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Calculus 1 **Written HW-2**

Section 1.5:

1. Explain in your own words what is meant by the equation

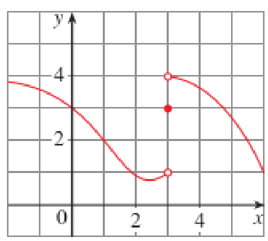
$$\lim_{x\to 2} f(x) = 5$$

Is it possible for this statement to be true and yet f(2) = 3? Explain.
Answer:

The equation indicates that as x approaches 2, the function of x will approach 5.

It is possible for this statement to be true and $f(2) = 3$, because it only implies that as x gets very close to 2, the function value f(x) gets very close to 5. It doesn't require f(x) to actually be 5.

5. For the function f whose graph is given, state the value of each quantity, if it exists. If it does not exist, explain why.



(a)

$$\boxed{\lim_{x\to 1} f(x) = 2}$$

(b)

$$\boxed{\lim_{x\to 3^-} f(x) = 1}$$

(c)

$$\boxed{\lim_{x\to 3^+} f(x) = 4}$$

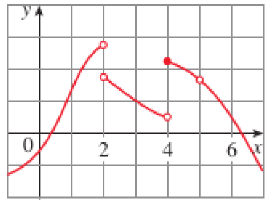
(d)

$$\boxed{\lim_{x\to 3} f(x) \text{ does not exist.}}$$

(e)

$$\boxed{f(3) = 3}$$

7. For the function f whose graph is shown, find a number a that satisfies the given description.



(a) $\lim_{x \to a} g(x)$ does not exist but g(a) is defined.

Answer: $\boxed{a = 4}$

(b) $\lim_{x \to a} g(x)$ exists but g(a) is not defined.

Answer: $\boxed{a = 5}$

(c) $\lim_{x \to a^-} g(x)$ and $\lim_{x \to a^+} g(x)$ both exist but $\lim_{x \to a} g(x)$ does not exist.

Answer: $\boxed{a = 2}$ and $\boxed{a = 4}$

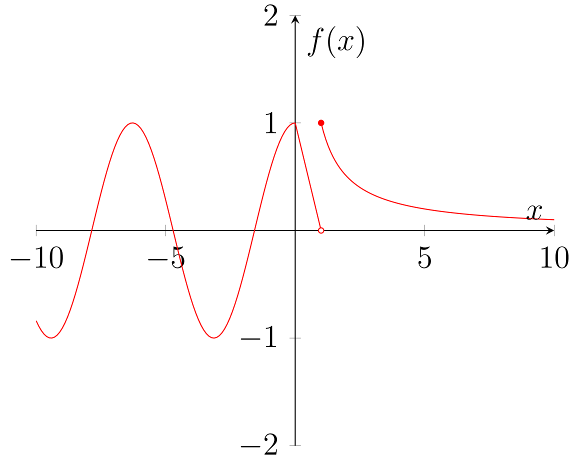
(d) $\lim_{x \to a^+} g(x) = g(a)$ but $\lim_{x \to a^-} g(x) \neq g(a)$.

Answer: $\boxed{a = 4}$

11. Sketch the graph of the function and use it to determine the values of a for which $\lim_{x\to a} f(x)$ exists.

$$f(x) = \begin{cases} \cos x \text{ if } x \leqslant 0 \\ 1 - x \text{ if } 0 < x < 1 \\ \dfrac{1}{x} \text{ if } x \geqslant 1 \end{cases}$$

Answer:



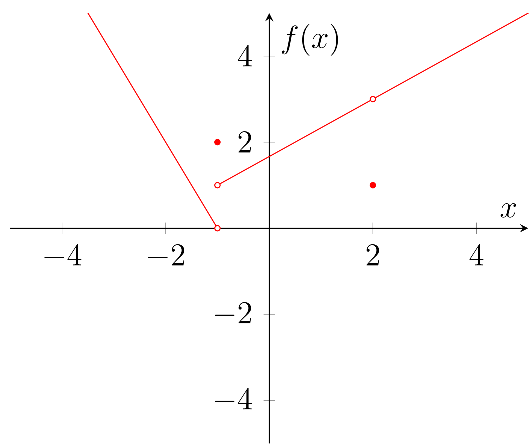
Because the function is discontinuous at $a = 1$, $\lim_{x\to a} f(x)$ exists for all value of a except for $a = 1$.

