

1. In the domain of integers, let $P(x, y)$ be the predicate " $x \cdot y = 12$." Tell whether each of the following statements is true or false.
 - (a) $P(3, 4) \rightarrow True$
 - (b) $P(3, 5) \rightarrow False$
 - (c) $[P(2, 6) \vee P(3, 7)] \rightarrow True$
 - (d) $[(\forall x)(\forall y)(P(x, y) \rightarrow P(y, x))] \rightarrow True$
 - (e) $[(\forall x)(\exists y)P(x, y)] \rightarrow True$
2. In the domain of all penguins, let $D(x)$ be the predicate " x is dangerous." Translate the following quantified statements into simple, everyday English.
 - (a) $(\forall x)D(x)$: All penguins are dangerous.
 - (b) $(\exists x)D(x)$: There exists at least one penguin that is dangerous.
 - (c) $\neg(\exists x)D(x) \equiv (\forall x)\neg D(x)$: All penguins are not dangerous.
 - (d) $(\exists x)\neg D(x)$: There exists at least one penguin that is not dangerous.
3. In the domain of all movies, let $V(x)$ be the predicate " x is violent." Write the following statements in the symbols of predicate logic.
 - (a) Some movies are violent: $(\exists x)V(x)$
 - (b) Some movies are not violent: $(\exists x)\neg V(x)$
 - (c) No movies are violent: $(\forall x)\neg V(x)$
 - (d) All movies are violent: $(\forall x)V(x)$
4. Let the following predicates be given. The domain is all mammals.

$$L(x) = \text{"x is a lion."}$$

$$F(x) = \text{"x is fuzzy."}$$

Translate the following statements into predicate logic.

- (a) All lions are fuzzy: $(\forall x)(L(x) \rightarrow F(x))$
 - (b) Some lions are fuzzy: $(\exists x)((L(x) \wedge F(x))$
5. In the domain of all books, consider the following predicates.

$$H(x) = \text{"x is heavy."}$$

$$C(x) = \text{"x is confusing."}$$

Translate the following statements in predicate logic into ordinary English.

- (a) $(\forall x)(H(x) \rightarrow C(x))$: All books that are heavy are confusing.
 - (b) $(\exists x)(C(x) \wedge H(x))$: There is at least one book that is both heavy and confusing.
 - (c) $(\forall x)(C(x) \vee H(x))$: All books are confusing or heavy .
 - (d) $(\exists x)(H(x) \wedge \neg C(x))$: There is at least one book that is heavy but not confusing.
6. The domain of the following predicates is the set of all plants.

$P(x)$ = "x is poisonous"

$Q(x)$ = "Jeff has eaten x."

Translate the following statements into predicate logic.

- (a) Some plants are poisonous: $(\exists x)P(x)$
- (b) Jeff has never eaten a poisonous plant: $(\forall x)[P(x) \rightarrow \neg Q(x)]$
- (c) There are some nonpoisonous plants that Jeff has never eaten:

$$(\exists x)[\neg P(x) \wedge \neg Q(x)]$$

7. In the domain of nonzero integers, let $I(x, y)$ be the predicate " x/y is an integer." Determine whether the following statements are true or false, and explain why.

- (a) $(\forall y)(\exists x)I(x, y)$: For all y , there exists at least one x that x/y is an integer.

For all nonzero integer y , we can always pick $x = 2y$, because $2y/y = 2$, which is an integer. Hence, this statement is true.

- (b) $(\exists x)(\forall y)I(x, y)$: There exists at least one x that with all y , x/y is an integer.

The only x that x/y with all y is an integer is 0. But since the domain is nonzero integers, this statement is false.

8. In the domain of integers, consider the following predicates: Let $N(x)$ be the statement " $x \neq 0$." Let $P(x, y)$ be the statement " $xy = 1$."

- (a) Translate the following statement into the symbols of predicate logic.

For all integers x , there is some integer y such that if $x \neq 0$, then $xy = 1$.

$$(\forall x)(\exists y)[N(x) \rightarrow P(x, y)]$$

- (b) Write the negation of your answer to part (a) in the symbols of predicate logic. Simplify your answer so that it uses the \wedge connective.

The negation of the answer to part (a) is:

$$\neg(\forall x)(\exists y)[N(x) \rightarrow P(x, y)]$$

$$(\exists x)\neg(\exists y)[N(x) \rightarrow P(x, y)]$$

$$(\exists x)(\forall y)\neg[N(x) \rightarrow P(x, y)]$$

$$(\exists x)(\forall y)\neg[\neg N(x) \vee P(x, y)]$$

$$(\exists x)(\forall y)[\neg(\neg N(x)) \wedge \neg P(x, y)]$$

$$(\exists x)(\forall y)[N(x) \wedge \neg P(x, y)]$$

- (c) Translate your answer from part (b) into an English sentence. The answer from part (b) is:

$$(\exists x)(\forall y)[N(x) \wedge \neg P(x, y)]$$

It can be translated: “There are at least one x that for all y , $x \neq 0$ and $xy \neq 1$.”

- (d) Which statement, (a) or (b), is true in the domain of integers? Explain.

