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Calculus 1 Assignment-7
Section: 04
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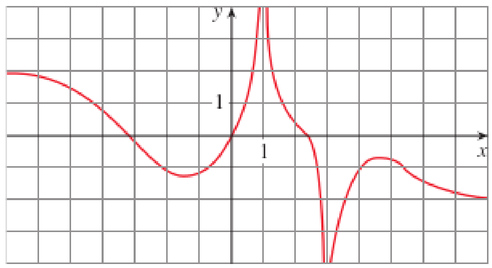
Section 3.4:

3. For the function f whose graph is given, state the following.



(a)

$$\boxed{\lim_{x\to\infty} f(x) = -2}$$

(b)

$$\boxed{\lim_{x\to-\infty} f(x) = 2}$$

(c)

$$\boxed{\lim_{x\to 1} f(x) = \infty}$$

(d)

$$\boxed{\lim_{x\to 3} f(x) = -\infty}$$

(e) The equations of the asymptotes

$$\boxed{x = 1, x = 3, y = -2, y = 2}$$

8. Evaluate the limit and justify each step by indicating the appropriate properties of limits.

$$\lim_{x\to\infty} \sqrt{\frac{9x^3 + 8x - 4}{3 - 5x + x^3}}$$

$$= \lim_{x\to\infty} \sqrt{\frac{x^3(9 + 8/x^2 - 4/x^3)}{x^3(3/x^3 - 5/x^2 + 1)}}$$

$$= \lim_{x\to\infty} \sqrt{\frac{9 + 8/x^2 - 4/x^3}{3/x^3 - 5/x^2 + 1}}$$

$$= \sqrt{\frac{9+0-0}{0-0+1}}$$

$$\boxed{= \sqrt{9} = 3}$$

11. Find the limit or show that it does not exist.

$$\lim_{t \to -\infty} \frac{3t^2 + t}{t^3 - 4t + 1}$$

$$= \lim_{t \to -\infty} \frac{t^2(3 + 1/t)}{t^3(1 - 4/t^2 + 1/t^3)}$$

$$= \lim_{t \to -\infty} \frac{(3 + 1/t)}{t(1 - 4/t^2 + 1/t^3)}$$

$$\boxed{= 0}$$

18. Find the limit or show that it does not exist.

$$\lim_{t \to \infty} \frac{t+3}{\sqrt{2t^2 - 1}}$$

$$= \lim_{t \to \infty} \frac{t(1 + 3/t)}{t\sqrt{2 - 1/t^2}}$$

$$= \lim_{t \to \infty} \frac{1 + 3/t}{\sqrt{2 - 1/t^2}}$$

$$= \frac{1+0}{\sqrt{2-0}}$$

$$\boxed{= \frac{1}{\sqrt{2}}}$$

26. Find the limit or show that it does not exist.

$$= \lim_{x \to -\infty} (\sqrt{4x^2 + 3x} + 2x)$$

$$= \lim_{x \to -\infty} (|x|\sqrt{4 + 3/x} + 2x)$$

Because x is approaching to $-\infty$. $|x| = -x$.

$$= \lim_{x \to -\infty} (-x\sqrt{4 + 3/x} + 2x)$$

$$= \lim_{x \to -\infty} x(-\sqrt{4 + 3/x} + 2)$$

$$= -\infty(-2 + 2)$$

$$= -\infty(0)$$

$$\boxed{= 0}$$

28. Find the limit or show that it does not exist.

$$\lim_{x\to\infty}(x-\sqrt{x})$$

$$= \lim_{x\to\infty} x(1-1/\sqrt{x})$$

$$= \infty(1-0)$$

$$\boxed{=\infty}$$

31. Find the limit or show that it does not exist.

$$\lim_{x\to\infty} x\sin\frac{1}{x}$$

$$= \infty \sin 0$$

$$\boxed{=0}$$

37. Find the horizontal and vertical asymptotes of each curve. You may want to use a graphing calculator (or computer) to check your work by graphing the curve and estimating the asymptotes.

