a})/(x-a)$$

$$(\tfrac{a-x}{ax})/(x-a)$$

$$\boxed{\boldsymbol{\frac{-1}{ax}}}$$

Section 1.2:

1. Classify each function as a power function, root function, polynomial (state its degree), rational function, algebraic function, trigonometric function, exponential function, or logarithmic function.

   (a) $f(x) = x^3 + 3x^2$

   Answer: **The function is a polynomial of degree 3.**

   (b) $g(t) = cos^2(t) - sin(t)$

   Answer: **The function is a trigonometric function.**

   (c) $r(t) = t^{\sqrt{3}}$

   Answer: **The function is a power function.**

   (d) $v(t) = 8^t$

   Answer: **The function is a exponential function.**

   (e) $y = \dfrac{\sqrt{x}}{x^2+1}$

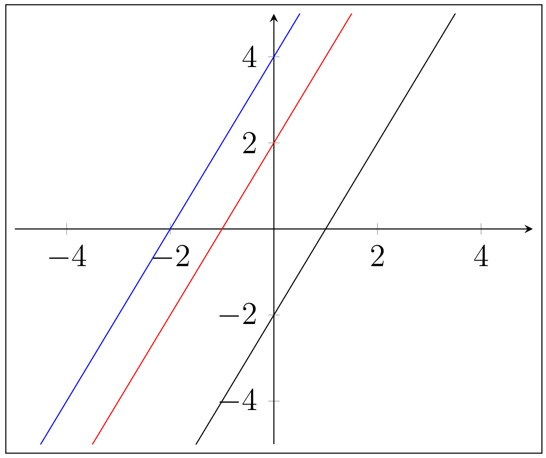
   Answer: **The function is a algebraic function.**

   (f) $f(u) = \log_{10} u$

   Answer: **The function is a logarithmic function.**

7. (a) Find an equation for the family of linear functions with slope 2 and sketch several members of the family.

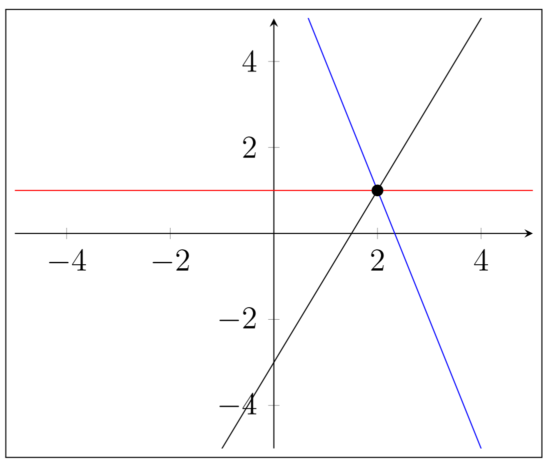
Answer: $y = 2x + b$ with $\boxed{\textbf{red: b = 2, blue: b = 4, black: b = -2.}}$



(b) Find an equation for the family of linear functions such that $f(2) = 1$. Sketch several members of the family.

Answer: $y - 1 = m(x - 2)$ where m is the *slope* with

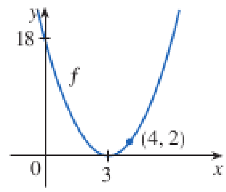
$\boxed{\textbf{red: m = 0, blue: m = -3, black: m = 2.}}$



(c) Which function belongs to both families?

Answer: The function $\boxed{\textbf{y = 2x - 3}}$ has slope 2 and also pass through coordinate (2,1). So, it belongs to both families.

11. Find a formula for the quadratic function whose graph is shown.



Answer: Since the graph is a parabola, we can infer that the function is a polynomial of degree 2.
The function we need to find is: $f(x) = ax^2 + bx + c$ where $a \neq 0$. The function also pass through three coordinates which are (0,18), (3,0), (4,2).

