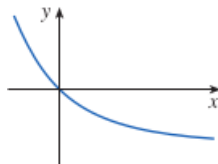


Section 6.1:

7. A function is given by a table of values, a graph, a formula, or a verbal description. Determine whether it is one-to-one.



Yes.

13. A function is given by a table of values, a graph, a formula, or a verbal description. Determine whether it is one-to-one.

$$g(x) = 1 - \sin x$$

No.

17. Assume that f is a one-to-one function.

(a) If $f(6) = 17$, what is $f^{-1}(17)$?

$$f^{-1}(17) = 6$$

(b) If $f^{-1}(3) = 2$, what is $f(2)$?

$$f(2) = 3$$

18. If $f(x) = x^5 + x^3 + x$, find $f^{-1}(3)$ and $f(f^{-1}(2))$.

Assume that $f(x)$ is a one-to-one function.

$$x^5 + x^3 + x = 3$$

$$x^5 + x^3 + x - 3 = 0$$

$$x = 1$$

Hence, $f^{-1}(3) = 1$.

$$f(f^{-1}(2)) = 2$$

24. Find a formula for the inverse of the function.

$$h(x) = \frac{6 - 3x}{5x + 7}$$

Assume that $f(x)$ is a one-to-one function.

$$y = \frac{6 - 3x}{5x + 7}$$

$$y(5x + 7) = 6 - 3x$$

$$5xy + 7y = 6 - 3x$$

$$5xy + 3x = 6 - 7y$$

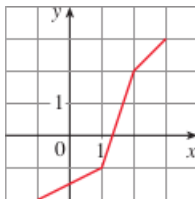
$$x(5y + 3) = 6 - 7y$$

$$x = \frac{6 - 7y}{5y + 3}$$

Hence the inverse function is:

$$f^{-1}(x) = \frac{6 - 7x}{5x + 3}$$

33. Use the given graph of f to sketch the graph of f^{-1} .



42. Find $(f^{-1})'(a)$.

$$f(x) = x^3 + 3 \sin x + 2 \cos x, a = 2$$

$$(f^{-1})'(a) = \frac{1}{f'(x)}, \text{ where } x \text{ satisfies } f(x) = 2$$

$$x^3 + 3 \sin x + 2 \cos x = 2$$

$$x = 0$$

Hence:

$$(f^{-1})'(a) = \frac{1}{f'(0)} = \frac{1}{(x^3 + 3 \sin x + 2 \cos x)'} = \frac{1}{3x^2 + 3 \cos x - 2 \sin x}$$

$$(f^{-1})'(a) = \frac{1}{3 \times 0^2 + 3 \cos 0 - 2 \sin 0} = \frac{1}{0 + 3 - 2} = \frac{1}{1} = 1$$

47. If $f(x) = \int_3^x \sqrt{1+t^3} dt$. Find $(f^{-1})'(0)$

$$(f^{-1})'(0) = \frac{1}{f'(x)}, \text{ where } x \text{ satisfies } f(x) = 0$$

$$\begin{aligned} f(x) &= 0 \\ \int_3^x \sqrt{1+t^3} dt &= 0 \\ x &= 3 \end{aligned}$$

By the Fundamental Theorem of Calculus:

$$f'(x) = \left(\int_3^x \sqrt{1+t^3} dt \right)' = \sqrt{1+x^3}$$

$$(f^{-1})'(0) = \frac{1}{\sqrt{1+0^3}} = \frac{1}{1} = 1$$

Section 6.2:

2. Let:

- (a) How is the e defined?
 e is the number such that:

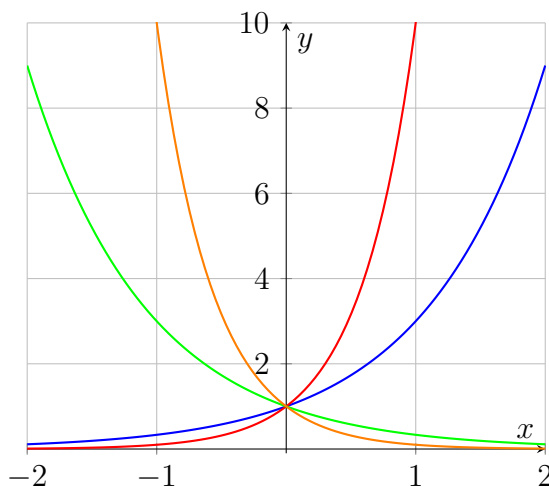
$$\lim_{h \rightarrow 0} \frac{e^h - 1}{h} = 1$$

