LECTURE

1

Basic Terms in Probability, Its Classical Definition

BASIC CONCEPTS

1. **Experiment** An operation which can produce some result or outcome is called an experiment. An experiment may consist of one or more trials.
2. **Trial** The performance of an experiment is called trial.
3. **Random Experiment** If an experiment can be repeated under the same conditions and the outcome of any such trial that cannot be decided in advance even after knowing the results of all previous trials, then such an experiment is called a random experiment. The outcome of a random experiment does not obey any rule. (Tossing a coin or die is a random experiment.)
4. **Event** The possible outcomes of a trial are called events. When a coin is tossed, the outcome of a head or tail is an event and the various possible results of a random experiment are called elementary events of that trial or simple events of that trial.

   Tossing a die whose six faces are numbered 1, 2, 3, 4, 5 and 6 respectively is example of a random experiment, and the results 1, 2, 3, 4, 5 or 6 coming up are the six simple events of this trial. (Here we rule out the possibility of balancing an edge.)
5. **Equally Likely Events** The events are said to be equally likely if there is no reason to expect any one in preference to any other.

Ex: When a die is thrown, all the six faces are equally likely to come. Also when a card is drawn from a well-shuffled deck all the 52 case are equally likely to come.

6. **Mutually Exclusive Events** Two or more events are said to be mutual exclusive if they cannot happen simultaneously in a trial. These are also called incompatible events.

Ex: In tossing a coin, the appearance of head and tail are mutually exclusive events. In throwing a die, the appearance of an even number and odd number are mutually exclusive events.

7. **Favourable Events** The cases which ensure the occurrence of the events are called favourable events.

   7.1 The following tables show number of favourable cases for the sum of digits when two or three dice are thrown together.

   For two dice:

Sum of digits	No. of favourable cases
2	1
3	2
4	3
5	4
6	5
7	6
8	5
9	4
10	3
11	2
12	1

7.2 **For three dice:**

Sum of digits	No. of favourable cases
3	1
4	3
5	6
6	10
7	15
8	21
9	25
10	27
11	27
12	25
13	21
14	15
15	10
16	6
17	3
18	1

7.3 **Playing cards:**

(i) Total 52 (26 red + 26 black)

(ii) Four suits: Heart, diamond, spade, club

(iii) Court (face) cards: 12 (4 kings, 4 queens, 4 jacks)

(iv) Honour cards: 16 (4 aces, 4 kings, 4 queens, 4 jacks)

(v) Numbered cards from 2 to 10 = 36

7.4 The favourable and unfavourable elementary events are called the successes and failures of that event.

8. Independent and Dependent Events Two or more events are said to be independent if the happening or non-happening of any one does not depend on the happening or non-happening of any other; otherwise they are said to be dependent.

Ex 1: When a card is drawn from a pack of well-shuffled cards and replaced before drawing the second card, the result of the second drawn is independent of the first one. However, if the first card is not replaced, the second draw is dependent on the first one.

Ex 2: In tossing two coins, let E_1 be the event of occurrence of head on the first coin and E_2 be the event of occurrence of head on the second coin. Then the occurrence of head on the second coin does not depend on the occurrence of head on the first and vice versa.

NOTE

Generally, mutually exclusiveness is used when the events are taken from the same experiment and independence is used when the events are taken from different experiments.

9. Simple Events An event containing only a single sample point is called an elementary or simple event.

10. Compound Events When two or more simple events occur in connection with each other, their joint occurrence is called a compound event.

11. Exhaustive Events The set of all possible outcomes of any trial:

Ex: When a coin is tossed, there are two exhaustive events, i.e., heads and tails. The set of all possible outcomes in a random experiment is called a sample space and every element of the set a sample point of that space.

11.1 Number of exhaustive cases of tossing n coins simultaneously (or of tossing a coin n times) = 2^n

11.2 Number of exhausitive cases of throwing n dice simultaneously (or throwing one die n times) = 6^n

12. Certain Event and Impossible Event

(i) Probability of occurrence of an impossible event is 0, i.e., $P(\phi) = 0$.

(ii) Probability of occurrence of a sure event is 1.

Ex: A multiple of 7 coming up is an impossibility and a number less than 7 coming up is a certainty when a die is rolled.

13. Odds in Favour of an Event and Odds against an Event It is defined as the ratio of the number of favourable cases to the number of failures. The odds against the event A is the ratio of the number of unfavourable cases to the number of favourable cases. If the number of ways in which an event can occur is m and the number of ways in which it does not occur is n, then

(i) odds in favour of the event = (m/n) and

(ii) odds against the event = (n/m)

If odds in favour of an event are $a : b$, then the probability of the occurrence of that event is $\frac{a}{a+b}$ and the probability of the non-occurrence of that event is $\frac{b}{a+b}$.

14. Complementary Event If A is an event, then the event which occurs if A does not occur, i.e., negation of A is called the complementary event of A. It is denoted by A', $\bar{A}$ or A^c and $P(\bar{A}) = 1 - P(A)$

NOTES

1. A real number between 0 and 1 is associated with each event E (i.e., a sample point), called the probability of that event and is denoted by $P(E)$. Thus, $0 \le P(E) \le 1$.
2. The sum of the probabilities of all simple (elementary) events constituting the sample space is 1. Thus, $P(S) = 1$.
3. The probability of a compound event (i.e., an event made up of two or more sample events) is the sum of the probabilities of simple events comprising the compound event.
4. If two events A and B are:
   (a) mutually exclusive events, then
   $P(A \cap B) = 0$,
   $\Rightarrow P(A \cup B) = P(A) + P(B) \le 1$
   (b) equally likely events $P(A) = P(B)$
   (c) exhaustive events $P(A \cup B) = 1$
   (d) independent events
   $P(A \cap B) = P(A)\, P(B)$
   $P(A \cup B) = P(A) + P(B) - P(A)\, P(B)$
   (e) exclusive and exhaustive event
   $P(A) + P(B) = 1$

15. Classical Definition of Probability Let there be n exhaustive, mutually exclusive and equally likely cases. Out of these let m be favourable to the happening of an event A, then the probability of occurrence of event A, denoted by $P(A)$, is defined as:

$$P(A) = \frac{\text{Number of favourable cases}}{\text{Number of exhaustive cases}} = \frac{m}{n}$$

$$P(\bar{A}) = \frac{\text{Number of unfavourable cases}}{\text{Number of exhaustive cases}} = \frac{n-m}{n}$$

$$0 \le P(A) \le 1; \qquad 0 \le P(\bar{A}) \le 1$$

NOTE

$P(A) + P(\bar{A}) = 1$

SOLVED SUBJECTIVE PROBLEMS (XII BOARD (C.B.S.E./STATE)): FOR BETTER UNDERSTANDING AND CONCEPT BUILDING OF THE TOPIC

1. Sanjay has 3 shares in a lottery, which has 3 prizes and 6 empty. Nitin has 1 share which has 1 prize and 2 empty. Prove that the ratio of the probabilities of winning Sanjay and Nitin is 16:7.

Solution

Sanjay can draw 3 tickets in ${}^9C_3$ ways. Again 3 tickets from 6 empty tickets can be drawn in ${}^6C_3$ ways.

$\therefore$ Probability of drawing all empty tickets by Sanjay

$$= \frac{{}^6C_3}{{}^9C_3} = \frac{5}{21}$$

Probability of winning the prize by Sanjay

$$= 1 - \frac{5}{21} = \frac{16}{21}$$

Nitin can draw 1 ticket in ${}^3C_1$ ways and 1 empty ticket out of 2 tickets in ${}^2C_1$ ways.

$\therefore$ Probability of drawing an empty ticket by Nitin

$$= \frac{{}^2C_1}{{}^3C_1} = \frac{2}{3}$$

$\therefore$ Probability of winning the prize by Nitin

$$= 1 - \frac{2}{3} = \frac{1}{3}$$

$\therefore$ Ratio of probabilities of winning the prize by Sanjay and Nitin $= \dfrac{16}{21} : \dfrac{1}{3} = 16:7$

Proved

2. From a group of 2 boys and 3 girls, 2 children are selected. Find the sample space associated to this random experiment. *[HPSB-1994]*

Solution

Let the 2 boys be taken as B_1 and B_2 and the 3 girls be taken as G_1, G_2 and G_3. Clearly, there are 5 children; out of which 2 children can be chosen in ${}^5C_2$ ways. So, there are ${}^5C_2 = 10$ elementary events associated to this experiments and are given by B_1B_2, B_1G_1, B_1G_2, B_1G_3, B_2G_1, B_2G_2, B_2G_3, G_1G_2, G_1G_3 and G_2G_3. Consequently, the sample space S associated to this random experiment is given by $S = \{B_1B_2, B_1G_1, B_1G_2, B_1G_3, B_2G_1, B_2G_2, B_2G_3, G_1G_2, G_1G_3, G_2G_3\}$

3. The probability of obtaining an even prime number on each die when a pair of dice is rolled is *[NCERT]*

(a) 0 (b) 1/3

(c) 1/12 (d) 1/36

Solution

(d) When a pair of dice is rolled once, the sample space contains $6 \times 6 = 36$ equally likely simple events of the type (x, y), where $x, y \in \{1, 2, 3, 4, 5, 6\}$

Required probability $= P$ (an even prime on each dice) $= 1/36$.