$$y = \frac{2x^2+x-1}{x^2+x-2}$$

Horizontal Asymptotes:

$$\lim_{x\to\infty}\frac{2x^2+x-1}{x^2+x-2}$$

$$= \lim_{x\to\infty}\frac{x^2(2+1/x-1/x^2)}{x^2(1+1/x-2/x^2)}$$

$$= \lim_{x\to\infty}\frac{2+1/x-1/x^2}{1+1/x-2/x^2}$$

$$\boxed{=2}$$

$$\lim_{x\to-\infty}\frac{2x^2+x-1}{x^2+x-2}$$

$$= \lim_{x\to-\infty}\frac{x^2(2+1/x-1/x^2)}{x^2(1+1/x-2/x^2)}$$

$$= \lim_{x\to-\infty}\frac{2+1/x-1/x^2}{1+1/x-2/x^2}$$

$$\boxed{=2}$$

$$\boxed{y=2}$$

Vertical Asymptotes;

$$x^2+x-2=0$$

$$(x-1)(x+2)=0$$

$$\boxed{x=1 \text{ or } x=-2}$$

$$\boxed{x=1, x=-2}$$

54. Find the limits as $x \to \infty$ and as $x \to -\infty$. Use this information, together with intercepts, to give a rough sketch of the graph as in Example 11.

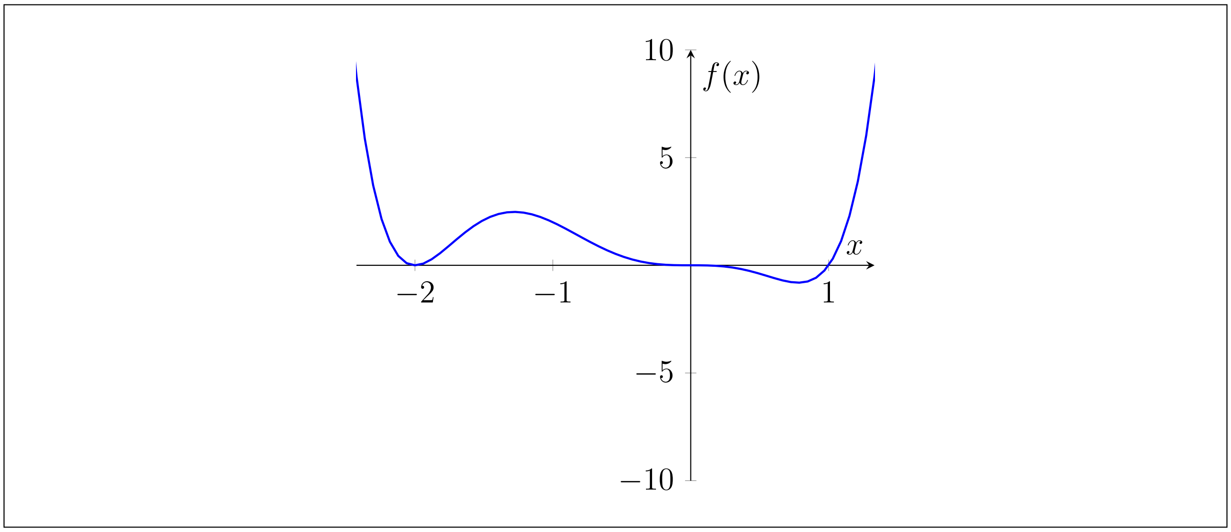
$$y = x^3(x+2)^2(x-1)$$

$$\lim_{x\to\infty} x^3(x+2)^2(x-1)$$

$$\boxed{=\infty}$$

$$\lim_{x\to-\infty} x^3(x+2)^2(x-1)$$

$$\boxed{=-\infty}$$



59. Sketch the graph of a function that satisfies all of the given conditions.

5. Use the guidelines of this section to sketch the curve.

$$y = x(x-4)^3 = x(x^3 - 3x^2 \times 4 + 3x \times 4^2 - 4^3)$$

$$y = x(x^3 - 12x^2 + 48x - 64) = x^4 - 12x^3 + 48x^2 - 64x$$

(a) Domain: $(-\infty, \infty)$

(b) Intercepts:

$$f(0) = 0(0-4)^3 = 0$$

$$y-\text{intercepts are } 0$$

$$f(x) = x(x-4)^3 = 0$$

$$x-\text{intercepts are } 0 \text{ and } 4$$

(c) Symmetry:

$$f(-x) = (-x)^4 - 12(-x)^3 + 48(-x)^2 - 64(-x)$$

$$f(-x) = x^4 + 12x^3 + 48x^2 + 64x$$

The function is not odd nor even.

(d) Asymptotes:
Since the function is a polynomial function. It will be defined everywhere and has no vertical asymptote.

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} x^4 - 12x^3 + 48x^2 - 64x = \infty$$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} x^4 - 12x^3 + 48x^2 - 64x = \infty$$

The function also doesn't have horizontal asymptote.