For the first coordinate (0,18):

$$18 = a(0)^2 + b(0) + c$$
$$c = 18$$

Thus, we can conclude that $c = 18$

For the second coordinate (3,0):

$$0 = a(3)^2 + b(3) + 18$$
$$a(3) + b = -6 \text{ (Equation 1)}$$

For the third coordinate (4,2):

$$2 = a(4)^2 + b(4) + 18$$
$$a(4) + b = -4 \text{ (Equation 2)}$$

Now we can solve the system of two equations:

$$a(3) + b = -6 \text{ (Equation 1)}$$
$$a(4) + b = -4 \text{ (Equation 2)}$$

We can subtract the second equation from the first one as follows:

$$-a = -2$$
$$a = 2$$

Input $a = 2$ into the first equation we get:

$$2(3) + b = -6$$
$$b = -12$$

Thus, the final equation is: $\boxed{\boldsymbol{f(x) = 2x^2 - 12x + 18}}$

25. The power output of a wind turbine depends on many factors. It can be shown using physical principles that the power P generated by a wind turbine is modeled by

$$P = kAv^3$$

where v is the wind speed, A is the area swept out by the blades, and k is a constant that depends on air density, efficiency of the turbine, and the design of the wind turbine blades.

(a) If only wind speed is doubled, by what factor is the power output increased?

Answer: If wind speed is doubled, the function becomes:

$$P = kA(2v)^3$$

$$P = kA8v^3$$

$\boxed{\textbf{Thus, the power output increases by a factor of 8.}}$

(b) If only the length of the blades is doubled, by what factor is the power output increased?

Answer: If the length of the blades is doubled, the function becomes:

$$P = k(2l * 2l)\pi v^3 \text{ where l is the length of the blade}$$

$$P = k4Av^3$$

$\boxed{\textbf{Thus, the power output increases by a factor of 4.}}$

(c) For a particular wind turbine, the length of the blades is $30m$ and $k = 0.214kg/m^3$. Find the power output (in watts, $W = m^2 * kg/s^3$) when the wind speed is $10m/s$, $15m/s$, and $25m/s$.

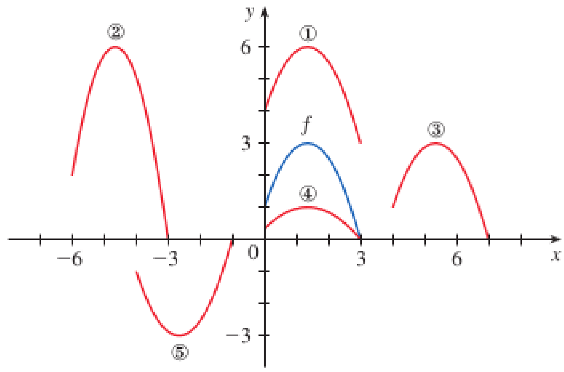
Answer:
Area of the blades is $30 \times 30 \times \pi = 900\pi(m^2)$

When the windspeed is $10m/s$: $P = 0.214 \times 900\pi \times (10)^3 \approx \boxed{\mathbf{605,071(W)}}$

When the windspeed is $15m/s$: $P = 0.214 \times 900\pi \times (15)^3 \approx \boxed{\mathbf{2,042,114(W)}}$

When the windspeed is $25m/s$: $P = 0.214 \times 900\pi \times (25)^3 \approx \boxed{\mathbf{9,454,230(W)}}$

3. The graph of $y = f(x)$ is given. Match each equation with its graph and given reasons for your choices.



(a) $y = f(x - 4)$

Answer: The graph will be shifted 4 digits to the right and will be the $\boxed{\textbf{3rd graph.}}$

(b) $y = f(x) + 3$

Answer: The graph will be shifted 3 digits upward and will be the $\boxed{\textbf{1st graph.}}$

(c) $y = \frac{1}{3} f(x)$

Answer: The graph will be shrunk by a factor of 3 and will be the $\boxed{\textbf{4th graph.}}$

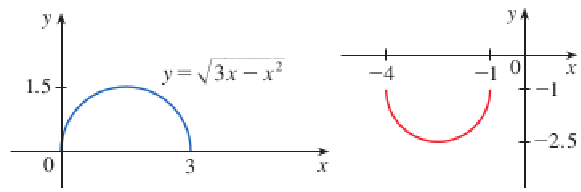
(d) $y = -f(x + 4)$

Answer: The graph will be inverted and shifted 4 digits to the left. It will be the $\boxed{\textbf{5th graph.}}$