($\because$ (2, 2) is the only favourable outcome).

$\therefore$ (d) is the correct option.

4. Five cards are drawn successively with replacement from a well-shuffled deck of 52 cards. What is the probability that *[NCERT]*

(i) all the 5 cards are spades

(ii) only 3 cards are spades

(iii) none is a spade

Solution

It is a case of Bernoullian trials with $n = 5$, where success is 'a spade is drawn'.

$p = P$ (a success) $= P$ (a spade is drawn)

$$= \frac{13}{52} = \frac{1}{4} \text{ and } q = 1 - p = 1 - \frac{1}{4} = \frac{3}{4}$$

(i) P (all the 5 cards are spades)

$$= P(X = 5) = {}^5C_5 p^5 q^0 = 1p^5 = \left(\frac{1}{4}\right)^5 = \frac{1}{1024}$$

(ii) P (only 3 cards are spades)

$$= P(X = 3) = {}^5C_3 p^3 q^2$$

$$= \frac{5\times4\times3}{3!}\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^2$$

$$= \frac{60}{1\times2\times3}\times\frac{3^2}{4^5} = \frac{90}{1024} = \frac{45}{512}$$

(iii) P (none is a spade) $= P(X = 0)$

$$= {}^5C_0 p^0 q^5 = 1 \times q^5$$

$$= \left(\frac{3}{4}\right)^5 = \frac{243}{1024}$$

5. A couple has 2 children. Find the probability that both children are females if it is known that the elder child is a female.

Solution

Sample space $= \{GB, GG\}$

$$\text{Probability} = \frac{1}{2}$$

UNSOLVED SUBJECTIVE PROBLEMS (XII BOARD (C.B.S.E./STATE)): SOLVE THESE PROBLEMS TO GRASP THE TOPIC

EXERCISE 1

1. If $P(A) = 0.42$, $P(B) = 0.48$ and $P(A \cap B) = 0.16$, calculate the following:
(i) P (not A)
(ii) P (not B)
(iii) $P(A \cup B)$

2. In a bag there are 5 white and 10 black balls. If ball is drawn at random from it, what is the probability that it is white?

3. A coin is tossed twice. If the second throw results in a tail, a die is thrown. Describe the sample space for this experiment. ***[CBSE-93]***

4. Two dice are thrown simultaneously. Find the probability of getting:
(i) an even number as the sum. ***[CBSE-95]***
(ii) the sum as a prime number. ***[CBSE-95]***

5. Three coins are tossed simultaneously. List the sample space of the random experiment. ***[CBSE-91]***

6. Find the probability of drawing a diamond card in each of the two consecutive draws from a well-shuffled pack of cards, if the card drawn is not replaced after the first draw. ***[CBSE-2002(C)]***

7. At least one event out of two events must occur. It is given that the probability of happening of the first event is 2/3 that of the other. Find the odds in favour of the second event. ***[Imp.]***

8. Find the probability of getting an odd number on the uppermost face in throwing a dice. ***[MP-88, 91, 93, 98]***

9. One ticket is drawn at random from a well-shuffled 12 ticket numbers 1 to 12. Find the probability that the number written on the face of this ticket is a multiple of 2 or 3. ***[MP-91, 94, 2000, 2001, 2008, 2009]***

10. Find the probability distribution of the number of 6 in 3 throws of a dice. ***[MP-2009]***

EXERCISE 2

1. Two dice are thrown simultaneously. Find the probability of getting:
(i) a total of at least 10. ***[CBSE-92]***
(ii) a doublet of even number. ***[HSB-91(C)]***

2. Two dice are thrown simultaneously. Find the probability of getting:
(i) a multiple of 2 on one dice and a multiple of 3 on the other dice. ***[HSB-93 (C)]***
(ii) the same number on both dice. ***[HSB-90]***
(iii) a multiple of 3 as the sum. ***[CBSE-95]***

3. An urn contains 7 red and 4 blue balls. Two balls are drawn at random with replacement. Find the probability of getting: ***[CBSE-2007]***
(i) 2 red balls.
(ii) 2 blue balls.
(iii) 1 red and 1 blue ball.

4. Four coins are tossed simultaneously. Find the chance to get at least one head. ***[MP-1993]***

5. A and B are two events such that $P(A) = 0.42$, $P(B) = 0.48$ and $P(AB) = 0.16$, find $P(A + B)$. ***[MP-1998]***

6. When an ordinary dice is thrown find the probability of getting a number greater than 3. ***[MP-93, 97, 2002, 2004 (A)]***

7. Two dice are thrown simultaneously. Find the probability of getting a sum 9 in a single throw. ***[MP-98, 2003, 2004 (C)]***

8. One card is drawn randomly from a pack of 52 cards. Find the probability of it being an ace or a king. ***[MP-2000, 2004 (C)]***

9. Two cards are drawn from a well-shuffled pack of cards. Find the probability that both of them are aces. ***[MP-95, 2000]***

10. A bag contains 3 red, 4 white and 5 blue balls. All balls are different. Two balls are drawn at random. Find the probability that they are of different colours. ***[MP-2008]***

ANSWERS

EXERCISE 1

1. (i) 0.58 (ii) 0.52 (iii) 0.74
2. 1/3
3. $S = \{HH, TH, (HT, 1), (HT, 2), (HT, 3), (HT, 4), (HT, 5), (HT, 6), (TT, 1), (TT, 2), (TT, 3), (TT, 4), (TT, 5), (TT, 6)\}$
4. (i) 1/2 (ii) 5/12
5. $\{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$
6. 1/17
7. 3:2
8. 1/2
9. 2/3
10. 1/216

EXERCISE 2

1. (i) 1/6 (ii) 1/12
2. (i) 11/36 (ii) 1/6 (iii) 1/3
3. (i) $\frac{7}{11} \times \frac{7}{11}$ (ii) $\frac{4}{11} \times \frac{4}{11}$ (iii) $2 \times \frac{7}{11} \times \frac{4}{11}$
4. 15/16
5. 0.74
6. 1/2
7. 1/9
8. 2/13
9. 1/221
10. 47/144

SOLVED OBJECTIVE PROBLEMS: HELPING HAND

1. A committee consists of 9 experts taken from 3 institutions A, B and C; of which 2 are from A, 3 from B and 4 from C. If 3 experts resign, then the probability that they belong to different institutions is

[Roorkee Qualifying 1998]

(a) 1/729 (b) 1/24 (c) 1/21 (d) 2/7

Solution

(d) Requiredp robability

$$= \frac{{}^2C_1 \times {}^3C_1 \times {}^4C_1}{{}^9C_3} = \frac{2 \times 3 \times 4}{\left(\frac{9 \times 8 \times 7}{3 \times 2}\right)} = \frac{2}{7}$$

2. Five-digit numbers are formed using the digits 1, 2, 3, 4, 5, 6 and 8. What is the probability that they have even digits at both the ends?

[RPET-1999]

(a) 2/7 (b) 3/7
(c) 4/7 (d) None

Solution

(a) By using digits 1, 2, 3, 4, 5, 6 and 8, total five-digit numbers $= {}^7P_5$

And number of ways to form the numbers, they have even digit at both ends = $4 \times 3 \times {}^5P_3$

$$\therefore \text{Probability} = \frac{4 \times 3 \times {}^5P_3}{{}^7P_5} = \frac{2}{7}$$

3. In a lottery there are 90 tickets numbered 1 to 90. Five tickets are drawn at random. The probability that 2 of the tickets drawn are numbers 15 and 89 is: *[AMU-2001]*

(a) $\frac{2}{801}$ (b) $\frac{2}{623}$ (c) $\frac{1}{267}$ (d) $\frac{1}{623}$

Solution

(a) Required probability $= \frac{{}^{88}C_3}{{}^{90}C_5} = \frac{2}{801}$

4. From 80 cards numbered 1 to 80, 2 cards are selected randomly. The probability that both the cards have the numbers divisible by 4 is given by: *[Pb. CET-2000]*

(a) 21/316 (b) 19/316
(c) 1/4 (d) None of these

Solution

(b) Total number of ways $= {}^{80}C_2$ and favourable ways $= {}^{20}C_2$

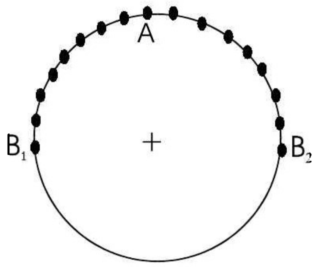
$$\therefore \text{Required probability } P = \frac{{}^{20}C_2}{{}^{80}C_2} = \frac{19}{316}$$

5. Fifteen persons among whom are A and B sit down at random at a round table. The probability that there are 4 persons between A and B is

(a) 1/3 (b) 2/3
(c) 2/7 (d) 1/7

Solution

(d) Let A occupy any seat at the round table. Then there are seats available for B. If there are to be 4 persons between A and B. Then B has only 2 ways to sit, as shown in the figure.



