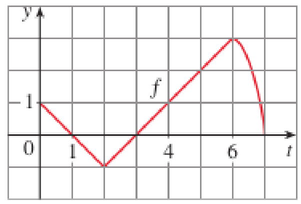
Minh Anh Nguyen
Calculus 1 Assignment-11
Section: 04
TA's name: Arthur Huey

Section 4.3:

2. Let $g(x) = \int_0^x f(t)dt$, where f is the function whose graph is shown.



(a) Evaluate $g(x)$ for $x = 0, 1, 2, 3, 4, 5, 6$.

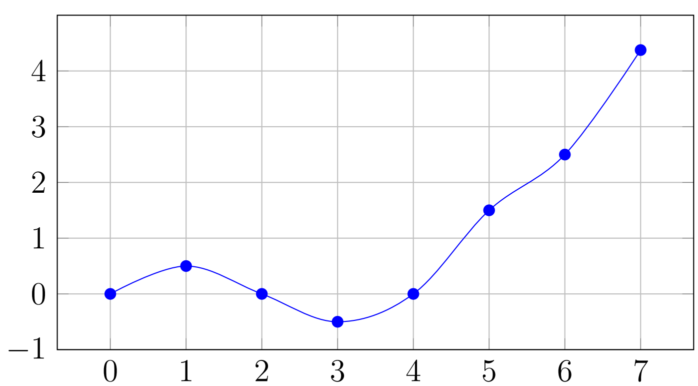
$$\begin{aligned} g(0) &= 0 \\ g(1) &= 0.5 \\ g(2) &= 0 \\ g(3) &= -0.5 \\ g(4) &= 0 \\ g(5) &= 1.5 \\ g(6) &= 2.5 \end{aligned}$$

(b) Estimate $g(7)$.

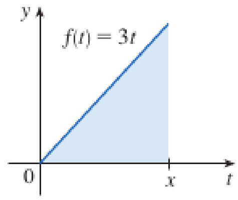
$$g(7) = 4.375$$

(c) Where does g have a maximum value? Where does it have a minimum value?
The function g has a maximum value at x = 7 and minimum value at x = 3.

(d) Sketch a rough graph of g.



6. The graph of a function f is shown. Let g be the function that represents the area under the graph of f between 0 and x.



(a) Use geometry to find a formula for $g(x)$.

$$g(x) = \frac{3x^2}{2}$$

(b) Verify that g is an antiderivative of f and explain how this confirms Part 1 of the Fundamental Theorem of Calculus for the function f.

$$g'(x) = (\frac{3x^2}{2})' = 3x = f(x)$$

Because g(x) is both continuous and differentiable everywhere, g'(x) = f(x) confirms Part 1 of the Fundamental Theorem of Calculus.

7. Sketch the area represented by g(x). Then find g'(x) in two ways:

$$g(x) = \int_1^x t^2 dt$$

(a) by using Part 1 of the Fundamental Theorem and

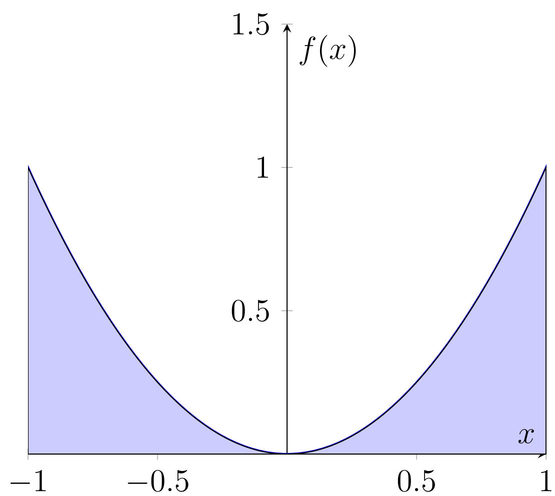
$$g(x) = \int_1^x t^2 dt, \text{ where} f(t) = t^2$$

Hence, $g'(x) = f(x) = x^2$.