- (b) What is an approximate value for e ?
 Approximate value of e is: 2.71828
 (c) What is the natural exponential function?

$$f(x) = e^x$$

5. Graph the given functions on a common screen. How are these graphs related?

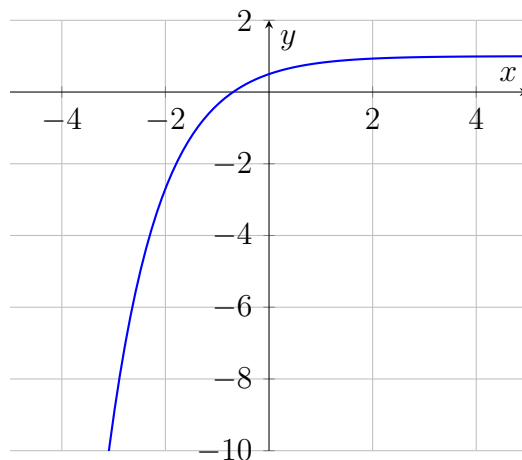
$$y = 3^x, y = 10^x, y = \left(\frac{1}{3}\right)^x, y = \left(\frac{1}{10}\right)^x$$



These graphs are symmetrical through the $y - axis$.

11. Make a rough sketch by hand of the graph of the function.

$$y = 1 - \frac{1}{2}e^{-x}$$



13. Starting with the graph of $y = e^x$, write the equation of the graph that results from

- (a) shifting 2 units downward.

$$y = e^x - 2$$

- (b) shifting 2 units to the right.

$$y = e^{x-2}$$

- (c) reflecting about the $x - axis$.

$$y = -e^x$$

- (d) reflecting about the $y - axis$.

$$y = e^{-x}$$

- (e) reflecting about the $x - axis$ and then about the $y - axis$.

$$y = -e^{-x}$$

15. Find the domain of each function.

(a) $f(x) = \frac{1-e^{x^2}}{1-e^{1-x^2}}$

$$1 - e^{1-x^2} \neq 0$$

$$e^{1-x^2} \neq 1$$

$$\log_e(e^{1-x^2}) \neq \log_e 1$$

$$(1 - x^2) \log_e(e) \neq 0$$

$$x^2 \neq 1$$

$$x \neq \pm 1$$

Hence the domain of the function is: $(-\infty, \infty) \setminus \{-1, 1\}$

(b) $f(x) = \frac{1+x}{e^{\cos x}}$

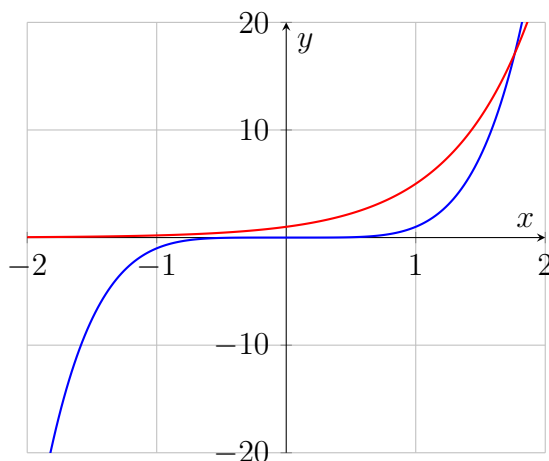
$$e^{\cos x} \neq 0$$

$$\log_e(e^{\cos x}) \neq \log_e 0$$

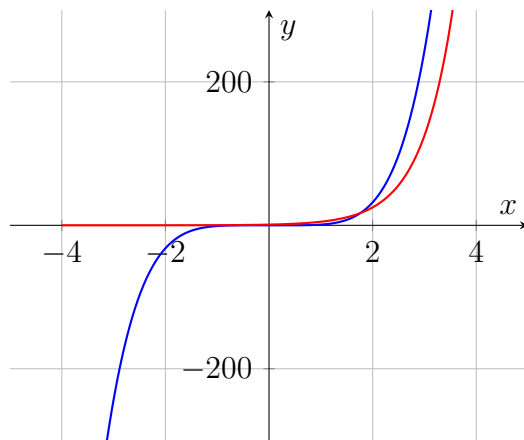
Because $\log_e 0$ does not exist. Hence the domain of the function is $(-\infty, \infty)$

