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Calculus 1 Assignment-6
Section: 04
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Section 3.1:

35. Find the critical numbers of the function.

$$g(y) = \frac{y-1}{y^2-y+1}$$
$$g'(y) = \frac{(y-1)'(y^2-y+1)-(y^2-y+1)'(y-1)}{(y^2-y+1)^2}$$
$$g'(y) = \frac{y^2-y+1-(2y-1)(y-1)}{(y^2-y+1)^2}$$
$$g'(y) = \frac{y^2-y+1-2y^2+2y+y-1}{(y^2-y+1)^2}$$
$$g'(y) = \frac{-y^2+2y}{(y^2-y+1)^2}$$
$$-y^2+2y = 0$$
$$-y(y-2) = 0$$
$$y = 0 \text{ or } y = 2$$

The critical numbers of the function are 0 and 2.

41. Find the critical numbers of the function.

$$F(x) = x^{4/5}(x-2)^2$$
$$F(x) = x^{4/5}(x^2-4x+4)$$
$$F(x) = x^{14/5} - 4x^{9/5} + 4x^{4/5}$$
$$F'(x) = \frac{14}{5}x^{9/5} - \frac{36}{5}x^{4/5} + \frac{16}{5}x^{-1/5}$$
$$\frac{14}{5}x^{9/5} - \frac{36}{5}x^{4/5} + \frac{16}{5}x^{-1/5} = 0$$
$$x = \frac{4}{7} \text{ or } x = 2$$

The critical numbers of the function are $\frac{4}{7}$ and 2.

45. Find the critical numbers of the function.

$$f(\theta) = 2\cos\theta + \sin^2\theta$$
$$f'(\theta) = -2\sin\theta + 2\sin\theta\cos\theta$$
$$0 = -2\sin\theta + 2\sin\theta\cos\theta$$
$$2\sin\theta = 2\sin\theta\cos\theta$$
$$\theta = n\pi \text{ with n is a integer.}$$

The critical numbers of the function are $n\pi$ with n is a integer.

53. Find the absolute maximum and absolute minimum values of f on the given interval.

$$f(x) = 3x^4 - 4x^3 - 12x^2 + 1, \text{ [-2,3]}$$
$$f'(x) = 12x^3 - 12x^2 - 24x$$
$$12x^3 - 12x^2 - 24x = 0$$
$$12x(x^2 - x - 2) = 0$$
$$12x(x-2)(x+1) = 0$$
$$x = 0 \text{ or } x = 2 \text{ or } x = -1$$
$$f(-2) = 33$$
$$f(-1) = -4$$
$$f(0) = 1$$
$$f(2) = -31$$
$$f(3) = 28$$

The absolute maximum value of f is 33 and the absolute minimum value of f is -31.

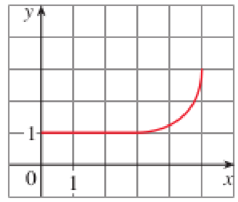
59. Find the absolute maximum and absolute minimum values of f on the given interval.

$$f(t) = 2\cos t + \sin 2t, [0, \frac{\pi}{2}]$$
$$f'(t) = -2\sin t + 2\cos 2t$$
$$0 = -2\sin t + 2\cos 2t$$
$$2\cos 2t = 2\sin t$$
$$\cos 2t = \cos(\frac{\pi}{2} - t)$$
$$2t = \frac{\pi}{2} - t$$
$$3t = \frac{\pi}{2}$$
$$t = \frac{\pi}{6}$$
$$f(0) = 2$$
$$f(\frac{\pi}{6}) = \frac{3\sqrt{3}}{2}$$
$$f(\frac{\pi}{2}) = 0$$

The absolute maximum value of f is $\frac{3\sqrt{3}}{2}$ and the absolute minimum value of f is 0.

Section 3.2:

7. The graph of a function f is shown. Does f satisfy the hypotheses of the Mean Value Theorem on the interval $[0, 5]$? If so, find a value c that satisfies the conclusion of the Mean Value Theorem on that interval.



Based on the graph, f is both continuous on the interval $[0, 5]$ and differentiable on the interval $(0, 5)$.

$$f'(c) = \frac{f(5) - f(0)}{5 - 0}$$

$$f'(c) = \frac{3 - 1}{5}$$

$$f'(c) = \frac{2}{5}$$

Based on the graph:

$$c \approx 4$$

11. Verify that the function satisfies the three hypotheses of Rolle's Theorem on the given interval. Then find all numbers c that satisfy the conclusion of Rolle's Theorem.

$$f(x) = \sin(x/2),\ [\frac{\pi}{2}, \frac{3\pi}{2}]$$

Because $f(x)$ is a trigonometric function, $f(x)$ is both continuous on the interval $[\frac{\pi}{2}, \frac{3\pi}{2}]$ and differentiable on the interval $(\frac{\pi}{2}, \frac{3\pi}{2})$.

$$f(\frac{\pi}{2}) = \sin(\pi/4) = \frac{\sqrt{2}}{2}$$

$$f(\frac{3\pi}{2}) = \sin(3\pi/4) = \frac{\sqrt{2}}{2}$$

Hence, $f(\frac{\pi}{2}) = f(\frac{3\pi}{2}) = \frac{\sqrt{2}}{2}$. Therefore, there exists c in $(\frac{\pi}{2}, \frac{3\pi}{2})$ such as:

$$f'(c) = 0$$

$$\frac{1}{2}\cos(c/2) = 0$$

$$\cos(c/2) = 0$$

$$c/2 = \pi/2 + k\pi \text{ with k is a integer.}$$