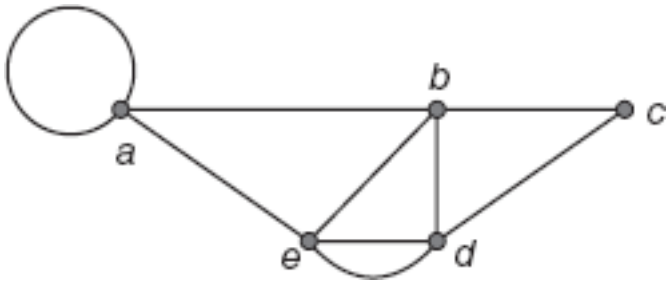
Minh Anh Nguyen
Discrete Mathematics **Assignment-4**

Section 2.1:

1. Consider the following undirected graph.



   (a) How many edges are there in this graph?

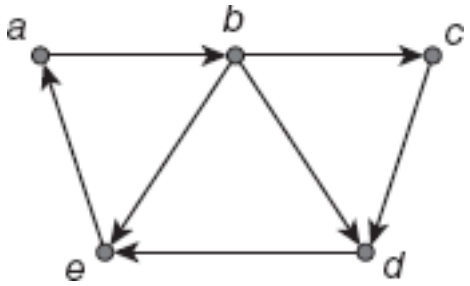
   There are 9 edges in this graph.

   (b) Give the degree of each vertex.
   The dergree of each vertex is:

   a has degree 4
   b has degree 4
   c has degree 2
   d has degree 4
   e has degree 4

   (c) Do these numbers agree with Euler's first observation?
   The sum of the degrees is $4 + 4 + 2 + 4 + 4 = 18$.
   There are 9 edges in the graph.
   The sum of the degrees is doubled the number of edges.
   Hence, these numbers agree with Euler's first observation.

2. Consider the following directed graph.



(a) Give the indegree of each vertex.

The indegree of a is 1.
The indegree of b is 1.
The indegree of c is 1.
The indegree of d is 2.
The indegree of e is 2.

(b) Give the outdegree of each vertex.

The outdegree of a is 1.
The outdegree of b is 3.
The outdegree of c is 1.
The outdegree of d is 1.
The outdegree of e is 1.

(c) Compute the sum of the indegrees and the sum of the outdegrees. What do you notice?
The sum of the indegrees is:

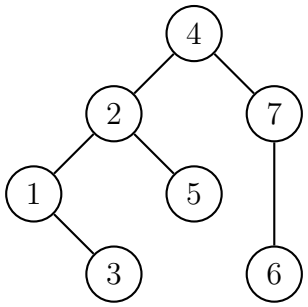
$$1 + 1 + 1 + 2 + 2 = 7$$

The sum of the outdegrees is:

$$1 + 3 + 1 + 1 + 1 = 7$$

I noticed that the sum of the indegrees is equal to the sum of outdegrees and also equal to the numbers of edges.

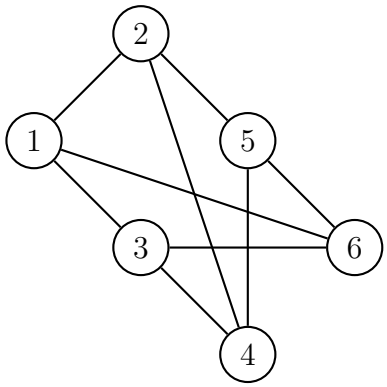
3. A circuit is *simple* if it has no repeated edges. Draw a connected, undirected graph with seven vertices and no simple circuits. How many edges does it have?



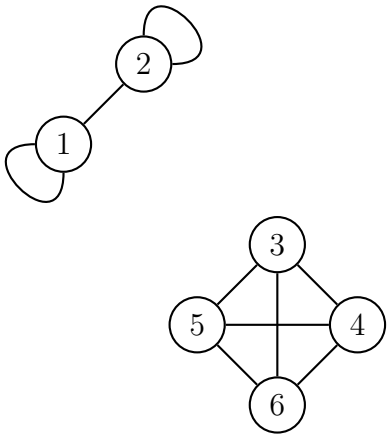
It has 6 edges.

