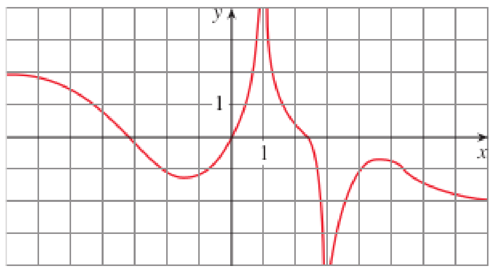
Minh Anh Nguyen
Calculus 1 Assignment-7
Section: 04
TA's name: Arthur Huey

Section 3.4:

3. For the function f whose graph is given, state the following.



(a)

$$\boxed{\lim_{x\to\infty} f(x) = -2}$$

(b)

$$\boxed{\lim_{x\to-\infty} f(x) = 2}$$

(c)

$$\boxed{\lim_{x\to 1} f(x) = \infty}$$

(d)

$$\boxed{\lim_{x\to 3} f(x) = -\infty}$$

(e) The equations of the asymptotes

$$\boxed{x = 1, x = 3, y = -2, y = 2}$$

8. Evaluate the limit and justify each step by indicating the appropriate properties of limits.

$$\lim_{x\to\infty} \sqrt{\frac{9x^3+8x-4}{3-5x+x^3}}$$

$$= \lim_{x\to\infty} \sqrt{\frac{x^3(9+8/x^2-4/x^3)}{x^3(3/x^3-5/x^2+1)}}$$

$$= \lim_{x\to\infty} \sqrt{\frac{9+8/x^2-4/x^3}{3/x^3-5/x^2+1}}$$

$$= \sqrt{\frac{9+0-0}{0-0+1}}$$

$$\boxed{= \sqrt{9} = 3}$$

11. Find the limit or show that it does not exist.

$$\lim_{t\to-\infty} \frac{3t^2+t}{t^3-4t+1}$$

$$= \lim_{t\to-\infty} \frac{t^2(3+1/t)}{t^3(1-4/t^2+1/t^3)}$$

$$= \lim_{t\to-\infty} \frac{(3+1/t)}{t(1-4/t^2+1/t^3)}$$

$$\boxed{= 0}$$

18. Find the limit or show that it does not exist.

$$\lim_{t\to\infty} \frac{t+3}{\sqrt{2t^2-1}}$$

$$= \lim_{t\to\infty} \frac{t(1+3/t)}{t\sqrt{2-1/t^2}}$$

$$= \lim_{t\to\infty} \frac{1+3/t}{\sqrt{2-1/t^2}}$$

$$= \frac{1+0}{\sqrt{2-0}}$$

$$\boxed{= \frac{1}{\sqrt{2}}}$$

26. Find the limit or show that it does not exist.

$$= \lim_{x\to-\infty} (\sqrt{4x^2+3x}+2x)$$

$$= \lim_{x\to-\infty} (|x|\sqrt{4+3/x}+2x)$$

Because x is approaching to $-\infty$. $|x| = -x$.

$$= \lim_{x\to-\infty} (-x\sqrt{4+3/x}+2x)$$

$$= \lim_{x\to-\infty} x(-\sqrt{4+3/x}+2)$$

$$= -\infty(-2+2)$$

$$= -\infty(0)$$

$$\boxed{= 0}$$

28. Find the limit or show that it does not exist.

$$\lim_{x\to\infty}(x-\sqrt{x})$$

$$=\lim_{x\to\infty}x(1-1/\sqrt{x})$$

$$=\infty(1-0)$$

$$\boxed{=\infty}$$

31. Find the limit or show that it does not exist.

$$\lim_{x\to\infty}x\sin\frac{1}{x}$$

$$=\infty\sin 0$$

$$\boxed{=0}$$

37. Find the horizontal and vertical asymptotes of each curve. You may want to use a graphing calculator (or computer) to check your work by graphing the curve and estimating the asymptotes.

$$y=\frac{2x^2+x-1}{x^2+x-2}$$

Horizontal Asymptotes:

$$\lim_{x\to\infty}\frac{2x^2+x-1}{x^2+x-2}$$

$$=\lim_{x\to\infty}\frac{x^2(2+1/x-1/x^2)}{x^2(1+1/x-2/x^2)}$$

$$=\lim_{x\to\infty}\frac{2+1/x-1/x^2}{1+1/x-2/x^2}$$

$$\boxed{=2}$$

$$\lim_{x\to-\infty}\frac{2x^2+x-1}{x^2+x-2}$$

$$=\lim_{x\to-\infty}\frac{x^2(2+1/x-1/x^2)}{x^2(1+1/x-2/x^2)}$$

$$=\lim_{x\to-\infty}\frac{2+1/x-1/x^2}{1+1/x-2/x^2}$$

$$\boxed{=2}$$

$$\boxed{y=2}$$

Vertical Asymptotes;

$$x^2+x-2=0$$

$$(x-1)(x+2)=0$$

$$\boxed{x=1\text{ or }x=-2}$$

$$\boxed{x=1,x=-2}$$

54. Find the limits as $x \to \infty$ and as $x \to -\infty$. Use this information, together with intercepts, to give a rough sketch of the graph as in Example 11.