17. Sketch the graph of an example of a function f that satisfies all of the given conditions.

$\lim_{x\to -1^-} f(x) = 0,\ \lim_{x\to -1^+} f(x) = 1,\ \lim_{x\to 2} f(x) = 3$

$f(-1) = 2,\ f(2) = 1$

Answer:



25. Use a table of values to estimate the value of the limit. If you have a graphing device, use it to confirm your result graphically.

$$\lim_{x\to 0^+} x^x$$

Answer:

x	$f(x)$
0.5	0.7071067812
0.1	0.7943282347
0.01	0.95492586
0.001	0.9931160484
0.0001	0.99907939

Therefore, $\boxed{\lim_{x\to 0^+} f(x) = 1}$

31. Determine the infinite limit.

$$\lim_{x \to -2^+} \frac{x-1}{x^2(x+2)}$$

Answer:

x	$f(x)$
-1.5	$-2.(2)$
-1.9	-8.33240997
-1.99	-75.50314386
-1.999	-750.5003127
-1.9999	-7500.500031

Therefore, $\boxed{\lim_{x \to -2^+} \frac{x-1}{x^2(x+2)} = -\infty}$

Section 1.6:

3. Evaluate the limit and justify each step by indicating the appropriate Limit Law(s).

$$\lim_{x \to 5}(4x^2 - 5x)$$

Answer:

$$\lim_{x \to 5}(4x^2 - 5x)$$

$$= (4(5)^2 - 5(5))$$

$$\boxed{= 75}$$

7. Evaluate the limit and justify each step by indicating the appropriate Limit Law(s).

$$\lim_{u \to -2} \sqrt{9 - u^3 + 2u^2}$$

Answer:

$$\lim_{u \to -2} \sqrt{9 - u^3 + 2u^2}$$

$$= \sqrt{9 - (-2)^3 + 2(2)^2}$$

$$\boxed{= 5}$$

11. Evaluate the limit, if it exists.

$$\lim_{x \to -2}(3x - 7)$$

Answer:

$$\lim_{x \to -2}(3x - 7)$$

$$= 3(-2) - 7$$

$$\boxed{= -13}$$

13. Evaluate the limit, if it exists.

$$\lim_{t\to 4} \frac{t^2-2t-8}{t-4}$$

Answer:

$$\begin{aligned}
&\lim_{t\to 4} \frac{t^2-2t-8}{t-4} \\
&= \lim_{t\to 4} \frac{(t-4)(t+2)}{t-4} \\
&= \lim_{t\to 4}(t+2) \\
&= (4+2) \\
&\boxed{=6}
\end{aligned}$$

17. Evaluate the limit, if it exists.

$$\lim_{x\to -2} \frac{x^2-x-6}{3x^2+5x-2}$$

Answer:

$$\begin{aligned}
&\lim_{x\to -2} \frac{x^2-x-6}{3x^2+5x-2} \\
&= \lim_{x\to -2} \frac{(x-3)(x+2)}{(3x-1)(x+2)} \\
&= \lim_{x\to -2} \frac{(x-3)}{(3x-1)} \\
&= \frac{(-2-3)}{(3(-2)-1)} \\
&\boxed{=\frac{5}{7}}
\end{aligned}$$

23. Evaluate the limit, if it exists.

$$\lim_{h\to 0} \frac{\sqrt{9+h}-3}{h}$$

Answer:

$$\begin{aligned}
&\lim_{h\to 0} \frac{\sqrt{9+h}-3}{h} \\
&= \lim_{h\to 0} \frac{(\sqrt{9+h}-3)(\sqrt{9+h}+3)}{h(\sqrt{9+h}+3)} \\
&= \lim_{h\to 0} \frac{h}{h(\sqrt{9+h}+3)} \\
&= \lim_{h\to 0} \frac{1}{\sqrt{9+h}+3} = \frac{1}{\sqrt{9+0}+3} \\
&\boxed{=\frac{1}{6}}
\end{aligned}$$

