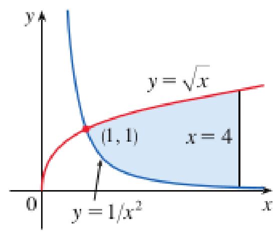
Minh Anh Nguyen
Calculus 1 Assignment-12
Section: 04
TA's name: Arthur Huey

Section 5.1:

2. Let:



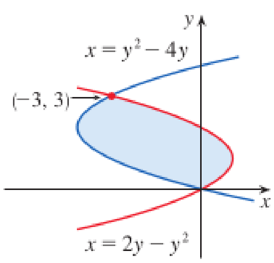
(a) Set up an integral for the area of the shaded region.

$$\int_1^4 (\sqrt{x} - \frac{1}{x^2})dx$$

(b) Evaluate the integral to find the area.

$$\int_1^4 (\sqrt{x} - \frac{1}{x^2})dx = \int_1^4 (x^{1/2} - x^{-2})dx = (\frac{2x^{3/2}}{3} + \frac{1}{x})|_1^4 = \frac{67}{12} - \frac{5}{3} = \frac{47}{12}$$

4. Let:



(a) Set up an integral for the area of the shaded region.

$$\int_0^3 (y^2 - 4y - 2y + y^2)dy = \int_0^3 (2y^2 - 6y)dy$$

(b) Evaluate the integral to find the area.

$$\int_0^3 (2y^2 - 6y)dy = (\frac{2y^3}{3} - 3y)|_0^3 = 9 - 0 = 9$$

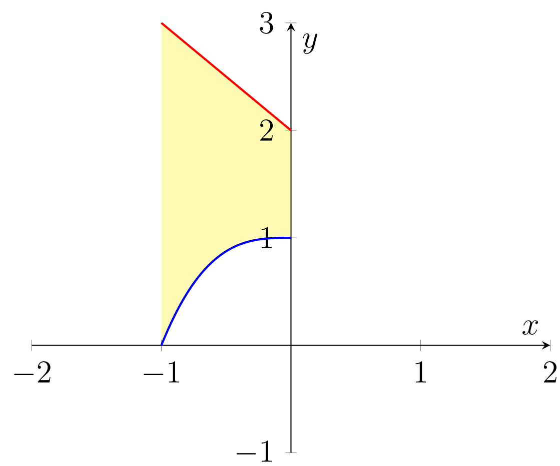
7. Set up, but do not evaluate, an integral representing the area of the region enclosed by the given curves.

$$y = \frac{1}{x}, y = \frac{1}{x^2}, x = 2$$

$$\int_0^2 |\frac{1}{x} - \frac{1}{x^2}|dx$$

12. Sketch the region enclosed by the given curves. Decide whether to integrate with respect to x or y. Draw a typical approximating rectangle and label its height and width. Then find the area of the region.

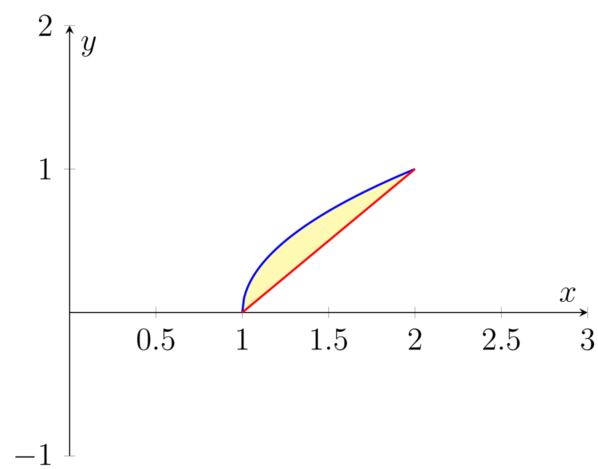
$$y = 1 + x^3, y = 2 - x, x = -1, x = 0$$



$$\int_{-1}^{0}(2 - x - 1 - x^3)dx = \int_{-1}^{0}(-x^3 - x + 1) = (-\frac{x^4}{4} - \frac{x^2}{2} + x)|_{-1}^{0} = \frac{7}{4}$$

22. Sketch the region enclosed by the given curves and find its area.

$$y = \sqrt{x - 1}, x - y = 1$$

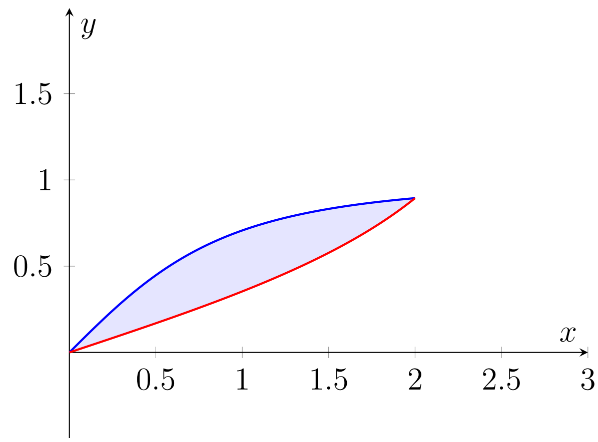


$$x - y = 1 \to y = x - 1$$

$$\int_1^2 (\sqrt{x-1} - x + 1)dx = \frac{1}{6}$$

36. Sketch the region enclosed by the given curves and find its area.

$$y = \frac{x}{\sqrt{1+x^2}}, y = \frac{x}{\sqrt{9-x^2}}, x \geq 0$$



$$\int_0^2 (\frac{x}{\sqrt{1+x^2}} - \frac{x}{\sqrt{9-x^2}})dx = \int_0^2 (\frac{x}{\sqrt{1+x^2}})dx - \int_0^2 (\frac{x}{\sqrt{9-x^2}})dx$$