Hence required probability $= \frac{2}{14} = \frac{1}{7}$.

6. In a horse race the odds in favour of 3 horses are 1:2, 1:3 and 1:4. The probability that one of the horses will win the race is

(a) 37/60 (b) 47/60
(c) 1/4 (d) 3/4

Solution

(b) Probabilities of winning the race by 3 horses are 1/3, 1/4 and 1/5. Hence required probability $= \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{47}{60}$.

7. In four schools B_1, B_2, B_3, B_4 the percentage of girls students is 12, 20, 13, 17, respectively. From a school selected at random, one student is picked up at random, and it is found that the student is a girl. The probability that the school selected is B_2 is ***[Pb.CET-2004]***

(a) $\frac{6}{31}$ (b) $\frac{10}{31}$

(c) $\frac{13}{62}$ (d) $\frac{17}{62}$

Solution

(b) Favourable number of cases $= {}^{20}C_1 = 20$
Samplesp ace $= {}^{62}C_1 = 62$

$\therefore$ Required probability $= \frac{20}{62} = \frac{10}{31}$.

8. Six boys and six girls sit in a row. What is the probability that the boys and girls sit alternatively? ***[IIT-1979]***

(a) $\frac{1}{462}$ (b) $\frac{1}{924}$

(c) $\frac{1}{2}$ (d) None of these

Solution

(a) Let n = total number of ways = 12! and m = favourable number of ways $= 2 \times 6! \times 6!$ Since the boys and girls can sit alternately in $6! \times 6!$ ways if we begin with a boy and similarly they can sit alternately in $6! \times 6!$ ways if we begin with a girl

Hence required probability =

$$\frac{m}{n} = \frac{2 \times 6!.6!}{12!} = \frac{1}{462}$$

9. Five persons entered the lift cabin on the ground floor of an 8-floor house. Suppose that each of them independently and with equal probability can leave the cabin at any floor beginning with the first. Find out the probability of all 5 persons leaving at different floors. ***[Roorkee 1990; Delhi (CEE), 98]***

(a) $\frac{1}{7^5}$ (b) $\frac{1}{{}^7P_5}$

(c) $\frac{{}^7P_5}{7^5}$ (d) None of these

Solution

(c) Besides the ground floor, there are 7 floors. The total number of ways in which each of the 5 persons can leave cabin at any of the 7 floors $= 7^5$. And the favourable number of ways, that is, the number of ways in which the 5 persons leave at different floors is ${}^7P_5$.

$\therefore$ The required probability $= {}^7P_5/7^5$.

10. Three numbers are selected one by one from whole numbers 1 to 20. The probability that they are consecutive integers is *[PET (Raj.)-98]*

(a) 1/380 (b) 3/190
(c) 3/20 (d) None of these

Solution

(a) Total number of sequences of 3 numbers selected one by one from whole numbers 1 to 20 = ${}^{20}P_3 = 20 \times 19 \times 18$

Now sequences which will contain 3 consecutive integers are (1, 2, 3), (2, 3, 4), (3, 4, 5),…, (18, 19, 20)

These are 18 sequences.

$\therefore$ Required probability $= \dfrac{18}{20 \times 19 \times 18} = \dfrac{1}{380}$

11. The odds against throwing 7 with 2 dice in a throw are: *[Ranchi-95]*

(a) 5:1 (b) 1:5
(c) 1:4 (d) 3:1

Solution

(a) Favourable cases = 6, non-favourable cases = 30.

$\therefore$ Odds against the event = 30:6 = 5:1.

12. A die is tossed. The event an even or a prime number occurs on the top of the die is *[MP PET-2007]*

(a) {2, 5} (b) {2, 3, 4, 5, 6}
(c) {1, 2, 3, 5} (d) None of these

Solution

(b) {2, 3, 4, 5, 6}

13. One number is selected from 1 to 100 integers. The probability that it is divisible by 6 or 8 (but not by 24) is *[Kerala (CEE)-2003]*

(a) 4/5 (b) 1/5
(c) 6/25 (d) 1/4

Solution

(b) Probability $= \dfrac{16 + 12 - 8}{100} = \dfrac{1}{5}$

14. Among 600 bolts, 20% are very large 10% are very small and the remaining are useful. One bolt is chosen at random. The probability that it is a useful bolt is *[UPSEAT-2005]*

(a) 1/10 (b) 3/10
(c) 7/10 (d) 8/10

Solution

(c) Total bolts = 600, useful bolts = 600 – (120 + 60) = 420

$\therefore$ Required probability $= \dfrac{420}{600} = \dfrac{7}{10}$

15. A book has 1000 pages, which are numbered from 1 to 1000. If a page is selected at random, then the probability that the sum of the digits of its number is 9 will be *[UPSEAT-2005]*

(a) 33/1000 (b) 44/1000
(c) 55/1000 (d) 66/1000

Solution

(c) Favourable numbers between 1 and 100 = 11

Favourable numbers between 101 and 200 = 9
Favourable numbers between 201 and 300 = 8
Favourable numbers between 301 and 400 = 6
Favourable numbers between 401 and 500 = 6
Favourable numbers between 501 and 600 = 5
Favourable numbers between 601 and 700 = 4
Favourable numbers between 701 and 800 = 3
Favourable numbers between 801 and 900 = 3

$\therefore$ Total number of favourable numbers = 55

$\therefore$ Required probability $= \dfrac{55}{1000}$.

16. One mapping is selected from all mappings which can be defined from a set $A = \{1, 2, 3, \ldots, n\}$ to A. The probability that it is one-one will be:

(a) $1/n!$ (b) $1/n^n$
(c) $n!/n^{n-1}$ (d) $(n-1)!/n^{n-1}$

Solution

(a) Total number of mappings defined from A to $A = n^n$. Out of them $n!$ will be one-one. ($\because A$ is finite with n elements.)

$\therefore$ Probability $= \dfrac{n!}{n^n} = \dfrac{(n-1)!}{n^{n-1}}$

17. A 4-digit number is formed using digits 0, 1, 2, 3, 4. The probability that this number is divisible by 6 is

(a) 1/4 (b) 7/48
(c) 5/48 (d) None of these

Solution

(a) Total number of numbers of 4 digits may be formed $= {}^5P_4 - {}^4P_3 = 96$. A number is divisible by 6 if its unit number is even and the sum of its digits is divisible by 3. Such numbers are 24.

$\therefore$ Probability $= \dfrac{24}{96} = \dfrac{1}{4}$

18. A dice is rolled three times, the probability of getting a larger number than the previous number each time is

(a) $\dfrac{15}{216}$ (b) $\dfrac{5}{54}$
(c) $\dfrac{13}{216}$ (d) $\dfrac{1}{18}$

Solution

(b) Exhaustive number of cases = 63 = 216. Obviously, the second number has to be greater than unity. If the second number is $i (i > 1)$, then the first can be chosen in $i - 1$ ways and the third in $6 - i$ ways and hence three numbers can be chosen in $(i - 1) \times 1 \times (6 - i)$ ways. But the second number can be 2, 3, 4, 5. Thus the favourable number of cases =

$$\sum_{i=2}^{5} (i-1)(6-i) = 1\times 4 + 2\times 3 + 3\times 2 + 4\times 1$$
$$= 20$$

Hence the required probability $= \dfrac{20}{216} = \dfrac{5}{54}$.

