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Discrete Mathematics Homework 1.1

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1. Let the following statements be given.

$$p = \text{“}\textit{There is water in the cylinders.}\text{”}$$
$$q = \text{“}\textit{The head gasket is blown.}\text{”}$$
$$r = \text{“}\textit{The car will start.}$$

   (a) Translate the following statement into symbols of formal logic.

   If the head gasket is blown and there's water in the cylinders, then the car won't start.

   Answer: $(q \wedge p) \to \neg r$

   (b) Translate the formal statement $r \to \neg(q \wedge p)$ into everyday English.
   Answer:

p	q	r	$p \wedge q$	$r \to \neg(q \wedge p)$	$r \to \neg q \vee \neg p$
T	T	T	T	F	F
T	T	F	T	T	T
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	F	T	T
F	T	F	F	T	T
F	F	T	F	T	T
F	F	F	F	T	T

   According to the truth table, $r \to \neg(q \wedge p) \equiv r \to \neg q \vee \neg p$.

   So we can translate $r \to \neg(q \wedge p)$ as if the car starts, then the head gasket is not blown or there is no water in the cylinders.

2. Let the following statements be given.

$$p = \text{“}\textit{You are in Seoul.}\text{”}$$
$$q = \text{“}\textit{You are in Kwangju.}\text{”}$$
$$r = \text{“}\textit{You are in South Korea.}\text{”}$$

   (a) Translate the following statement into symbols of formal logic.

   If you are not in South Korea, then you are not in Seoul or Kwangju.

   Answer: $\neg r \to \neg(p \vee q)$

   (b) Translate the formal statement $q \to (r \wedge \neg p)$ into everyday English.
   Answer: If you are in Kwangju, then you are in South Korea and not in Seoul.

3. Let the following statements be given.

$$p = \text{"You can vote."}$$
$$q = \text{"You are under 18 years old."}$$
$$r = \text{"You are from Mars."}$$

(a) Translate the following statement into symbols of formal logic.

You can't vote if you are under 18 years old or you are from Mars.

Answer: $(q \vee r) \to \neg p$

(b) Give the contrapositive of this statement in the symbols of formal logic.

Answer: $p \to \neg(q \vee r)$

(c) Give the contrapositive in English.

Answer: If you can vote, then you are not under 18 years old or from Mars.

4. Let s be the following statement.

If you are studying hard, then you are staying up late at night.

(a) Give the converse of s.

Answer: If you are staying up late at night, then you are studying hard.

(b) Give the contrapositive of s

Answer: If you are not staying up late at night, then you are not studying hard.

5. Let s be the following statement.

If it is raining, then the ground is wet.

(a) Give the converse of s.

Answer: If the ground is wet, then it is raining.

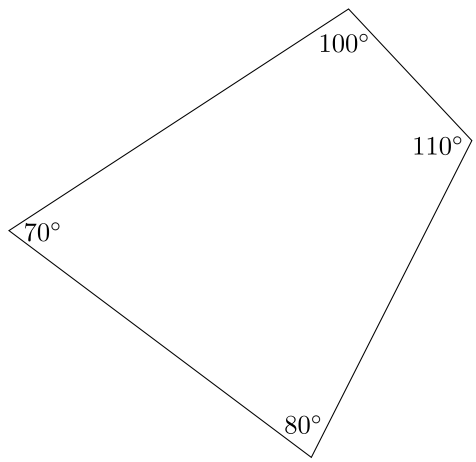
(b) Give the contrapositive of s.

Answer: If the ground is not wet, then it is not raining.

6. Give an example of a quadrilateral that shows that the converse of the following statement is false.

If a quadrilateral has a pair of parallel sides, then it has a pair of supplementary angles.

Answer:
The converse of the following statement is if a quadrilateral has a pair of supplementary angles, it has a pair of parallel sides.
An example of a quadrilateral that has a pair of supplementary angles but does not have a pair of parallel sides is one with the following angles:



7. We say that two ordered pairs (a, b) and (c, d) are *equal* when $a = c$ and $b = d$. Let s be the following statement.

$$\text{If } (a, b) = (c, d), \text{ then } a = c.$$

   (a) Is this statement true?
   Answer: According to the information given in the question, the statement is true.

   (b) Write down the converse of s.
   Answer: If $\boldsymbol{a = c}$, then $\boldsymbol{(a, b) = (c, d)}$.

   (c) Is the converse of s true? Explain.
   Answer: The converse of s is not always true. Because if $\boldsymbol{a = c}$ but $\boldsymbol{b \neq d}$, then $\boldsymbol{(a, b) \neq (c, d)}$.

