



Counting Beyond 10

A Journey Through Numbers & Their
Hidden Languages

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problem solving



Unary
(base-1)

binary
(base-2)

binary digit

bi

t

bit

0

1

decimal
(base-10)

123

1

123

10 1

123

100 10 1

123

100 10 1

123

100×1

100 10 1

123

100×1 $+$ 10×2

100 10 1

123

100×1 $+$ 10×2 $+$ 1×3

100 10 1

123

100 + 20 + 3

123

100 10 1

#

10^2 10^1 10^0

#

2^2 2^1 2^0

#

4 2 1

#

4 2 1

000

4 2 1

001

4 2 1

010

4 2 1

011

4 2 1

100

4 2 1

101

4 2 1

110

4 2 1

111

4 2 1

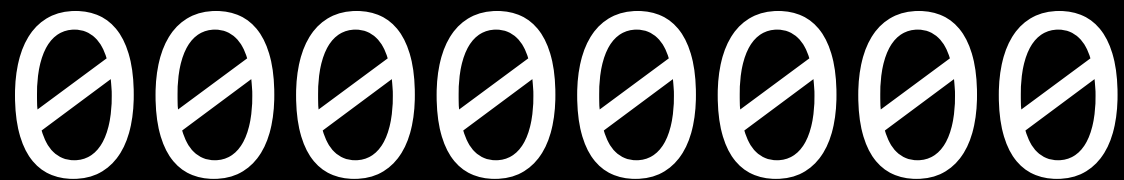
000

8 4 2 1

1000

byte

128 64 32 16 8 4 2 1



128 64 32 16 8 4 2 1

11111111

A

128 64 32 16 8 4 2 1

01000001

65

ASCII

0	<u>NUL</u>	16	<u>DLE</u>	32	<u>SP</u>	48	0	64	@	80	P	96	`	112	p
1	<u>SOH</u>	17	<u>DC1</u>	33	!	49	1	65	A	81	Q	97	a	113	q
2	<u>STX</u>	18	<u>DC2</u>	34	"	50	2	66	B	82	R	98	b	114	r
3	<u>ETX</u>	19	<u>DC3</u>	35	#	51	3	67	C	83	S	99	c	115	s
4	<u>EOT</u>	20	<u>DC4</u>	36	\$	52	4	68	D	84	T	100	d	116	t
5	<u>ENQ</u>	21	<u>NAK</u>	37	%	53	5	69	E	85	U	101	e	117	u
6	<u>ACK</u>	22	<u>SYN</u>	38	&	54	6	70	F	86	V	102	f	118	v
7	<u>BEL</u>	23	<u>ETB</u>	39	'	55	7	71	G	87	W	103	g	119	w
8	<u>BS</u>	24	<u>CAN</u>	40	(56	8	72	H	88	X	104	h	120	x
9	<u>HT</u>	25	<u>EM</u>	41)	57	9	73	I	89	Y	105	i	121	y
10	<u>LF</u>	26	<u>SUB</u>	42	*	58	:	74	J	90	Z	106	j	122	z
11	<u>VT</u>	27	<u>ESC</u>	43	+	59	;	75	K	91	[107	k	123	{
12	<u>FF</u>	28	<u>FS</u>	44	,	60	<	76	L	92	\	108	l	124	
13	<u>CR</u>	29	<u>GS</u>	45	-	61	=	77	M	93]	109	m	125	}
14	<u>SO</u>	30	<u>RS</u>	46	.	62	>	78	N	94	^	110	n	126	~
15	<u>SI</u>	31	<u>US</u>	47	/	63	?	79	O	95	_	111	o	127	<u>DEL</u>