(b) by evaluating the integral using Part 2 and then differentiating.

$$g(x) = \int_1^x x^2 = (\frac{x^3}{3} + C)|_1^x = \frac{x^3}{3} - \frac{1}{3}$$

$$g'(x) = (\frac{x^3}{3} - \frac{1}{3})' = x^2$$



9. Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.

$$g(x) = \int_0^x \sqrt{t + t^3} dt$$

Due to Part 1 of the Fundamental Theorem of Calculus:

$$g'(x) = \sqrt{x + x^3}$$

14. Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.

$$R(y) = \int_y^2 t^3 \sin(t) dt = -\int_2^y t^3 \sin(t) dt = \int_2^y -t^3 \sin(t) dt$$

Due to Part 1 of the Fundamental Theorem of Calculus:

$$R'(y) = -t^3 \sin(t)$$

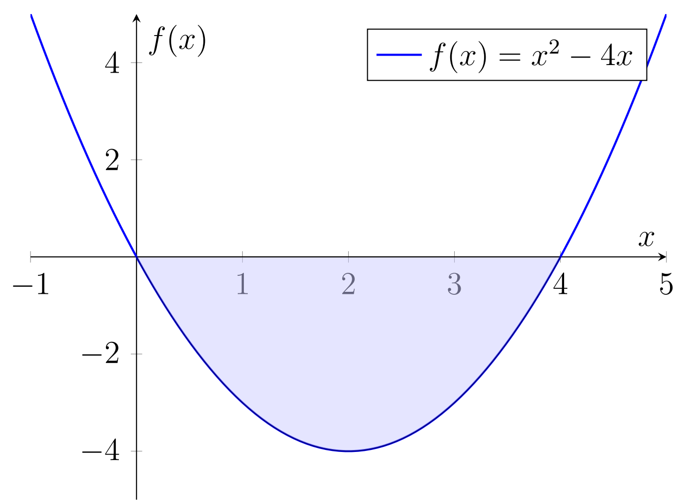
22. Use Part 2 of the Fundamental Theorem of Calculus to evaluate the integral and interpret the result as an area or a difference of areas. Illustrate with a sketch.

$$\int_0^4 (x^2 - 4x) dx$$

Let F be the antiderivative of the function $f(x) = x^2 - 4x$.

$$F(x) = \frac{x^3}{3} - 2x^2 + C$$

$$\int_0^4 (x^2 - 4x) dx = F(4) - F(0) = (\frac{x^3}{3} - 2x^2)|_0^4 = \frac{4^3}{3} - 2(4)^2 - 0 = -\frac{32}{3}$$



27. Evaluate the integral

$$\int_0^2 (\frac{4}{5}t^3 - \frac{3}{4}t^2 + \frac{2}{5}t) dt = (\frac{1}{5}t^4 - \frac{1}{4}t^3 + \frac{1}{5}t^2)|_0^2$$

$$= \frac{1}{5}(2)^4 - \frac{1}{4}(2)^3 + \frac{1}{5}(2)^2 - 0 = 2$$

35. Evaluate the integral

$$\int_0^1 (u+2)(u-3)du = \int_0^1 (u^2-u-6)du = (\frac{u^3}{3} - \frac{u^2}{2} - 6u)|_0^1$$

$$= \frac{1^3}{3} - \frac{1^2}{2} - 6(1) - 0 = -\frac{37}{6}$$

45. Evaluate the integral

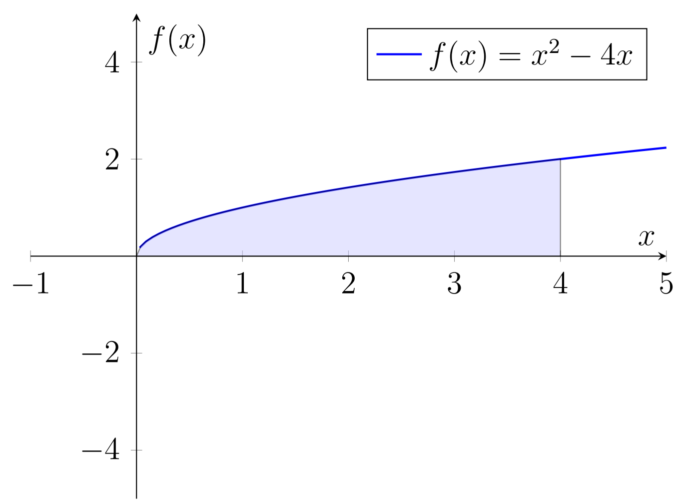
$$\int_0^\pi f(x)\,dx \quad \text{where} \quad f(x) = \begin{cases} \sin x & \text{if } 0 \le x < \frac{\pi}{2} \\ \cos x & \text{if } \frac{\pi}{2} \le x \le \pi \end{cases}$$

$$= \int_0^{\pi/2} f(x)dx + \int_{\pi/2}^{\pi} f(x)dx = \int_0^{\pi/2} \sin x dx + \int_{\pi/2}^{\pi} \cos x dx$$

$$= (-\cos x)|_0^{\pi/2} + (\sin x)|_{\pi/2}^{\pi} = (-\cos \pi/2 + \cos 0) + (\sin \pi - \sin \pi/2) = 1 - 1 = 0$$