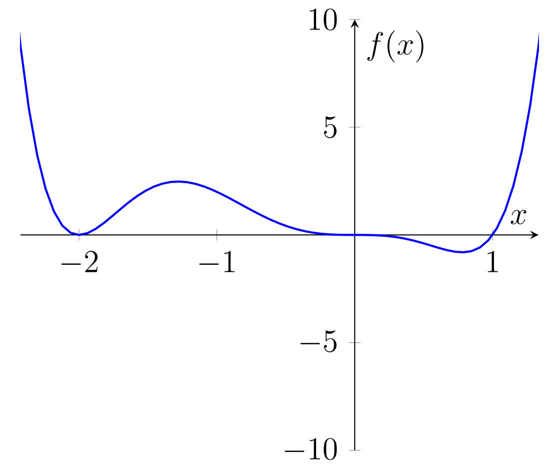
$$y = x^3(x+2)^2(x-1)$$

$$\lim_{x \to \infty} x^3(x+2)^2(x-1)$$

$$\boxed{= \infty}$$

$$\lim_{x \to -\infty} x^3(x+2)^2(x-1)$$

$$\boxed{= \infty}$$



59. Sketch the graph of a function that satisfies all of the given conditions.

5. Use the guidelines of this section to sketch the curve.

$$y = x(x-4)^3 = x(x^3 - 3x^2 \times 4 + 3x \times 4^2 - 4^3)$$

$$y = x(x^3 - 12x^2 + 48x - 64) = x^4 - 12x^3 + 48x^2 - 64x$$

(a) Domain: $(-\infty, \infty)$

(b) Intercepts:

$$f(0) = 0(0-4)^3 = 0$$

$$y - \text{intercepts are } 0$$

$$f(x) = x(x-4)^3 = 0$$

$$x - \text{intercepts are } 0 \text{ and } 4$$

(c) Symmetry:

$$f(-x) = (-x)^4 - 12(-x)^3 + 48(-x)^2 - 64(-x)$$

$$f(-x) = x^4 + 12x^3 + 48x^2 + 64x$$

The function is not odd nor even.

(d) Asymptotes:
Since the function is a polynomial function. It will be defined everywhere and has no vertical asymptote.

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} x^4 - 12x^3 + 48x^2 - 64x = \infty$$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} x^4 - 12x^3 + 48x^2 - 64x = \infty$$

The function also doesn't have horizontal asymptote.

(e) Intervals of Increase or Decrease:

$$f'(x) = 4x^3 - 36x^2 + 96x - 64 = 0$$

$$x = 1 \text{ or } x = 4$$

x	$-\infty$		1		4		∞
$f'(x)$		$-$	0	$+$	0	$+$	
$f(x)$		$\downarrow$	-27	$\uparrow$	0	$\uparrow$	

The function f(x) increases on the intervals $(1, 4)$ and $(4, \infty)$.
The function f(x) decreases on the interval $(-\infty, 1)$.

(f) Local Maximum and Minimum Values:
Local Minimum Values is $f(1) = -27$.

(g) Concavity and Points of Inflection:

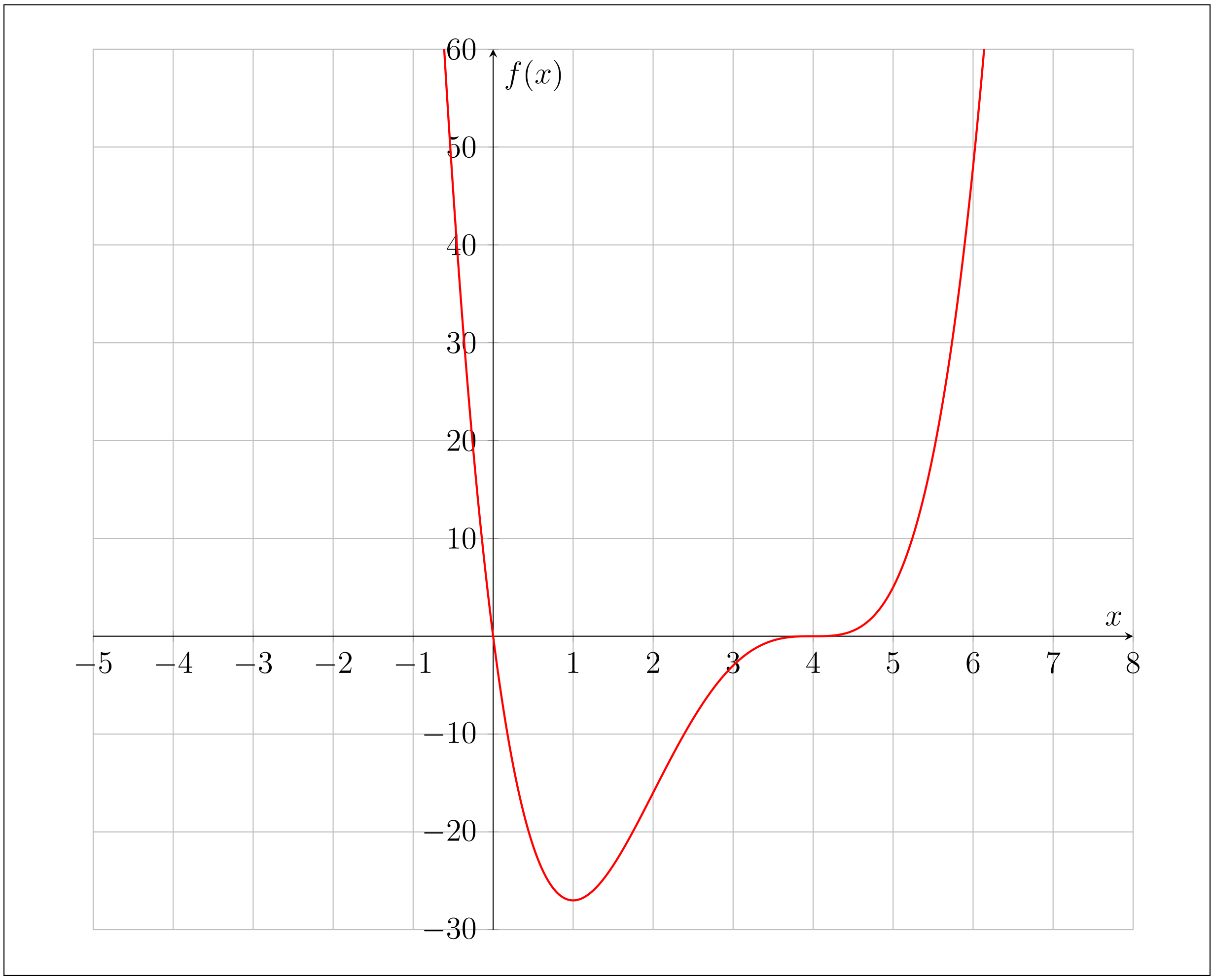
$$f''(x) = 12x^2 - 72x + 96 = 0$$

$$x = 2 \text{ or } x = 4$$

x	$-\infty$		2		4		∞
$f''(x)$		+	0	–	0	+	

The function f(x) is concave up on the intervals $(-\infty, 2)$ and $(4, \infty)$.
The function f(x) is concave down on the intervals $(2, 4)$.
Points of Inflection are (2, 16) and (4, 0).