(e) $y = 2f(x + 6)$

Answer: The graph will be expanded by a factor of 2 and shifted 6 digits to the left. It will be the $\boxed{\textbf{2nd graph.}}$

7. The graph of $y = \sqrt{3x - x^2}$ is given. Use transformations to create a function whose graph is as shown.

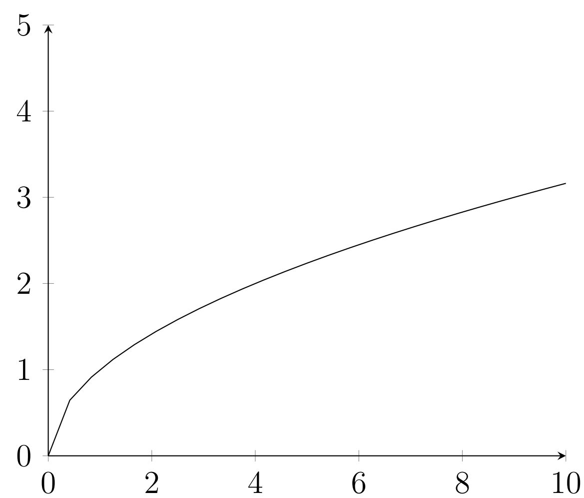


Answer:
Let say that $f(x) = y = \sqrt{3x - x^2}$.
First, the graph is inverted so the new graph will be $-f(x)$.
And then, the graph is shift 1 digit downward and 4 digits to the left.
So, the new graph will be $\boxed{\boldsymbol{-f(x+4)-1}}$.

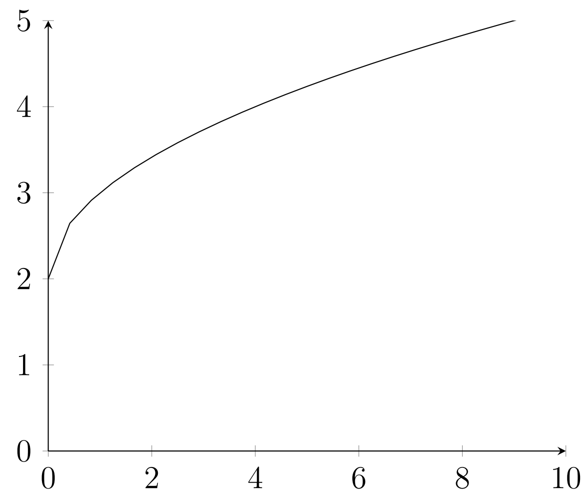
17. Graph the function by hand, not by plotting points, but by starting with the graph of one of the standard functions given in Table 1.2.3, and then applying the appropriate transformations.

$$y = 2 + \sqrt{x+1}$$

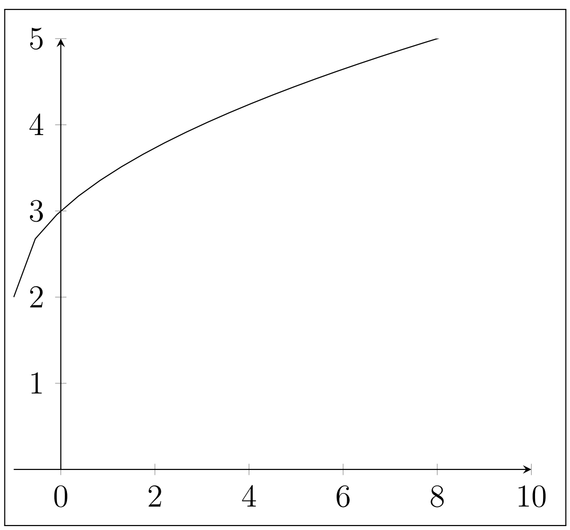
Answer: First we draw $f(x) = \sqrt{x}$:



Then we draw $f(x) + 2$:



Finally, we draw $\boxed{\boldsymbol{f(x+1)+2}}$

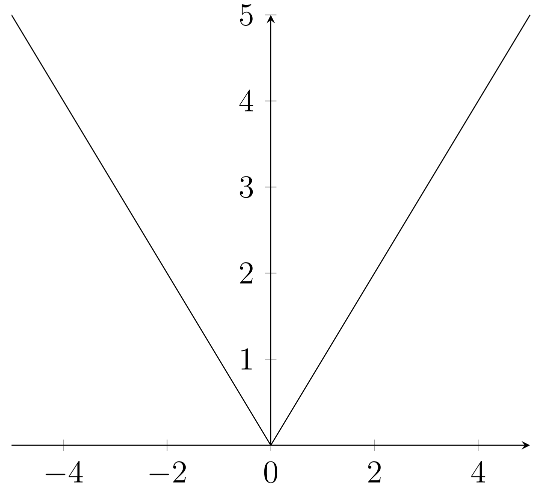


Thus, the final graph is the graph of function $y = 2 + \sqrt{x+1}$.

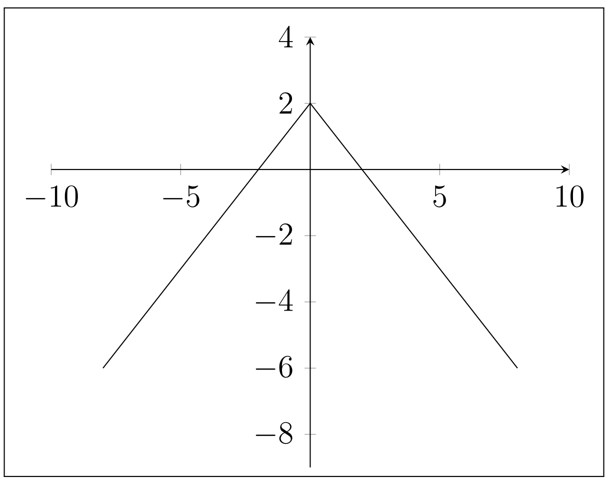
21. Graph the function by hand, not by plotting points, but by starting with the graph of one of the standard functions given in Table 1.2.3, and then applying the appropriate transformations.

$$y = 2 - |x|$$

Answer: First we draw $f(x) = |x|$:



Finally we draw $\boxed{\boldsymbol{2 - f(x)}}$



Thus, the final graph is the graph of function $y = 2 - |x|$.

33. Given $f(x) = \sqrt{25 - x^2}$, $g(x) = \sqrt{x+1}$. Find

(a) $(f+g)(x)$

Answer: $(f+g)(x) = \sqrt{25-x^2} + \sqrt{x+1}$

The domain of $f(x)$ is x such that:

$$\begin{aligned} 25 - x^2 &\geq 0 \\ x^2 &\leq 25 \\ -5 \leq x &\leq 5 \end{aligned}$$

Thus, the domain of $f(x)$ is: $[-5, 5]$

The domain of $g(x)$ is x such that:

$$\begin{aligned} x + 1 &\geq 0 \\ x &\geq -1 \end{aligned}$$

Thus, the domain of $g(x)$ is: $[-1, \infty)$

$\boxed{\textbf{The domain of } (\boldsymbol{f}+\boldsymbol{g})(\boldsymbol{x}) \textbf{ is: } [-1, 5]}$

(b) $(f-g)(x)$

Answer: $(f-g)(x) = \sqrt{25-x^2} - \sqrt{x+1}$

$\boxed{\textbf{The domain of } (\boldsymbol{f}-\boldsymbol{g})(\boldsymbol{x}) \textbf{ is: } [-1, 5]}$

(c) $(f \times g)(x)$

Answer: $(f \times g)(x) = \sqrt{25-x^2} \times \sqrt{x+1} = \sqrt{-x^3 - x^2 + 25x + 25}$

