**Minh Anh Nguyen**
**Calculus 1 Assignment-14**
**Section: 04**
**TA's name: Arthur Huey**

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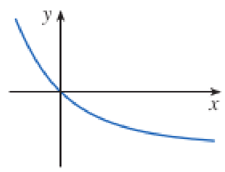
Section 6.1:

7. A function is given by a table of values, a graph, a formula, or a verbal description. Determine whether it is one-to-one.




Yes.

13. A function is given by a table of values, a graph, a formula, or a verbal description. Determine whether it is one-to-one.

$$g(x) = 1 - \sin x$$

No.

17. Assume that f is a one-to-one function.

 (a) If $f(6) = 17$, what is $f^{-1}(17)$?

$$f^{-1}(17) = 6$$

 (b) If $f^{-1}(3) = 2$, what is $f(2)$?

$$f(2) = 3$$

18. If $f(x) = x^5 + x^3 + x$, find $f^{-1}(3)$ and $f(f^{-1}(2))$.
Assume that $f(x)$ is a one-to-one function.

$$x^5 + x^3 + x = 3$$

$$x^5 + x^3 + x - 3 = 0$$

$$x = 1$$

Hence, $f^{-1}(3) = 1$.

$$f(f^{-1}(2)) = 2$$

24. Find a formula for the inverse of the function.

$$h(x) = \frac{6-3x}{5x+7}$$

Assume that $f(x)$ is a one-to-one function.

$$y = \frac{6-3x}{5x+7}$$

$$y(5x+7) = 6-3x$$

$$5xy + 7y = 6-3x$$

$$5xy + 3x = 6-7y$$

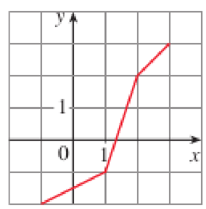
$$x(5y+3) = 6-7y$$

$$x = \frac{6-7y}{5y+3}$$

Hence the inverse function is:

$$f^{-1}(x) = \frac{6-7x}{5x+3}$$

33. Use the given graph of $f$ to sketch the graph of $f^{-1}$.




42. Find $(f^{-1})'(a)$.

$$f(x) = x^3 + 3\sin x + 2\cos x, a = 2$$

$$(f^{-1})'(a) = \frac{1}{f'(x)}, \text{ where } x \text{ satisfies } f(x) = 2$$

$$x^3 + 3\sin x + 2\cos x = 2$$

$$x = 0$$

Hence:

$$(f^{-1})'(a) = \frac{1}{f'(0)} = \frac{1}{(x^3 + 3\sin x + 2\cos x)'} = \frac{1}{3x^2 + 3\cos x - 2\sin x}$$

$$(f^{-1})'(a) = \frac{1}{3 \times 0^2 + 3\cos 0 - 2\sin 0} = \frac{1}{0+3-2} = \frac{1}{1} = 1$$

47. If $f(x) = \int_3^x \sqrt{1+t^3}dt$. Find $(f^{-1})'(0)$

$$(f^{-1})'(0) = \frac{1}{f'(x)}, \text{ where } x \text{ satisfies } f(x) = 0$$

$$f(x) = 0$$

$$\int_3^x \sqrt{1+t^3}dt = 0$$

$$x = 3$$

By the Fundamental Theorem of Calculus:

$$f'(x) = (\int_3^x \sqrt{1+t^3}dt)' = \sqrt{1+x^3}$$

$$(f^{-1})'(0) = \frac{1}{\sqrt{1+0^3}} = \frac{1}{1} = 1$$

Section 6.2:

2. Let:

(a) How is the $e$ defined?
$e$ is the number such that:

$$lim_{h\to 0}\frac{e^h - 1}{h} = 1$$

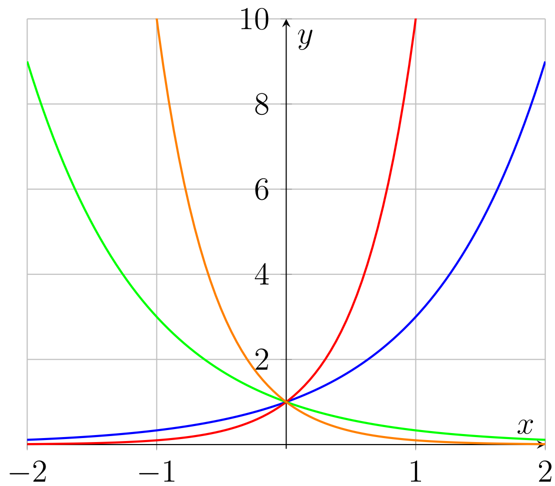
(b) What is an approximate value for $e$?
Approximate value of $e$ is: 2.71828

(c) What is the natural exponential function?

$$f(x) = e^x$$

5. Graph the given functions on a common screen. How are these graphs related?

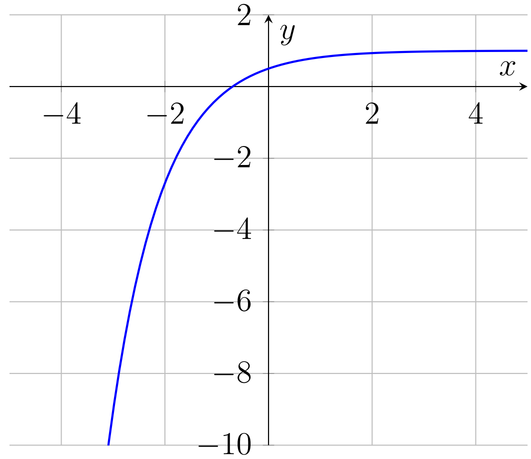
$$y = 3^x,\ y = 10^x,\ y = (\frac{1}{3})^x,\ y = (\frac{1}{10})^x$$



These graphs are symmentrical through the $y - axis$.

11. Make a rough sketch by hand of the graph of the function.

$$y = 1 - \frac{1}{2}e^{-x}$$




13. Starting with the graph of $y = e^x$, write the equation of the graph that results from

    (a) shifting 2 units downward.

    $$y = e^x - 2$$

    (b) shifting 2 units to the right.

    $$y = e^{x-2}$$

    (c) reflecting about the $x - axis$.

    $$y = -e^x$$

    (d) reflecting about the $y - axis$.

    $$y = e^{-x}$$

    (e) reflecting about the $x - axis$ and then about the $y - axis$.

    $$y = -e^{-x}$$

15. Find the domain of each function.

    (a) $f(x) = \frac{1-e^{x^2}}{1-e^{1-x^2}}$

    $$\begin{aligned} 1 - e^{1-x^2} &\neq 0 \\ e^{1-x^2} &\neq 1 \\ \log_e(e^{1-x^2}) &\neq \log_e 1 \\ (1 - x^2)\log_e(e) &\neq 0 \\ x^2 &\neq 1 \\ x &\neq \pm 1 \end{aligned}$$