Aliter:

Favourable case $= {}^6C_3$

Sample space $= 6 \times 6 \times 6$

Probability $= \dfrac{{}^6C_3}{6^3} = \dfrac{5}{54}$

OBJECTIVE PROBLEMS: IMPORTANT QUESTIONS WITH SOLUTIONS

1. A card is drawn from a pack of cards. Find the probability that the card will be a queen or a heart. ***[RPET-99]***

(a) 4/3 (b) 16/3
(c) 4/13 (d) 21/3

2. The chance of throwing a total of 7 or 12 with 2 dice is ***[Kurukshetra CEE-2002]***

(a) 2/9 (b) 5/9
(c) 5/36 (d) 7/36

3. If $P(A) = 2/3$, $P(B) = 1/2$ and $P(A \cup B) = 5/6$, then events A and B are ***[Kerala (Engg.)-02]***

(a) mutuallye xclusive
(b) independent as well as mutually exclusive
(c) independent
(d) dependent only on A

4. Two cards are drawn without replacement from a well-shuffled pack. Find the probability that one of them is an ace or a heart: ***[UPSEAT-2002]***

(a) 1/25 (b) 1/26
(c) 1/52 (d) None of these

5. Find the probability that the two digit number formed by digits 1, 2, 3, 4, 5 is divisible by 4 (while repetition of digit is allowed): ***[UPSEAT-2002]***

(a) 1/30 (b) 1/20
(c) 1/40 (d) 1/5

6. The probability that at least one of the events A and B occurs is 3/5. If A and B occur simultaneously with probability 1/5, then $P(A') + P(B')$ is ***[DCE-2002]***

(a) 2/5 (b) 4/5
(c) 6/5 (d) 7/5

7. From a pack of 52 cards 2 cards are drawn in succession one by one without replacement. The probability that both are aces is *[MNR-88; UPSEAT-2000]*
(a) 2/13 (b) 1/51
(c) 1/221 (d) 2/21

8. What is the probability that when one die is thrown, the number appearing on top is even? *[AMU-2000]*
(a) 1/6 (b) 1/3
(c) 1/2 (d) None of these

9. A bag contains 3 red, 4 white and 5 black balls. Three balls are drawn at random. The probability of being their different colours is *[RPET-99]*
(a) 3/11 (b) 2/11
(c) 8/11 (d) None of these

10. The probability that the 3 cards drawn from a pack of 52 cards are all red is *[MPPET-99]*
(a) 1/17 (b) 3/19
(c) 2/19 (d) 2/17

11. For an event, odds against is 6 : 5. The probability that event does not occur is
(a) 5/6 (b) 6/11
(c) 5/11 (d) 1/6

12. Let A and B be two events such that $P(A) = 0.3$ and $P(A \cup B) = 0.8$. If A and B are independent events, then $P(B)$ is *[IIT-1990; UPSEAT-2001, 2002]*
(a) 5/6 (b) 5/7
(c) 3/5 (d) 2/5

13. If A and B are two independent events such that $P(A \cap B') = 3/25$ and $P(A' \cap B) = 8/25$, then $P(A)$ is *[IIT Screening]*
(a) 1/5 (b) 3/8
(c) 2/5 (d) 4/5

14. From the word POSSESSIVE, a letter is chosen at random. The probability of it to be S is *[SCRA-1987]*
(a) 3/10 (b) 4/10
(c) 3/6 (d) 4/6

15. If A and B are two events such that $P(A \cup B) + P(A \cap B) = 7/8$ and $P(A) = 2P(B)$, then $P(A)$ is
(a) 7/12 (b) 7/24
(c) 5/12 (d) 17/24

16. Three fair coins are tossed. If both heads and tails appears, then the probability that exactly one head appears is
(a) 3/8 (b) 1/6
(c) 1/2 (d) 1/3

17. Two cards are drawn from a pack of 52 cards. What is the probability that one of them is a queen and the other is an ace?
(a) 2/663 (b) 2/13
(c) 4/663 (d) 8/663

18. The probability of getting a number greater than 2 in throwing a die is *[MPPET-1988]*
(a) 1/3 (b) 2/3
(c) 1/2 (d) 1/6

19. Two dice are thrown together. The probability that sum of the two numbers will be a multiple of 4 is *[MPPET-1990]*
(a) 1/9 (b) 1/3
(c) 1/4 (d) 5/9

20. One of the two events must occur. If the chance of one is 2/3 of the other, then odds in favour of the other are
(a) 2:3 (b) 1:3
(c) 3:1 (d) 3:2

21. Two dice are thrown. The probability that the sum of numbers appearing is more than 10 is
(a) 1/18 (b) 1/12
(c) 1/6 (d) None of these

22. A coin is tossed twice. The probability of getting head both the times is *[MNR-1978]*
(a) 1/2 (b) 1/4
(c) 3/4 (d) 1

23. If two balanced dice are tossed once, the probability of the event that the sum of the integers coming on the upper sides of the two dice is 9 is *[MPPET-1987, 2008]*
(a) 7/18 (b) 5/36
(c) 1/9 (d) 1/6

24. From 10,000 lottery tickets numbered from 1 to 10,000, one ticket is drawn at random. What is the probability that the number marked on the drawn ticket is divisible by 20?
(a) 1/100 (b) 1/50
(c) 1/20 (d) 1/10

25. Two dice are thrown. What is the probability that the sum of the faces equals or exceeds 10? *[NDA-2009]*
(a) 1/12 (b) 1/4
(c) 1/3 (d) 1/6

26. When a card is drawn from a well-shuffled pack of cards, what is the probability of getting a queen? *[NDA-2009]*
(a) 2/13 (b) 1/13
(c) 1/26 (d) 1/52

27. The probability that the 3 cards drawn from a pack of 52 cards are all black is *[MPPET-2009]*
(a) $\frac{1}{17}$ (b) $\frac{2}{17}$
(c) $\frac{3}{17}$ (d) $\frac{2}{19}$

SOLUTIONS

1. (c) Probability that the card will be a given $P(A) = A/52$ and probability 6 that the card will be a heart = 13/52. Both events are mutually exclusive so required probability

$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{4}{13}$$

2. (d) Total number of outcomes = 36. For sum = 7, favourable outcomes are 6, i.e., (6, 1); (5, 2); (4, 3); (3, 4); (2, 5); (1, 6). For sum = 12, favourable outcomes is only 1, i.e., (6, 6).

$\therefore$ Probability $= \frac{6}{36} + \frac{1}{36} = \frac{7}{36}$

3. (c) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\frac{5}{6} = \frac{2}{3} + \frac{1}{2} - P(A \cap B)$$

$\therefore$ $P(A \cap B) = \frac{2}{6} = \frac{1}{3}$

Also, $P(A) \times P(B) = \frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$

$\therefore$ $P(A \cap B) = P(A) \times P(B)$

$\therefore$ Events A and B are independent.

4. (b) $\therefore$ Required probability $= \frac{1 \times {}^5C_1}{{}^{52}C_2} = \frac{1}{26}$

5. (d) Total number of numbers $= (5)^2$

Favourable cases = [12, 24, 32, 44, 52]

$\therefore$ Required probability $= \frac{5}{25} = \frac{1}{5}$

6. (c) $P(A') + P(B') = 1 - P(A) + 1 - P(B)$

$= 2 - \{P(A) + P(B)\}$

$= 2 - \{P(A \cup B) + P(A \cap B)\}$

$= 2 - \left(\frac{3}{5} + \frac{1}{5}\right) = \frac{6}{5}$

7. (c) Required probability $= \frac{4}{52} \times \frac{3}{51} = \frac{1}{221}$

8. (c) Required probability $= \frac{3}{6} = \frac{1}{2}$

9. (a) Probability $= \frac{{}^3C_1 \times {}^4C_1 \times {}^5C_1}{{}^{12}C_3} = \frac{3}{11}$

10. (d) Total ways to draw 3 cards $= {}^{52}C_3$

Favourable ways to draw 3 red cards $= {}^{26}C_3$

P (all red) $= \frac{{}^{26}C_3}{{}^{52}C_3} = \frac{26 \times 25 \times 24}{52 \times 51 \times 50} = P = \frac{2}{17}.$

11. (b) Required probability $= \frac{6}{6+5} = \frac{6}{11}$

12. (b) $P(A \cup B) = P(A) + P(B) = P(A \cap B)$

$0.8 = P(A) + P(B) - P(A)\,P(B)$

($\because$ events are independent.)

$0.8 = 0.3 + P(B) - 0.3\,P(B)$

or $0.5 = (1 - 0.3)\,P(B) = 0.7\,P(B)$

$\therefore P(B) = \dfrac{0.5}{0.7} = \dfrac{5}{7}$

13. (a) Since events are independent.