$\boxed{\textbf{The domain of } (\boldsymbol{f}\times\boldsymbol{g})(\boldsymbol{x}) \textbf{ is: } [-1, 5]}$

(d) $(\frac{f}{g})(x)$

Answer: $(\frac{f}{g})(x) = \sqrt{\frac{25-x^2}{x+1}}$

The domain of $f(x)$ and $g(x)$ is: $[-1, 5] \setminus \{1\}$

$\boxed{\textbf{Thus, the domain of } \frac{\boldsymbol{f}}{\boldsymbol{g}} \textbf{ is: } (-1, 5]}$

37. Given $f(x) = \dfrac{1}{\sqrt{x}}$, $g(x) = x + 1$. Find

(a) $(f \circ g)(x)$

Answer: $(f \circ g)(x) = \dfrac{1}{\sqrt{x+1}}$

The domain of $f(x)$ is x such that:

$$x > 0$$

Thus, the domain of $f(x)$ is: $(0, \infty)$

The domain of $(f \circ g)(x)$ is x such that:

$$\begin{aligned} x + 1 &> 0 \\ x &> -1 \end{aligned}$$

The domain of $(f \circ g)(x)$ is: $(0, \infty) \cap (-1, \infty)$

$\boxed{\textbf{Thus, the domain of } \boldsymbol{(f \circ g)(x)} \textbf{ is: } \boldsymbol{(-1, \infty)}}$

(b) $(g \circ f)(x)$

Answer: $(g \circ f)(x) = \dfrac{1}{\sqrt{x}} + 1$

The domain of $(g \circ f)(x)$ is x such that:

$$x > 0$$

$\boxed{\textbf{The domain of} \boldsymbol{(g \circ f)(x)} \textbf{ is: } \boldsymbol{(0, \infty)}}$

(c) $(f \circ f)(x)$

Answer: $(f \circ f)(x) = \sqrt[4]{x}$

$\boxed{\textbf{Thus, the domain of } \boldsymbol{(f \circ f)(x)} \textbf{ is: } \boldsymbol{(0, \infty)}}$

(d) $(g \circ g)(x)$

Answer: $(g \circ g)(x) = x + 1 + 1 = x + 2$

$\boxed{\textbf{Thus, the domain of } \boldsymbol{(g \circ g)(x)} \textbf{ is: } \boldsymbol{(-\infty, \infty)}}$

Section 1.4:

1. A tank holds 1000 gallons of water, which drains from the bottom of the tank in half an hour. The values in the table show the volume V of water remaining in the tank (in gallons) after t minutes.

$t(min)$	5	10	15	20	25	30
$V(gal)$	694	444	250	111	28	0

(a) If P is the point $(15, 250)$ on the graph of V, find the slopes of the secant lines PQ when Q is the point on the graph with $t = 5, 10, 20, 25$, and 30.

Answer:

The slopes of the secant lines when $t = 5$ is: $m = \dfrac{250 - 694}{15 - 5} = \boxed{\mathbf{-44.4}}$

The slopes of the secant lines when $t = 10$ is: $m = \dfrac{250 - 444}{15 - 10} = \boxed{\mathbf{-38.8}}$

The slopes of the secant lines when $t = 20$ is: $m = \dfrac{111 - 250}{20 - 15} = \boxed{\mathbf{-27.8}}$

The slopes of the secant lines when $t = 25$ is: $m = \dfrac{25 - 250}{25 - 15} = \boxed{\mathbf{-22.5}}$

The slopes of the secant lines when $t = 30$ is: $m = \dfrac{0 - 250}{30 - 15} = \boxed{\mathbf{-16.(6)}}$

(b) Estimate the slope of the tangent line at P by averaging the slopes of two secant lines.