    Hence the domain of the function is: $(-\infty, \infty) \setminus \{-1, 1\}$

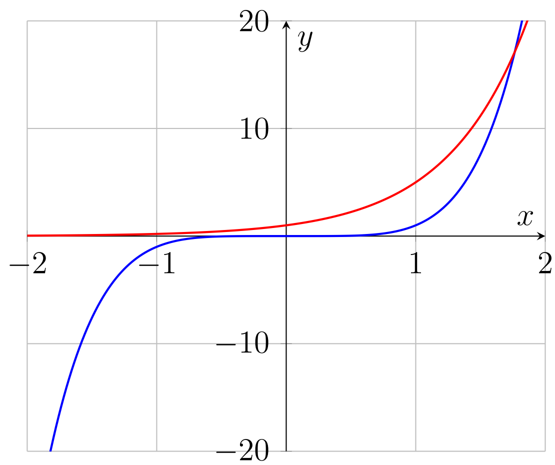
(b) $f(x) = \frac{1+x}{e^{\cos x}}$

$$e^{\cos x} \neq 0$$

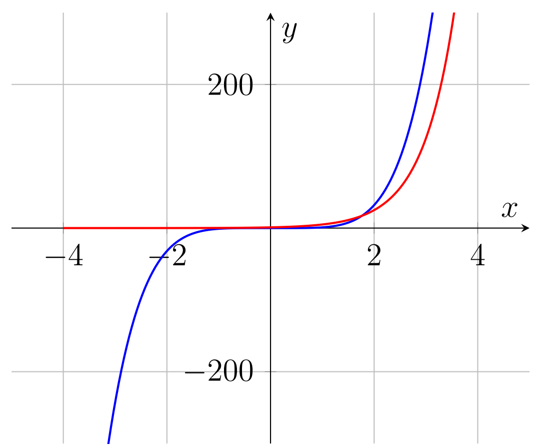
$$\log_e(e^{\cos x}) \neq \log_e 0$$

Because $\log_e 0$ does not exist. Hence the domain of the function is $(-\infty, \infty)$

20. Compare the functions $f(x) = x^5$ and $g(x) = 5^x$ by graphing both functions in several viewing rectangles. Find all points of intersection of the graphs correct to one decimal place. Which function grows more rapidly when $x$ is large?
A small viewing rectangle with $x \in [-2, 2]$ and $y \in [-20, 20]$:




A larger viewing rectangle with $x \in [-5, 5]$ and $y \in [-300, 300]$:




There are two points of intersections which are approximately 1.8 and 5.
The function $g(x) = 5^x$ grows more rapidly when $x$ is large.

23. Find the limit:

$$\lim_{x\to\infty} (1.001)^x = \infty$$

Because $(1.001)^x$ will become infinitely larger as $x$ becomes infinitely larger.

29. Find the limit:

$$\lim_{x\to\infty}(e^{-2x}\cos x)$$

Because $e^{-2x}$ will come closer to 0 as $x$ becomes larger and $\cos x$ only fluctuates between -1 and 1.
Hence:

$$\lim_{x\to\infty}(e^{-2x}\cos x) = 0$$

33. Differentiate the function.

$$f(x) = (3x^2 - 5x)e^x$$

$$f'(x) = [(3x^2 - 5x)e^x]' = (3x^2 - 5x)'e^x + (e^x)'(3x^2 - 5x) = e^x(6x - 5) + e^x(3x^2 - 5x)$$

$$f'(x) = e^x(6x - 5 + 3x^2 - 5x) = e^x(3x^2 + x - 5)$$

39. Differentiate the function

$$y = \sqrt[3]{e^x + 1}$$

$$y = (e^x + 1)^{1/3}$$

$$y' = \frac{1}{3}(e^x + 1)^{-2/3}e^x = \frac{e^x}{3}(e^x + 1)^{-2/3}$$

54. Find an equation of the tangent line to the curve $xe^y + ye^x = 1$ at the point (0,1).
The slope of the tangent line is:

$$(xe^y + ye^x)' = (1)'$$

$$e^y + xe^y\frac{dy}{dx} + \frac{dy}{dx}e^x + ye^x = 0$$

$$\frac{dy}{dx}(xe^y + e^x) = -e^y - ye^x$$

$$\frac{dy}{dx} = \frac{-e^y - ye^x}{xe^y + e^x}$$

$$\frac{dy}{dx}(0) = \frac{-e^1 - 1 \times e^0}{0 \times e^1 + e^0} = \frac{-e - 1}{1} = -(e + 1)$$

Hence, the tangent line is:

$$y = -x(e + 1) + b$$

Because the tangent line pass the point (0,1):

$$1 = -0(e + 1) + b$$

$$b = 1$$

Hence, the equation of the tangent line is:

$$y = -x(e + 1) + 1$$

57. For what value of $r$ does the function $y = e^{rx}$ satisfy the differential equation:

$$y'' + 6y' + 8y = 0$$

$$(e^{rx})'' + 6(e^{rx})' + 8(e^{rx}) = 0$$
$$(re^{rx})' + 6re^{rx} + 8e^{rx} = 0$$
$$r^2e^{rx} + 6re^{rx} + 8e^{rx} = 0$$