47. Sketch the region enclosed by the given curves and calculate its area.

$$y = \sqrt{x}, y = 0, x = 4$$



If y = 0, x = 0. Hence, the region enclosed by x = 0 and x = 4, and the arae of the region is $\int_0^4 \sqrt{x}$.

$$\int_0^4 \sqrt{x} = (\frac{2x^{3/2}}{3})|_0^4 = \frac{2(4)^{3/2}}{3} - 0 = \frac{16}{3}$$

55. What is wrong with the equation?

$$\int_{-2}^1 x^{-4}dx = (\frac{x^{-3}}{-3})|_{-2}^1 = -\frac{3}{8}$$

Because the function x^{-4} is not continuous at x = 0. Hence, the integral is undefined.

60. Find the derivative of the function

$$g(x) = \int_{1-2x}^{1+2x} t \sin t dt$$

Using the Leibniz rule:

$$g'(x) = ((1+2x)\sin(1+2x))(1+2x)' - ((1-2x)\sin(1-2x))(1-2x)'$$

$$= 2((1+2x)\sin(1+2x)) + 2((1-2x)\sin(1-2x))$$

67. If $f(1) = 12$, f' is continuous, and $\int_1^4 f'(x)dx = 17$, what is the value of $f(4)$?

$$\int_1^4 f'(x)dx = f(4) - f(1) = 17$$

$$f(4) - 12 = 17$$

$$f(4) = 29$$

74. Evaluate the limit by first recognizing the sum as a Riemann sum for a function defined on [0,1].

$$\lim_{n\to\infty} \frac{1}{n}(\sqrt{\frac{1}{n}} + \sqrt{\frac{2}{n}} + \sqrt{\frac{3}{n}} + ... + \sqrt{\frac{n}{n}})$$

$$= \lim_{n\to\infty} \frac{1}{n}(\frac{\sqrt{1} + \sqrt{2} + \sqrt{3} + ... + \sqrt{n}}{\sqrt{n}})$$

$$= \lim_{n\to\infty} \Sigma_{i=1}^{n} \frac{1}{n}(\frac{\sqrt{i}}{\sqrt{n}})$$

$$= \lim_{n\to\infty} \Sigma_{i=1}^{n} \frac{1}{n}(\sqrt{\frac{i}{n}})$$

$$\Delta x = \frac{1}{n}$$

$$x_i = \frac{i}{n}$$

Hence,

$$a = 0, b = 1$$

$$f(x) = \sqrt{x}$$

$$\lim_{n\to\infty} \Sigma_{i=1}^{n} \frac{1}{n}(\sqrt{\frac{i}{n}}) = \int_0^1 \sqrt{x} dx = (\frac{2x^{3/2}}{3})|_0^1 = \frac{2}{3}$$

Section 4.4:

2. Verify by differentiation that the formula is correct.

$$\int \tan^2 x dx = \tan x - x + C$$

$$(\tan x - x + C)' = \sec^2 x - 1 = \tan^2 x$$

Hence, the formula is correct.

6. Find the general indefinite integral.

$$\int (5 + 2\sqrt{x}) dx = 5x + \frac{4x^{3/2}}{3} + C$$

18. Find the general indefinite integral.

$$\int \sec t(\sec t + \tan t) dt = \int (\sec^2 t + \sec t \tan t) dt = \tan t + \sec t + C$$

33. Evaluate the definite integral.

$$\int_{\pi/6}^{\pi/3} (4\sec^2 y) dy = 4\int_{\pi/6}^{\pi/3} (\sec^2 y) dy = 4 \times (\tan y)|_{\pi/6}^{\pi/3} = 4(\sqrt{3} - \frac{1}{\sqrt{3}}) = 4(\frac{3-1}{\sqrt{3}}) = \frac{8}{\sqrt{3}}$$

39. Evaluate the definite integral.

$$\int_0^{\pi/4} \frac{1 + \cos^2\theta}{\cos^2\theta} d\theta = \int_0^{\pi/4} (\frac{1}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta}) d\theta$$

$$= \int_0^{\pi/4} (\sec^2\theta + 1) d\theta = (\tan x + x)|_0^{\pi/4} = 1 + \pi/4 - 0 - 0 = \frac{4+\pi}{4}$$

43. Evaluate the definite integral.

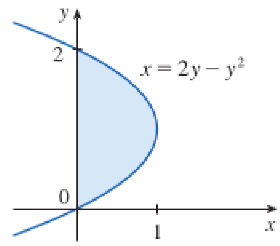
$$\int_2^5 |x - 3| dx$$

Let $f(x) = |x-3|$. The function $f(x)$ is negative with $x < 3$ and positive with $x > 3$. Hence

$$\int_2^5 |x-3| dx = -\int_2^3 (x-3) dx + \int_3^5 (x-3) dx$$

$$= -(\frac{x^2}{2} - 3x)|_2^3 + (\frac{x^2}{2} - 3x)|_3^5 = -(\frac{3^2}{2} - 3(3) - \frac{2^2}{2} + 3(2)) + (\frac{5^2}{2} - 3(5) - \frac{3^2}{2} + 3(3)) = \frac{5}{2}$$

49. The area of the region that lies to the right of the y-axis and to the left of the parabola $x = 2y - y^2$ (the shaded region in the figure) is given by the integral $\int_0^2 (2y - y^2)dy$. (Turn your head clockwise and think of the region as lying below the curve $x = 2y - y^2$ from $y = 0$ to $y = 2$.) Find the area of the region.



The area of the region is:

$$\int_0^2 (2y - y^2)dy = (y^2 - \frac{y^3}{3})|_0^2 = 2^2 - \frac{2^3}{3} - 0 + 0 = \frac{4}{3}$$

51. If $w'(t)$ is the rate of growth of a child in pounds per year, what does $\int_5^{10} w'(t)dt$ represent?
It represents the change of the child's weight between the age of 5 with the age of 10.

52. The current in a wire is defined as the derivative of the charge: $I(t) = Q'(t)$. (See Example 2.7.3.) What does $\int_a^b I(t)dt$ represent?
This represents the total change of the current flowing through the wire between time a to b.

63. The acceleration function (in m/s^2) and the initial velocity are given for a particle moving along a line. Find

$$a(t) = t + 4, v(0) = 5, 0 \le t \le 10$$

(a) the velocity at time t and

$$v(t) = \int (t + 4)dt = \frac{t^2}{2} + 4t + C$$

Because $v(0) = 5$:

$$v(0) = \frac{(0)^2}{2} + 4(0) + C = 5$$
$$C = 5$$

Hence, the velocity at time t is:

$$v(t) = \frac{t^2}{2} + 4t + 5$$

(b) the distance traveled during the given time integral,
The distance traveled during the given time is:

$$D = \int_0^{10} v(t)dt = \int_0^{10} (\frac{t^2}{2} + 4t + 5)dt = (\frac{t^3}{6} + 2t^2 + 5t)|_0^{10}$$
$$= \frac{10^3}{6} + 2(10)^2 + 5(10) - 0 = \frac{1250}{3}$$

Section 4.5:

2. Evaluate the integral by making the given substitution.

$$\int x(2x^2+3)^4dx, u = 2x^2+3$$

$$u = 2x^2+3$$
$$du = 4xdx$$
$$\frac{du}{4} = xdx$$

$$\int x(2x^2+3)^4dx = \frac{1}{4}\int u^4du = \frac{1}{4}\frac{u^5}{5} + C = \frac{u^5}{20} + C = \frac{(2x^2+3)^5}{20} + C$$

6. Evaluate the integral by making the given substitution.

$$\int \frac{1}{x^2}\sqrt{1+\frac{1}{x}}dx, u = 1+\frac{1}{x}$$

$$u = 1+\frac{1}{x}$$
$$du = -\frac{1}{x^2}dx$$
$$-du = \frac{1}{x^2}dx$$

$$\int \frac{1}{x^2}\sqrt{1+\frac{1}{x}}dx = -\int \sqrt{u}du = -\frac{2u^{3/2}}{3} + C = -\frac{2(1+\frac{1}{x})^{3/2}}{3} + C$$

9. Evaluate the indefinite integral.

$$\int x\sqrt{1-x^2}dx$$

Let:

$$u = 1-x^2$$
$$du = -2xdx$$
$$-\frac{du}{2} = xdx$$

Hence:

$$\int x\sqrt{1-x^2}dx = -\frac{1}{2}\int \sqrt{u}du = -\frac{1}{2}\frac{2u^{3/2}}{3} + C = -\frac{u^{3/2}}{3} + C = -\frac{(1-x^2)^{3/2}}{3} + C$$

15. Evaluate the indefinite integral.

$$\int \sec 3t \tan 3t dt$$

Let:

$$u = 3t$$
$$du = 3dx$$
$$\frac{du}{3} = dx$$

Hence:

$$\int \sec 3t \tan 3t dt = \frac{1}{3} \int \sec u \tan u du = \frac{\sec u}{3} + C = \frac{\sec 3t}{3} + C$$