20. Compare the functions $f(x) = x^5$ and $g(x) = 5^x$ by graphing both functions in several viewing rectangles. Find all points of intersection of the graphs correct to one decimal place. Which function grows more rapidly when x is large?

A small viewing rectangle with $x \in [-2, 2]$ and $y \in [-20, 20]$:



A larger viewing rectangle with $x \in [-5, 5]$ and $y \in [-300, 300]$:



There are two points of intersections which are approximately 1.8 and 5.
The function $g(x) = 5^x$ grows more rapidly when x is large.

23. Find the limit:

$$\lim_{x \rightarrow \infty} (1.001)^x = \infty$$

Because $(1.001)^x$ will become infinitely larger as x becomes infinitely larger.

29. Find the limit:

$$\lim_{x \rightarrow \infty} (e^{-2x} \cos x)$$

Because e^{-2x} will come closer to 0 as x becomes larger and $\cos x$ only fluctuates between -1 and 1.

Hence:

$$\lim_{x \rightarrow \infty} (e^{-2x} \cos x) = 0$$

33. Differentiate the function.

$$f(x) = (3x^2 - 5x)e^x$$

$$f'(x) = [(3x^2 - 5x)e^x]' = (3x^2 - 5x)'e^x + (e^x)'(3x^2 - 5x) = e^x(6x - 5) + e^x(3x^2 - 5x)$$

$$f'(x) = e^x(6x - 5 + 3x^2 - 5x) = e^x(3x^2 + x - 5)$$

39. Differentiate the function

$$y = \sqrt[3]{e^x + 1}$$

$$y = (e^x + 1)^{1/3}$$

$$y' = \frac{1}{3}(e^x + 1)^{-2/3}e^x = \frac{e^x}{3}(e^x + 1)^{-2/3}$$

54. Find an equation of the tangent line to the curve $xe^y + ye^x = 1$ at the point (0,1).

The slope of the tangent line is:

$$(xe^y + ye^x)' = (1)'$$

$$e^y + xe^y \frac{dy}{dx} + \frac{dy}{dx} e^x + ye^x = 0$$

$$\frac{dy}{dx}(xe^y + e^x) = -e^y - ye^x$$

$$\frac{dy}{dx} = \frac{-e^y - ye^x}{xe^y + e^x}$$

$$\frac{dy}{dx}(0) = \frac{-e^1 - 1 \times e^0}{0 \times e^1 + e^0} = \frac{-e - 1}{1} = -(e + 1)$$

Hence, the tangent line is:

$$y = -x(e + 1) + b$$

Because the tangent line pass the point (0,1):

$$1 = -0(e + 1) + b$$

$$b = 1$$

Hence, the equation of the tangent line is:

$$y = -x(e + 1) + 1$$

57. For what value of r does the function $y = e^{rx}$ satisfy the differential equation:

$$y'' + 6y' + 8y = 0$$

$$(e^{rx})'' + 6(e^{rx})' + 8(e^{rx}) = 0$$

$$(re^{rx})' + 6re^{rx} + 8e^{rx} = 0$$

$$r^2e^{rx} + 6re^{rx} + 8e^{rx} = 0$$

Because e^{rx} is not equal to 0 with every x :

$$r^2 + 6r + 8 = 0$$

$$r = -2 \text{ or } r = -4$$

61. Let:

- (a) Use the Intermediate Value Theorem to show that there is a solution of the equation

$$e^x + x = 0$$

Let:

$$f(x) = e^x + x$$

This function is continuous with every x :

Because of the IVT.

Choose $x = -1$:

$$f(-1) = e^{-1} - 1 = \frac{1}{e} - 1 < 0 \text{ Because } e > 1.$$

Choose $x = 1$:

$$f(1) = e^1 + 1 > 0 \text{ Because } e > 0.$$

Hence, there is at least one solutions in the interval $(-1,1)$.

69. Find the absolute maximum value of the function $f(x) = x - e^x$.

$$f'(x) = 1 - e^x = 0$$

$$e^x = 1$$

$$\log_e e^x = \log_e 1$$

$$x = 0$$

Hence, the absolute maximum value of the function is $f(0) = -1$.

71. Find

$$f(x) = xe^{2x}$$

(a) the intervals of increase or decrease,

$$f'(x) = e^{2x} + 2xe^{2x} = 0$$

$$e^{2x} + 2xe^{2x} = 0$$

$$e^{2x}(1 + 2x) = 0$$

Because e^{2x} will not be equal to 0 with every x.

Hence:

$$1 + 2x = 0$$

$$x = -\frac{1}{2}$$

Therefore, the function increases on the interval $(-\frac{1}{2}, \infty)$.

The function decrease on the interval $(-\infty, -\frac{1}{2})$.

(b) the intervals of concavity, and

$$f''(x) = 2e^{2x} + 2e^{2x} + 4xe^{2x} = 0$$

$$4e^{2x} + 4xe^{2x} = 0$$

$$4e^{2x}(1 + x) = 0$$

Because $4e^{2x}$ will not equal 0 for every x.

$$1 + x = 0$$

$$x = -1$$

Hence, the function concaves down on the interval $(-\infty, -1)$ and concaves up on the interval $(-1, \infty)$.

(c) the points of inflection.

$$f(-1) = (-1)e^{-2} = -\frac{1}{e^2}$$

Hence, the point of inflection is $(-1, -\frac{1}{e^2})$.

81. Evaluate the integral.

$$\begin{aligned}\int_0^1 (x^e + e^x) dx &= \left(\frac{x^{e+1}}{e+1} + e^x \right) \Big|_0^1 = \left(\frac{1^{e+1}}{e+1} + e^1 - \frac{0^{e+1}}{e+1} - e^0 \right) = \frac{1}{e+1} + e - 1 \\ &= \frac{1 + e^2 - 1}{e+1} = \frac{e^2}{e+1}\end{aligned}$$

84. Evaluate the integral.

$$\int t^3 e^{-t^4} dt$$

Let:

$$u = -t^4$$

$$du = -4t^3 dt$$

$$-\frac{du}{4} = t^3 dt$$

$$-\frac{1}{4} \int e^u du = -\frac{1}{4} e^u + C = -\frac{1}{4} e^{-t^4} + C$$

90. Evaluate the integral.

$$\int e^{\sin \theta} \cos \theta \, d\theta$$

Let:

$$u = \sin \theta$$

$$du = \cos \theta d\theta$$

$$\int e^u du = e^u + C = e^{\sin \theta} + C$$

Section 6.3:

3. Find the exact value of each expression.

(a) $\log_3 81$

$$\log_3 81 = 4$$

(b) $\log_3 \frac{1}{81}$

$$\log_3 \frac{1}{81} = -4$$

(c) $\log_9 3$

$$\log_9 3 = \frac{1}{2}$$

5. Find the exact value of each expression.

(a) $\log_2 30 - \log_2 15$

$$\log_2 30 - \log_2 15 = \log_2 \frac{30}{15} = \log_2 2 = 1$$

(b) $\log_3 10 - \log_3 5 - \log_3 18$

$$\log_3 10 - \log_3 5 - \log_3 18 = \log_3 \frac{10}{5} - \log_3 18 = \log_3 2 - \log_3 18$$

$$= \log_3 \frac{2}{18} = \log_3 \frac{1}{9} = -2$$

(c) $2\log_5 100 - 4\log_5 50$

$$2\log_5 100 - 4\log_5 50 = \log_5 100^2 - \log_5 50^4 = \log_5 \frac{100^2}{50^4} = \log_5 \frac{10000}{6250000}$$

$$= \log_5 \frac{1}{625} = -4$$

6. Find the exact value of each expression.

(a) $e^{3\ln 2}$

$$e^{3\ln 2} = e^{\ln 2^3} = 2^3 = 8$$

(b) $e^{-2 \ln 5}$

$$e^{-2 \ln 5} = e^{\ln 5^{-2}} = 5^{-2} = \frac{1}{25}$$

(c) $e^{\ln(\ln e^3)}$

$$e^{\ln(\ln e^3)} = \ln e^3 = 3 \ln e = 3$$

8. Use the laws of logarithms to expand each expression

(a) $\ln \sqrt{\frac{3x}{x-3}}$

$$\ln \sqrt{\frac{3x}{x-3}} = \ln \left(\frac{3x}{x-3} \right)^{1/2} = \frac{1}{2} \ln \left(\frac{3x}{x-3} \right) = \frac{1}{2} \ln 3x - \frac{1}{2} \ln(x-3)$$

(b) $\log_2[(x^3 + 1)\sqrt[3]{(x-3)^2}]$

$$\begin{aligned} \log_2[(x^3 + 1)\sqrt[3]{(x-3)^2}] &= \log_2(x^3 + 1) + \log_2 \sqrt[3]{(x-3)^2} \\ &= \log_2(x^3 + 1) + \log_2(x-3)^{2/3} = \log_2(x^3 + 1) + \frac{2}{3} \log_2(x-3) \end{aligned}$$

10. Express as a single logarithm.

(a) $\ln 10 + 2 \ln 5$

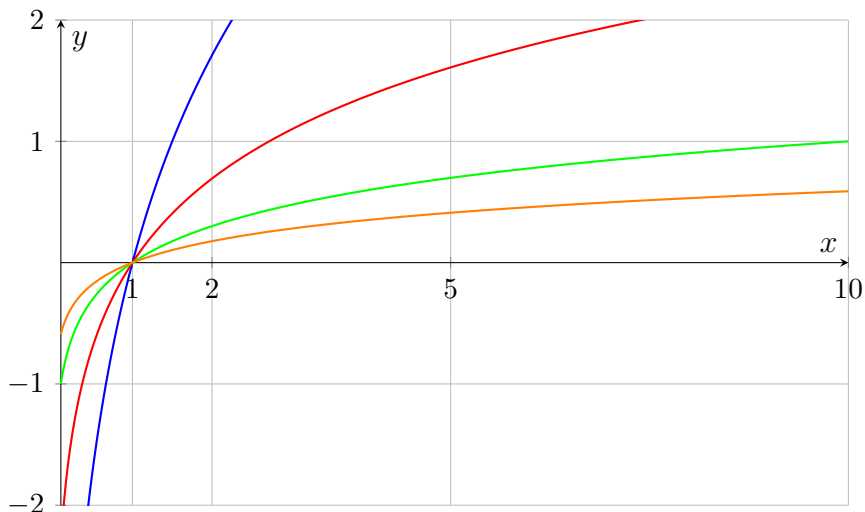
$$\ln 10 + 2 \ln 5 = \ln 10 + \ln 5^2 = \ln(10 \times 5^2) = \ln(10 \times 25) = \ln 250$$

(b) $\log_{10} 4 + \log_{10} a - \frac{1}{3} \log_{10}(a+1)$

$$\log_{10} 4 + \log_{10} a - \frac{1}{3} \log_{10}(a+1) = \log_{10} 4a - \log_{10}(a+1)^{1/3} = \log_{10} \frac{4a}{(a+1)^{1/3}}$$

