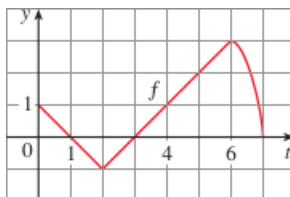


Section 4.3:

2. Let $g(x) = \int_0^x f(t)dt$, where f is the function whose graph is shown.



- (a) Evaluate $g(x)$ for $x = 0, 1, 2, 3, 4, 5, 6$.

$$g(0) = 0$$

$$g(1) = 0.5$$

$$g(2) = 0$$

$$g(3) = -0.5$$

$$g(4) = 0$$

$$g(5) = 1.5$$

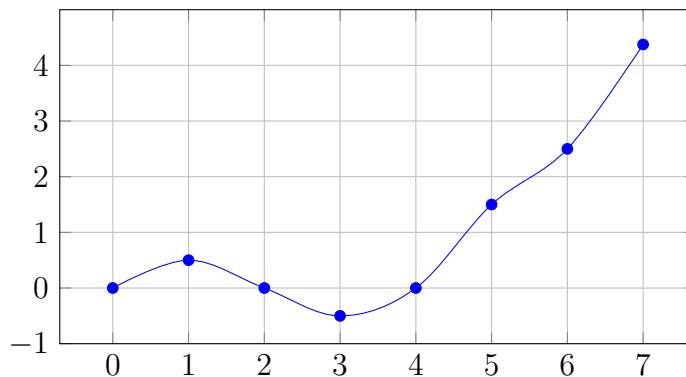
$$g(6) = 2.5$$

- (b) Estimate $g(7)$.

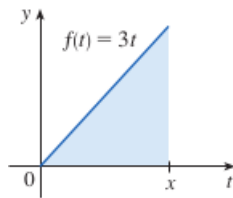
$$g(7) = 4.375$$

- (c) Where does g have a maximum value? Where does it have a minimum value?
 The function g has a maximum value at $x = 7$ and minimum value at $x = 3$.

- (d) Sketch a rough graph of g .



6. The graph of a function f is shown. Let g be the function that represents the area under the graph of f between 0 and x .



- (a) Use geometry to find a formula for $g(x)$.

$$g(x) = \frac{3x^2}{2}$$

- (b) Verify that g is an antiderivative of f and explain how this confirms Part 1 of the Fundamental Theorem of Calculus for the function f .

$$g'(x) = \left(\frac{3x^2}{2}\right)' = 3x = f(x)$$

Because $g(x)$ is both continuous and differentiable everywhere, $g'(x) = f(x)$ confirms Part 1 of the Fundamental Theorem of Calculus.

7. Sketch the area represented by $g(x)$. Then find $g'(x)$ in two ways:

$$g(x) = \int_1^x t^2 dt$$

- (a) by using Part 1 of the Fundamental Theorem and

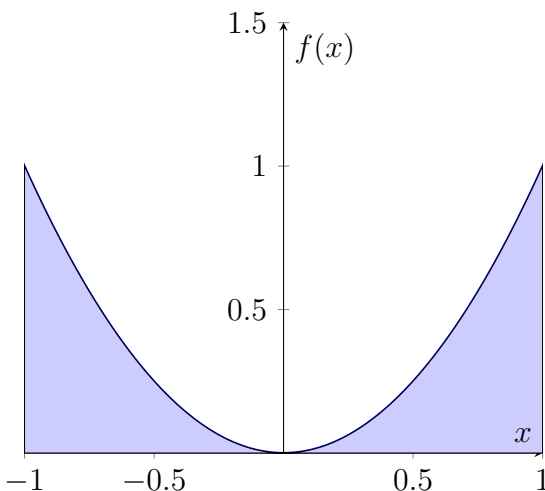
$$g(x) = \int_1^x t^2 dt, \text{ where } f(t) = t^2$$

Hence, $g'(x) = f(x) = x^2$.

- (b) by evaluating the integral using Part 2 and then differentiating.

$$g(x) = \int_1^x x^2 = \left(\frac{x^3}{3} + C\right)\Big|_1^x = \frac{x^3}{3} - \frac{1}{3}$$

$$g'(x) = \left(\frac{x^3}{3} - \frac{1}{3}\right)' = x^2$$



9. Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.

$$g(x) = \int_0^x \sqrt{t + t^3} dt$$

Due to Part 1 of the Fundamental Theorem of Calculus:

$$g'(x) = \sqrt{x + x^3}$$

14. Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.

$$R(y) = \int_y^2 t^3 \sin(t) dt = - \int_2^y t^3 \sin(t) dt = \int_2^y -t^3 \sin(t) dt$$