0	<u>NUL</u>	16	<u>DLE</u>	32	<u>SP</u>	48	0	64	@	80	P	96	`	112	p
1	<u>SOH</u>	17	<u>DC1</u>	33	!	49	1	65	A	81	Q	97	a	113	q
2	<u>STX</u>	18	<u>DC2</u>	34	"	50	2	66	B	82	R	98	b	114	r
3	<u>ETX</u>	19	<u>DC3</u>	35	#	51	3	67	C	83	S	99	c	115	s
4	<u>EOT</u>	20	<u>DC4</u>	36	\$	52	4	68	D	84	T	100	d	116	t
5	<u>ENQ</u>	21	<u>NAK</u>	37	%	53	5	69	E	85	U	101	e	117	u
6	<u>ACK</u>	22	<u>SYN</u>	38	&	54	6	70	F	86	V	102	f	118	v
7	<u>BEL</u>	23	<u>ETB</u>	39	'	55	7	71	G	87	W	103	g	119	w
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9	<u>HT</u>	25	<u>EM</u>	41)	57	9	73	I	89	Y	105	i	121	y
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11	<u>VT</u>	27	<u>ESC</u>	43	+	59	;	75	K	91	[107	k	123	{
12	<u>FF</u>	28	<u>FS</u>	44	,	60	<	76	L	92	\	108	l	124	
13	<u>CR</u>	29	<u>GS</u>	45	-	61	=	77	M	93]	109	m	125	}
14	<u>SO</u>	30	<u>RS</u>	46	.	62	>	78	N	94	^	110	n	126	~
15	<u>SI</u>	31	<u>US</u>	47	/	63	?	79	O	95	_	111	o	127	<u>DEL</u>

01001000

01001001

00100001

72

73

33

0	<u>NUL</u>	16	<u>DLE</u>	32	<u>SP</u>	48	0	64	@	80	P	96	`	112	p
1	<u>SOH</u>	17	<u>DC1</u>	33	!	49	1	65	A	81	Q	97	a	113	q
2	<u>STX</u>	18	<u>DC2</u>	34	"	50	2	66	B	82	R	98	b	114	r
3	<u>ETX</u>	19	<u>DC3</u>	35	#	51	3	67	C	83	S	99	c	115	s
4	<u>EOT</u>	20	<u>DC4</u>	36	\$	52	4	68	D	84	T	100	d	116	t
5	<u>ENQ</u>	21	<u>NAK</u>	37	%	53	5	69	E	85	U	101	e	117	u
6	<u>ACK</u>	22	<u>SYN</u>	38	&	54	6	70	F	86	V	102	f	118	v
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9	<u>HT</u>	25	<u>EM</u>	41)	57	9	73	I	89	Y	105	i	121	y
10	<u>LF</u>	26	<u>SUB</u>	42	*	58	:	74	J	90	Z	106	j	122	z
11	<u>VT</u>	27	<u>ESC</u>	43	+	59	;	75	K	91	[107	k	123	{
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2	<u>STX</u>	18	<u>DC2</u>	34	"	50	2	66	B	82	R	98	b	114	r
3	<u>ETX</u>	19	<u>DC3</u>	35	#	51	3	67	C	83	S	99	c	115	s
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6	<u>ACK</u>	22	<u>SYN</u>	38	&	54	6	70	F	86	V	102	f	118	v
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13	<u>CR</u>	29	<u>GS</u>	45	-	61	=	77	M	93]	109	m	125	}
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15	<u>SI</u>	31	<u>US</u>	47	/	63	?	79	O	95	_	111	o	127	<u>DEL</u>

H
72

I
73

!
33

0	<u>NUL</u>	16	<u>DLE</u>	32	<u>SP</u>	48	0	64	@	80	P	96	`	112	p
1	<u>SOH</u>	17	<u>DC1</u>	33	!	49	1	65	A	81	Q	97	a	113	q
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10	<u>LF</u>	26	<u>SUB</u>	42	*	58	:	74	J	90	Z	106	j	122	z
11	<u>VT</u>	27	<u>ESC</u>	43	+	59	;	75	K	91	[107	k	123	{
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13	<u>CR</u>	29	<u>GS</u>	45	-	61	=	77	M	93]	109	m	125	}
14	<u>SO</u>	30	<u>RS</u>	46	.	62	>	78	N	94	^	110	n	126	~
15	<u>SI</u>	31	<u>US</u>	47	/	63	?	79	O	95	_	111	o	127	<u>DEL</u>



