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Calculus 1 Assignment-
Section: 04
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Section 2.6:

7. Find $\frac{dy}{dx}$ by implicit differentiation.

$$x^4 + x^2y^2 + y^3 = 5$$

$$\frac{d}{dx}(x^4 + x^2y^2 + y^3) = \frac{d}{dx}(5)$$

$$4x^3 + 2xy^2 + 2x^2y\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$$

$$4x^3 + 2xy^2 + 2x^2y\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(2x^2y + 3y^2) = -4x^3 - 2xy^2$$

$$\boxed{\frac{dy}{dx} = \frac{-4x^3 - 2xy^2}{2x^2y + 3y^2}}$$

11. Find $\frac{dy}{dx}$ by implicit differentiation.

$$\sin x + \cos y = 2x - 3y$$

$$\frac{d}{dx}(\sin x + \cos y) = \frac{d}{dx}(2x - 3y)$$

$$\cos x - \sin y\frac{dy}{dx} = 2 - 3\frac{dy}{dx}$$

$$\cos x - 2 = \sin y\frac{dy}{dx} - 3\frac{dy}{dx}$$

$$\cos x - 2 = (\sin y - 3)\frac{dy}{dx}$$

$$\boxed{\frac{dy}{dx} = \frac{\cos x - 2}{\sin y - 3}}$$

15. Find $\frac{dy}{dx}$ by implicit differentiation.

$$\tan(\frac{x}{y}) = x + y$$

$$\frac{d}{dx}\tan(\frac{x}{y}) = \frac{d}{dx}(x + y)$$

$$\sec^2(\frac{x}{y})\frac{d}{dx}(\frac{x}{y}) = 1 + \frac{dy}{dx}$$

$$\sec^2(\frac{x}{y})\frac{y - x\frac{dy}{dx}}{y^2} = 1 + \frac{dy}{dx}$$

$$\frac{y - x\frac{dy}{dx}}{y^2} = \cos^2(\frac{x}{y}) + \cos^2(\frac{x}{y})\frac{dy}{dx}$$

$$\frac{y - x\frac{dy}{dx}}{y^2} - \cos^2(\frac{x}{y})\frac{dy}{dx} = \cos^2(\frac{x}{y})$$

$$\frac{y - x\frac{dy}{dx} - \cos^2(\frac{x}{y})y^2\frac{dy}{dx}}{y^2} = \cos^2(\frac{x}{y})$$

$$y - x\frac{dy}{dx} - \cos^2(\frac{x}{y})y^2\frac{dy}{dx} = \cos^2(\frac{x}{y})y^2$$

$$-\frac{dy}{dx}(x - \cos^2(\frac{x}{y})y^2) = \cos^2(\frac{x}{y})y^2 - y$$

$$\boxed{\frac{dy}{dx} = \frac{y - \cos^2(\frac{x}{y})y^2}{x - \cos^2(\frac{x}{y})y^2}}$$

24. Regard y as the independent variable and x as the dependent variable and use implicit differentiation to find $\frac{dx}{dy}$.

$$y \sec x = x \tan y$$

$$\frac{d}{dy}(y \sec x) = \frac{d}{dy}(x \tan y)$$

$$\sec x + y \sec x \tan x \frac{dx}{dy} = \frac{dx}{dy} \tan y + x \sec y \tan y$$

$$\sec x - x \sec y \tan y = \frac{dx}{dy} \tan y - y \sec x \tan x \frac{dx}{dy}$$

$$\frac{dx}{dy}(\tan y - y \sec x \tan x) = \sec x - x \sec y \tan y$$

$$\boxed{\frac{dx}{dy} = \frac{\sec x - x \sec y \tan y}{\tan y - y \sec x \tan x}}$$

25. Use implicit differentiation to find an equation of the tangent line to the curve at the given point.

$$y \sin 2x = x \cos 2y \;\; (\pi/2, \pi/4)$$

$$\frac{d}{dx}(y \sin 2x) = \frac{d}{dx}(x \cos 2y)$$

$$\frac{dy}{dx} \sin 2x + 2y \cos 2x = \cos 2y - 2xsin2y\frac{dy}{dx}$$

$$\frac{dy}{dx} \sin 2x + 2xsin2y\frac{dy}{dx} = \cos 2y - 2y \cos 2x$$

$$\frac{dy}{dx}(\sin 2x + 2xsin2y) = \cos 2y - 2y\cos 2x$$

$$\frac{dy}{dx} = \frac{\cos 2y - 2y\cos 2x}{\sin 2x + 2xsin2y}$$

$$\frac{dy}{dx}|_{\pi/2} = \frac{\cos \pi/2 - \pi/2 \cos \pi}{\sin \pi + \pi sin\pi/2}$$

$$\frac{dy}{dx}|_{\pi/2} = \frac{0 - \pi/2(-1)}{0 + \pi(1)}$$

$$\frac{dy}{dx}|_{\pi/2} = \frac{1}{2}$$

The tangent line:

$$y - \frac{\pi}{4} = \frac{1}{2}(x - \frac{\pi}{2})$$

$$\boxed{y = \frac{1}{2}x}$$

41. If $xy + y^3 = 1$, find the value of y'' at the point where $x = 0$.

$$xy + y^3 = 1$$

$$y + x\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(x + 3y^2) = -y$$

$$\frac{dy}{dx} = -\frac{y}{x + 3y^2}$$

$$y'' = -\frac{\frac{dy}{dx}(x + 3y^2) - y(1 + 6y\frac{dy}{dx})}{(x + 3y^2)^2}$$

$$y'' = \frac{y(1 + 6y\frac{dy}{dx}) - \frac{dy}{dx}(x + 3y^2)}{(x + 3y^2)^2}$$

$$y'' = \frac{y(1 - 6y\frac{y}{x+3y^2}) + \frac{y}{x+3y^2}(x + 3y^2)}{(x + 3y^2)^2}$$

$$y'' = \frac{y - \frac{6y^3}{x+3y^2} + \frac{yx}{x+3y^2} + \frac{3y^3}{x+3y^3}}{(x + 3y^2)^2}$$

$$y'' = \frac{y + \frac{-6y^3+yx+3y^3}{x+3y^2}}{(x + 3y^2)^2}$$

$$y'' = \frac{yx + 3y^3 - 6y^3 + yx + 3y^3}{(x + 3y^2)(x + 3y^2)^2}$$

$$y'' = \frac{2xy}{(x + 3y^2)^3}$$

If $x = 0$ then $y = 1$.

$$y''|_{x=0,y=1} = \frac{2xy}{(x+3y^2)^3}$$

$$y''|_{x=0,y=1} = \frac{2(0)(1)}{((0)+3(1)^2)^3}$$

$$\boxed{y''|_{x=0,y=1} = 0}$$

63. Use implicit differentiation to find dy/dx for the equation

$$\frac{x}{y} = y^2 + 1 \qquad y \neq 0$$

and for the equivalent equation

$$x = y^3 + y \qquad y \neq 0$$

Show that although the expressions you get for dy/dx look different, they agree for all points that satisfy the given equation.

$\frac{x}{y} = y^2 + 1$ because $y \neq 0$ so we can multiply all by y.

$$x = y^3 + y$$

$$\frac{d}{dx}x = \frac{d}{dx}(y^3 + y)$$

$$\boxed{1 = 3y^2\frac{dy}{dx} + \frac{dy}{dx}}$$

Because $y \neq 0$, the first and the second are the same. Hence, their derivatives are the same.

Section 2.8:

3. Each side of a square is increasing at a rate of $6cm/s$. At what rate is the area of the square increasing when the area of the square is $16cm^2$?

Let:

A: Area of the square (cm^2)
x: Length of the square (cm)

For $A = 16(cm^2)$

$$A = x^2$$

$$16 = x^2$$

$$x = \pm 4$$

Because length cannot be negative: $x = 4(cm)$

$$A(t) = x^2(t)$$

$$\frac{dA}{dt} = 2x\frac{dx}{dt}$$

$$\frac{dA}{dt} = 2(4)(6)$$

$$\boxed{\frac{dA}{dt} = 48(cm^2/s)}$$

The area is increasing at the rate $48cm^2/s$ when the area of the square is $16cm^2$.

7. A cylindrical tank with radius $5m$ is being filled with water at a rate of $3m^3/min$. How fast is the height of the water increasing?

Let:

r: the radius of the cylindrical tank (m).
h: the height of the cylindrical tank (m).
V: the volume of the cylindrical tank (m^3)

$$V = \pi r^2 h$$

$$V(t) = \pi r(t)^2 h(t)$$

$$\frac{dV}{dt} = \pi(2r(t)\frac{dr}{dt}h(t) + r^2\frac{dh}{dt})$$

$$3 = \pi(2(5)(0)h(t) + 5^2\frac{dh}{dt})$$

$$3 = \pi 5^2\frac{dh}{dt}$$

$$3 = \pi 25\frac{dh}{dt}$$

$$\boxed{\frac{dh}{dt} = \frac{3}{\pi 25}(m/min)}$$

The height of the cylindrical tank is increasing at the rate $\frac{3}{\pi 25}(m/min)$.

10. If $x^2 + y^2 + z^2 = 9$, $dx/dt = 5$, and $dy/dt = 4$, find dz/dt when $(x, y, z) = (2, 2, 1)$.

$$x^2 + y^2 + z^2 = 9$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} + 2z\frac{dz}{dt} = 0$$

$$2(2)(5) + 2(2)(4) + 2(1)\frac{dz}{dt} = 0$$

$$20 + 16 + 2\frac{dz}{dt} = 0$$

$$2\frac{dz}{dt} = -36$$

$$\boxed{\frac{dz}{dt} = -18}$$

25. Water is leaking out of an inverted conical tank at a rate of $10,000cm^3/min$ at the same time that water is being pumped into the tank at a constant rate. The tank has height $6m$ and the diameter at the top is $4m$. If the water level is rising at a rate of $20cm/min$ when the height of the water is $2m$, find the rate at which water is being pumped into the tank.

    Let:

    r: the radius of the tank (m)
    h: the height of the tank (m)
    V: the volume of the tank (m^3)
    x: the volume of water is pumped into the tank $(m^3)/min$

    The radius is: $r = 4/2 = 2(m)$
    And also:

$$\frac{r}{h} = 2/6$$

$$\text{new } r = \frac{1}{3}h(m)$$

    The volume of the tank is:

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi(\frac{h}{3})^2 h$$

$$V = \frac{1}{3}\pi\frac{h^3}{9}$$

$$\frac{dV}{dt} = \frac{1}{3}\pi 3\frac{h^2}{9}\frac{dh}{dt}$$

$$(x - 10,000) = \pi\frac{(200)^2}{9} \times 20$$

$$x - 10,000 = \frac{800,000\pi}{9}$$

$$\boxed{x = \frac{800,000\pi}{9} + 10,000}$$

Section 2.9

2. Find the linearization $L(x)$ of the function at a.

$$f(x) = cos2x,\ a = \pi/6$$
$$f'(x) = -2sin2x$$
$$f'(\pi/6) = -2sin(\pi/3) = -\sqrt{3}$$

The linearization $L(x)$ is:

$$L(x) = f(a) + f'(a)(x - a)$$
$$L(x) = f(\pi/6) + f'(\pi/6)(x - \pi/6)$$
$$L(x) = \frac{1}{2} - \sqrt{3}(x - \pi/6)$$
$$\boxed{L(x) = -\sqrt{3}x + \frac{\sqrt{3}\pi}{6} + \frac{1}{2}}$$

5. Find the linear approximation of the function $f(x) = \sqrt{1-x}$ at $a = 0$ and use it to approximate the numbers $\sqrt{0.9}$ and $\sqrt{0.99}$. Illustrate by graphing f and the tangent line.

$$f'(x) = -\frac{1}{2\sqrt{1-x}}$$
$$f'(0) = -\frac{1}{2\sqrt{1-0}} = -1/2$$

Linearization of the equation at $a = 0$ is:

$$L(x) = f(a) + f'(a)(x - a)$$
$$L(x) = 1 + -1/2(x - 0)$$
$$L(x) = -\frac{1}{2}x + 1$$

$$f(x) = \sqrt{1-x}$$
$$\sqrt{0.9} = \sqrt{1-x}$$
$$x = 0.1$$

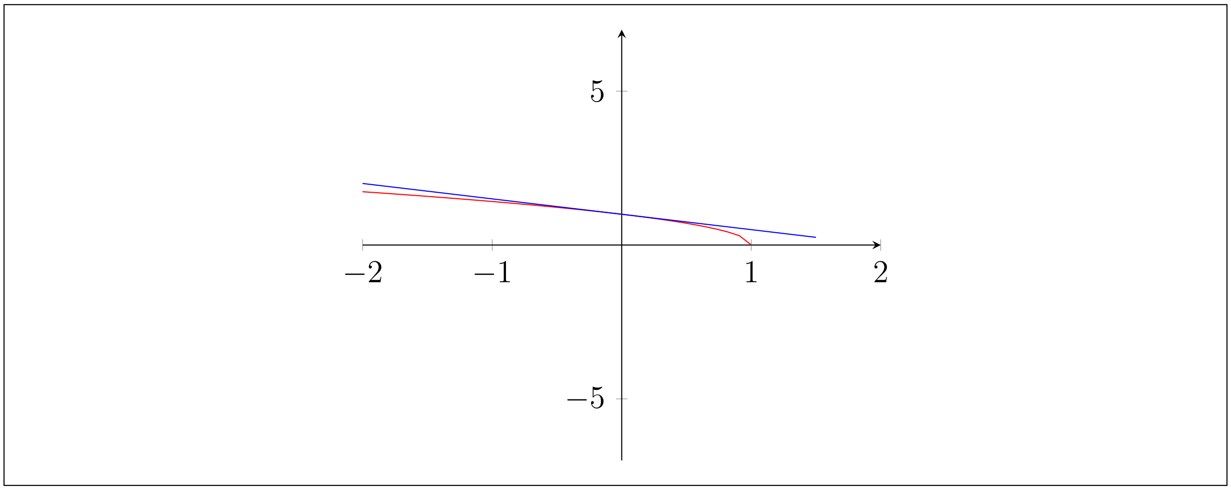
For the approximation at $x = 0.1$ is:

$$L(0.1) = -0.1(1/2) + 1$$
$$\boxed{L(0.1) = 0.95}$$

$$f(x) = \sqrt{1-x}$$
$$\sqrt{0.99} = \sqrt{1-x}$$
$$x = 0.01$$

For the approximation at $x = 0.01$ is:

$$L(0.01) = -0.01(1/2) + 1$$
$$\boxed{L(0.01) = 0.995}$$



14. Find the differential of the function.

$$y = \theta^2 \sin 2\theta$$

$$\boxed{y' = 2\theta \sin 2\theta + 2\theta^2 cos2\theta}$$

19. Find the differential dy and evaluate dy for the given values of x and dx.

$$y = \tan x,\ x = \frac{\pi}{4},\ dx = -0.1$$

$$\frac{dy}{dx} = \sec^2 x$$

$$dy = \sec^2 x \times dx$$

$$dy = \sec^2(\pi/4) \times (-0.1)$$

$$dy = \sec^2(\pi/4) \times (-0.1)$$

$$dy = 2 \times (-0.1)$$

$$\boxed{dy = -0.2}$$

23. Compute Δy and y for the given values of x and $dx = \Delta x$. Then sketch a diagram like Figure 5 showing the line segments with lengths dx, dy, and Δy.

$$y = x^2 - 4x,\ x = 3,\ \Delta x = 0.5$$

$$\Delta y = f(x + \Delta x) - f(x)$$

$$\Delta y = f(3 + 0.5) - f(3)$$

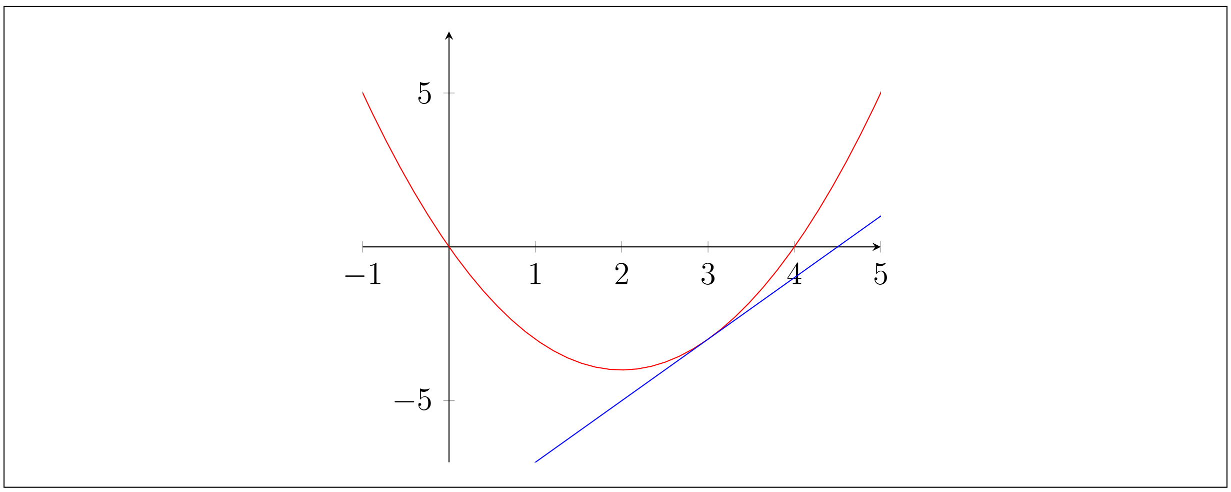
$$\Delta y = f(3.5) - f(3)$$

$$\boxed{\Delta y = (3.5)^2 - 4(3.5) - 3^2 + 4(3) = 1.25}$$

$$\frac{dy}{dx} = 2x - 4$$

$$dy = dx(2x - 4)$$

$$\boxed{dy = 0.5(2(3) - 4) = 1}$$



5
−5
−1
1
2
3
4
5