$P(A \cap B') = P(A) \times P(B') = \dfrac{3}{25}$

$\Rightarrow \quad P(A) \times \{1 - P(B)\} = \dfrac{3}{25}$ ……(i)

Similarly,

$P(B) \times \{1 - P(A)\} = \dfrac{8}{25}$ ……(ii)

On solving (i) and (ii),

we get $P(A) = \dfrac{1}{5}$ and $\dfrac{3}{5}$

14. (b) Total ways $= {}^{10}C_3$

Favourable cases $= {}^{4}C_1$ ($\because S = 4$)

$$\text{Required probability} = \frac{\text{Favourable case}}{\text{Total ways}}$$

$$= \frac{4C_1}{10C_1} = \frac{4}{10}$$

15. (a) Since we have

$$P(A \cup B) + P(A \cap B) = P(A) + P(B)$$

$$= P(A) + \frac{P(A)}{2}$$

$$\Rightarrow \quad \frac{7}{8} = \frac{3P(A)}{2}$$

$$\Rightarrow \quad P(A) = \frac{7}{12}$$

16. (c) Since both heads and tails appears, $n(S) = \{HHT, HTH, THH, HTT, THT, TTH\}$

$n(E) = \{HTT,\ THT,\ TTH\}$

Hence required probability $= \dfrac{3}{6} = \dfrac{1}{2}$

17. (d) Probability $= \dfrac{{}^4C_1 \times {}^4C_1}{{}^{52}C_2} = \dfrac{16.2}{52.51} = \dfrac{8}{663}$

18. (b) Required probability $= \dfrac{4}{6} = \dfrac{2}{3}$

19. (c) $S = \{(3, 1), (2, 2), (1, 3), (6, 2), (5, 3), (4, 4), (3, 5), (2, 6), (6, 6)\}$

Hence required probability $= \dfrac{9}{36} = \dfrac{1}{4}$

20. (d) Let p be the probability of the other event. Then the probability of the first event is $\dfrac{2}{3}p$. Since two events are totally exclusive, we have $p + (2/3)\,p = 1$ or $p = 3/5$

Hence odds in favour of the other are 3 : 5 – 3, i.e., 3 : 2.

21. (b) Required probability $= \dfrac{2+1}{36} = \dfrac{1}{12}$

22. (b) Required probability $= \left(\dfrac{1}{2}\right)\left(\dfrac{1}{2}\right) = \dfrac{1}{4}$

23. (c) Required probability $= \dfrac{4}{36} = \dfrac{1}{9}$

24. (c) Number of tickets numbered such that it is divisible by 20 are $\dfrac{10000}{20} = 500.$

Hence required probability $= \dfrac{500}{10000} = \dfrac{1}{20}.$

25. (d) $\because n(S) = 36$

$E =$ Sum of the faces equals or exceeds

$= \{(5, 5), (4, 6), (6, 4), (5, 6), (6, 5), (6, 6)\}$

$\therefore n(E) = 6$

Hence, $P(E) = \dfrac{n(E)}{n(S)} = \dfrac{6}{36} = \dfrac{1}{6}$

26. (b) $\because n(S) = 52$ and $n(E) = 4$

$\therefore P(E) = \dfrac{n(E)}{n(S)} = \dfrac{4}{52} = \dfrac{1}{13}$

27. (b) In a pack of 52 cards, there are 26 black cards.

$\therefore$ Required probability $= \dfrac{{}^{26}C_3}{{}^{52}C_3}$

$$= \frac{26 \times 25 \times 24}{3 \times 2 \times 1} \times \frac{3 \times 2 \times 1}{52 \times 51 \times 50} = \frac{2}{17}$$

UNSOLVED OBJECTIVE PROBLEMS (IDENTICAL PROBLEMS FOR PRACTICE): FOR IMPROVING SPEED WITH ACCURACY

1. In two events $P(A \cup B) = 5/6$, $P(A^c) = 5/6$, $P(B) = 2/3$, A and B are **[UPSEAT-2001]**
(a) independent
(b) mutually exclusive
(c) mutually exhaustive
(d) dependent

2. From a pack of 52 cards, 2 cards are drawn one by one without replacement. The probability that first drawn card is a king and the second is queen is **[MPPET-1997]**
(a) 2/13 (b) 8/663
(c) 4/663 (d) 103/663

3. A bag contains 5 white, 7 black and 4 red balls. Three balls are drawn from the bag at random. The probability that all the 3 balls are white is **[MPPET-1997]**
(a) 3/16 (b) 3/5
(c) 1/60 (d) 1/56

4. The chances of throwing a total of 3 or 5 or 11 with 2 dice is **[Kurukshetra CEE-1996]**
(a) 5/36 (b) 1/9
(c) 2/9 (d) 19/36

5. If $P(A) = 0.65$, $P(B) = 0.15$, then $P(\overline{A}) + P(\overline{B}) =$ **[Pb. CET-89; EAMCET-88]**
(a) 1.5 (b) 1.2
(c) 0.8 (d) None of these

6. One card is drawn from a pack of 52 cards. The probability that it is a king or a diamond is **[MPPET-1990, 94; RPET-96]**
(a) 1/26 (b) 3/26
(c) 4/13 (d) 3/13

7. Given two mutually exclusive events A and B such that $P(A) = 0.45$ and $P(B) = 0.35$, then $P(A \text{ or } B) =$ **[AICBSE-1979]**
(a) 0.1 (b) 0.25 (c) 0.15 (d) 0.8

8. A pair has 2 children. If one of them is a boy, then the probability that other is also a boy is
(a) 1/2 (b) 1/4
(c) 1/3 (d) None of these

9. A bag contains 6 red, 4 white and 8 blue balls. If 3 balls are drawn at random, then the probability that 2 are white and 1 is red is
(a) 5/204 (b) 7/102
(c) 3/68 (d) 1/13

10. If in a lottery there are 5 prizes and 20 blanks, then the probability of getting a prize is
(a) 1/5 (b) 2/5
(c) 4/5 (d) None of these

11. If $P(A) = 1/4$, $P(B) = 5/8$ and $P(A \cup B) = 3/4$, then $P(A \cap \text{B}) =$
(a) 1/8 (b) 0
(c) 3/4 (d) 1

12. The probability of getting number 5 in throwing a dice is **[MPPET-1988]**
(a) 1 (b) 1/3
(c) 1/6 (d) 5/6

13. Three cards are drawn at random from a pack of 52 cards. What is the chance of drawing 3 aces?
(a) 3/5525 (b) 2/5525
(c) 1/5525 (d) None of these

14. Two coins are tossed. Let A be the event that the first coin shows head and B be the event that the second coin shows a tail. Two events A and B are
(a) mutually exclusive
(b) independent
(c) independent and mutually exclusive
(d) None of these

15. A single letter is selected at random from the word PROBABILITY. The probability that the selected letter is a vowel is
(a) 2/11 (b) 3/11
(c) 4/11 (d) 0

16. If throwing of two dice, what is the number of exhaustive events? **[NDA-2006]**

(a) 6 (b) 12
(c) 36 (d) 18

17. Two dice are thrown. The probability that the sum of the points on 2 dice will be 7 is
[IIT 1974; MNR-1981, 91; RPET-1995, 97, 2002; UPSEAT-2000]
(a) 5/36 (b) 6/36
(c) 7/36 (d) 8/36

18. In a single throw of 2 dice the probability of obtaining an odd sum is
(a) 1/6 (b) 1/2
(c) 1/3 (d) None of these

19. From a pack of 52 cards two are drawn with replacement. The probability that the first is a diamond and the second is a king is
[MNR-1979]
(a) 1/26 (b) 17/2704
(c) 1/52 (d) None of these

20. If two events A and B are such that $P(A + B) = \frac{5}{6}$, $P(AB) = \frac{1}{3}$ and $P(\bar{A}) = \frac{1}{2}$, then the events A and B are
(a) independent
(b) mutually exclusive
(c) mutually exclusive and independent
(d) None of these

21. Two cards are drawn at random from a pack of 52 cards. The probability that both are the cards of spade is
(a) 1/26 (b) 1/4
(c) 1/17 (d) None of these

22. A box contains 10 red and 15 green balls. If 2 balls are drawn in succession then the probability that one is red and other is green is
(a) 1/3 (b) 1/2
(c) 1/4 (d) None of these

23. A coins is tossed twice. If events A and B are defined as: A = head on first toss, B = head on second toss. Then the probability of $A \cup B$ =
(a) 1/4 (b) 1/2
(c) 1/8 (d) 3/4

24. A coin is tossed and a dice is rolled. The probability that the coin shows the head and the dice shows 6 is ***[MPPET-94; Pb. CET-2001]***
(a) 1/8 (b) 1/12
(c) 1/2 (d) 1

25. A bag contains 6 red, 5 white and 4 black balls. Two balls are drawn. The probability that none of them is red is
(a) 12/35 (b) 6/35
(c) 4/35 (d) None of these