First Integral: $\int_0^2 \frac{x}{\sqrt{1+x^2}}dx$ Let $u = 1 + x^2$, then $du = 2x\,dx$ or $dx = \frac{du}{2x}$.

$$\int \frac{x}{\sqrt{1+x^2}}dx = \int \frac{x}{\sqrt{u}} \cdot \frac{du}{2x} = \int \frac{1}{2\sqrt{u}}\,du = \frac{1}{2}\int u^{-\frac{1}{2}}\,du$$

$$= \frac{1}{2} \cdot 2\sqrt{u} = \sqrt{u} = \sqrt{1+x^2}$$

$$\int_0^2 \frac{x}{\sqrt{1+x^2}}dx = \left[\sqrt{1+x^2}\right]_0^2 = \sqrt{5} - 1$$

Second Integral: $\int_0^2 \frac{x}{\sqrt{9-x^2}}dx$ Let $u = 9 - x^2$, then $du = -2x\,dx$ or $dx = \frac{-du}{2x}$.

$$\int \frac{x}{\sqrt{9-x^2}}dx = \int \frac{x}{\sqrt{u}} \cdot \frac{-du}{2x} = -\frac{1}{2}\int u^{-\frac{1}{2}}\,du$$

$$= -\frac{1}{2} \cdot 2\sqrt{u} = -\sqrt{u} = -\sqrt{9-x^2}$$

$$\int_0^2 \frac{x}{\sqrt{9-x^2}}dx = \left[-\sqrt{9-x^2}\right]_0^2 = -(\sqrt{5} - 3) = 3 - \sqrt{5}$$

Final Answer

$$\int_0^2 \left(\frac{x}{\sqrt{1+x^2}} - \frac{x}{\sqrt{9-x^2}}\right) dx = (\sqrt{5} - 1) - (3 - \sqrt{5}) = 2\sqrt{5} - 4$$

64. Question:

(a) Find the number a such that the line $x = a$ bisects the area under the curve $y = \frac{1}{x^2}$, $1 \le x \le 4$.
The area of the region under the curve $y = \frac{1}{x^2}$, $1 \le x \le 4$ is:

$$\int_1^4 \frac{1}{x^2} = (-\frac{1}{x})|_1^4 = (-\frac{1}{4} + \frac{1}{1}) = \frac{3}{4}$$

$$\int_1^a \frac{1}{x^2} = (-\frac{1}{x})|_1^a = (-\frac{1}{a} + \frac{1}{1}) = \frac{3}{8}$$

$$-\frac{1}{a} = -\frac{5}{8}$$

$$a = \frac{8}{5}$$

(b) Find the number b such that the line $y = b$ bisects the area in part (a).
The area of the region under the curve $x = \frac{1}{\sqrt{y}}$, $1 \ge y \ge \frac{1}{16}$ is:

$$\int_{1/16}^1 \frac{1}{\sqrt{y}} dy = (2\sqrt{y})|_{1/16}^1 = (2 - \frac{1}{4}) = \frac{3}{4}$$

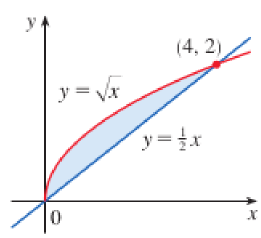
$$\int_{1/16}^b \frac{1}{\sqrt{y}} dy = (2\sqrt{y})|_{1/16}^b = (2 - \frac{1}{\sqrt{b}}) = \frac{3}{8}$$

$$-\frac{1}{\sqrt{b}} = -\frac{13}{8}$$

$$b = \frac{64}{169}$$

Section 5.2:

2. A solid is obtained by revolving the shaded region about the specified line.



About the x-axis.

(a) Sketch the solid and a typical disk or washer.

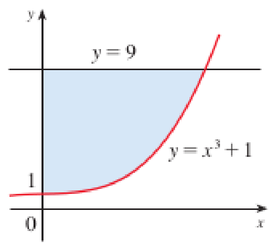
(b) Set up an integral for the volume of the solid.

$$\int_0^4 (\sqrt{x} - \frac{1}{2}x)dx$$

(c) Evaluate the integral to find the volume of the solid.

$$\int_0^4 (\sqrt{x} - \frac{1}{2}x)dx = (\frac{2x^{3/2}}{3} - \frac{1}{4}x^2)|_0^4 = \frac{4}{3}$$

3. A solid is obtained by revolving the shaded region about the specified line.



About the y-axis.

(a) Sketch the solid and a typical disk or washer.

(b) Set up an integral for the volume of the solid.

$$\int_1^9 (9 - \sqrt[3]{y-1})dy$$

(c) Evaluate the integral to find the volume of the solid.

$$\int_0^4 (\sqrt{x} - \frac{1}{2}x)dx = (\frac{2x^{3/2}}{3} - \frac{1}{4}x^2)|_0^4 = \frac{4}{3}$$