4. Draw an undirected graph with six vertices, each of degree 3, such that the graph is:

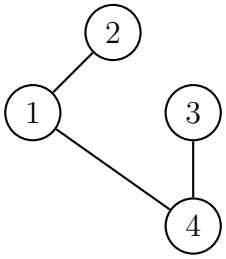
   (a) Connected.



   (b) Not connected.



5. A graph is called simple if it has no multiple edges or loops. Draw five different connected, simple, undirected graphs with four vertices.
   1.



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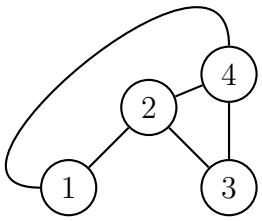

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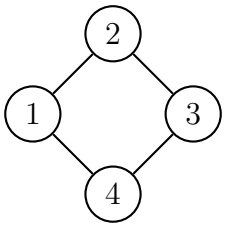
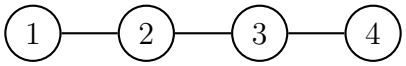
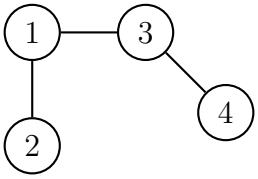


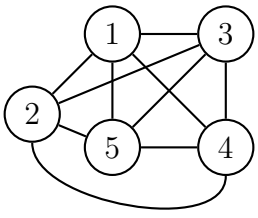
4.



5.



6. An undirected graph is called *complete* if every vertex shares an edge with every other vertex. Draw a complete graph on five vertices. How many edges does it have?

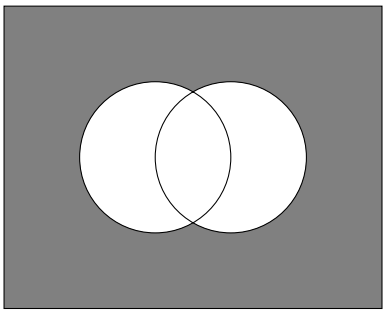


It has 10 edges.

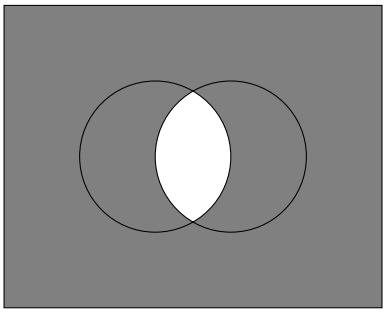
Section 2.2:

1. Draw Venn diagrams to illustrate De Morgan's laws for sets (Theorem 2.1).

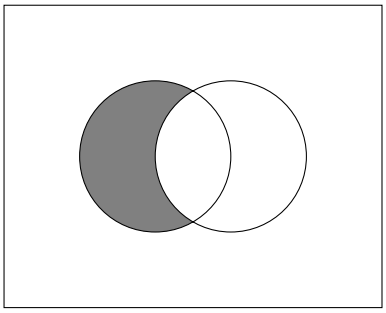
   1. $(A \cup B)' = A' \cap B'$



   2. $(A \cap B)' = A' \cup B'$



2. Draw a Venn diagram to show the region $A \cap B'$. This region is also denoted $A \setminus B$, and is called the set difference.



3. Let $A = \{2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, and suppose the universal set is $U = \{1, 2, \ldots, 9\}$. List all the elements in the following sets.

   (a) $(A \cup B)' = \{1, 7, 8, 9\}$

   (b) $(A \cap B) \times A = \{3, 4\} \times A = \{(3, 2), (3, 3)(3, 4), (4, 2), (4, 3), (4, 4)\}$

   (c) $P(B \setminus A) = \{\phi, \{4\}, \{5\}, \{4, 5\}\}$

4. Let the following sets be given.

$$\text{G} = \text{the set of all good citizen.}$$

$$\text{C} = \text{the set of all charitable people.}$$

$$\text{P} = \text{the set of all polite people.}$$

Write the statement, "Everyone who is charitable and polite is a good citizen," in the language of set theory.

$$C \cap P \subseteq G$$

5. Consider the following sets. The universal set for this problem is **N**.

$$\text{A} = \text{The set of all even numbers.}$$

$$\text{B} = \text{The set of all prime numbers.}$$

$$\text{C} =$$