26. Given $P(A \cup B) = \frac{5}{6}, P(A \cap B) = \frac{1}{3}$ and $P(\bar{B}) = \frac{1}{2}$. What is P($\bar{A}$)? ***[NDA-2008]***
(a) 1/6 (b) 1/3
(c) 2/3 (d) 1/2

27. A bag contains 3 white and 5 black balls. If one ball is drawn, then the probability that it is black is
(a) 3/8 (b) 5/8
(c) 6/8 (d) 10/20

28. A card is drawn at random from a pack of cards. The probability of this card being a red or a queen is
(a) 1/13 (b) 1/26
(c) 1/2 (d) 7/13

29. If 3 distinct numbers are chosen randomly from $\{1, 2, \ldots, 100\}$, then probability that all are divisible by both 2 and 3 is
[IIT Screening 2004]
(a) 4/25 (b) 4/35
(c) 4/33 (d) 4/1155

30. A bag contains 4 white, 5 red and 6 green balls. Three balls are picked up randomly. The probability that a white, a red and a green ball is drawn is
(a) 15/91 (b) 30/91
(c) 20/91 (d) 24/91

31. Two cards are drawn successively with replacement from a pack of 52 cards. The probability of drawing 2 aces is
[MNR-1988; UPSEAT-2000]
(a) 1/169 (b) 1/221
(c) 1/2652 (d) 4/663

WORKSHEET: TO CHECK THE PREPARATION LEVEL

Important Instructions

1. The answer sheet is immediately below the worksheet.
2. The test is of 14 minutes.
3. The worksheet consists of 14 questions. The maximum marks are 42.
4. Use Blue/Black Ball point pen only for writing particulars/marking responses. Use of pencil is strictly prohibited.

1. In a single throw of 2 dice, the probability of obtaining a total of 7 or 9 is *[AISSE-1979]*

(a) 5/18 (b) 1/6
(c) 1/9 (d) None of these

2. The chance of getting a doublet with 2 dice is *[Kurukshetra CEE-2002]*

(a) 2/3 (b) 1/6
(c) 5/6 (d) 5/36

3. One card is drawn randomly from pack of 52 cards, then the probability that it is a king or a spade is *[RPET-2001]*

(a) 1/26 (b) 3/26
(c) 4/13 (d) 3/13

4. An event has odds in favour 4 : 5, then the probability that event occurs is

(a) 1/5 (b) 4/5
(c) 4/9 (d) 5/9

5. The probability of happening of an impossible event, i.e., $P(\phi)$ is *[MPPET-93]*

(a) 1 (b) 0
(c) 2 (d) -1

6. A bag contains 3 white and 7 red balls. If a ball is drawn at random, then what is the probability that the drawn ball is either white or red?

(a) 0 (b) 3/10
(c) 7/10 (d) 10/10

7. A card is drawn from a well-shuffled pack of cards. The probability of getting a queen of club or a king of heart is *[MPPET-1988]*

(a) 1/52 (b) 1/26
(c) 1/18 (d) None of these

8. A die has 3 yellow, 2 red and 1 blue faces. The die is projected three times. The probability of getting yellow, red and blue face in the first, second and third projection, respectively, is *[IIT, 92]*

(a) 1/18 (b) 1/36
(c) 1/9 (d) 1/7

9. From a class of 12 girls and 18 boys, 2 students are chosen randomly. What is the probability that both of them are girls?

(a) 22/145 (b) 13/15
(c) 1/18 (d) None of these

10. If A and B are arbitrary events, then *[DCE-2002]*

(a) $P(A \cap B) \geq P(A) + P(B)$
(b) $P(A \cup B) \leq P(A) + P(B)$
(c) $P(A \cap B) = P(A) + P(B)$
(d) None of these

11. A card is drawn from a pack of 52 cards. A gambler bets that it is a spade or an ace. What are the odds against his winning this bet? *[NDA-2007]*

(a) 17:52 (b) 52:17
(c) 9:4 (d) 4:9

12. A card is drawn at random from a well-shuffled pack of 52 cards. The probability of getting a 2 of heart or diamond is

(a) 1/26 (b) 1/52
(c) 1/13 (d) None of these

13. The outcomes of 5 tosses of a coin are recorded in a single sequence as H (head) and T (tail) for each toss. What is the number of elementary events in the sample space? *[NDA-2008]*

(a) 5 (b) 10
(c) 25 (d) 32

14. From a pack of 52 cards, 2 cards are drawn at random. Then the probability that one is a king and the other is a queen is *[MPPET-2008]*

(a) 4/663 (b) 6/663
(c) 2/663 (d) 8/663

ANSWER SHEET

1. ⓐ ⓑ ⓒ ⓓ	6. ⓐ ⓑ ⓒ ⓓ	11. ⓐ ⓑ ⓒ ⓓ
2. ⓐ ⓑ ⓒ ⓓ	7. ⓐ ⓑ ⓒ ⓓ	12. ⓐ ⓑ ⓒ ⓓ
3. ⓐ ⓑ ⓒ ⓓ	8. ⓐ ⓑ ⓒ ⓓ	13. ⓐ ⓑ ⓒ ⓓ
4. ⓐ ⓑ ⓒ ⓓ	9. ⓐ ⓑ ⓒ ⓓ	14. ⓐ ⓑ ⓒ ⓓ
5. ⓐ ⓑ ⓒ ⓓ	10. ⓐ ⓑ ⓒ ⓓ	

HINTS AND EXPLANATIONS

1. (a) Seven can be thrown as

(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 2)

Nine can be thrown as

(3, 6), (4, 5), (5, 4), (6, 5) = 4 ways

Favourable methods = 6 + 4 = 10

Total manners = 36

Probability $= \frac{10}{36} = \frac{5}{18}$

4. (c) An event has odd is favour = 4 : 5, then probability that event occurs $= \frac{4}{9}$

8. (b) Probability of getting yellow face $= \frac{3}{6} = \frac{1}{2}$

Red face $= \frac{2}{6} = \frac{1}{3}$

Blue face $= \frac{1}{6}$

The events are independent, hence order does not matter.

So required probability $= \frac{1}{2} \times \frac{1}{3} \times \frac{1}{6} = \frac{1}{36}$

11. (c) Probability of a spade $= \frac{13}{52}$

Probability of an ace $= \frac{4}{52}$

and required probability

$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52}$

$= \frac{4}{13}$

Odds against his winning $= \dfrac{1 - \frac{4}{13}}{\frac{4}{13}} = \dfrac{\frac{9}{13}}{\frac{4}{13}}$

$= \frac{9}{4}$

13. (b) Required number of elements in sample space = 10.

LECTURE

2

Theorems of Probability

BASIC CONCEPTS

1. Important Notations

(i) $P(A)$ denotes the probability for an event A to happen.

(ii) $P(\bar{A})$ probability of the non-occurrence of E.

(iii) $P(A + B)$ or $P(A \cup B)$ = occurrence of at least one of the events A and B.

(iv) $P(A \times B)$ or $P(A \cap B)$ = occurrence of both the events A and B simultaneously.

(v) $P(A\bar{B})$ or $P(A \cap \bar{B})$ = happening of A and not of B (A occurs but B does not occur).

(vi) $P(\bar{A}B)$ or $P(B \cap \bar{A})$ = happening of B and not of A (A does not occur but B occurs).

(vii) $P(\bar{A}\,\bar{B})$ or $P(\bar{A} \cap \bar{B})$ = non-occurrence of both A and B. Neither A nor B occurs, i.e., none of A and B occurs or denotes the probability for neither of A and B.

(viii) $P(\bar{A} \cup \bar{B})$ or $P(\bar{A} + \bar{B})$ = non-occurrence of at least one of the events A and B.

(ix) $A \subseteq B$ = occurrence of A implies the occurrence of B. (x) $A \cap B = A$

(xi) $P(A/B)$ = denotes the probability of the happening of A after the happening of B is already known or probability of occurrence of A with the condition that B has already occurred.

(xii) $A' = U - A$, U = universal set = sample space. $A - B = A \cap B'$, $U' = \phi$, $A \cup A' = U$, $A \cap A' = \phi$

(xiii) $A \cup B \cup C$ denotes the occurrence of at least one event A, B or C.

(xiv) $A \cap B \cap C$ denotes the occurrence of all three events A, B and C.

(xv) $(A \cap \bar{B}) \cup (\bar{A} \cap B)$ denotes the occurrence of exactly one of A and B.

(xvi) $(A \cap B) \cup (B \cap C) \cup (A \cap C) - (A \cap B \cap C)$ denotes not more than two occurs.

(xvii) $(A \cap \bar{B} \cap \bar{C}) \cup (\bar{A} \cap B \cap \bar{C}) \cup (\bar{A} \cap \bar{B} \cap C)$ denotes one and only occurs.