(h) Sketch the graph:



9. Use the guidelines of this section to sketch the curve.

$$y = \frac{2x+3}{x+2}$$

(a) Domain:

$$x + 2 \neq 0$$

$$x \neq -2$$

Hence, the domain of f(x) is: $\mathbb{R} \setminus \{-2\}$

(b) Intercepts:

$$f(0) = \frac{2 \times 0 + 3}{0 + 2} = \frac{3}{2}$$

$$f(x) = \frac{2x+3}{x+2} = 0$$
$$x = -\frac{3}{2}$$

The y-intercepts of the function is $\frac{3}{2}$.
The x-intercepts of the function is $-\frac{3}{2}$.

(c) Symmetry:

$$f(-x) = \frac{2(-x)+3}{(-x)+2}$$
$$f(-x) = \frac{-2x+3}{-x+2}$$

The function is not odd nor even.

(d) Asymptotes:

$$\lim_{x \to -2^-} \frac{2x+3}{x+2} = \infty$$
$$\lim_{x \to -2^+} \frac{2x+3}{x+2} = -\infty$$

The function has a vertical asymptote x = -2.

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{2x+3}{x+2} = \lim_{x\to\infty} \frac{x(2+3/x)}{x(1+2/x)} = \lim_{x\to\infty} \frac{2+3/x}{1+2/x} = \frac{2+0}{1+0} = 2$$
$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{2x+3}{x+2} = \lim_{x\to-\infty} \frac{x(2+3/x)}{x(1+2/x)} = \lim_{x\to-\infty} \frac{2+3/x}{1+2/x} = \frac{2+0}{1+0} = 2$$

The function has a horizontal asymptote y = 2.

(e) Intervals of Increase or Decrease:

$$f'(x) = \frac{(2x+3)'(x+2) - (x+2)'(2x+3)}{(x+2)^2}$$
$$f'(x) = \frac{2(x+2) - (2x+3)}{(x+2)^2}$$
$$f'(x) = \frac{2x+4-2x-3}{(x+2)^2}$$
$$f'(x) = \frac{1}{(x+2)^2} > 0 \ \forall x \in (\mathbb{R} \setminus \{-2\})$$

Hence, the function always increases on the domain $\mathbb{R} \setminus \{-2\}$.

(f) Local Maximum and Minimum Values:
The function doesn't have any local maximum nor minimum.

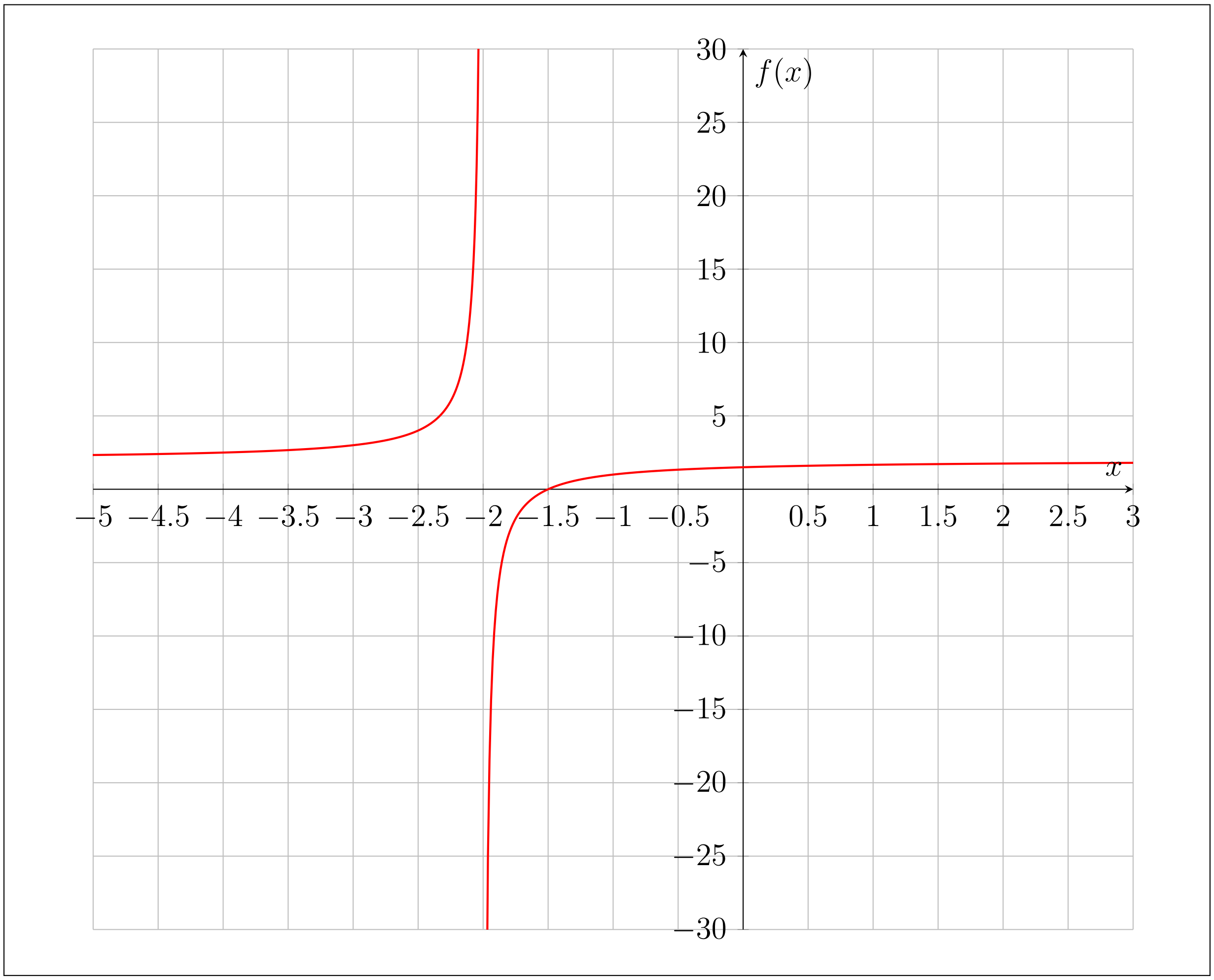
(g) Concavity and Points of Inflection:

$$f''(x) = -\frac{[(x+2)^2]'}{(x+2)^4}$$
$$f''(x) = -\frac{2(x+2)}{(x+2)^4}$$
$$f''(x) = -\frac{2}{(x+2)^3}$$

x	$-\infty$		-2		∞
$f''(x)$		$+$	0	$-$	

The function f(x) is concave up in the interval $(-\infty, -2)$.
The function f(x) is concave up in the interval $(-2, \infty)$.

(h) Sketch the graph:



21. Use the guidelines of this section to sketch the curve.

$$y = (x-3)\sqrt{x}$$

(a) Domain:

$$x \geq 0$$

The domain of the function f(x) is $[0, \infty)$.

(b) Intercepts:

$$f(0) = (0-3)\sqrt{0} = 0$$
$$f(x) = (x-3)\sqrt{x} = 0$$
$$(x-3)\sqrt{x} = 0$$

$$x = 0 \text{ or } x = 3$$

The y-intercept of the function is 0.
The x-intercepts of the function are 0 and 3.

(c) Symmetry:
Because the function has the domain $[0, \infty)$ so this function is not symmetrical.

(d) Asymptotes:

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} (x-3)\sqrt{x} = \infty$$

$$\lim_{x\to-\infty} f(x) \text{ is undefined.}$$

So the function doesn't have any horizontal nor vertical asymptote.

(e) Interval of Increase or Decrease:

$$f'(x) = (x-3)'\sqrt{x} + (x-3)\sqrt{x}'$$

$$f'(x) = \sqrt{x} + \frac{x-3}{2\sqrt{x}}$$

$$f'(x) = \frac{2x + x - 3}{2\sqrt{x}}$$

$$f'(x) = \frac{3x-3}{2\sqrt{x}} = 0$$

$$x = 1$$

x	0		1		∞
$f'(x)$	||	–	0	+	
$f(x)$	0	$\downarrow$	-2	$\uparrow$	

Hence, the function f increases on the interval $(1, \infty)$.
The function f decreases on the interval $(0, 1)$.

(f) Local Maximum and Minimum Values:
The function f has local minimum at x = 1, f(1) = -2.

(g) Concavity and Inflection Point

$$f''(x) = \frac{(3x-3)'2\sqrt{x} - (2\sqrt{x})'(3x-3)}{4x}$$

$$f''(x) = \frac{6\sqrt{x} - \frac{1}{\sqrt{x}}(3x-3)}{4x}$$

$$f''(x) = \frac{6x - 3x + 3}{4x\sqrt{x}}$$

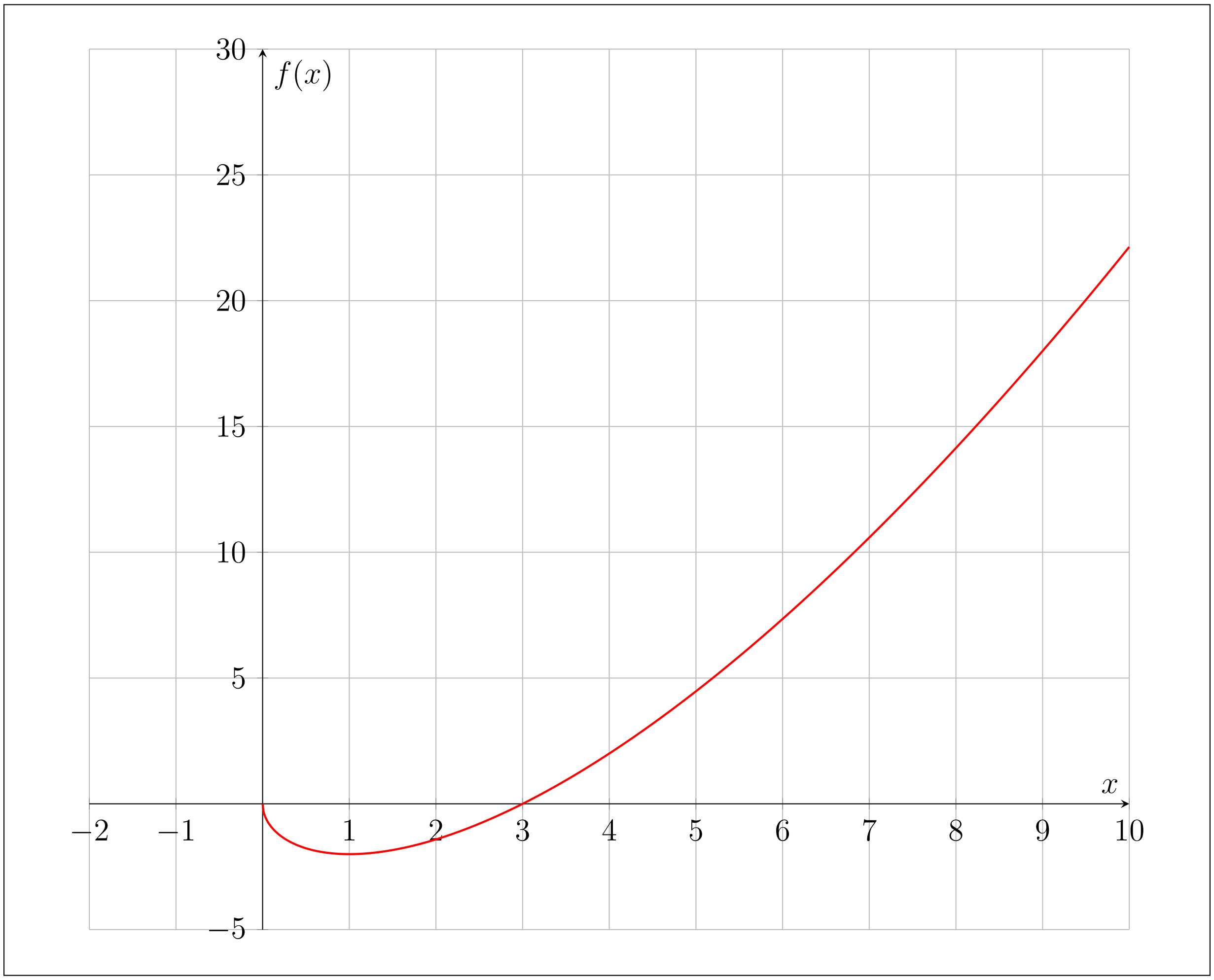
$$f''(x) = \frac{3x+3}{4x\sqrt{x}} = 0$$