Due to Part 1 of the Fundamental Theorem of Calculus:

$$R'(y) = -t^3 \sin(t)$$

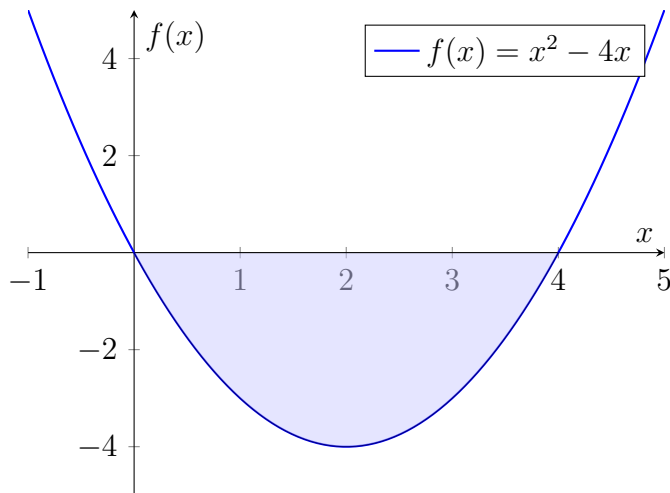
22. Use Part 2 of the Fundamental Theorem of Calculus to evaluate the integral and interpret the result as an area or a difference of areas. Illustrate with a sketch.

$$\int_0^4 (x^2 - 4x) dx$$

Let F be the antiderivative of the function $f(x) = x^2 - 4x$.

$$F(x) = \frac{x^3}{3} - 2x^2 + C$$

$$\int_0^4 (x^2 - 4x) dx = F(4) - F(0) = \left(\frac{x^3}{3} - 2x^2\right)\Big|_0^4 = \frac{4^3}{3} - 2(4)^2 - 0 = -\frac{32}{3}$$



27. Evaluate the integral

$$\begin{aligned} \int_0^2 \left(\frac{4}{5}t^3 - \frac{3}{4}t^2 + \frac{2}{5}t\right) dt &= \left(\frac{1}{5}t^4 - \frac{1}{4}t^3 + \frac{1}{5}t^2\right)\Big|_0^2 \\ &= \frac{1}{5}(2)^4 - \frac{1}{4}(2)^3 + \frac{1}{5}(2)^2 - 0 = 2 \end{aligned}$$

35. Evaluate the integral

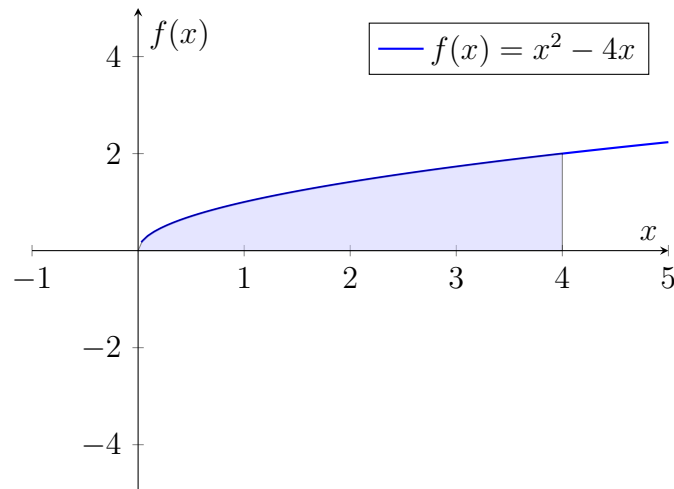
$$\begin{aligned}\int_0^1 (u+2)(u-3)du &= \int_0^1 (u^2 - u - 6)du = \left(\frac{u^3}{3} - \frac{u^2}{2} - 6u\right)\Big|_0^1 \\ &= \frac{1^3}{3} - \frac{1^2}{2} - 6(1) - 0 = -\frac{37}{6}\end{aligned}$$

45. Evaluate the integral

$$\begin{aligned}\int_0^\pi f(x) dx \quad \text{where} \quad f(x) &= \begin{cases} \sin x & \text{if } 0 \leq x < \frac{\pi}{2} \\ \cos x & \text{if } \frac{\pi}{2} \leq x \leq \pi \end{cases} \\ &= \int_0^{\pi/2} f(x)dx + \int_{\pi/2}^\pi f(x)dx = \int_0^{\pi/2} \sin x dx + \int_{\pi/2}^\pi \cos x dx \\ &= (-\cos x)\Big|_0^{\pi/2} + (\sin x)\Big|_{\pi/2}^\pi = (-\cos \pi/2 + \cos 0) + (\sin \pi - \sin \pi/2) = 1 - 1 = 0\end{aligned}$$