(xviii) $(A \cap B \cap \bar{C}) \cup (A \cap \bar{B} \cap C) \cup (\bar{A} \cap B \cap C)$ denotes two and no more occurs.

(xix) $(A \cap B) \cup (B \cap C) \cup (A \cap C)$ denotes at least two of A, B, C occur.

2. DeMorgan's Law $(A \cup B)' = A' \cap B'$, $(A \cap B)' = A' \cup B'$

3. Addition theorem of Probability

3.1 **Case I: When events are mutually exclusive** The probability of happening of any one of several mutually exclusive events is equal to the sum of their probabilities, i.e., if $A_1, A_2, \ldots, A_n$ are mutually exclusive events, then $P(A_1 + A_2 + \ldots + A_n) = P(A_1) + P(A_2) + \ldots + P(A_n)$, i.e., $P\left(\sum A_i\right) = \sum P(A_i)$.

3.2 $P(A \cap B) = 0$

3.3 $P(A \cup B) = P(A) + P(B)$

3.4 **Case II: When events are not mutually exclusive** In this we have $P(A + B) = P(A) + P(B) - P(AB)$.

$P(A + B + C) = P(A) + P(B) + P(C) + P(ABC) - P(BC) - P(CA) - P(AB)$

4. **Multiplication Theorem of Probability**

   4.1 **Case I: When events are not independent** The probability of simultaneous happening of two events A and B is equal to the probability of A multiplied by the conditional probability of B with respect to A, i.e.,

   $P(AB) = P(A)\, P(B/A)$ or
   $P(AB) = P(B)\, P(A/B)$

   Similarly, we shall have
   $P(ABC) = P(A)\, P(B/A)\, P(C/AB)$, etc.

   4.2 **Case II: When events are independent** If $A_1, A_2, \ldots, A_n$ are independent events, then $P(A_1, A_2, \ldots, A_n) = P(A_1)\, P(A_2) \ldots P(A_n)$

NOTE

$\because \quad \overline{A_1 + A_2 + \ldots\ldots + A_n} = \overline{A}_1 \times \overline{A}_2 \ldots\ldots \overline{A}_n$

$P(A_1 + A_2 + \ldots + A_n) = 1 - P(\overline{A}_1 \times \overline{A}_2 \ldots \overline{A}_n)$
$= 1 - P(\overline{A}_1)\, P(\overline{A}_2) \ldots P(\overline{A}_n)$
i.e., Probability at least one of the events to happen = 1 – P (none of the events happens)

5. **Some Important Results**

   (i) Sample space of two events A and B is: $S = \{AB, \overline{A}B, A\overline{B}, \overline{A}\,\overline{B}\}$

   (ii) $P(AB) + P(\overline{A}B) + P(A\overline{B}) + P(\overline{A}\,\overline{B}) = P(S) = 1$

   (iii) P(at least one of A and B to happen) = $P(A \cup B) = P(A) + P(B) - P(A \cap B) = P(A + B) = P(A \text{ or } B)$

   (iv) P(exactly one of A or B to happen) $= P(A\overline{B}) + P(\overline{A}B) = P(A) + P(B) - 2P(A \cap B)$

   (v) $P(A \cup B) \le P(A) + P(B)$

   (vi) $P(A \cap B) \ge P(A) + P(B) - 1$

   (vii) $P(AB) \le P(A)\, P(B) \le P(A + B) \le P(A) + P(B)$

   (viii) P(at least two of them (A, B and C) occur) $= P(A \cap B) + P(A \cap C) + P(B \cap C) - 2P(A \cap B \cap C)$

   (ix) P(exactly one of A, B, C occur) $= P(A) + P(B) + P(C) - 2P(A \cap B) - 2P(A \cap C) - 2P(B \cap C) + 3P(A \cap B \cap C)$

   (x) P(at least one of A, B, C occur) $= P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

   (xi) P(exactly two of them (A, B and C) occur) $= P(A \cap B) + P(A \cap C) + P(B \cap C) - 3P(A \cap B \cap C)$

6. **Probability of At least One of the Independent Events**

   Probability of success of an event in one trial is P.

   Probability of success of an event in m trials $= P.P.P = ..m$ times $= P^m$

   Probability of failure of an event in one trial $= (1 - P)$

   Probability of failure of an event in m trial = $(1 - P)^m$

   Probability of at least one success = $1 - (1 - P)^m = 1 - P$(no-event)

   6.1 Let there be n mutually independent events $E_1, E_2, E_3, \ldots, E_n$, with respective probabilities $P_1, P_2, P_3, \ldots, P_n$. The probability of m events $E_1, E_2, \ldots, E_n$ to occur and the remaining $n - m$ events $E_{m+1}, E_{m+2}, \ldots, E_n$ not to occur is generally, $P_1, P_2, \ldots, P_m (1 - P_{m+1})(1 - P_{m+2}) \ldots (1 - P_n)$

   The probability of the failure of all the n-event is given by:

   $(1 - P_1)(1 - P_2) \ldots (1 - P_n)$

   The probability that at least one of the n-event must occur is equal to $[1 - \{(1 - P_1)(1 - P_2) \ldots (1 - P_n)\}]$

SOLVED SUBJECTIVE PROBLEMS (XII BOARD (C.B.S.E./STATE)): FOR BETTER UNDERSTANDING AND CONCEPT BUILDING OF THE TOPIC

1. A bag contains 50 tickets numbered 1, 2, 3,..., 50. Of which 5 are drawn at random and arranged in ascending order of magnitude ($x_1 < x_2 < x_3 < x_4 < x_5$). Find the probability that $x_3 = 30$. ***[CBSE-2002]***

Solution

Five ticket out of 50 can be drawn in ${}^{50}C_5$ ways.

$\therefore$ Total number of elementary events $= {}^{50}C_5$

Since $x_1 < x_2 < x_3 < x_4 < x_5$ and $x_3 = 30$, x_1, $x_2 < 30$, i.e., x_1 and x_2 should come from tickets numbered 1 to 29, and this may happen in ${}^{29}C_2$ ways. The remaining two, i.e., x_4, $x_5 > 30$, should come from 20 tickets numbered from 31 to 50 in ${}^{20}C_2$ ways.

$\therefore$ Favourable number of elementary events

$$= {}^{29}C_2 \times {}^{20}C_2$$

Hence, required probability

$$= \frac{{}^{29}C_2 \times {}^{20}C_2}{{}^{50}C_5} = \frac{551}{15{,}134}$$

2. Four cards are drawn at a time from a pack of 52 playing cards. Find the probability of getting all the 4 cards of the same suit. ***[CBSE-1993]***

Solution

Since 4 cards can be drawn at a time from a pack of 52 cards in ${}^{52}C_4$ ways, total number of elementary events $= {}^{52}C_4$.

Consider the following events:

A = Getting all spade cards;

B = Getting all club cards;

C = Getting all diamond cards and

D = Getting all heart cards.

Then A, B, C and D are mutually exclusive events such that

$$P(A) = \frac{{}^{13}C_4}{{}^{52}C_4}, P(B) = \frac{{}^{13}C_4}{{}^{52}C_4}, P(C) = \frac{{}^{13}C_4}{{}^{52}C_4}$$

and $P(D) = \dfrac{{}^{13}C_4}{{}^{52}C_4}$

Now, required probability $= P(A \cup B \cup C \cup D) = P(A) + P(B) + P(C) + P(D)$

[by add. theorem]

$$= 4\left(\frac{{}^{13}C_4}{{}^{52}C_4}\right) = \frac{44}{4165}$$

3. Assume that each child born is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that: ***[NCERT]***

(i) the youngest is a girl?
(ii) at least one is a girl?

Solution

Let the first child be denoted by capital letter and the second (younger one) by a small letter.

The sample space in this case is $S = \{Bb, Bg, Gb, Gg\}$, which contains four equally likely sample points. Let E: 'both children are girls', then $E = \{Gg\}$.

(i) Let F: 'the youngest is a girl', then $F = \{Bg, Gg\}$

$\Rightarrow \quad E \cap F = \{Gg\} = E$

$\therefore$ Required probability $= P(E/F)$

$$= \frac{P(E \cap F)}{P(F)} = \frac{P(E)}{P(F)} = \frac{1/4}{2/4} = \frac{1}{2}$$

(ii) Let F: 'at least one is a girl', then $F = \{Bg, Gb, Gg\}$

$\Rightarrow \quad E \cap F = \{Gg\} = E$

$\therefore$ Required probability $= P(E/F)$

$$= \frac{P(E \cap F)}{P(F)} = \frac{P(E)}{P(F)} = \frac{1/4}{3/4} = \frac{1}{3}$$

4. A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event, 'number is even' and B be the event 'number is red'. Are A and B independent? *[NCERT]*

Solution

Here, the sample space is $S = \{1, 2, 3, 4, 5, 6\}$

Also, A: 'number is even'

and B: 'number is red'

i.e., $A = \{2, 4, 6\}$, $B = \{1, 2, 3\}$ and $A \cap B = \{2\}$.