(e) Intervals of Increase or Decrease:

$$f'(x) = 4x^3 - 36x^2 + 96x - 64 = 0$$

$$x = 1 \text{ or } x = 4$$

x	$-\infty$		1		4		∞
$f'(x)$		–	0	+	0	+	
$f(x)$		↓	-27	↑	0	↑	

The function f(x) increases on the intervals $(1,4)$ and $(4,\infty)$.
The function f(x) decreases on the interval $(-\infty, 1)$.

(f) Local Maximum and Minimum Values:
Local Minimum Values is $f(1) = -27$.

(g) Concavity and Points of Inflection:

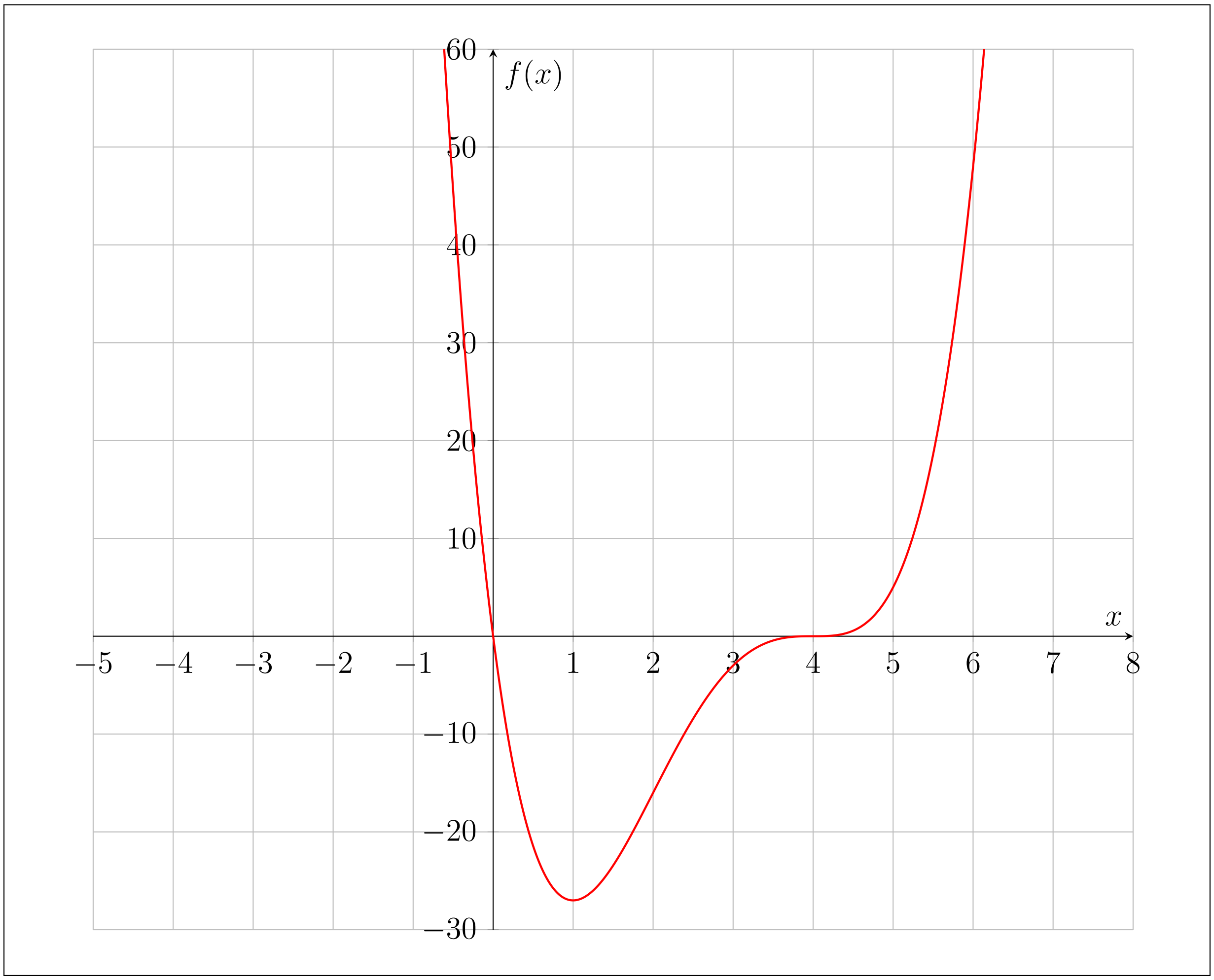
$$f''(x) = 12x^2 - 72x + 96 = 0$$

$$x = 2 \text{ or } x = 4$$

x	$-\infty$		2		4		∞
$f''(x)$		+	0	–	0	+	

The function f(x) is concave up on the intervals $(-\infty, 2)$ and $(4, \infty)$.
The function f(x) is concave down on the intervals $(2, 4)$.
Points of Inflection are (2, 16) and (4, 0).