47. Sketch the region enclosed by the given curves and calculate its area.

$$y = \sqrt{x}, y = 0, x = 4$$



If $y = 0$, $x = 0$. Hence, the region enclosed by $x = 0$ and $x = 4$, and the area of the region is $\int_0^4 \sqrt{x}$.

$$\int_0^4 \sqrt{x} = \left(\frac{2x^{3/2}}{3}\right)\Big|_0^4 = \frac{2(4)^{3/2}}{3} - 0 = \frac{16}{3}$$

55. What is wrong with the equation?

$$\int_{-2}^1 x^{-4} dx = \left(\frac{x^{-3}}{-3}\right)\Big|_{-2}^1 = -\frac{3}{8}$$

Because the function x^{-4} is not continuous at $x = 0$. Hence, the integral is undefined.

60. Find the derivative of the function

$$g(x) = \int_{1-2x}^{1+2x} t \sin t dt$$

Using the Leibniz rule:

$$\begin{aligned} g'(x) &= ((1+2x) \sin(1+2x))(1+2x)' - ((1-2x) \sin(1-2x))(1-2x)' \\ &= 2((1+2x) \sin(1+2x)) + 2((1-2x) \sin(1-2x)) \end{aligned}$$

67. If $f(1) = 12$, f' is continuous, and $\int_1^4 f'(x) dx = 17$, what is the value of $f(4)$?

$$\int_1^4 f'(x) dx = f(4) - f(1) = 17$$

$$f(4) - 12 = 17$$

$$f(4) = 29$$

74. Evaluate the limit by first recognizing the sum as a Riemann sum for a function defined on $[0,1]$.

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{1}{n} (\sqrt{\frac{1}{n}} + \sqrt{\frac{2}{n}} + \sqrt{\frac{3}{n}} + \dots + \sqrt{\frac{n}{n}}) \\ = \lim_{n \rightarrow \infty} \frac{1}{n} (\frac{\sqrt{1} + \sqrt{2} + \sqrt{3} + \dots + \sqrt{n}}{\sqrt{n}}) \end{aligned}$$

$$= \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{1}{n} (\frac{\sqrt{i}}{\sqrt{n}})$$

$$= \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{1}{n} (\sqrt{\frac{i}{n}})$$

$$\Delta x = \frac{1}{n}$$

$$x_i = \frac{i}{n}$$

Hence,

$$a = 0, b = 1$$

$$f(x) = \sqrt{x}$$

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{1}{n} (\sqrt{\frac{i}{n}}) = \int_0^1 \sqrt{x} dx = (\frac{2x^{3/2}}{3})|_0^1 = \frac{2}{3}$$

Section 4.4:

2. Verify by differentiation that the formula is correct.

$$\int \tan^2 x dx = \tan x - x + C$$

$$(\tan x - x + C)' = \sec^2 x - 1 = \tan^2 x$$

Hence, the formula is correct.

6. Find the general indefinite integral.

$$\int (5 + 2\sqrt{x})dx = 5x + \frac{4x^{3/2}}{3} + C$$

18. Find the general indefinite integral.

$$\int \sec t(\sec t + \tan t)dt = \int (\sec^2 t + \sec t \tan t)dt = \tan t + \sec t + C$$

33. Evaluate the definite integral.

$$\int_{\pi/6}^{\pi/3} (4 \sec^2 y)dy = 4 \int_{\pi/6}^{\pi/3} (\sec^2 y)dy = 4 \times (\tan y)|_{\pi/6}^{\pi/3} = 4(\sqrt{3} - \frac{1}{\sqrt{3}}) = 4(\frac{3-1}{\sqrt{3}}) = \frac{8}{\sqrt{3}}$$

39. Evaluate the definite integral.

$$\begin{aligned} \int_0^{\pi/4} \frac{1 + \cos^2 \theta}{\cos^2 \theta} d\theta &= \int_0^{\pi/4} (\frac{1}{\cos^2 \theta} + \frac{\cos^2 \theta}{\cos^2 \theta}) d\theta \\ &= \int_0^{\pi/4} (\sec^2 \theta + 1) d\theta = (\tan \theta + \theta)|_0^{\pi/4} = 1 + \pi/4 - 0 - 0 = \frac{4 + \pi}{4} \end{aligned}$$

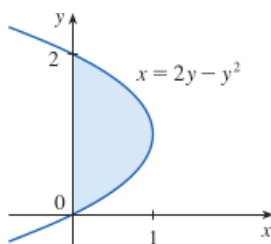
43. Evaluate the definite integral.

$$\int_2^5 |x - 3| dx$$

Let $f(x) = |x - 3|$. The function $f(x)$ is negative with $x < 3$ and positive with $x > 3$. Hence

$$\begin{aligned} \int_2^5 |x - 3| dx &= - \int_2^3 (x - 3) dx + \int_3^5 (x - 3) dx \\ &= -(\frac{x^2}{2} - 3x)|_2^3 + (\frac{x^2}{2} - 3x)|_3^5 = -(\frac{3^2}{2} - 3(3) - \frac{2^2}{2} + 3(2)) + (\frac{5^2}{2} - 3(5) - \frac{3^2}{2} + 3(3)) = \frac{5}{2} \end{aligned}$$

49. The area of the region that lies to the right of the y-axis and to the left of the parabola $x = 2y - y^2$ (the shaded region in the figure) is given by the integral $\int_0^2 (2y - y^2) dy$. (Turn your head clockwise and think of the region as lying below the curve $x = 2y - y^2$ from $y = 0$ to $y = 2$.) Find the area of the region.



The area of the region is:

$$\int_0^2 (2y - y^2) dy = (y^2 - \frac{y^3}{3}) \Big|_0^2 = 2^2 - \frac{2^3}{3} - 0 + 0 = \frac{4}{3}$$

51. If $w'(t)$ is the rate of growth of a child in pounds per year, what does $\int_5^{10} w'(t) dt$ represent?
It represents the change of the child's weight between the age of 5 with the age of 10.
52. The current in a wire is defined as the derivative of the charge: $I(t) = Q'(t)$. (See Example 2.7.3.) What does $\int_a^b I(t) dt$ represent?
This represents the total change of the current flowing through the wire between time a to b .
63. The acceleration function (in m/s^2) and the initial velocity are given for a particle moving along a line. Find

$$a(t) = t + 4, v(0) = 5, 0 \leq t \leq 10$$

- (a) the velocity at time t and

$$v(t) = \int (t + 4) dt = \frac{t^2}{2} + 4t + C$$

Because $v(0) = 5$:

$$v(0) = \frac{(0)^2}{2} + 4(0) + C = 5$$

$$C = 5$$

Hence, the velocity at time t is:

$$v(t) = \frac{t^2}{2} + 4t + 5$$

- (b) the distance traveled during the given time integral,
The distance traveled during the given time is:

$$D = \int_0^{10} v(t) dt = \int_0^{10} (\frac{t^2}{2} + 4t + 5) dt = (\frac{t^3}{6} + 2t^2 + 5t) \Big|_0^{10}$$

$$= \frac{10^3}{6} + 2(10)^2 + 5(10) - 0 = \frac{1250}{3}$$

Section 4.5:

3. Evaluate the integral by making the given substitution.

$$\int x(2x^2 + 3)^4 dx, u = 2x^2 + 3$$

$$du = 4x dx$$

$$\frac{du}{4} = x dx$$

Hence,

$$\int x(2x^2 + 3)^4 dx = \frac{1}{4} \int u^4 du = \frac{u^5}{20} + C = \frac{(2x^2 + 3)^5}{20} + C$$

4. Evaluate the integral by making the given substitution.

$$\int \sin^2 \theta \cos \theta d\theta, u = \sin \theta$$

$$du = \cos \theta d\theta$$

Hence,

$$\int \sin^2 \theta \cos \theta d\theta = \int u^2 du = \frac{u^3}{3} + C = \frac{(\sin x)^3}{3} + C$$

9. Evaluate the indefinite integral.

$$\int x\sqrt{1-x^2} dx$$

Let:

$$u = 1 - x^2$$

$$du = -2x dx$$

$$-\frac{du}{2} = x dx$$

Hence:

$$\int x\sqrt{1-x^2} dx = -\frac{1}{2} \int \sqrt{u} du = -\frac{1}{2} \frac{2u^{3/2}}{3} + C = -\frac{u^{3/2}}{3} + C = -\frac{(1-x^2)^{3/2}}{3} + C$$

15. Evaluate the indefinite integral.

$$\int \sec 3t \tan 3t dt$$

Let:

$$u = 3t$$

$$du = 3 dt$$

$$\frac{du}{3} = dt$$

Hence:

$$\int \sec 3t \tan 3t dt = \frac{1}{3} \int \sec u \tan u du = \frac{\sec u}{3} + C = \frac{\sec 3t}{3} + C$$