Because $e^{rx}$ is not equal to 0 with every x:

$$r^2 + 6r + 8 = 0$$

$$r = -2 \text{ or } r = -4$$

61. Let:

   (a) Use the Intermediate Value Theorem to show that there is a solution of the equation
   $$e^x + x = 0$$
   Let:
   $$f(x) = e^x + x$$
   This function is continuous with every $x$:
   Because of the IVT.
   Choose x = -1:
   $$f(-1) = e^{-1} - 1 = \frac{1}{e} - 1 < 0 \text{ Because } e > 1.$$
   Choose x = 1:
   $$f(1) = e^1 + 1 > 0 \text{ Because } e > 0.$$
   Hence, there is at least one solutions in the interval (-1,1).

69. Find the absolute maximum value of the function $f(x) = x - e^x$.

$$f'(x) = 1 - e^x = 0$$

$$e^x = 1$$

$$\log_e e^x = \log_e 1$$

$$x = 0$$

Hence, the absolute maximum value of the function is $f(0) = -1$.

71. Find

$$f(x) = xe^{2x}$$

(a) the intervals of increase or decrease,

$$f'(x) = e^{2x} + 2xe^{2x} = 0$$
$$e^{2x} + 2xe^{2x} = 0$$
$$e^{2x}(1 + 2x) = 0$$

Because $e^{2x}$ will not be equal to 0 with every x.
Hence:

$$1 + 2x = 0$$
$$x = -\frac{1}{2}$$

Therefore, the function increases on the interval $(-\frac{1}{2}, \infty)$.
The function decrease on the interval $(-\infty, -\frac{1}{2})$.

(b) the intervals of concavity, and

$$f''(x) = 2e^{2x} + 2e^{2x} + 4xe^{2x} = 0$$
$$4e^{2x} + 4xe^{2x} = 0$$
$$4e^{2x}(1 + x) = 0$$

Because $4e^{2x}$ will not equal 0 for every x.

$$1 + x = 0$$
$$x = -1$$

Hence, the function concaves down on the interval $(-\infty, -1)$ and concaves up on the interval $(-1, \infty)$.

(c) the points of inflection.

$$f(-1) = (-1)e^{-2} = -\frac{1}{e^2}$$

Hence, the point of inflection is $(-1, -\frac{1}{e^2})$.

81. Evaluate the integral.

$$\int_0^1 (x^e + e^x)dx = (\frac{x^{e+1}}{e+1} + e^x)|_0^1 = (\frac{1^{e+1}}{e+1} + e^1 - \frac{0^{e+1}}{e+1} - e^0) = \frac{1}{e+1} + e - 1$$
$$= \frac{1 + e^2 - 1}{e+1} = \frac{e^2}{e+1}$$

84. Evaluate the integral.

$$\int t^3 e^{-t^4} dt$$

Let:

$$u = -t^4$$
$$du = -4t^3 dt$$
$$-\frac{du}{4} = t^3 dt$$
$$-\frac{1}{4}\int e^u du = -\frac{1}{4}e^u + C = -\frac{1}{4}e^{-t^4} + C$$

90. Evaluate the integral.

$$\int e^{\sin\theta}\cos\theta\ d\theta$$

Let:

$$u = \sin\theta$$

$$du = \cos\theta d\theta$$

$$\int e^u du = e^u + C = e^{\sin\theta} + C$$

Section 6.3:

3. Find the exact value of each expression.

   (a) $\log_3 81$

   $$\log_3 81 = 4$$

   (b) $\log_3 \frac{1}{81}$

   $$\log_3 \frac{1}{81} = -4$$

   (c) $\log_9 3$

   $$\log_9 3 = \frac{1}{2}$$

5. Find the exact value of each expression.

   (a) $\log_2 30 - \log_2 15$

   $$\log_2 30 - \log_2 15 = \log_2 \frac{30}{15} = \log_2 2 = 1$$

   (b) $\log_3 10 - \log_3 5 - \log_3 18$

   $$\log_3 10 - \log_3 5 - \log_3 18 = \log_3 \frac{10}{5} - \log_3 18 = \log_3 2 - \log_3 18$$

   $$= \log_3 \frac{2}{18} = \log_3 \frac{1}{9} = -2$$

   (c) $2\log_5 100 - 4\log_5 50$

   $$2\log_5 100 - 4\log_5 50 = \log_5 100^2 - \log_5 50^4 = \log_5 \frac{100^2}{50^4} = \log_5 \frac{10000}{6250000}$$

   $$= \log_5 \frac{1}{625} = -4$$

6. Find the exact value of each expression.

   (a) $e^{3\ln 2}$

   $$e^{3\ln 2} = e^{\ln 2^3} = 2^3 = 8$$

(b) $e^{-2\ln 5}$

$$e^{-2\ln 5} = e^{\ln 5^{-2}} = 5^{-2} = \frac{1}{25}$$

(c) $e^{\ln(\ln e^3)}$

$$e^{\ln(\ln e^3)} = \ln e^3 = 3\ln e = 3$$

8. Use the laws of logarithms to expand each expression

(a) $\ln\sqrt{\frac{3x}{x-3}}$

$$\ln\sqrt{\frac{3x}{x-3}} = \ln(\frac{3x}{x-3})^{1/2} = \frac{1}{2}\ln(\frac{3x}{x-3}) = \frac{1}{2}\ln 3x - \frac{1}{2}\ln(x-3)$$

(b) $\log_2[(x^3+1)\sqrt[3]{(x-3)^2}]$

$$\log_2[(x^3+1)\sqrt[3]{(x-3)^2}] = \log_2(x^3+1) + \log_2\sqrt[3]{(x-3)^2}$$

$$= \log_2(x^3+1) + \log_2(x-3)^{2/3} = \log_2(x^3+1) + \frac{2}{3}\log_2(x-3)$$

10. Express as a single logarithm.

(a) $\ln 10 + 2\ln 5$

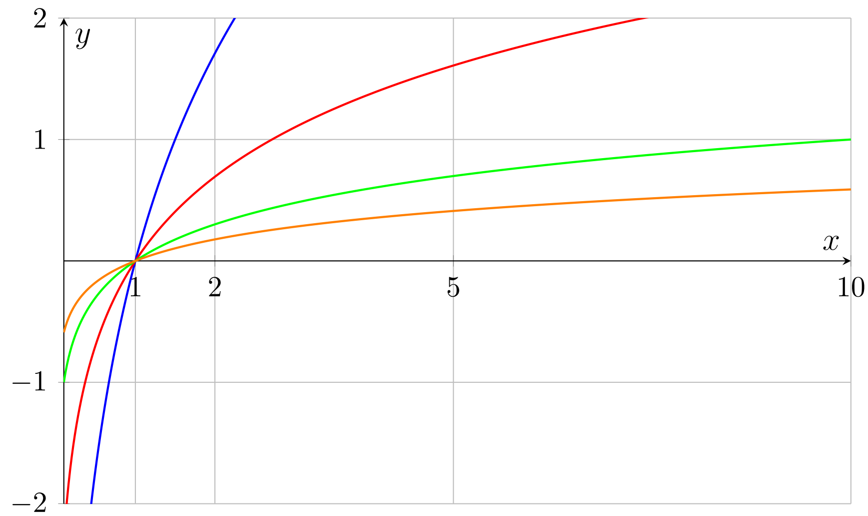
$$\ln 10 + 2\ln 5 = \ln 10 + \ln 5^2 = \ln(10\times 5^2) = \ln(10\times 25) = \ln 250$$