8. Give an example of a true if–then statement whose converse is also true.

Answer:

"If a number is divisible by 2 and 3, then it is divisible by 6."

The converse of this statement is:

"If a number is divisible by 6, then is divisible by 2 and 3."

Both of this statements is correct.

9. Show that $p \leftrightarrow q$ is logically equivalent to $(p \to q) \wedge (q \to p)$ using truth tables.

Answer:

p	q	$p \to q$	$q \to p$	$(p \to q) \wedge (q \to p)$	$p \leftrightarrow q$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

According to the truth tables, the columns of $\boldsymbol{p \leftrightarrow q}$ and $\boldsymbol{(p \to q) \wedge (q \to p)}$ have the same T/F values for all cases, so they are logically equivalent.

10. Use truth tables to establish the following equivalences.

    (a) Show that $\neg(p \vee q)$ is logically equivalent to $\neg p \wedge \neg q$
    Answer:

p	q	$\neg(p \vee q)$	$\neg p \wedge \neg q$
T	T	F	F
T	F	F	F
F	T	F	F
F	F	T	T

    According to the truth table given, $\boldsymbol{\neg(p \vee q) \equiv \neg p \wedge \neg q}$.

    (b) Show that $\neg(p \wedge q)$ is logically equivalent to $\neg p \vee \neg q$ **Answer:**

p	q	$\neg(p \wedge q)$	$\neg p \vee \neg q$
T	T	F	F
T	F	T	T
F	T	T	T
F	F	T	T

    According to the truth table given, $\boldsymbol{\neg(p \wedge q) \equiv \neg p \vee \neg q}$.

11. Are the statements $\neg(p \to q)$ and $\neg p \to \neg q$ logically equivalent?
Justify your answer using truth tables.

Answer:

p	q	$\neg(p \to q)$	$\neg p \to \neg q$
T	T	F	T
T	F	T	T
F	T	F	F
F	F	F	T

According to the truth table given, $\boldsymbol{\neg(p \to q) \not\equiv \neg p \to \neg q}$.

12. Use truth tables to show that $(a \vee b) \wedge (\neg(a \wedge b))$ is logically equivalent to $a \leftrightarrow \neg b$. (This arrangement of T/F values is sometimes called the exclusive or of a and b.)

Answer:

a	b	$a \vee b$	$a \wedge b$	$(a \vee b) \wedge (\neg(a \wedge b))$	$a \leftrightarrow \neg b$
T	T	T	T	F	F
T	F	T	F	T	T
F	T	T	F	T	T
F	F	F	F	F	F

According to the truth given, $(a \vee b) \wedge (\neg(a \wedge b)) \equiv a \leftrightarrow \neg b$.

15. Let A be the statement $p \to (q \wedge \neg r)$. Let B be the statement $q \leftrightarrow r$.

(a) Construct truth tables for A and B.
Answer:

p	q	r	$q \wedge \neg r$	$A = p \to (q \wedge \neg r)$	$B = q \leftrightarrow r$
T	T	T	F	F	T
T	T	F	T	T	F
T	F	T	F	F	F
T	F	F	F	F	T
$\boldsymbol{F}$	$\boldsymbol{T}$	$\boldsymbol{T}$	$\boldsymbol{F}$	$\boldsymbol{T}$	$\boldsymbol{T}$
F	T	F	T	T	F
F	F	T	F	T	F
$\boldsymbol{F}$	$\boldsymbol{F}$	$\boldsymbol{F}$	$\boldsymbol{F}$	$\boldsymbol{T}$	$\boldsymbol{T}$

(b) Suppose statements A and B are both true.
What can you conclude about statement p?
Explain your answer using the truth table.
If statement A and B are both true, p is False based on both rows 5 and 8 (highlighted rows).

16. Use truth tables to prove the following *distributive properties* for propositional logic.

(a) $p \wedge (q \vee r)$ is logically equivalent to $(p \wedge q) \vee (p \wedge r)$
Answer:

p	q	r	$q \vee r$	$p \wedge q$	$p \wedge r$	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T
T	F	T	T	F	T	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

According to the truth table given, $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

(b) $p \vee (q \wedge r)$ is logically equivalent to $(p \vee q) \wedge (p \vee r)$
Answer:

p	q	r	$q \wedge r$	$p \vee q$	$p \vee r$	$p \vee (q \wedge r)$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	T	F	F	F
F	F	T	F	F	T	F	F
F	F	F	F	F	F	F	F

17. Use truth tables to prove the *associative properties* for propositional logic.

(a) $p \vee (q \vee r)$ is logically equivalent to $(p \vee q) \vee r$
Answer:

p	q	r	$q \vee r$	$p \vee q$	$p \vee (q \vee r)$	$(p \vee q) \vee r$
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	T	T	T	T
T	F	F	F	T	T	T
F	T	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	T	F	T	T
F	F	F	F	F	F	F

According to the truth table given, $\boldsymbol{p \vee (q \vee r) \equiv (p \vee q) \vee r}$.