20. Evaluate the indefinite integral.

$$\int \sin x \sin(\cos x) dx$$

Let:

$$u = \cos x$$

$$du = -\sin x dx$$

$$-du = \sin x dx$$

Hence:

$$\int \sin x \sin(\cos x) dx = - \int \sin u du = -(-\cos u) + C = \cos u + C = \cos(\cos x) + C$$

24. Evaluate the indefinite integral.

$$\int \frac{\sec^2 x}{\tan^2 x} dx$$

Let:

$$u = \tan^2 x$$

$$du = \sec^2 x dx$$

Hence:

$$\int \frac{\sec^2 x}{\tan^2 x} dx = \int \frac{1}{u} du = \ln|u| + C = \ln|\tan^2 x| + C$$

42. Evaluate the definite integral.

$$\int_1^4 \frac{\sqrt{2+\sqrt{x}}}{\sqrt{x}} dx$$

Let:

$$u = \sqrt{x}$$

$$du = \frac{1}{2\sqrt{x}} dx$$

$$2du = \frac{1}{\sqrt{x}} dx$$

Hence:

$$\int_1^4 \frac{\sqrt{2+\sqrt{x}}}{\sqrt{x}} dx = 2 \int_1^2 \sqrt{2+u} du$$

Let:

$$t = 2 + u$$

$$dt = du$$

Hence:

$$2 \int_1^2 \sqrt{2+u} du = 2 \int_3^4 \sqrt{t} dt = 2 \times \left(\frac{2t^{3/2}}{3} \right) \Big|_3^4 = 2 \times \left(\frac{2(4)^{3/2}}{3} - \frac{2(3)^{3/2}}{3} \right)$$

48. Evaluate the definite integral.

$$\int_{-\pi/3}^{\pi/3} x^4 \sin x dx$$

Because $(-x)^4 = x^4$ and $\sin(-x) = -\sin(x)$.

Hence, their multiply will be an odd function and:

$$\int_{-\pi/3}^{\pi/3} x^4 \sin x dx = 0$$

55. Evaluate $\int_{-2}^2 (x+3)\sqrt{4-x^2}$ by writing it as a sum of two integrals and interpreting one of those integrals in terms of an area.

$$\int_{-2}^2 (x+3)\sqrt{4-x^2} = \int_{-2}^2 x\sqrt{4-x^2} + \int_{-2}^2 3\sqrt{4-x^2}$$

Because $-(x\sqrt{4-x^2}) = (-x)\sqrt{4-(-x)^2}$, the function $x\sqrt{4-x^2}$ is a odd function.
Hence,

$$\int_{-2}^2 x\sqrt{4-x^2} = 0$$

Because $3\sqrt{4-x^2} = 3\sqrt{4-(-x)^2}$, the function $3\sqrt{4-x^2}$ is an even function.
Hence,

$$\int_{-2}^2 3\sqrt{4-x^2} = 2 \int_0^2 3\sqrt{4-x^2} = 6 \int_0^2 \sqrt{4-x^2}$$

The definite integral $\int_0^2 \sqrt{4-x^2}$ is one fourth of a circle with a radius of 2.
Hence,

$$6 \int_0^2 \sqrt{4-x^2} = 6 \times \frac{1}{4}(\pi 2^2) = 6\pi$$

Therefore,

$$\int_{-2}^2 x\sqrt{4-x^2} + \int_{-2}^2 3\sqrt{4-x^2} = 0 + 6\pi = 6\pi$$

59. If f is continuous and $\int_0^4 f(x)dx = 10$, find $\int_0^2 f(2x)dx$.

Let:

$$u = 2x$$

$$du = 2dx$$

$$\frac{du}{2} = dx$$

Hence,

$$\int_0^2 f(2x)dx = \frac{1}{2} \int_0^4 f(u)du = \frac{1}{2}10 = 5$$

69. Evaluate the integral.

$$\int \frac{(\ln x)^2}{x} dx$$

Let

$$u = \ln x$$

$$du = \frac{1}{x} dx$$

Hence,

$$\int \frac{(\ln x)^2}{x} dx = \int u^2 du = \frac{u^3}{3} + C = \frac{(\ln x)^3}{3} + C$$

80. Evaluate the integral.

$$\int \frac{x}{1+x^4} dx$$

Let

$$u = x^2$$

$$du = 2x dx$$

$$\frac{du}{2} = x dx$$

Hence,

$$\int \frac{x}{1+x^4} dx = \frac{1}{2} \int \frac{1}{1+u^2} du = \frac{\tan^{-1} u}{2} + C = \frac{\tan^{-1}(x^2)}{2} + C$$