

Section 2.3

1. Differentiate the function.

$$g(x) = 4x + 7$$
$$\frac{d}{dx}g(x) = \frac{d}{dx}(4x + 7)$$
$$\boxed{\frac{d}{dx}g(x) = 4}$$

7. Differentiate the function.

$$f(x) = x^{3/2} + x^{-3}$$
$$\frac{d}{dx}f(x) = \frac{d}{dx}(x^{3/2} + x^{-3})$$
$$\boxed{\frac{d}{dx}f(x) = \frac{3}{2}x^{1/2} - 3x^{-4}}$$

11. Differentiate the function.

$$y = 2x + \sqrt{x}$$
$$\frac{dy}{dx} = \frac{d}{dx}(2x + \sqrt{x})$$
$$\boxed{\frac{dy}{dx} = 2 + \frac{1}{2\sqrt{x}}}$$

15. Differentiate the function after first rewriting the function in a different form. (Do not use the Product or Quotient Rules.)

$$f(x) = x^3(x + 3)$$
$$f(x) = x^4 + 3x^3$$
$$\frac{d}{dx}f(x) = \frac{d}{dx}(x^4 + 3x^3)$$
$$\boxed{\frac{d}{dx}f(x) = 4x^3 + 9x^2}$$

18. Differentiate the function after first rewriting the function in a different form. (Do not use the Product or Quotient Rules.)

$$y = \frac{\sqrt{x} + x}{x^2}$$

$$y = \frac{\sqrt{x}}{x^2} + \frac{x}{x^2}$$

$$y = x^{-\frac{3}{4}} + x^{-1}$$

$$\frac{dy}{dx} = \frac{d}{dx}(x^{-\frac{3}{4}} + x^{-1})$$

$$\boxed{\frac{dy}{dx} = -\frac{3}{4}x^{-\frac{7}{4}} - x^{-2}}$$

30. Use the Product Rule to find the derivative of the function.

$$y = (10x^2 + 7x - 2)(2 - x^2)$$

$$\frac{d}{dx}y = \frac{d}{dx}[(10x^2 + 7x - 2)(2 - x^2)]$$

$$\frac{d}{dx}y = \frac{d}{dx}(10x^2 + 7x - 2)(2 - x^2) + (10x^2 + 7x - 2)\frac{d}{dx}(2 - x^2)$$

$$\frac{d}{dx}y = (20x + 7)(2 - x^2) + (10x^2 + 7x - 2)(-2x)$$

$$\frac{d}{dx}y = 40x - 20x^3 + 14 - 7x^2 - 20x^3 - 14x^2 + 4x$$

$$\boxed{\frac{d}{dx}y = -40x^3 - 21x^2 + 44x + 14}$$

35. Use the Quotient Rule to find the derivative of the function.

$$g(t) = \frac{3 - 2t}{5t + 1}$$

$$\frac{d}{dx}g(t) = \frac{d}{dx}\left(\frac{3 - 2t}{5t + 1}\right)$$

$$\frac{d}{dx}g(t) = \frac{\frac{d}{dx}(3 - 2t)(5t + 1) - (3 - 2t)\frac{d}{dx}(5t + 1)}{(5t + 1)^2}$$

$$\frac{d}{dx}g(t) = \frac{-2(5t + 1) - 5(3 - 2t)}{(5t + 1)^2}$$

$$\frac{d}{dx}g(t) = \frac{-10t - 2 - 15 + 10t}{(5t + 1)^2}$$

$$\boxed{\frac{d}{dx}g(t) = \frac{-17}{(5t + 1)^2}}$$

42. Differentiate.

$$\begin{aligned}
 y &= \frac{(u+2)^2}{1-u} \\
 \frac{d}{du}y &= \frac{d}{du}\left(\frac{(u+2)^2}{1-u}\right) \\
 \frac{d}{du}y &= \frac{\frac{d}{du}(u+2)^2(1-u) - (u+2)^2\frac{d}{du}(1-u)}{(1-u)^2} \\
 \frac{d}{du}y &= \frac{2(u+2)(1-u) + (u+2)^2}{(1-u)^2} \\
 \frac{d}{du}y &= \frac{(2u+4)(1-u) + u^2 + 4u + 4}{(1-u)^2} \\
 \frac{d}{du}y &= \frac{2u - 2u^2 + 4 - 4u + u^2 + 4u + 4}{1 - 2u + u^2} \\
 \boxed{\frac{d}{du}y} &= \boxed{\frac{-u^2 + 2u + 8}{u^2 - 2u + 1}}
 \end{aligned}$$