$$x = -1$$

x	$-\infty$	-1	0		∞
$f''(x)$			||	+	

Hence, the function is concave up on the interval $(0, \infty)$

(h) Sketch the graph:



28. Use the guidelines of this section to sketch the curve.

$$y = \frac{x}{\sqrt{x^2 - 1}}$$

(a) Domain:

$$x^2 - 1 > 0$$

$$x^2 > 1$$

$$x < 1 \text{ or } x > 1$$

Hence, the domain of the function is $(-\infty, 1) \cup (1, \infty)$.

(b) Intercepts:

$$f(0) = \frac{0}{\sqrt{0^2 - 1}} \text{ is not defined}$$

$$f(x) = \frac{x}{\sqrt{x^2 - 1}} = 0$$

$$\frac{x}{\sqrt{x^2 - 1}} = 0 \text{ doesn't have any solution.}$$

Hence, there is no x nor y intercept.

(c) Symmetry:

$$f(-x) = \frac{-x}{\sqrt{(-x)^2 - 1}} = -\frac{x}{\sqrt{x^2 - 1}}$$

Hence, $f(-x) = -f(x)$. The graph is odd.

(d) Asymptotes:

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} \frac{x}{\sqrt{x^2 - 1}} = \lim_{x \to 1^+} \frac{x}{|x|\sqrt{1 - 1/x^2}}$$

Since x > 1 and is positive.

$$\lim_{x \to 1^+} \frac{1}{\sqrt{1 - 1/x^2}} = \infty$$

$$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} \frac{x}{\sqrt{x^2 - 1}} = \lim_{x \to -1^-} \frac{x}{|x|\sqrt{1 - 1/x^2}}$$

Since x < 1 and is negative.

$$\lim_{x \to -1^-} \frac{-1}{\sqrt{1 - 1/x^2}} = -\infty$$

$$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{x}{\sqrt{x^2 - 1}} = \lim_{x \to \infty} \frac{1}{\sqrt{1 - 1/x^2}} = \lim_{x \to \infty} \frac{x}{|x|\sqrt{1 - 1/x^2}}$$

Since x to ∞ is positive.

$$\lim_{x \to \infty} \frac{1}{\sqrt{1 - 1/x^2}} = 1$$

$$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \frac{x}{\sqrt{x^2 - 1}} = \lim_{x \to -\infty} \frac{-1}{\sqrt{1 - 1/x^2}} = \lim_{x \to -\infty} \frac{x}{|x|\sqrt{1 - 1/x^2}}$$

Since x to $-\infty$ is negative.

$$\lim_{x \to -\infty} -\frac{1}{\sqrt{1 - 1/x^2}} = -1$$

Hence, the vertical asymptotes of the function are x = -1, x = 1.
The horizontal asymptotes of the function are y = -1, y = 1.

(e) Intervals of Increase or Decrease:

$$f'(x) = \frac{x'\sqrt{x^2 - 1} - (\sqrt{x^2 - 1})'x}{(x^2 - 1)}$$

$$f'(x) = \frac{\sqrt{x^2-1} - \frac{x}{\sqrt{x^2-1}}x}{(x^2-1)}$$

$$f'(x) = \frac{x^2-1-x^2}{\sqrt{x^2-1}(x^2-1)}$$

$$f'(x) = \frac{-1}{\sqrt{x^2-1}(x^2-1)} < 0 \ \forall x \in [(-\infty, 1) \cup (1, \infty)]$$

Hence, the function decreases on its interval $(-\infty, 1) \cup (1, \infty)$.

(f) Local Maximum and Minimum Values:
The function doesn't have any maximum or minimum values since it always decreases.

(g) Concavity and Inflection Point:

$$f''(x) = -\frac{(\sqrt{x^2-1}(x^2-1))'}{(x^2-1)^3}$$

$$f''(x) = -\frac{\frac{x}{\sqrt{x^2-1}}(x^2-1) + 2x\sqrt{x^2-1}}{(x^2-1)^3}$$

$$f''(x) = -\frac{x(x^2-1) + 2x(x^2-1)}{\sqrt{x^2-1}(x^2-1)^3}$$

$$f''(x) = -\frac{x^3 - x + 2x^3 - 2x}{\sqrt{x^2-1}(x^2-1)^3}$$

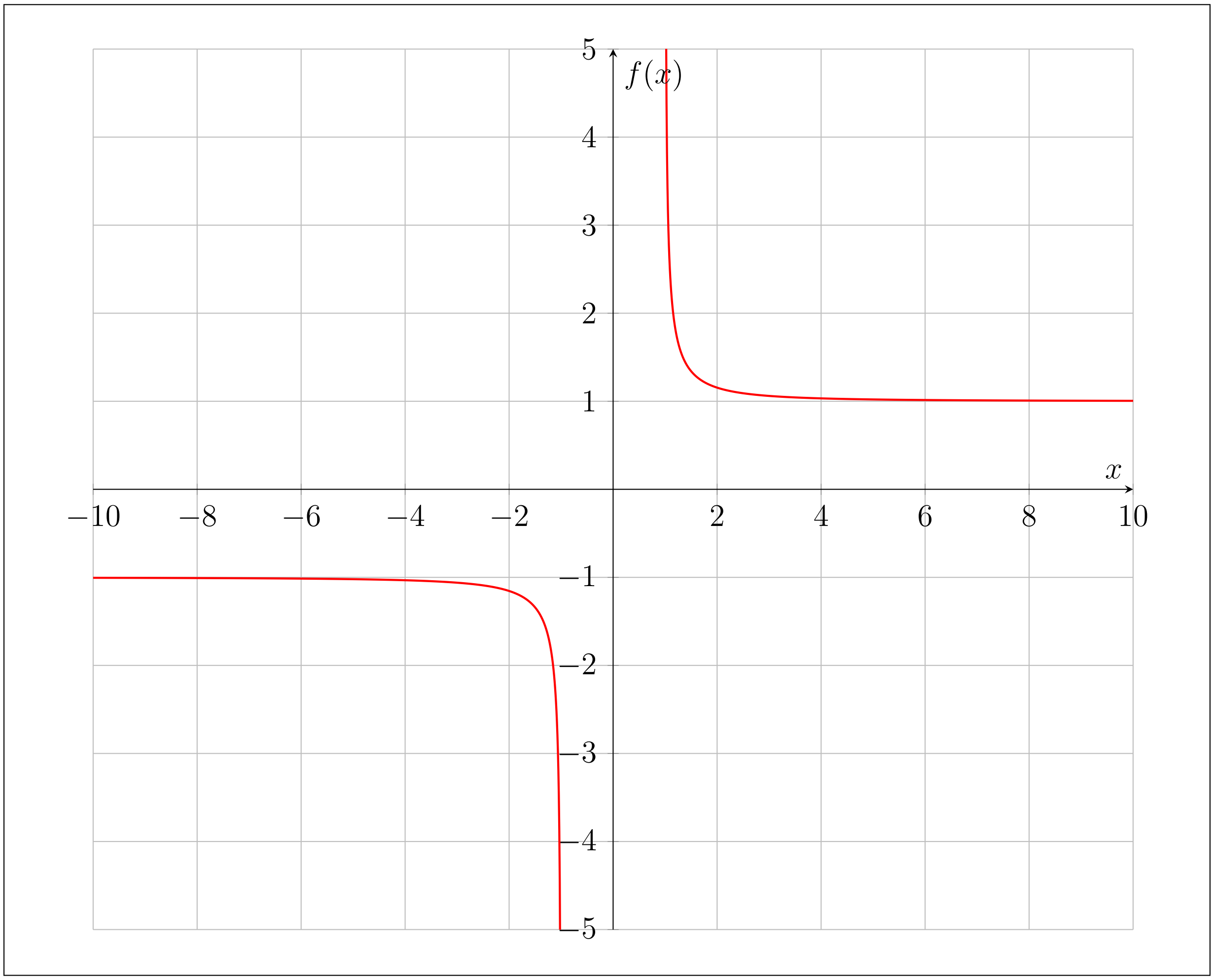
$$f''(x) = -\frac{3x^3 - 3x}{\sqrt{x^2-1}(x^2-1)^3} = 0$$

$$x = 0 \text{ or } x = \pm 1$$

x	$-\infty$		-1	0	1		∞
$f''(x)$		$-$	$\|\|$	0	$\|\|$	$+$	

Hence, the function is concave up on the interval $(1, \infty)$.
The function is concave down on the interval $(-\infty, -1)$.

(h) Sketch the graph:



33. Use the guidelines of this section to sketch the curve.

$$y = \sin^3 x$$

(a) Domain:
Since this function is a trigonometric function. It will be continuous everywhere.
The domain of this function is $(-\infty, \infty)$.

(b) Intercepts:

$$f(0) = \sin^3(0) = 0$$

$$f(x) = \sin^3(x) = 0$$

$$x = n\pi \text{ with n is a integer.}$$

The x-intercept is $n\pi$ with n is a integer.
The y-intervals is 0.

(c) Symmetry:

$$f(-x) = \sin^3(-x) = -\sin^3(x)$$
$$f(-x) = -f(x)$$

Hence, the function is odd.

(d) Asymptotes:
The function doesn't have any asymptote since it is a trigonometric function.

(e) Intervals of Increase or Decrease:

$$f'(x) = \sin^3(x)' = 3\sin^2(x)\cos(x) = 0$$
$$\sin(x) = 0 \text{ or } \cos(x) = 0$$
$$x = \frac{\pi}{2}n \text{ with n is a integer.}$$

x	$-\infty$	...	0		$\pi/2$		π		$\pi 3/2$	...	∞
$f'(x)$		...	0	+	0	-	0	+	0	...	

The function repeated itself each $\frac{\pi}{2}$.

(f) Local Maximum and Minimum Values:

$$-1 \leq \sin(x) \leq 1$$
$$(-1)^3 \leq \sin^3(x) \leq (1)^3$$
$$-1 \leq \sin^3(x) \leq 1$$