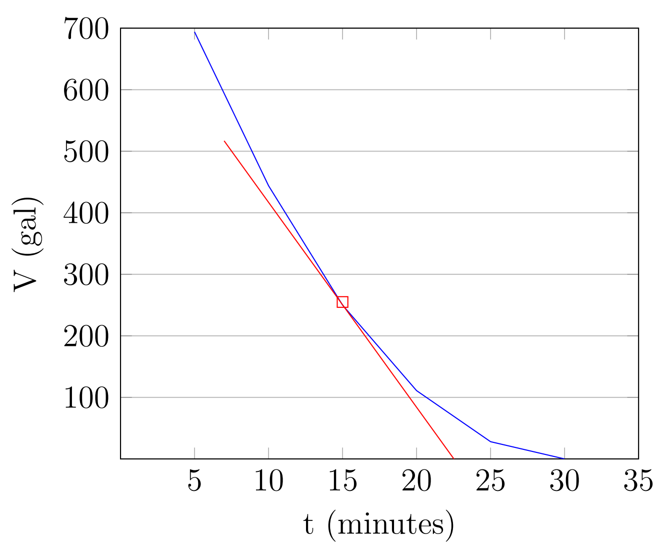
Answer:

The slope of the tangent line at P is the average of the slopes of two secant lines PQ with Q when $t = 10$ and with Q when $t = 20$:

$$m = \frac{-38.8 - 27.8}{2} = \boxed{\mathbf{-33.3}}$$

(c) Use a graph of V to estimate the slope of the tangent line at P. (This slope represents the rate at which the water is flowing from the tank after 15 minutes.)

Answer:



2. A student bought a smartwatch that tracks the number of steps she walks throughout the day. The table shows the number of steps recorded t minutes after 3:00PM on the first day she wore the watch.

$t(min)$	0	10	20	30	40
$V(gal)$	3438	4559	5622	6536	7398

(a) Find the slopes of the secant lines corresponding to the given intervals of t. What do these slopes represent?

i. $[0, 40]$
Answer:

$$m = \frac{7398 - 3438}{40 - 0} = \boxed{\mathbf{99}}$$

ii. $[10, 20]$
Answer:

$$m = \frac{5622 - 4559}{20 - 10} = \boxed{\mathbf{106.3}}$$

iii. $[20, 30]$
Answer:

$$m = \frac{6536 - 5622}{30 - 20} = \boxed{\mathbf{91.4}}$$

These slopes represent the average number of steps per minute for the corresponding time intervals.

(b) Estimate the student's walking pace, in steps per minute, at 3:20PM by averaging the slopes of two secant lines.

Answer: The student's walking pace, in steps per minute, at 3:20PM is the average of the slopes of $[10, 20]$ and $[20, 30]$:

$$\frac{106.3 + 91.4}{2} = 98.85$$

Thus, at 3: 20 PM, the student's walking pace of the student is 98.85 steps per

7. The table shows the position of a motorcyclist after accelerating from rest.

$t(second)$	0	1	2	3	4	5	6
$s(feet)$	0	4.9	20.6	46.5	79.2	124.8	176.7

(a) Find the average velocity for each time period:

i. $[2, 4]$
Answer:

$$m = \frac{79.2 - 20.6}{4 - 2} = \boxed{\mathbf{29.3}}$$

ii. $[3, 4]$
Answer:

$$m = \frac{79.2 - 46.5}{4 - 3} = \boxed{\mathbf{32.7}}$$

iii. $[4, 5]$
Answer:

$$m = \frac{124.8 - 79.2}{5 - 4} = \boxed{\mathbf{45.6}}$$

iv. $[4, 6]$
Answer:

$$m = \frac{176.7 - 79.2}{6 - 4} = \boxed{\mathbf{48.75}}$$

(b) Use the graph of s as a function of t to estimate the instantaneous velocity when $t = 3$.
Answer:

The average velocity for [2,3] is:

$$m = \frac{46.5 - 20.6}{3 - 2} = \boxed{\mathbf{25.9}}$$

The average velocity for [3,4] is:

$$m = \frac{79.2 - 46.5}{4 - 3} = \boxed{\mathbf{32.7}}$$

The instantaneous velocity at $t = 3$ is:

$$m = \frac{25.9 + 32.7}{2} = \boxed{\mathbf{29.3}}$$

Thus, the instantaneous velocity at $t = 3$ is $\boxed{\boldsymbol{29.3 ft/s}}$.