(h) Sketch the graph:



9. Use the guidelines of this section to sketch the curve.

$$y = \frac{2x+3}{x+2}$$

(a) Domain:

$$x + 2 \neq 0$$

$$x \neq -2$$

Hence, the domain of f(x) is: $\mathbb{R} \setminus \{-2\}$

(b) Intercepts:

$$f(0) = \frac{2 \times 0 + 3}{0 + 2} = \frac{3}{2}$$

$$f(x) = \frac{2x+3}{x+2} = 0$$
$$x = -\frac{3}{2}$$

The y-intercepts of the function is $\frac{3}{2}$.
The x-intercepts of the function is $-\frac{3}{2}$.

(c) Symmetry:

$$f(-x) = \frac{2(-x)+3}{(-x)+2}$$
$$f(-x) = \frac{-2x+3}{-x+2}$$

The function is not odd nor even.

(d) Asymptotes:

$$\lim_{x\to-2^-} \frac{2x+3}{x+2} = \infty$$
$$\lim_{x\to-2^+} \frac{2x+3}{x+2} = -\infty$$

The function has a vertical asymptote x = -2.

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{2x+3}{x+2} = \lim_{x\to\infty} \frac{x(2+3/x)}{x(1+2/x)} = \lim_{x\to\infty} \frac{2+3/x}{1+2/x} = \frac{2+0}{1+0} = 2$$
$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{2x+3}{x+2} = \lim_{x\to-\infty} \frac{x(2+3/x)}{x(1+2/x)} = \lim_{x\to-\infty} \frac{2+3/x}{1+2/x} = \frac{2+0}{1+0} = 2$$

The function has a horizontal asymptote y = 2.

(e) Intervals of Increase or Decrease:

$$f'(x) = \frac{(2x+3)'(x+2)-(x+2)'(2x+3)}{(x+2)^2}$$
$$f'(x) = \frac{2(x+2)-(2x+3)}{(x+2)^2}$$
$$f'(x) = \frac{2x+4-2x-3}{(x+2)^2}$$
$$f'(x) = \frac{1}{(x+2)^2} > 0 \ \forall x \in (\mathbb{R} \setminus \{-2\})$$

Hence, the function always increases on the domain $\mathbb{R} \setminus \{-2\}$.

(f) Local Maximum and Minimum Values:
The function doesn't have any local maximum nor minimum.

