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**Calculus 1 Assignment-8**
**Section: 04**
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2. Find two numbers whose difference is 100 and whose product is a minimum.
   Let calls that two numbers x and y.

$$x + y = 100$$

$$y = 100 - x$$

   Product is a minimum:

$$P = xy$$

$$P = x(100 - x)$$

$$P = 100x - x^2$$

$$P' = 100 - 2x = 0$$

$$x = 50$$

   Since x = 50:

$$y = 50$$

   Hence, these two numbers are both 50.

5. What is the maximum vertical distance between the line $y = x + 2$ and the parabola $y = x^2$ for $-1 \leq x \leq 2$?
   Vertical Distance:

$$D = |y_{parabola} - y_{line}| = |x^2 - x - 2|$$

$$D = |x^2 - x - 2|$$

$$D' = |2x - 1| = 0$$

$$x = \frac{1}{2}$$

   Hence, the maximum vertical distance is:

$$D = |\frac{1}{2}^2 - \frac{1}{2} - 2| = \frac{9}{4}$$

9. A model used for the yield $Y$ of an agricultural crop as a function of the nitrogen level $N$ in the soil (measured in appropriate units) is

$$Y = \frac{kN}{1 + N^2}$$

where $k$ is a positive constant. What nitrogen level gives the best yield?

$$Y' = \frac{(kN)'(1+N^2) - (1+N^2)'(kN)}{(1+N^2)^2}$$

$$Y' = \frac{k(1+N^2) - 2N(kN)}{(1+N^2)^2}$$

$$Y' = \frac{k + kN^2 - 2kN^2}{(1+N^2)^2}$$

$$Y' = \frac{k - kN^2}{(1+N^2)^2} = 0$$

$$k - kN^2 = 0$$

$$kN^2 = k$$

$$N^2 = 1$$

$$N = \pm 1$$

Nitrogen level cannot go below 0.
Hence, $N = 1$ is the best nitrogen level for the best yield.

13. A farmer wants to fence in an area of 1.5 million square feet in a rectangular field and then divide it in half with a fence parallel to one of the sides of the rectangle. How can he do this so as to minimize the cost of the fence?