~ `	! 1	@ 2	# 3	\$ 4	% 5	^ 6	& 7	* 8	(9) 0	- _	+ =	← Backspace
Tab ⇐ ⇒	Q	W	E	R	T	Y	U	I	O	P	{ [}]	 \ _
Caps Lock ⬆	A	S	D	F	G	H	J	K	L	: ;	" '	Enter ↵	
Shift ⬆	Z	X	C	V	B	N	M	< ,	> .	? /	Shift ⬆		
Ctrl	Win Key	Alt								Alt	Win Key	Menu	Ctrl

à	á	â	ä	æ	ã	å	ā
1	2	3	4	5	6	7	8

a



SMILEYS & PEOPLE



Unicode

1111000010011111001100010000010

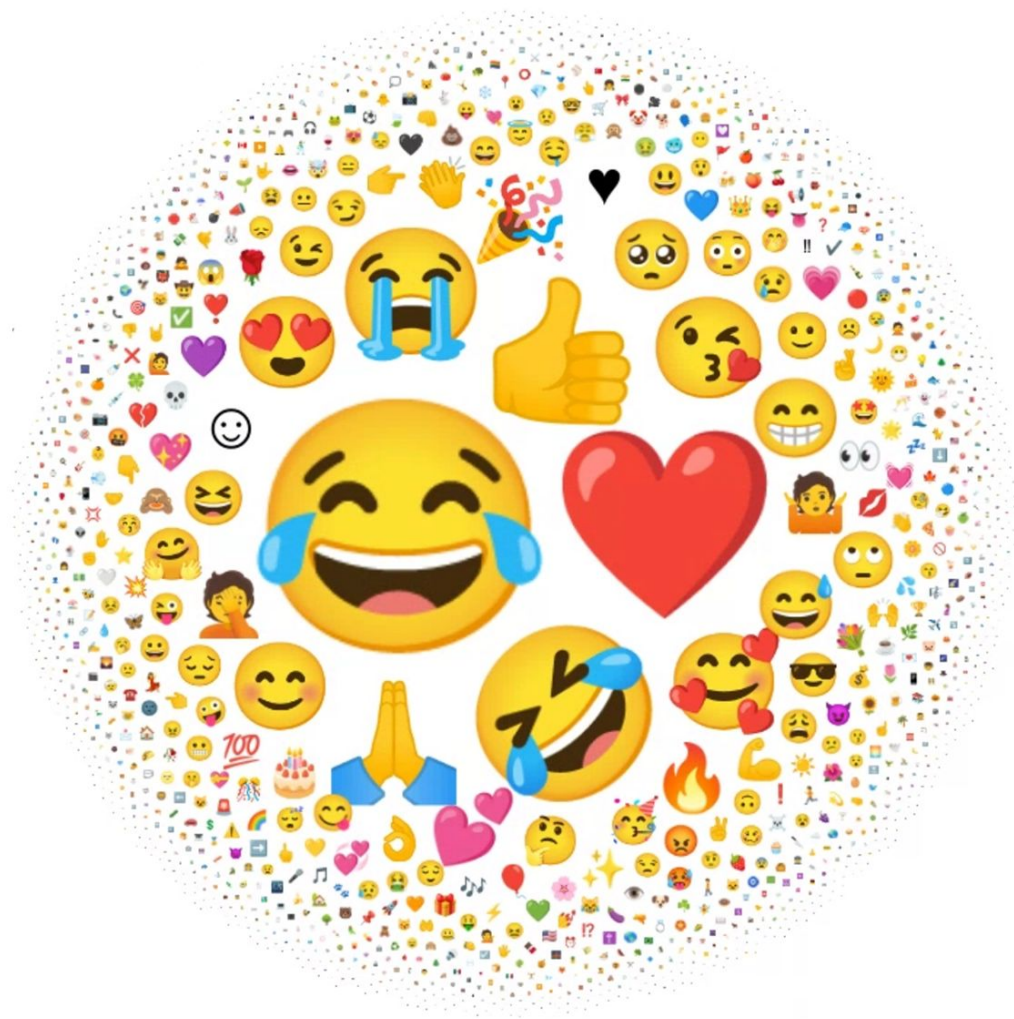
4036991106

U+1F602

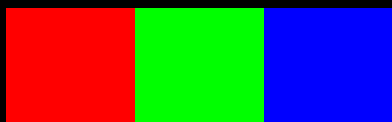








RGB

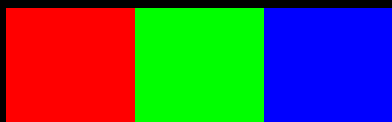


72 73 33

72

73

33







face-with-tears-of-joy_1f602.png



Search





face-with-tears-of-joy_1f602.png



Search





face-with-tears-of-joy_1f602.png



Search





```
print("Hello World")
```


[illegible]



Image Credit: BBC

Base Conversion

Denary	Binary	Hex
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

Binary to Hex

0101101

010 1101

0010 1101

2 D

Hex to Binary

F92

F 9 2

1111 1001 0010

111110010010

Arithmetic

Addition

11111

1110

11111

1110

1

1

11111

1110

01

1 1
11111

1110

101

1 1 1
11111

1110

1101

1 1 1 1

11111

1110

01101

11111

1110

101101

Multiplication

11010

10

11010

10

00000

11010

10

00000

0

Shift (x2)

11010

10

00000

110100

11010

10

00000

+ 110100

110100

Subtraction

11110

1101

0 2
11110
1101

0 2
111~~10~~

1101

1

0 2
111~~10~~

1101

01

0 2
111~~10~~

1101

001

0 2
11110

1101

0001

0 2
111~~10~~

1101

10001

Division

11111 | 101

$$\begin{array}{r} 11111 \\ 101 \overline{) 11111} \\ \underline{101} \\ 10 \end{array}$$