(b) $\log_{10} 4 + \log_{10} a - \frac{1}{3}\log_{10}(a+1)$

$$\log_{10} 4 + \log_{10} a - \frac{1}{3}\log_{10}(a+1) = \log_{10} 4a - \log_{10}(a+1)^{1/3} = \log_{10}\frac{4a}{(a+1)^{1/3}}$$

15. Use Formula 8 to graph the given functions on a common screen. How are these graphs related?

$$y = \log_{1.5} x,\ y = \ln x,\ y = \log_{10} x,\ y = \log_{50} x$$

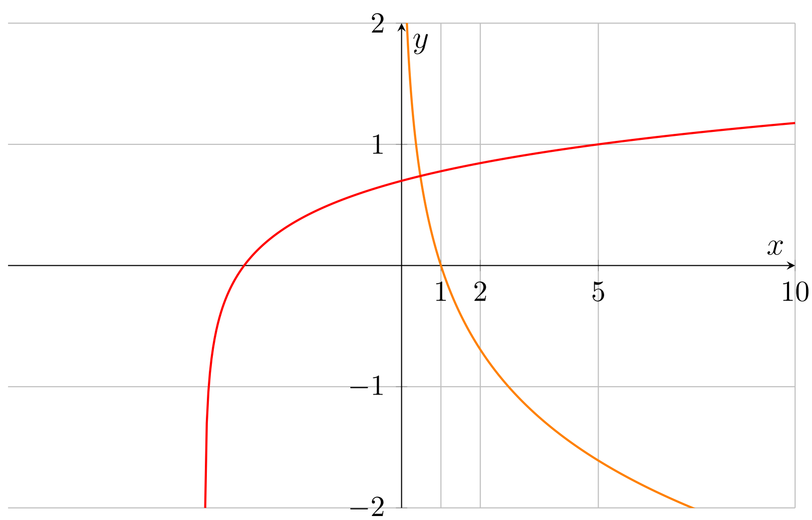


All of these graphs are defined over $x > 0$.

19. Make a rough sketch by hand of the graph of each function. Use the graphs given in Figures 2, and 3 and, if necessary, the transformations of Section 1.3.

    (a) $y = \log_{10}(x + 5)$

    (b) $y = -\ln x$




22. Let:

$$f(x) = \ln(x - 1) - 1$$

   (a) What are the domain and range of $f$?

$$x - 1 > 0$$

$$x > 1$$

   The domain of the function $f$ is $(1, \infty)$.
   The range of the function $f$ is $(-\infty, \infty)$.

   (b) What is the $x$-intercept of the graph of $f$?

$$\ln(x - 1) - 1 = 0$$

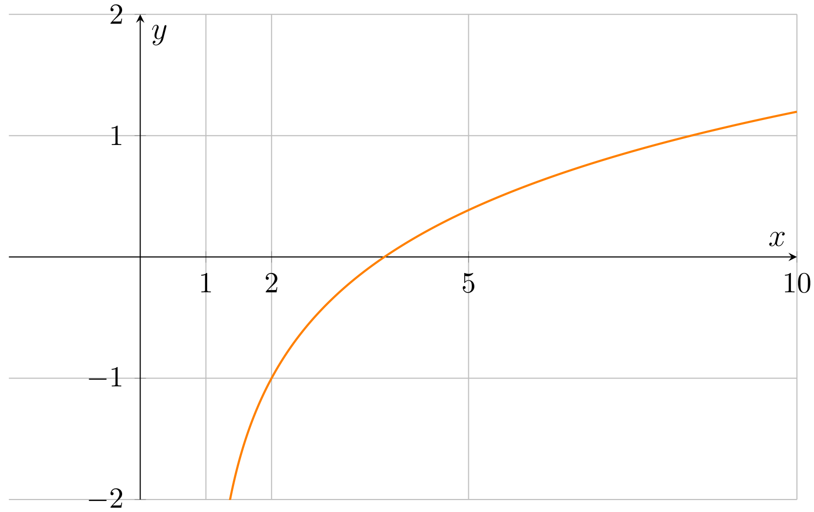
$$\ln(x - 1) = 1$$

$$x - 1 = e$$

$$x = e + 1$$

   Hence, the $x$-intercept of the graph $f$ is $e + 1$.