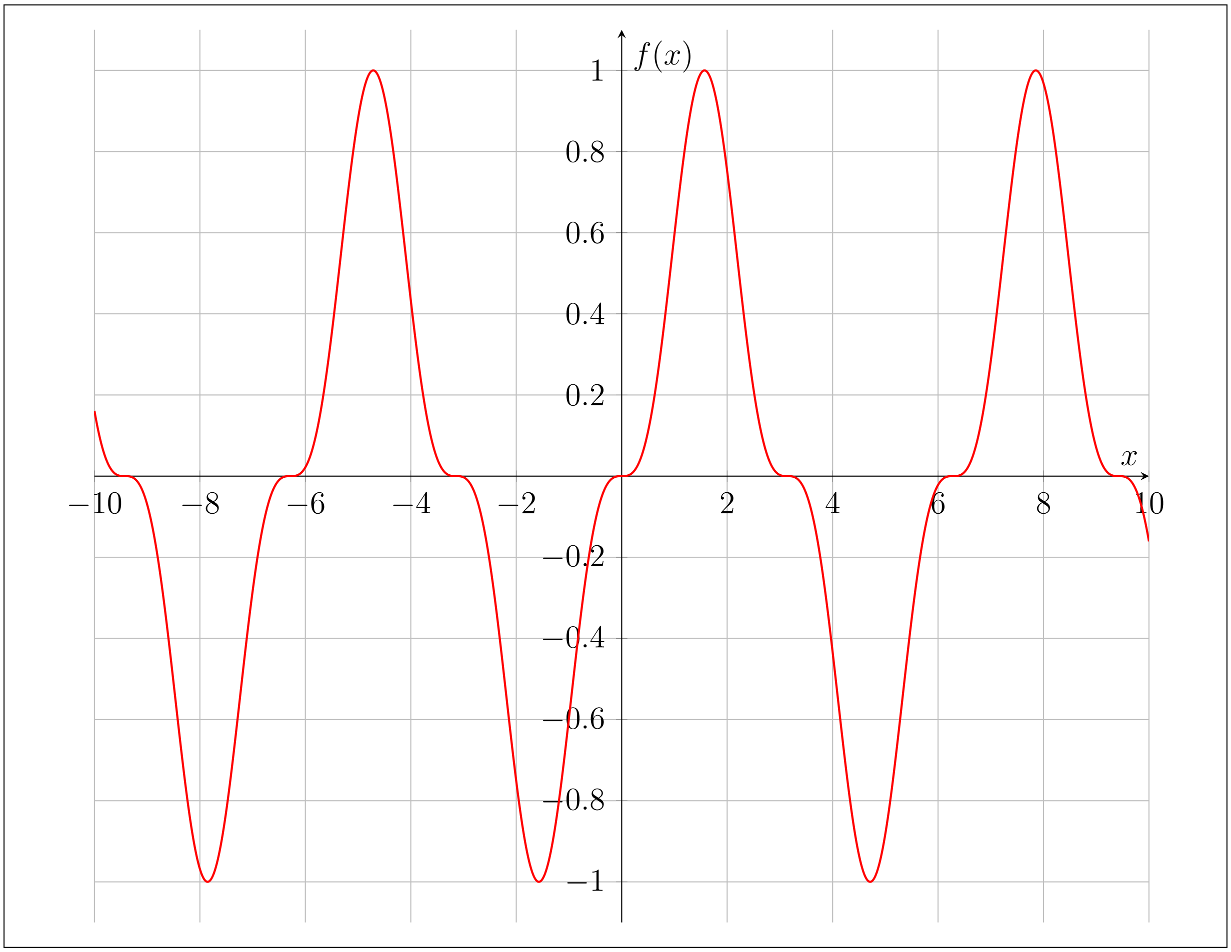
Hence, the function local maximum is 1 and local minimum is -1.

(g) Concavity and Inflection Points:

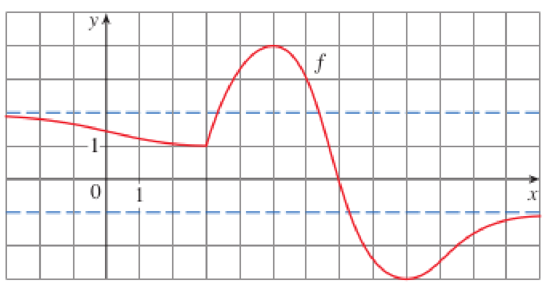
$$f''(x) = 3\sin^2(x)'\cos(x) + \cos(x)'3\sin^2(x)$$
$$f''(x) = 6\sin(x)\cos^2(x) - 3\sin^3(x) = 0$$
$$6\sin(x)\cos^2(x) = 3\sin^3(x)$$
$$2 = \frac{\sin^2(x)}{\cos^2(x)}$$
$$tan(x) = \sqrt{2}$$
$$x = \tan^{-1}\sqrt{2} + n\pi \text{ with n is an integer.}$$

The function is concave up in $(0, \tan^{-1}\sqrt{2})$
then concave down in $(\tan^{-1}\sqrt{2}, \tan^{-1}\sqrt{2} + \pi)$ and repeats itself.

(h) Sketch the graph:



41. The graph of a function f is shown. (The dashed lines indicate horizontal asymptotes.) Find each of the following for the given function g.



$$g(x) = \sqrt{f(x)}$$

(a) The domains of g and g'.
The domain of g is $(-\infty, 7]$.

$$g'(x) = \frac{f'(x)}{2\sqrt{f(x)}}$$

The domain of g' is $(-\infty, 3) \cup (3, 7)$.

(b) The critical numbers of g.

$$g'(x) = \frac{f'(x)}{2\sqrt{f(x)}} = 0$$

$$f'(x) = 0$$

$$x = 3 \text{ or } x = 5$$

(c) The approximate value of $g'(6)$.

$$g'(6) = \frac{f'(6)}{2\sqrt{f(6)}} \approx \frac{-2}{4} = -\frac{1}{2}$$

(d) All vertical and horizontal asymptotes of g.

$$g(x) = \sqrt{f(x)}$$

Because $f(x)$ has horizontal asymptotes f(x) = 2 and f(x) = -1. But f(x) = -1 is not defined for g(x).
Therefore, $g(x) = \sqrt{f(x)} = \sqrt{2}$. Hence the horizontal asymptote of g is $y = \sqrt{2}$.

53. Use the guidelines of this section to sketch the curve. In guideline D, find an equation of the slant asymptote.

$$y = \frac{x^2}{x-1}$$

(a) Domain:

$$x - 1 \neq 0$$

$$x \neq 1$$

Hence the domain of the function is $(-\infty, 1) \cup (1, \infty)$.

(b) Intercepts:

$$f(0) = \frac{0^2}{0-1} = 0$$

$$f(x) = \frac{x^2}{x-1} = 0$$

$$x = 0$$

Hence, the y-intercept is 0. The x-intercept is 0.

(c) Symmetry:

$$f(-x) = \frac{(-x)^2}{-x-1} = \frac{x^2}{-x-1}$$

Hence the function is not odd nor even.