$$\begin{array}{r} 11111 \\ 101 \overline{) 11111} \\ \underline{101} \\ 101 \end{array}$$

$$\begin{array}{r} 11111 \\ 101 \overline{) } \\ \hline 101 \end{array} \quad \begin{array}{r} 101 \\ 11 \overline{) } \\ \hline 11 \end{array}$$

11111	101
101	11
101	
101	

11111	101
101	11
101	
101	
00	

$$\begin{array}{r}
 11111 \\
 \underline{101} \\
 101 \\
 \underline{101} \\
 001
 \end{array}
 \quad
 \begin{array}{r}
 101 \\
 \hline
 11
 \end{array}$$

11111	101
101	110
101	
101	
001	

$$\begin{array}{r}
 11111 \\
 101 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 101 \\
 101 \\
 \hline
 001
 \end{array}$$

$$\begin{array}{r}
 101 \\
 \hline
 110
 \end{array}$$

Bitwise Logical Operators



(AND)

$$\begin{array}{r} 101 \\ \& 011 \\ \hline \end{array}$$

$$\begin{array}{r} 101 \\ \& 011 \\ \hline 001 \end{array}$$

|
(OR)

$$\begin{array}{r} 101 \\ | 011 \\ \hline \end{array}$$

$$\begin{array}{r} 101 \\ | 011 \\ \hline 111 \end{array}$$



(XOR)

$$\begin{array}{r} 101 \\ \wedge 011 \\ \hline \end{array}$$

$$\begin{array}{r} 101 \\ \wedge 011 \\ \hline 110 \end{array}$$



(left shift)









01011010

010110100

010110100

10110100



(right shift)











01011010

001011010

001011010

00101101

References

- Patterson, D. A., & Hennessy, J. L. (2013). *Computer Organization and Design: The Hardware/Software Interface*. Elsevier Inc. DOI: [10.1016/B978-0-12-407726-3.00001-1](https://doi.org/10.1016/B978-0-12-407726-3.00001-1)
- Harvard University, Introduction to Computer Science.
- FreeCodeCamp.org
- geeksforgeeks.org