(b) $p \wedge (q \wedge r)$ is logically equivalent to $(p \wedge q) \wedge r$
Answer:

p	q	r	$q \wedge r$	$p \wedge q$	$p \wedge (q \wedge r)$	$(p \wedge q) \wedge r$
T	T	T	T	T	T	T
T	T	F	F	T	F	F
T	F	T	F	F	F	F
T	F	F	F	F	F	F
F	T	T	T	F	F	F
F	T	F	F	F	F	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

18. Mathematicians say that "statement P is stronger than statement Q" if Q is true whenever P is true, but not conversely. (In other words, "P is stronger than Q" means that P $\to$ Q is always true, but Q $\to$ P is not true, in general.) Use truth tables to show the following.

(a) $a \wedge b$ is stronger than a

Answer:

a	b	$a \wedge b$	$(a \wedge b) \to a$	$a \to (a \wedge b)$
T	T	T	T	T
T	F	F	T	F
F	T	F	T	T
F	F	F	T	T

According to the truth table given, $(a \wedge b) \to a$ is always True while $a \to (a \wedge b)$ is not always True. So, $a \wedge b$ is stronger than a.

(b) a is stronger than $a \vee b$

Answer:

a	b	$a \vee b$	$a \to (a \vee b)$	$(a \vee b) \to a$
T	T	T	T	T
T	F	T	T	T
F	T	T	T	F
F	F	F	T	T

According to the truth table given, $a \to (a \vee b)$ is always True while $(a \vee b) \to a$ is not always True. So, a is stronger than $a \vee b$.

(c) $a \wedge b$ is stronger than $a \vee b$

Answer:

a	b	$a \wedge b$	$a \vee b$	$(a \wedge b) \to (a \vee b)$	$(a \vee b) \to (a \wedge b)$
T	T	T	T	T	T
T	F	F	T	T	F
F	T	F	T	T	F
F	F	F	F	T	T

According to the truth table given, $(a \wedge b) \to (a \vee b)$ is always True while $(a \vee b) \to (a \wedge b)$ is not always True. So, $a \wedge b$ is stronger than $a \vee b$.

(d) b is stronger than $a \to b$

Answer:

a	b	$a \to b$	$b \to (a \to b)$	$(a \to b) \to b$
T	T	T	T	T
T	F	F	T	T
F	T	T	T	T
F	F	T	T	F

According to the truth table given, $b \to (a \to b)$ is always True while $(a \to b) \to b$ is not always True. So, b is stronger than $(a \to b)$.

19. Suppose Q is a quadrilateral. Which statement is stronger? Explain.

- Q is a square.
- Q is a rectangle.

Answer: The statement "Q is a square" is stronger than "Q is a rectangle" because if Q is a square, it automatically satisfies the conditions to be a rectangle. However, while Q can be a rectangle, it is not necessarily a square in all cases.

20. Which statement is stronger? Explain.

- Manchester United is the best football team in England.
- Manchester United is the best football team in Europe.

Answer: The statement "Manchester United is the best football team in Europe" is stronger than "Manchester United is the best football team in England" because if Manchester United is the best in Europe, it also qualifies as the best in England. However, while Manchester United may be the best football team in England, this doesn't necessarily mean it will be the best in Europe.

21. Which statement is stronger? Explain.

- n is divisible by 3.
- n is divisible by 12.

Answer: The statement "n is divisible by 12" is stronger than "n is divisible by 3" because if n is divisible by 12, it is automatically divisible by 3. However, while n maybe divisible by 3, this doesn't necessarily mean it will be divisible by 12.

26. Often a complicated expression in formal logic can be simplified. For example, consider the statement $S = (p \wedge q) \vee (p \wedge \neg q)$.

(a) Construct a truth table for S.
Answer:

p	q	$p \wedge q$	$p \wedge \neg q$	$S = (p \wedge q) \vee (p \wedge \neg q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	F	F
F	F	F	F	F

(b) Find a simpler expression that is logically equivalent to S.
Answer: A simpler expression is p, as both expressions have the same truth table values.