15. Use Formula 8 to graph the given functions on a common screen. How are these graphs related?

$$y = \log_{1.5} x, y = \ln x, y = \log_{10} x, y = \log_{50} x$$

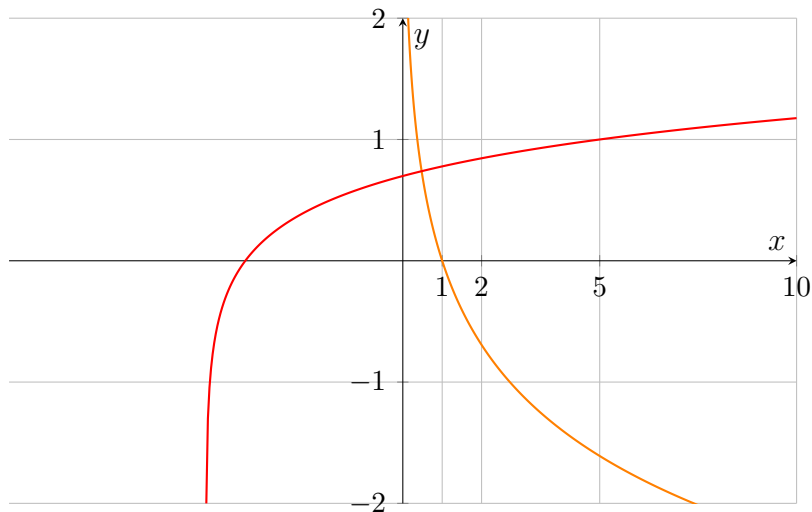


All of these graphs are defined over $x > 0$.

19. Make a rough sketch by hand of the graph of each function. Use the graphs given in Figures 2, and 3 and, if necessary, the transformations of Section 1.3.

(a) $y = \log_{10}(x + 5)$

(b) $y = -\ln x$



22. Let:

$$f(x) = \ln(x - 1) - 1$$

- (a) What are the domain and range of f ?

$$x - 1 > 0$$

$$x > 1$$

The domain of the function f is $(1, \infty)$.

The range of the function f is $(-\infty, \infty)$.

- (b) What is the x -intercept of the graph of f ?

$$\ln(x - 1) - 1 = 0$$

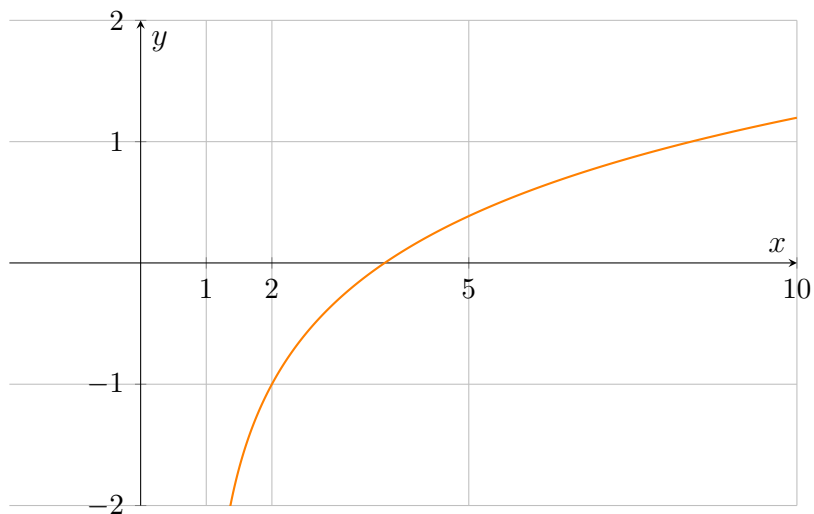
$$\ln(x - 1) = 1$$

$$x - 1 = e$$

$$x = e + 1$$

Hence, the x -intercept of the graph f is $e + 1$.

- (c) Sketch the graph of f .



25. Solve each equation for x . Give both an exact value and a decimal approximation, correct to three decimal places.

(a) $\ln x + \ln(x - 1) = 0$

$$\ln x + \ln(x - 1) = 0$$

$$\ln(x - 1) = -\ln x$$

$$\ln(x - 1) = \ln x^{-1}$$

$$x - 1 = x^{-1}$$

$$x - 1 = \frac{1}{x}$$

$$x^2 - x - 1 = 0$$

$$x = \frac{1 + \sqrt{5}}{2} \approx 1.618 \text{ or } x = \frac{1 - \sqrt{5}}{2} \approx -0.618$$

Because $x > 1$ so that the equation can be defined.

$$x = \frac{1 + \sqrt{5}}{2} \approx 1.618$$

(b) $5^{1-2x} = 9$

$$\log_5 5^{1-2x} = \log_5 9$$

$$(1 - 2x) \log_5 5 = \log_5 9$$

$$-2x = \log_5(9) - 1$$

$$x = \frac{1 - \log_5(9)}{2} \approx -0.183$$