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Discrete Math Homework 2

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1. Use truth table to establish the *modus tollens* tautology:

$$\left.\begin{array}{r} \neg q \\ p \to q \end{array}\right\} \Longrightarrow \neg p$$

Answer:

p	q	$\neg q$	$p \to q$	$(\neg q) \wedge (p \to q)$	$\neg p$	$[(\neg q) \wedge (p \to q)] \to \neg p$
T	T	F	T	F	F	T
T	F	T	F	F	F	T
F	T	F	T	F	T	T
F	F	T	T	T	T	T

Since all the values in the last row are True, the argument is a tautology.

2. Fill in the reasons in the following proof sequence. Make sure you indicate which step(s) each derivation rule refers to.

Statements	*Reasons*
1.$q \wedge r$	*Given*
2.$\neg(\neg p \wedge q)$	*Given*
3.$\neg\neg p \vee \neg q$	**De Morgan's Laws, 2**
4.$p \vee \neg q$	**Double Negation, 3**
5.$\neg q \vee p$	**Commutative Laws, 4**
6.$q \to p$	**Implication, 5**
7.q	**Simplification, 1**
8.p	**Modus Ponens, 6, 7**

3. Fill in the reasons in the following proof sequence. Make sure you indicate which step(s) each derivation rule refers to.

Statements	*Reasons*
1.$(p \wedge q) \to r$	*Given*
2.$\neg(p \wedge q) \vee r$	**Conjunction, 1**
3.$(\neg p \vee \neg q) \vee r$	**De Morgan's Laws, 2**
4.$\neg p \vee (\neg q \vee r)$	**Associative Laws, 3**
5.$p \to (\neg q \vee r)$	**Implication, 4**

4. Is the proof in Exercise 2 reversible? Why or why not?
   Answer: No, the proof is not reversible because it used Modus Ponens and Simplification, which are one-way operations.

5. Is the proof in Exercise 3 reversible? Why or why not?
   Answer: Yes, the proof is reversible because it only used Conjunction, De Morgan's Laws, Associative, and Implication, which are reversible operations.

6. Fill in the reasons in the following proof sequence. Make sure you indicate which step(s) each derivation rule refers to.

Statements	*Reasons*
1.$p \wedge (q \vee r)$	*Given*
2.$\neg(p \wedge q)$	*Given*
3.$\neg p \vee \neg q$	**De Morgan's Laws, 2**
4.$\neg q \vee \neg p$	**Commutative Laws, 3**
5.$q \rightarrow \neg p$	**Implication, 4**
6.p	**Simplification, 1**
7.$\neg(\neg p)$	**Double Negation, 6**
8.$\neg q$	**Modus Tollens, 5, 7**
9.$(q \vee r) \wedge p$	**Commutative Laws, 1**
10.$q \vee r$	**Simplification, 9**
11.$r \vee q$	**Commutative Laws, 10**
12.$\neg(\neg r) \vee q$	**Double Negation, 11**
13.$\neg r \rightarrow q$	**Implication, 12**
14.$\neg(\neg r)$	**Modus Tollens, 8, 13**
15.r	**Double Negation, 14**
16.$p \wedge r$	**Conjunction, 6, 15**

7. Justify each conclusion with a derivation rule.

   (a) If Joe is artistic, he must also be creative. Joe is not creative. Therefore, Joe is not artistic.
   Answer: Modus Tollens

   (b) Lingli is both athletic and intelligent. Therefore, Lingli is athletic.
   Answer: Simplification

   (c) If Monique is 18 years old, then she may vote. Monique is 18 years old. Therefore, Monique may vote.
   Answer: Modus Ponens

   (d) Marianne has never been north of Saskatoon or south of Santo Domingo. In other words, she has never been north of Saskatoon and she has never been south of Santo Domingo.
   Answer: De Morgan's Laws

8. Which derivation rule justifies the following argument?

   If n is a multiple of 4, then n is even.
   However, n is not even. Therefore, n is not a multiple of 4.

   Answer: Modus Tollens

9. Let x and y be integers. Given the statement. What statement follows by the Implication rule?

   x > y or x is odd.

   Answer: The statement follows by Implication Law is:
   "If x is not larger than y, then x is odd."

10. Let Q be a quadrilateral. Given the statements. What statement follows by modus tollens?

    If Q is a rhombus, then Q is a parallelogram.
    Q is not a parallelogram.

    Answer: The statement follows by Modus Tollens Law is:
    "Q is not a rhombus."

11. Let x and y be numbers. Simplify the following statement using De Morgan's laws and double negation.

    It is not the case that x is not greater than 3 and y is not found.

    Answer: Assign the following statement:

    p = *"x is greater than 3."*
    q = *"y is found."*

Statements	*Reasons*
1.$\neg(\neg p \wedge \neg q)$	*Given*
2.$\neg(\neg \boldsymbol{p}) \vee \neg(\neg \boldsymbol{q})$	**De Morgan's Laws, 1**
3.$\boldsymbol{p} \vee \boldsymbol{q}$	**Double Negation, 2**

17. Write a proof sequence for the following assertion. Justify each step.

$$\left.\begin{array}{r} p \to q \\ p \wedge r \end{array}\right\} \Longrightarrow q \wedge r$$

Answer:
Proof:

Statements	*Reasons*
1.$p \to q$	*Given*
2.$p \wedge r$	*Given*
3.p	Simplification, 2
4.q	Modus Ponens, 1, 3
5.r	Simplification, 2
6.$q \wedge r$	Conjunction, 4, 5

18. Write a proof sequence for the following assertion. Justify one of the steps in your proof using the result of Example 1.8.

$$\left.\begin{array}{r} \neg(a \wedge \neg b) \\ \neg b \end{array}\right\} \Longrightarrow \neg a$$

Answer:
Proof:

Statements	*Reasons*
1.$\neg(a \wedge \neg b)$	*Given*
2.$\neg b$	*Given*
3.$\neg a \vee \neg(\neg b)$	De Morgan's Laws, 1
4.$\neg a \vee b$	Double Negation, 3
5.$a \to b$	Implication, 4
6.$\neg a$	Modus Tollens, 2, 5