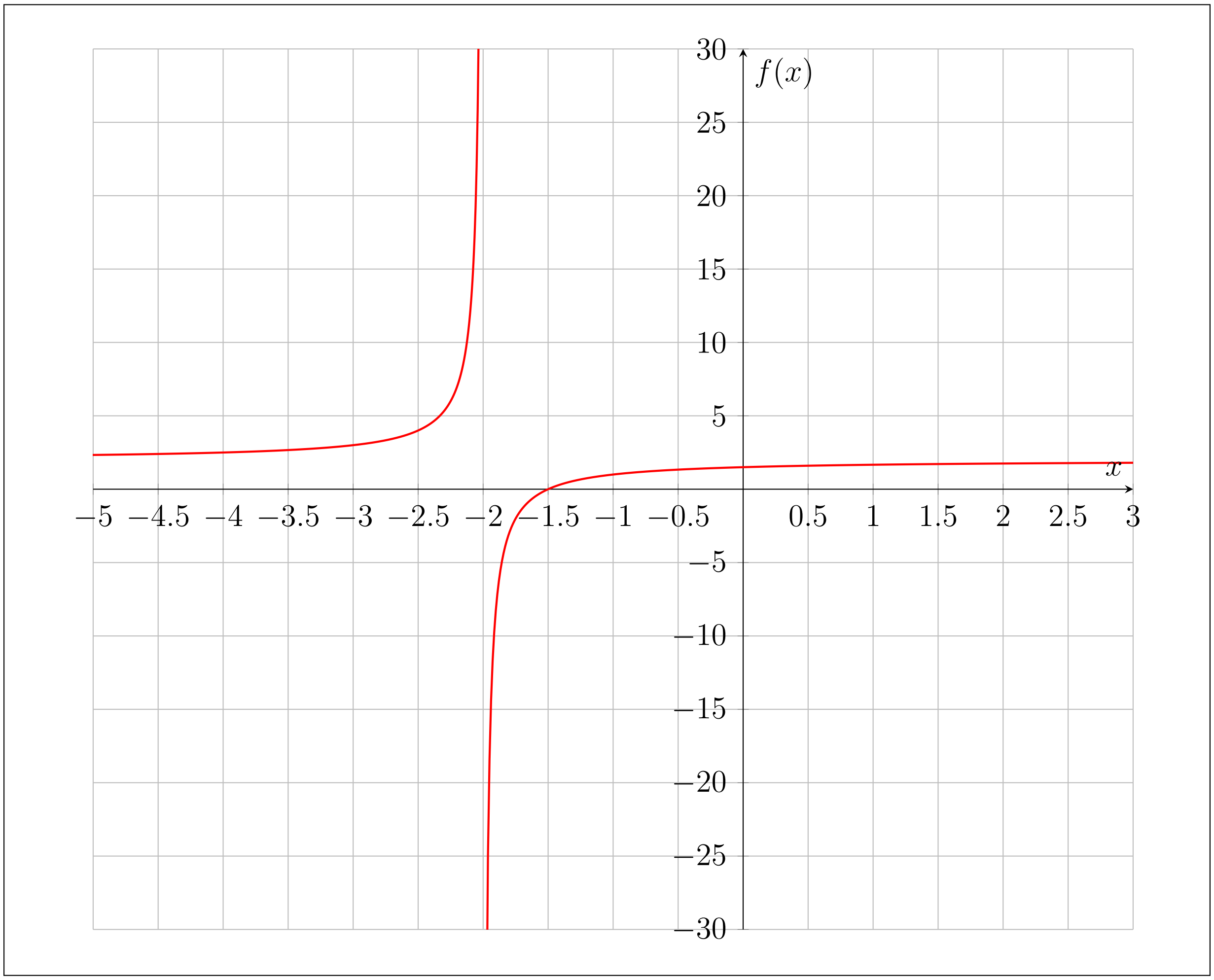
(g) Concavity and Points of Inflection:

$$f''(x) = -\frac{[(x+2)^2]'}{(x+2)^4}$$
$$f''(x) = -\frac{2(x+2)}{(x+2)^4}$$
$$f''(x) = -\frac{2}{(x+2)^3}$$

x	$-\infty$		-2		∞
$f''(x)$		$+$	0	$-$	

The function f(x) is concave up in the interval $(-\infty, -2)$.
The function f(x) is concave up in the interval $(-2, \infty)$.

(h) Sketch the graph:



21. Use the guidelines of this section to sketch the curve.

$$y = (x-3)\sqrt{x}$$

(a) Domain:

$$x \geq 0$$

The domain of the function f(x) is $[0, \infty)$.

(b) Intercepts:

$$f(0) = (0-3)\sqrt{0} = 0$$
$$f(x) = (x-3)\sqrt{x} = 0$$
$$(x-3)\sqrt{x} = 0$$

$$x = 0 \text{ or } x = 3$$

The y-intercept of the function is 0.
The x-intercepts of the function are 0 and 3.

(c) Symmetry:
Because the function has the domain $[0, \infty)$ so this function is not symmetrical.

(d) Asymptotes:

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} (x-3)\sqrt{x} = \infty$$

$$\lim_{x\to-\infty} f(x) \text{ is undefined.}$$

So the function doesn't have any horizontal nor vertical asymptote.

(e) Interval of Increase or Decrease:

$$f'(x) = (x-3)'\sqrt{x} + (x-3)\sqrt{x}'$$

$$f'(x) = \sqrt{x} + \frac{x-3}{2\sqrt{x}}$$

$$f'(x) = \frac{2x + x - 3}{2\sqrt{x}}$$

$$f'(x) = \frac{3x-3}{2\sqrt{x}} = 0$$

$$x = 1 \text{ or } x = 0$$

x	0		1		∞
$f'(x)$	||	–	0	+	
$f(x)$	0	$\downarrow$	-2	$\uparrow$	

Hence, the function f increases on the interval $(1, \infty)$.
The function f decreases on the interval $(0, 1)$.

(f) Local Maximum and Minimum Values:
The function f has local minimum at x = 1, f(1) = -2.

(g) Concavity and Inflection Points: