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Discrete Math **Assignment-7**

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1. Refer to Definition 1.10. Show that the divisibility relation — makes the set N of natural numbers a partially ordered set.
   Reflexivity:
   Because every number $x \in N$ can divides itself. Hence, the divisibility relation is reflexive.
   Transitivity:
   If $a|b$ and $b|c$ for $a, b, c \in N$. Then $b = a.k$ and $c = b.m$ and $c = a.k.m$. Therefore, c can divides a. Hence, the relation | is transitivity.
   Antisymmetry:
   If $a|b$ with $a, b \in N$, $a < b$. Hence, a cannot divide a. Therefore, the relation | is antisymmetric.
   Hence, the relation | is a partially order set.

2. Explain why the divisibility relation | does not define a partially ordering on the set Z of integers.
   For $x = -1$ and $y = 1$. $x|y$ and also $y|x$. Hence, the relation is not antisymmetric. Therefore, the relation is not a partially ordering set.

3. Consider the poset $(N, |)$. Are there any minimal elements? Are there any maximal elements? Explain.
   Because N = {1,2,3,4,...∞}. The minimal element is 1 and there is no maximal elements.

4. Let A = {a,b,c,...z}. In the poset(P(A), ⊂), find a pair of incomparable elements.
   A pair of incomparable elements is ({a,b,c},{d,e,f}).

5. Let W be the set of all web pages. For $x, y \in W$, let xRy if you can navigate from x to y by following links (Let's say it always possible to "navigate" from a page to itself; just do nothing.) Explain why R is not a partial ordering.
   Let $x, y \in W$, it is possible to navigate from x to y and from y to x. Hence, xRy and yRx. Therefore, R is not antisymmetric and not a partially ordering set.

6. Let a relation R be defined on the set of real numbers as follows:

$$xRy \Leftrightarrow 2x + y = 3$$

   Prove that this relation is antisymmetric.
   Let: $y = 3 - 2x$
   For yRx:

$$yRx \Leftrightarrow 2y + x = 3$$

$$2(3 - 2x) + x = 3$$

$$6 - 4x + x = 3$$

$$-3x = -3$$

$$x = 1$$

$$y = 3 - 2(1) = 1$$

Hence, x = y.
Therefore, the relation is antisymmetric.

7. Explain why the relation R on {0, 1, 2, 3} given by

$$R = \{(0,0),(1,1),(2,2),(3,3),(0,1),(1,2),(2,3),(0,2)\}$$

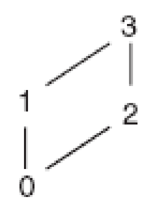
is not a partial ordering on {0, 1, 2, 3}. Be specific.
Because 1R2 and 2R3 but there is no relation between 1 and 3. Hence, the relation R is not transitive. Therefore, the relation is not a partially ordering set.

8. Explain why the relation R on {0, 1, 2, 3} given by

$$R = \{(0,0),(1,1),(2,2),(3,3),(0,1),(1,2),(0,2),(2,1)\}$$

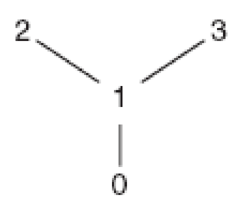
is not a partial ordering on {0, 1, 2, 3}. Be specific.
Because 1R2 and 2R1, the relation is not fully antisymmetric. Hence, the relation is not partial ordering.

9. The Hasse diagram below defines a partial ordering on the set {0, 1, 2, 3}. Give the set of ordered pairs corresponding to this relation.



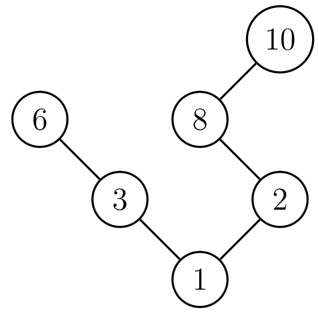
$$R = \{(0,1),(1,3),(0,2),(2,3),(0,3),(0,0),(1,1),(2,2),(3,3)\}$$

10. The Hasse diagram below defines a partial ordering on the set {0, 1, 2, 3}. Give the set of ordered pairs corresponding to this relation.



$$R = \{(0,1),(0,2),(0,3),(1,2),(1,3),(0,0),(1,1),(2,2),(3,3)\}$$

11. The divides relation "|" defines a partial ordering on the set $\{1, 2, 3, 6, 8, 10\}$. Draw the Hasse diagram for this poset. What are the maximal elements?



The maximal elements are 6 and 10.

12. Let S = $\{1, 2, 3, 5, 10, 15, 20\}$. It is a fact that (S, |) is a poset. Draw its Hasse diagram.