

Section 3.1:

35. Find the critical numbers of the function.

$$\begin{aligned}g(y) &= \frac{y-1}{y^2-y+1} \\g'(y) &= \frac{(y-1)'(y^2-y+1) - (y^2-y+1)'(y-1)}{(y^2-y+1)^2} \\g'(y) &= \frac{y^2-y+1 - (2y-1)(y-1)}{(y^2-y+1)^2} \\g'(y) &= \frac{y^2-y+1 - 2y^2+2y+y-1}{(y^2-y+1)^2} \\g'(y) &= \frac{-y^2+2y}{(y^2-y+1)^2} \\-y^2+2y &= 0 \\-y(y-2) &= 0 \\y &= 0 \text{ or } y = 2\end{aligned}$$

The critical numbers of the function are 0 and 2.

41. Find the critical numbers of the function.

$$\begin{aligned}F(x) &= x^{4/5}(x-2)^2 \\F(x) &= x^{4/5}(x^2-4x+4) \\F(x) &= x^{14/5} - 4x^{9/5} + 4x^{4/5} \\F'(x) &= \frac{14}{5}x^{9/5} - \frac{36}{5}x^{4/5} + \frac{16}{5}x^{-1/5} \\\frac{14}{5}x^{9/5} - \frac{36}{5}x^{4/5} + \frac{16}{5}x^{-1/5} &= 0 \\x &= \frac{4}{7} \text{ or } x = 2\end{aligned}$$

The critical numbers of the function are  $\frac{4}{7}$  and 2.

45. Find the critical numbers of the function.

$$f(\theta) = 2 \cos \theta + \sin^2 \theta$$

$$f'(\theta) = -2 \sin \theta + 2 \sin \theta \cos \theta$$

$$0 = -2 \sin \theta + 2 \sin \theta \cos \theta$$

$$2 \sin \theta = 2 \sin \theta \cos \theta$$

$$\theta = n\pi \text{ with } n \text{ is a integer.}$$

The critical numbers of the function are  $n\pi$  with  $n$  is a integer.

53. Find the absolute maximum and absolute minimum values of  $f$  on the given interval.

$$f(x) = 3x^4 - 4x^3 - 12x^2 + 1, [-2, 3]$$

$$f'(x) = 12x^3 - 12x^2 - 24x$$

$$12x^3 - 12x^2 - 24x = 0$$

$$12x(x^2 - x - 2) = 0$$

$$12x(x - 2)(x + 1) = 0$$

$$x = 0 \text{ or } x = 2 \text{ or } x = -1$$

$$f(-2) = 33$$

$$f(-1) = -4$$

$$f(0) = 1$$

$$f(2) = -31$$

$$f(3) = 28$$

The absolute maximum value of  $f$  is 33 and the absolute minimum value of  $f$  is -31.

59. Find the absolute maximum and absolute minimum values of  $f$  on the given interval.

$$f(t) = 2 \cos t + \sin 2t, [0, \frac{\pi}{2}]$$

$$f'(t) = -2 \sin t + 2 \cos 2t$$

$$0 = -2 \sin t + 2 \cos 2t$$

$$2 \cos 2t = 2 \sin t$$

$$\cos 2t = \cos(\frac{\pi}{2} - t)$$

$$2t = \frac{\pi}{2} - t$$

$$3t = \frac{\pi}{2}$$

$$t = \frac{\pi}{6}$$

$$f(0) = 2$$

$$f(\frac{\pi}{6}) = \frac{3\sqrt{3}}{2}$$

$$f(\frac{\pi}{2}) = 0$$

The absolute maximum value of  $f$  is  $\frac{3\sqrt{3}}{2}$  and the absolute minimum value of  $f$  is 0.