(d) Asymptotes:

$$\lim_{x\to 1^+} \frac{x^2}{x-1} = \infty$$

$$\lim_{x\to 1^-} \frac{x^2}{x-1} = -\infty$$

$$\lim_{x\to \infty} \frac{x^2}{x-1} = \lim_{x\to \infty} \frac{x}{1-1/x} = \infty$$

$$\lim_{x\to -\infty} \frac{x^2}{x-1} = -\infty$$

Hence, the function has vertical asymptotes x = 1.
The slant asymptote after doing long division is $y = x + 1$.

(e) Intervals of Increase or Decrease:

$$f'(x) = \frac{(x^2)'(x-1) - (x-1)'(x^2)}{x-1}$$

$$f'(x) = \frac{2x(x-1) - (x^2)}{x-1}$$

$$f'(x) = \frac{2x^2 - 2x - x^2}{x-1}$$

$$f'(x) = \frac{(x^2 - 2x)}{x-1} = 0$$

$$x = 0 \text{ or } x = 2$$

x	$-\infty$		0		1		2		∞
$f'(x)$		–	0	+	$\vert\vert$	–	0	+	

Hence, the function increases on the interval (0,1) and $(2, \infty)$.
The function decreases on the interval $(-\infty, 0)$ and (1,2).

(f) Local Maximum and Minimum Values:
Local Minimum Values:

$$f(0) = \frac{0^2}{0-1} = 0$$

$$f(2) = \frac{2^2}{2-1} = 4$$

(g) Concavity and Inflection Points:

$$f''(x) = \frac{(x^2-2x)'(x-1) - (x-1)'(x^2-2x)}{(x-1)^2}$$

$$f''(x) = \frac{(2x-2)(x-1) - (x^2-2x)}{(x-1)^2}$$

$$f''(x) = \frac{2x^2 - 2x - 2x + 2 - x^2 + 2x}{(x-1)^2}$$

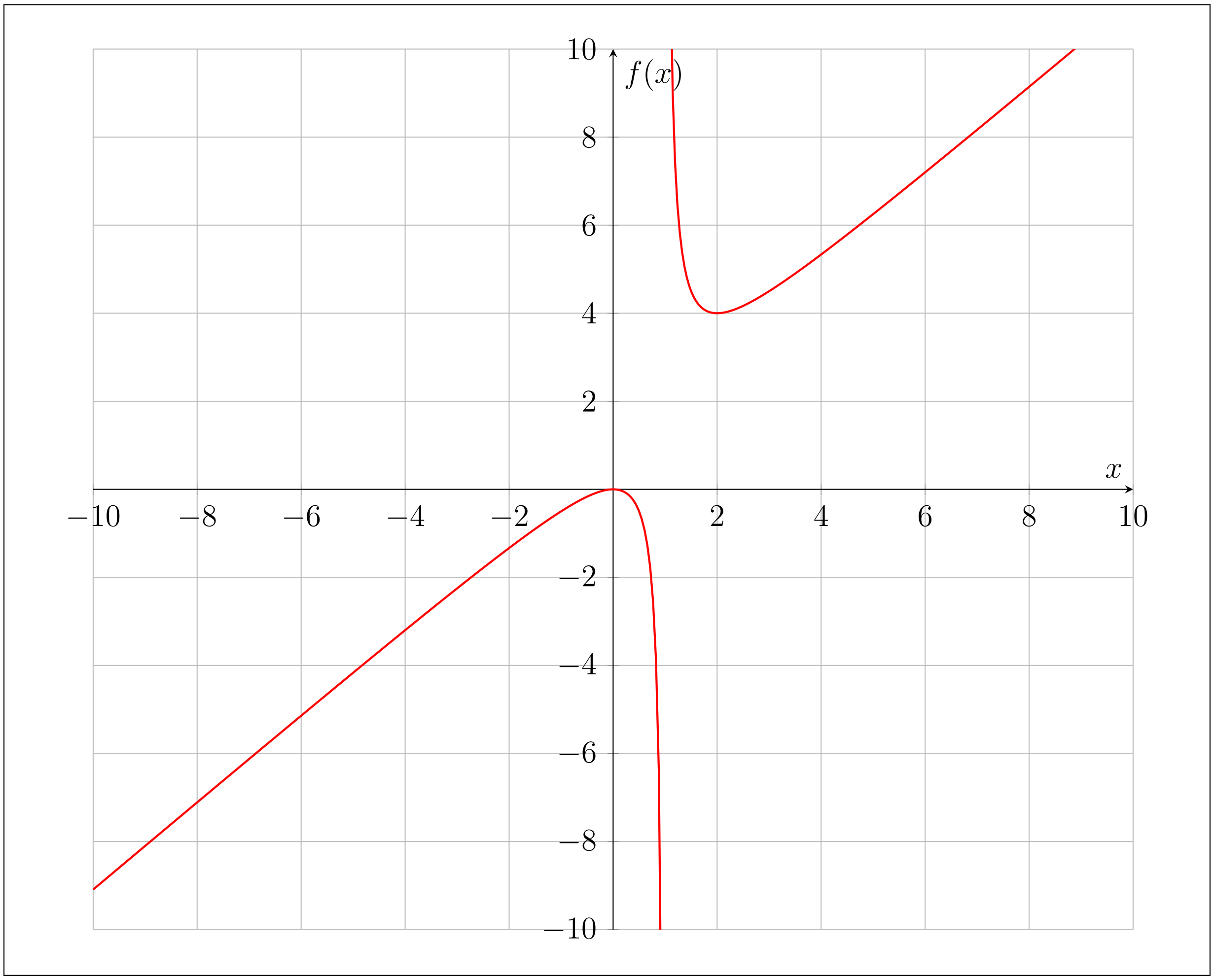
$$f''(x) = \frac{x^2 - 2x + 2}{(x-1)^2} = 0$$

There is no solution for this function.

x	$-\infty$		1	∞
$f''(x)$		$-$	$\|\|$	$+$

The function is concave up on the interval $(1, \infty)$.
The function is concave down on the interval $(-\infty, 1)$.

(h) Sketch the graph:



59. Show that the curve $y = \sqrt{4x^2 + 9}$ has two slant asymptotes: $y = 2x$ and $y = --2x$. Use this fact to help sketch the curve.