The statement of (a) is false and (b) is true in the domain of integers.

$$P(x, y) : xy = 1$$

$$y = \frac{1}{x}$$

Because for all x different from 0, you cannot pick a y that $y = \frac{1}{x}$ that y is in the domain of integers (except for $x = 1$). Hence, the statement (a) is false.

The statement (b) is the negation of the statement (a). Therefore, the statement (b) is true.

9. Let $P(x, y, z)$ be the predicate “ $x + y = z$.”

- (a) Simplify the statement $\neg(\forall x)(\forall y)(\exists z)P(x, y, z)$ so that no quantifier lies within the scope of a negation.

$$\neg(\forall x)(\forall y)(\exists z)P(x, y, z)$$

$$(\exists x)\neg(\forall y)(\exists z)P(x, y, z)$$

$$(\exists x)(\exists y)\neg(\exists z)P(x, y, z)$$

$$(\exists x)(\exists y)(\forall z)\neg P(x, y, z)$$

- (b) Is the statement $(\forall x)(\forall y)(\exists z)P(x, y, z)$ true in the domain of all integers? Explain why or why not.

The statement is true because for all x and y in the domain of all integers, we can always pick a z that $z = x + y$ and z is also a integer.

- (c) Is the statement $(\forall x)(\forall y)(\exists z)P(x, y, z)$ true in the domain of all integers between 1 and 100? Explain why or why not.

The statement is false because for $x = 99$ and $y = 98$, we cannot pick a z in the domain of all integers between 1 and 100 because $z = x + y = 99 + 98 = 197$.

10. The domain of the following predicates is the set of all traders who work at the Tokyo Stock Exchange.

$$P(x, y) = \text{"x makes more money than y"}$$

$$Q(x, y) = \text{"x} \neq \text{y"}$$

Translate the following predicate logic statements into ordinary, everyday English. (Don't simply give a word-for-word translation; try to write sentences that make sense.)

- (a) $(\forall x)(\exists y)P(x, y)$: For every trader working at the Tokyo Stock Exchange, there is at least one trader that make less money than them.
- (b) $(\exists y)(\forall x)[Q(x, y) \rightarrow P(x, y)]$: There is a trader that every other traders make more money than.
- (c) Which statement is impossible in this context? Why?
The statement "For every trader working at the Tokyo Stock Exchange, there is at least one trader that make more money than them." is impossible since there must be at least a trader that make more money than everyone else.
11. Translate the following statements into predicate logic using the two common constructions in Section 1.3.5. State what your predicates are, along with the domain of each.

- (a) All natural numbers are integers.

$$F(x) = \text{"x is an integer."}$$

$$P(x) = \text{"x is a natural number."}$$

The domain is all integers.

The statement can be translated into: $(\forall x)[P(x) \rightarrow F(x)]$

- (b) Some integers are natural numbers.

$$F(x) = \text{"x is a natural number."}$$

The domain is all integers.

The statement can be translated into: $(\exists x)F(x)$

- (c) All the streets in Cozumel, Mexico, are one-way.

$$F(x) = \text{"x is one-way."}$$

The domain is all the streets in Cozumel, Mexico.

The statement can be translated into: $(\forall x)F(x)$

- (d) Some streets in London don't have modern curb cuts.

$$F(x) = \text{"x has modern curb cuts."}$$

The domain is all the streets in London.

The statement can be translated into: $(\exists x)\neg F(x)$

12. Write the following statements in predicate logic. Define your predicates. Use the domain of all quadrilaterals.

- (a) All rhombuses are parallelograms.

$$F(x) = \text{"x is a rhombus."}$$

$$P(x) = \text{"x is a parallelogram."}$$

The statement can be translated into: $(\forall x)(F(x) \rightarrow P(x))$

- (b) Some parallelograms are not rhombuses.

$$F(x) = \text{"x is a rhombus."}$$

$$P(x) = \text{"x is a parallelogram."}$$

The statement can be translated into: $(\exists x)(P(x) \wedge \neg F(x))$

13. Let the following predicates be given. The domain is all people.

$$R(x) = \text{"x is rude."}$$

$$\neg R(x) = \text{"x is pleasant."}$$

$$C(x) = \text{"x is a child."}$$

- (a) Write the following statement in predicate logic.

There is at least one rude child.

The statement can be translated into: $(\exists x)[C(x) \wedge (R(x))]$

- (b) Formally negate your statement from part (a).

The negation of my statement from part (a) is:

$$\neg(\exists x)[C(x) \wedge R(x)]$$

$$(\forall x)\neg(C(x) \wedge R(x))$$

$$(\forall x)(\neg C(x) \vee \neg R(x))$$

- (c) Write the English translation of your negated statement. The negated statement can be translated into:

There is no rude child.

or

All the children are pleasant.

14. In the domain of all people, consider the following predicate.

$$P(x, y) = \text{"x needs to love y."}$$

- (a) Write the statement "Everybody needs somebody to love" in predicate logic.

$$(\forall x)(\exists y)(P(x, y))$$

(b) Formally negate your statement from part (a).

$$\neg(\forall x)(\exists y)(P(x, y))$$

$$(\exists x)\neg(\exists y)(P(x, y))$$

$$(\exists x)(\forall y)(\neg P(x, y))$$

(c) Write the English translation of your negated statement. The negated statement can be translated into:

There exists at least a person that doesn't need to love anyone.