   (c) Sketch the graph of $f$.




25. Solve each equation for $x$. Give both an exact value and a decimal approximation, correct to three decimal places.

   (a) $\ln x + \ln(x-1) = 0$

$$\begin{aligned} \ln x + \ln(x-1) &= 0 \\ \ln(x-1) &= -\ln x \\ \ln(x-1) &= \ln x^{-1} \\ x-1 &= x^{-1} \\ x-1 &= \frac{1}{x} \\ x^2 - x - 1 &= 0 \\ x = \frac{1+\sqrt{5}}{2} \approx 1.618 &\text{ or } x = \frac{1-\sqrt{5}}{2} \approx -0.618 \end{aligned}$$

   Because $x > 1$ so that the equation can be defined.

$$x = \frac{1+\sqrt{5}}{2} \approx 1.618$$

   (b) $5^{1-2x} = 9$

$$\begin{aligned} \log_5 5^{1-2x} &= \log_5 9 \\ (1-2x)\log_5 5 &= \log_5 9 \\ -2x &= \log_5(9) - 1 \\ x &= \frac{1-\log_5(9)}{2} \approx -0.183 \end{aligned}$$

27. Find the limit.

$$\lim_{x\to 2^+} e^{3/(2-x)} = 0$$

31. Differentiate the function.

$$\begin{aligned} f(t) &= -2e^t \\ f'(t) &= -2e^t \end{aligned}$$

41. Differentiate the function.

$$y = x^2 e^{-3x}$$
$$y' = (x^2)' e^{-3x} + (e^{-3x})'(x^2)$$
$$y' = 2xe^{-3x} - 3e^{-3x}x^2$$

43. Differentiate the function.

$$f(t) = e^{at} \sin bt$$
$$f'(t) = (e^{at})' \sin bt + e^{at}(\sin bt)'$$
$$f'(t) = ae^{at} \sin bt + be^{at} \cos bt$$

45. Differentiate the function.

$$F(t) = e^{t \sin 2t}$$
$$F'(t) = e^{t \sin 2t}(t \sin 2t)'$$
$$F'(t) = e^{t \sin 2t}(\sin 2t + 2t \cos 2t)$$

49. Differentiate the function.

$$g(x) = \sin(\frac{e^x}{1+e^x})$$
$$g'(x) = \cos(\frac{e^x}{1+e^x})(\frac{e^x}{1+e^x})'$$
$$g'(x) = \cos(\frac{e^x}{1+e^x})(\frac{(e^x)'(1+e^x) - (1+e^x)'e^x}{(1+e^x)^2})$$
$$g'(x) = \cos(\frac{e^x}{1+e^x})(\frac{e^x(1+e^x) - e^{2x}}{(1+e^x)^2})$$
$$g'(x) = \cos(\frac{e^x}{1+e^x})(\frac{e^x + e^{2x} - e^{2x}}{(1+e^x)^2})$$
$$g'(x) = \cos(\frac{e^x}{1+e^x})(\frac{e^x}{(1+e^x)^2})$$

55. Show that the function $y = e^x + e^{-x/2}$ satisfies the differential equation $2y'' - y' - y = 0$.

$$y' = e^x - \frac{1}{2}e^{-x/2}$$
$$y'' = e^x + \frac{1}{4}e^{-x/2}$$

Hence:

$$2y'' - y' - y = 2(e^x + \frac{1}{4}e^{-x/2}) - e^x + \frac{1}{2}e^{-x/2} - e^x - e^{-x/2}$$
$$= 2e^x + \frac{1}{2}e^{-x/2} - e^x + \frac{1}{2}e^{-x/2} - e^x - e^{-x/2} = 2e^x + e^{-x/2} - 2e^x - e^{-x/2} = 0$$