59. Find an equation of the tangent line to the curve at the given point.

$$\begin{aligned}
 y &= \frac{2x}{x+1}, (1, 1) \\
 \frac{d}{dx}y &= \frac{d}{dx}\left(\frac{2x}{x+1}\right) \\
 \frac{d}{dx}y &= \frac{\frac{d}{dx}(2x)(x+1) - (2x)\frac{d}{dx}(x+1)}{(x+1)^2} \\
 \frac{d}{dx}y &= \frac{2(x+1) - (2x)}{(x+1)^2} \\
 \frac{d}{dx}y &= \frac{2x + 2 - 2x}{x^2 + 2x + 1} \\
 \boxed{\frac{d}{dx}y} &= \boxed{\frac{2}{x^2 + 2x + 1}}
 \end{aligned}$$

75. Biologists have proposed a cubic polynomial to model the length L of Alaskan rockfish at age A :

$$L = 0.0155A^3 - 0.372A^2 + 3.95A + 1.21$$

where L is measured in inches and A in years. Calculate $\frac{dL}{dA}|_{A=12}$ and interpret your answer.

$$\begin{aligned}
 \frac{dL}{dA} &= \frac{dL}{dA}(0.0155A^3 - 0.372A^2 + 3.95A + 1.21) \\
 \frac{dL}{dA} &= 0.0465A^2 - 0.744A + 3.95
 \end{aligned}$$

$$\boxed{\frac{dL}{dA}|_{A=12} = 0.0465 \times 12^2 - 0.744 \times 12 + 3.95 = 1.718}$$

The length of Alaskan rockfish at age 12 is increasing by 1.718 inches per year.

87. Find the points on the curve $y = x^3 + 3x^2 - 9x + 10$ where the tangent is horizontal.

$$\frac{dy}{dx} = \frac{d}{dx}(x^3 + 3x^2 - 9x + 10)$$

$$\frac{dy}{dx} = 3x^2 + 6x - 9$$

The tangent line is horizontal when the slope is zero.

$$3x^2 + 6x - 9 = 0$$

$$x = 1 \vee x = -3$$

For $x = 1$, then:

$$y = 1^3 + 3 \times 1^2 - 9 \times 1 + 10 = 5$$

Therefore, the first point is $(1, 5)$

For $x = -3$, then:

$$y = (-3)^3 + 3 \times (-3)^2 - 9 \times (-3) + 10 = 37$$

Therefore, the second point is $(-3, 37)$.

Hence, there are two points in which the tangent line is horizontal:

$$\boxed{(1, 5), (-3, 37)}$$

Section 2.4:

1. Differentiate.

$$f(x) = 3 \sin(x) - 2 \cos(x)$$

$$\frac{d}{dx}f(x) = \frac{d}{dx}(3 \sin(x) - 2 \cos(x))$$

$$\boxed{\frac{d}{dx}f(x) = 3 \cos(x) + 2 \sin(x)}$$

3. Differentiate.

$$y = x^2 + \cot(x)$$

$$\frac{d}{dx}y = \frac{d}{dx}(x^2 + \cot(x))$$

$$\frac{d}{dx}y = 2x - \csc^2(x)$$

13. Differentiate.

$$f(\theta) = \frac{\sin(\theta)}{1 + \cos(\theta)}$$

$$\frac{d}{d\theta}f(\theta) = \frac{d}{d\theta}\left(\frac{\sin(\theta)}{1 + \cos(\theta)}\right)$$

$$\frac{d}{d\theta}f(\theta) = \frac{\frac{d}{d\theta}\sin(\theta)(1 + \cos(\theta)) - (\sin(\theta))\frac{d}{d\theta}(1 + \cos(\theta))}{(1 + \cos(\theta))^2}$$

$$\frac{d}{d\theta}f(\theta) = \frac{\cos(\theta)(1 + \cos(\theta)) + \sin(\theta)\sin(\theta)}{(1 + \cos(\theta))^2}$$

$$\frac{d}{d\theta}f(\theta) = \frac{\cos(\theta) + \cos^2(\theta) + \sin^2(\theta)}{(1 + \cos(\theta))^2}$$

$$\frac{d}{d\theta}f(\theta) = \frac{1 + \cos(\theta)}{(1 + \cos(\theta))^2}$$

$$\boxed{\frac{d}{d\theta}f(\theta) = \frac{1}{1 + \cos(\theta)}}$$

19. Differentiate.

$$y = \frac{t \sin(t)}{1 + t}$$

$$\frac{d}{dt}y = \frac{d}{dt}\left(\frac{t \sin(t)}{1 + t}\right)$$

$$\frac{d}{dt}y = \frac{\frac{d}{dt}(t \sin(t))(1 + t) - (t \sin(t))\frac{d}{dt}(1 + t)}{(1 + t)^2}$$