31. Evaluate the limit, if it exists.

$$\lim_{t\to 0} \frac{1}{t\sqrt{1+t}} - \frac{1}{t}$$

Answer:

$$\lim_{t\to 0} \frac{1}{t\sqrt{1+t}} - \frac{1}{t}$$

$$= \lim_{t\to 0} \frac{1}{t\sqrt{1+t}} - \frac{\sqrt{1+t}}{t\sqrt{1+t}}$$

$$= \lim_{t\to 0} \frac{1-\sqrt{1+t}}{t\sqrt{1+t}}$$

$$= \lim_{t\to 0} \frac{1-1-t}{t\sqrt{1+t}(1+\sqrt{1+t})}$$

$$= \lim_{t\to 0} \frac{-1}{\sqrt{1+t}(1+\sqrt{1+t})}$$

$$= \frac{-1}{\sqrt{1+0}(1+\sqrt{1+0})}$$

$$\boxed{= -\frac{1}{2}}$$

43. Evaluate the limit, if it exists.

$$\lim_{x\to -4} (|x+4| - 2x)$$

Answer:

$$\lim_{x\to -4} (|x+4| - 2x)$$

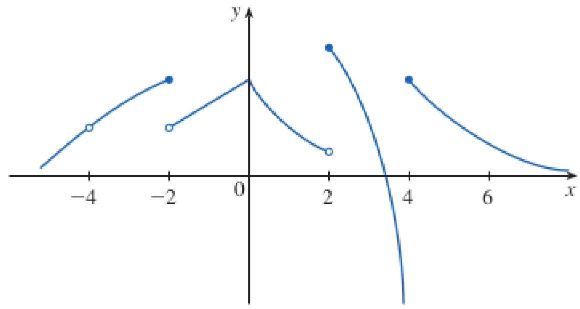
$$= \lim_{x\to -4^-} (x+4-2x) \text{ and } \lim_{x\to -4^+} (-x-4-2x)$$

$$= \lim_{x\to -4^-} (-4+4-2(-4)) \text{ and } \lim_{x\to -4^+} (-(-4)-4-2(-4))$$

$$= 8 \text{ and } 8$$

So, $\boxed{\lim_{x\to -4^-} (|x+4| - 2x) = \lim_{x\to -4^+} (|x+4| - 2x) = \lim_{x\to -4} (|x+4| - 2x) = 8}$

3. (a) From the given graph of f, state the numbers at which f is discontinuous and explain why.



Answer:

$\boxed{\text{The graph is discontinuous at -4, -2, 2, 4 because f(-4) does not exist and } \lim_{x \to a} f(x) \text{ with } a = -2, 2, 4.}$

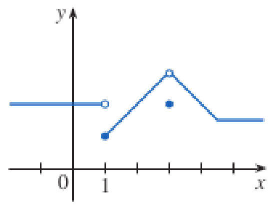
(b) For each of the numbers stated in part (a), determine whether f is continuous from the right, or from the left, or neither.
Answer:

$$\boxed{\lim_{t \to -2} f(x) \text{ is continuous from the left.}}$$

$$\boxed{\lim_{t \to 2} f(x) \text{ is continuous from the right.}}$$

$$\boxed{\lim_{t \to 4} f(x) \text{ is continuous from the right.}}$$

5. The graph of a function f is given.



(a) At what numbers a does $\lim_{t \to a} f(x)$ not exist?
Answer:

$\boxed{\text{At 1, } \lim_{t \to 1} f(x) \text{ does not exist.}}$

(b) At what numbers a is f not continuous?
Answer:

$\boxed{\text{At 1 and 3, } f \text{ is not continuous.}}$

(c) At what numbers a does $\lim_{x \to a} f(a)$ exist but f is not continuous at a?
Answer:

$\boxed{\text{At 3, } \lim_{x \to 1} f(x) \text{ exists but } f \text{ is not continuous at 1.}}$

13. Use the definition of continuity and the properties of limits to show that the function is continuous at the given number a.

$$f(x) = 3x^2 + (x+2)^5,\ a = -1$$

Answer:

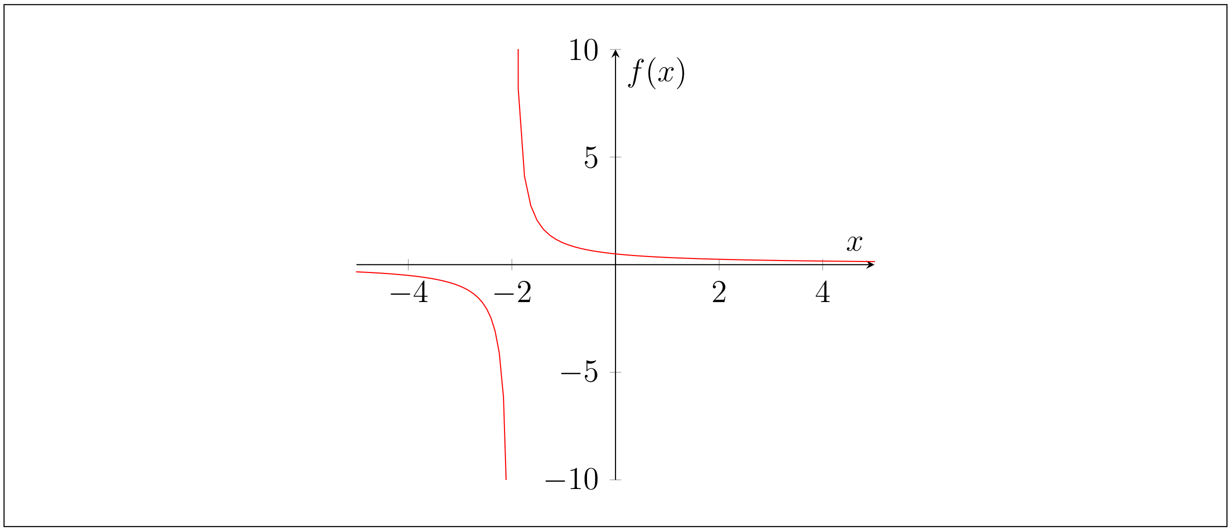
Because $f(x)$ is a polynomial function, $f(x)$ is continuous on $\mathbb{R}$. So it will also be continuous on -1.

19. Explain why the function is discontinuous at the given number a. Sketch the graph of the function.

$$f(x) = \frac{1}{x+2} \qquad a = -2$$

Answer:

Because $f(x)$ is not defined at $x = -2$, so it will not be continuous at $x = -2$.



25. Given the equation:

$$f(x) = \frac{x-3}{x^2-9}$$

(a) Show that f has a removable discontinuity at $x = 3$.
Answer:
$x^2 - 9 \neq 0$
$x = -3 \vee x = 3$
Domain of $f(x)$ is $\mathbb{R} \setminus \{-3, 3\}$.
$\boxed{\text{Therefore, } f(x) \text{ has a removable continuity at } x = 3.}$

(b) Redefine $f(3)$ so that f is continuous at $x = 3$ (and thus the discontinuity is "removed").

$$\boxed{f(x) = \begin{cases} \dfrac{x-3}{x^2-9} & \text{if } x \neq \pm 3 \\ \dfrac{1}{x+3} & \text{if } x = \pm 3 \end{cases}}$$

27. Explain, using Theorems 4, 5, 7, and 9, why the function is continuous at every number in its domain. State the domain.

$$f(x) = \frac{x^2}{\sqrt{x^4+2}}$$

Answer:

$\boxed{\text{Because the function } f \text{ is a rational function, so it will be continuous at every number in its domain.}}$