20. Evaluate the indefinite integral.

$$\int \sin x \sin(\cos x) dx$$

Let:

$$u = \cos x$$
$$du = -\sin x dx$$
$$-du = \sin x dx$$

Hence:

$$\int \sin x \sin(\cos x) dx = -\int \sin u du = -(-\cos u) + C = \cos u + C = \cos(\cos x) + C$$

24. Evaluate the indefinite integral.

$$\int \frac{\sec^2 x}{\tan^2 x} dx$$

Let:

$$u = \tan^2 x$$
$$du = \sec^2 x dx$$

Hence:

$$\int \frac{\sec^2 x}{\tan^2 x} dx = \int \frac{1}{u} du = len|u| + C = len|\tan^2 x| + C$$

42. Evaluate the definite integral.

$$\int_1^4 \frac{\sqrt{2+\sqrt{x}}}{\sqrt{x}}$$

Let:

$$u = \sqrt{x}$$
$$du = \frac{1}{2\sqrt{x}} dx$$

$$2du = \frac{1}{\sqrt{x}}dx$$

Hence:

$$\int_1^4 \frac{\sqrt{2+\sqrt{x}}}{\sqrt{x}} = 2\int_1^2 \sqrt{2+u}du$$

Let:

$$t = 2 + u$$

$$dt = du$$

Hence:

$$2\int_1^2 \sqrt{2+u}du = 2\int_3^4 \sqrt{t}dt = 2 \times (\frac{2t^{3/2}}{3})|_3^4 = 2 \times (\frac{2(4)^{3/2}}{3} - \frac{2(3)^{3/2}}{3})$$

48. Evaluate the definite integral.

$$\int_{-\pi/3}^{\pi/3} x^4 \sin x dx$$

Because $(-x)^4 = x^4$ and $\sin(-x) = -\sin(x)$.
Hence, their multiply will be an odd function and:

$$\int_{-\pi/3}^{\pi/3} x^4 \sin x dx = 0$$

55. Evaluate $\int_{-2}^{2}(x+3)\sqrt{4-x^2}$ by writing it as a sum of two integrals and interpreting one of those integrals in terms of an area.

$$\int_{-2}^{2}(x+3)\sqrt{4-x^2} = \int_{-2}^{2} x\sqrt{4-x^2} + \int_{-2}^{2} 3\sqrt{4-x^2}$$

Because $-(x\sqrt{4-x^2}) = (-x)\sqrt{4-(-x)^2}$, the function $x\sqrt{4-x^2}$ is a odd function.
Hence,

$$\int_{-2}^{2} x\sqrt{4-x^2} = 0$$

Because $3\sqrt{4-x^2} = 3\sqrt{4-(-x)^2}$, the function $3\sqrt{4-x^2}$ is an even function.
Hence,

$$\int_{-2}^{2} 3\sqrt{4-x^2} = 2\int_0^2 3\sqrt{4-x^2} = 6\int_0^2 \sqrt{4-x^2}$$

The definite integral $\int_0^2 \sqrt{4-x^2}$ is one fourth of a circle with a radius of 2.
Hence,

$$6\int_0^2 \sqrt{4-x^2} = 6 \times \frac{1}{4}(\pi 2^2) = 6\pi$$

Therefore,

$$\int_{-2}^{2} x\sqrt{4-x^2} + \int_{-2}^{2} 3\sqrt{4-x^2} = 0 + 6\pi = 6\pi$$

59. If f is continuous and $\int_0^4 f(x)dx = 10$, find $\int_0^2 f(2x)dx$.
Let:

$$u = 2x$$
$$du = 2dx$$
$$\frac{du}{2} = dx$$

Hence,

$$\int_0^2 f(2x)dx = \frac{1}{2}\int_0^4 f(u)du = \frac{1}{2}10 = 5$$

69. Evaluate the integral.

$$\int \frac{(lnx)^2}{x}dx$$

Let

$$u = lnx$$
$$du = \frac{1}{x}dx$$

Hence,

$$\int \frac{(lnx)^2}{x}dx = \int u^2 du = \frac{u^3}{3} + C = \frac{(lnx)^3}{3} + C$$

80. Evaluate the integral.

$$\int \frac{x}{1+x^4}dx$$

Let

$$u = x^2$$
$$du = 2xdx$$
$$\frac{du}{2} = xdx$$

Hence,

$$\int \frac{x}{1+x^4}dx = \frac{1}{2}\int \frac{1}{1+u^2}du = \frac{\tan^{-1}u}{2} + C = \frac{\tan^{-1}(x^2)}{2} + C$$