$x^4 + 2 > 0$

$x^4 > -2$ (True with all x)

$\boxed{\text{Domain: } (-\infty, \infty)}$

37. Use continuity to evaluate the limit.

$$\lim_{x \to \frac{\pi}{4}} x^2 \tan(x)$$

Answer:
$\cos(x) \neq 0$

Domain: $x \neq \dfrac{n}{2} + n\pi$ with all $n \in \mathbb{N}$

Because $\tan(x)$ is a trignometric function, it will be continuous with all x in its domain.
Moreover, x^2 is a polynomial, it will be continuous with all x.
Therefore, $f(x)$ will be continuous with all x in its domain.

Because $f(x)$ is continuous, $\boxed{\lim_{x \to \frac{\pi}{4}} f(x) = f(\frac{\pi}{4}) = 0.6168502751}$

41. Show that f is continuous on $(-\infty, \infty)$.

$$f(x) = \begin{cases} 1 - x^2 \text{ if } x \leqslant 1 \\ \sqrt{x-1} \text{ if } x > 1 \end{cases}$$

Answer:
For $x \neq 1$:

For $x < 1$: $f(x) = 1 - x^2$ is a polynomial, so it will be continuous on the intervals $(-\infty, 1)$.
For $x > 1$: $f(x) = \sqrt{x-1}$ is continuous on its domain which is $(1, \infty)$.

So, the function will be continuous everywhere except 1.

For $x = 1$:

$$\lim_{x \to 1} f(x)$$

$$= \begin{cases} \lim_{x \to 1^-} (1 - x^2) = 0 \\ \lim_{x \to 1^+} \sqrt{x-1} = 0 \end{cases}$$

Therefore, $\lim_{x \to 1^-} (1 - x^2) = \lim_{x \to 1^+} \sqrt{x-1} = \lim_{x \to 1} f(x) = 0$.

We also calculate that $f(1) = 0$.

We can conclude that $f(1) = \lim_{x \to 1} f(x)$.

$\boxed{\text{Since } f(x) \text{ is already proved to be continuous on } \mathbb{R} \setminus \{1\}, \text{ we can conclude that } f(x) \text{ is continuous on } (-\infty, \infty)}$

43. Find the numbers at which f is discontinuous. At which of these numbers is f continuous from the right, from the left, or neither? Sketch the graph of f.

$$f(x) = \begin{cases} x^2 \text{ if } x < -1 \\ x \text{ if } -1 \leqslant x < 1 \\ \dfrac{1}{x} \text{ if } x \geqslant 1 \end{cases}$$

Answer:
For $x = -1$:

$$\lim_{x \to -1^-} f(x)$$
$$= \lim_{x \to -1^-} x^2$$
$$= \lim_{x \to -1^-} (-1)^2$$
$$= 1$$

$$\lim_{x \to -1^+} f(x)$$
$$= \lim_{x \to -1^+} x$$
$$= -1$$

Therefore, $\lim_{x \to -1^-} f(x) \neq \lim_{x \to -1^+} f(x)$, so $\lim_{x \to -1} f(x)$ does not exist.

$\boxed{\text{Hence, we can conclude that } f(x) \text{ is discontinuous at } x = -1. \text{ Because if } x = 1, \text{ then } f(x) = x, \text{ we can conclude that } f(x) \text{ is continuity from the right at } x = 1}$

For $x = 1$:

$$\lim_{x \to 1^-} f(x)$$
$$= \lim_{x \to 1^-} x$$
$$= 1$$

$$\lim_{x \to 1^+} f(x)$$
$$= \lim_{x \to 1^+} \frac{1}{x}$$
$$= 1$$

Therefore, $\lim_{x \to -1^-} f(x) = \lim_{x \to -1^+} f(x)$, so $\lim_{x \to -1} f(x) = 1$.
The value of the function at $x = 1$ is: $f(1) = 1$.

$$\boxed{\text{Therefore, } f(x) \text{ is continuous at } x = 1.}$$