Now $P(A) = \frac{3}{6} = \frac{1}{2}$, $P(B) = \frac{3}{6} = \frac{1}{2}$

and $P(A \cap B) = \frac{1}{6} \neq \frac{1}{2} \times \frac{1}{2}$

$\Rightarrow$ $P(A \cap B) \neq P(A)\, P(B)$

$\Rightarrow$ A and B are not independent.

5. Given two independent events A and B such that $P(A) = 0.3$, $P(B) = 0.6$. Find: *[NCERT]*

(i) $P(A$ and $B)$ (ii) $P(A$ and not $B)$

(iii) $P(A$ or $B)$ (iv) $P($neither A nor $B)$

Solution

(i) $P(A \text{ and } B) = P(A \cap B) = P(A) \times P(B)$
$= 0.3 \times 0.6 = 0.18$

(ii) $P(A \text{ and not } B) = P(A \cap B^c)$
$= P(A) \times P(B^c)$

($\because$ A and B are independent,

$\therefore$ A and B^c are also independent)

$= (0.3)(1 - P(B)) = (0.3)(1 - 0.6)$

$= 0.3 \times 0.4 = 0.12$

(iii) $P(A \text{ or } B) = P(A \cup B)$
$= P(A) + P(B) - P(A \cap B)$
$= P(A) + P(B) - P(A) \times P(B)$
$= 0.3 + 0.6 - 0.3 \times 0.6$
$= 0.9 - 0.18 = 0.72$

(iv) $P(\text{neither } A \text{ nor } B) = P(A^c \text{ and } B^c)$
$= P(A^c \cap B^c) = P((A \cup B)^c) = 1 - P(A \cup B)$
$= 1 - 0.72 = 0.28$ (using part (iii))

Alternatively, $P($neither A nor $B)$

$= P(A^c \cap B^c)$

$= P(A^c)\, P(B^c)$

($\because$ A and B are independent,

$\therefore$ A^c and B^c are also independent)

$= (1 - P(A))(1 - P(B))$

$= (1 - 0.3)(1 - 0.6)$

$= 0.7 \times 0.4 = 0.28$

6. One card is drawn at random from a well-shuffled deck of 52 cards. In which of the following cases are the events E and F independent? *[NCERT]*

(i) E: 'The card drawn is a spade.'
F: 'The card drawn in an ace.'

(ii) E: 'The card drawn is black.'
F: 'The card drawn is a king.'

(iii) E: 'The card drawn in a king or queen.'
F: 'The card drawn is a queen or jack.'

Solution

(i) Here, $P(E) = P($card drawn is a spade$)$

$= \frac{13}{52} = \frac{1}{4}$ and

$P(F) = P(\text{card drawn is an ace}) = \frac{4}{52} = \frac{1}{13}$

Also, $E \cap F$: 'card drawn is an ace of spade S'

$\Rightarrow$ $P(E \cap F) = \frac{1}{52} = \frac{1}{4} \times \frac{1}{13} = P(E)\, P(F)$

$\Rightarrow$ E and F are independent.

(ii) Here, $P(E) = P($card drawn is black$)$

$= \frac{26}{52} = \frac{1}{2}$ and

$P(F) = P(\text{card drawn is a king}) = \frac{4}{52} = \frac{1}{13}$

Also, $E \cap F$: 'card drawn is a black king'

$\Rightarrow$ $P(E \cap F) = \frac{2}{52} = \frac{1}{26} = \frac{1}{2} \times \frac{1}{13}$

$\Rightarrow$ $P(E \cap F) = P(E)\, P(F)$

$\Rightarrow$ E and F are independent

(iii) Here, $P(E) = P($card drawn is a king or a queen$)$

$= \frac{4+4}{52} = \frac{8}{52} = \frac{2}{13}$ and

$P(F) = P(\text{card drawn is a queen or a jack})$

$$= \frac{4+4}{52} = \frac{8}{52} = \frac{2}{13}$$

Also, $E \cap F$: 'card drawn is a queen'

$$\Rightarrow \quad P(E \cap F) = \frac{4}{52} = \frac{1}{13} \neq \frac{2}{13} \times \frac{2}{13}$$

$\Rightarrow \quad P(E \cap F) \neq P(E)\, P(F)$

$\Rightarrow \quad E$ and F are not independent.

7. If each element of a second-order determinant is either 0 or 1, what is the probability that the value of the determinant is positive? (Assume that the individual entries of the determinant are chosen independently, each value being assumed with probability 1/2.) *[NCERT]*

Solution

The only determinants of the said type are

$$\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} \text{ and } \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}$$

Since each entry of the above determinant can be selected with probability 1/2, required probability

$$= 3\left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) = \frac{3}{16}$$

8. A couple has two children. Find the probability that both children are males if it is known that at least one of the children is male.

Solution

Sample space = $\{MF, FM, MN\}$
Probability = 1/3

UNSOLVED SUBJECTIVE PROBLEMS (XII BOARD (C.B.S.E./STATE)): SOLVE THESE PROBLEMS TO GRASP THE TOPIC

EXERCISE 1

1. A man who is 40 years old, odds against his living up to 65 years is 9 : 7 and B who is 50 years and odds against his living up to 75 years is 3 : 2. Find the probability that at least one of them is alive 25 years hence.

2. Probability of Mohan's failing in C.A. is 20% and in M.Com. is 10%. What is the probability of Mohan's failing in at least one in C.A. and M. Com.

3. If the probability of happening of one event is 0.40 and that of the other event is 0.30, then find the probability that both of them happen simultaneously.

4. The probability of A winning a race is 1/6 and that of B winning the same race is 1/8. Find the probability that neither A nor B wins the race.

5. A coin is tossed six times. Find the probability of getting head in even number. (O is taken an even number.) *[CBSE-91]*

6. In a simultaneous throw of a pair of dice, find the probability of getting: *[MP-2001, CBSE-96, 98, 2007, HSB-97]*

(i) neither 9 or 11 as the sum of the numbers on the aces.

(ii) a sum less than 6.

7. If the probability of a horse A winning a race is 1/7 and the probability of a horse B winning the same race is 1/4, what is the probability that one of the horse will win? *[MP-2001]*

8. A policeman fires four bullets on a dacoit. The probability that the dacoit will be killed by one bullet is 0.6. What is the probability that the dacoit is still alive? *[HSB-92]*

9. Eight letters to each of which corresponds an envelope are placed in the envelopes at random. What is the probability that all letters are not placed in the right envelopes?

EXERCISE 2

1. A and B are two independent events. Probability of happening of both the events simultaneously is 1/6. Probability of happening of none of the events is 1/3. Find the probability that A will happen. *[IIT-84]*

2. Two dice are thrown together. What is the probability that the sum of the numbers on the two faces is neither divisible by 3 nor by 4. *[CBSE-96, 2005 (Foreign)-I]*
3. A bag contains 5 white, 7 red and 4 black balls. If 4 balls are drawn one by one with replacement, what is the probability that none is white? *[CBSE-93]*
4. Two unbiased dice are thrown. Find the probability that: *[CBSE-98]*
   (i) neither a doublet nor a total of 8 will appear.
   (ii) the sum of the numbers obtained on the two dice is neither a multiple of 2 nor a multiple of 3.
5. A and B are two horses participating in a race. The probability of A's win is 1/5 and that of B's win is 1/6. Find the probability that one of them will win. *[MP-2000, 2003]*
6. Two cubical dice are thrown simultaneously. Find the probability of getting an odd number on the first dice or a sum 9. *[MP-93, 97, 99, 2001]*
7. Two cards are drawn at random from a well-shuffled pack of 52 cards. What is the probability that either both red or both are aces? *[MP-2004 (A),2007]*
8. Two cards are drawn from a well-shuffled pack of 52 cards without replacement. What is the probability that one is a queen of red colour and the other is a king of black colour. *[CBSE-99]*

ANSWERS

EXERCISE 1

1. 53/80
2. 28%
3. 0.12
4. $\frac{17}{24}$
5. 1/2
6. (i) 5/6
   (ii) 5/18
7. 11/28
8. 0.0256
9. $1-\frac{1}{8!}$

EXERCISE 2

1. 1/3 or 1/2
2. 4/9
3. $(11/16)^2$
4. (i) $\frac{13}{18}$
   (ii) $\frac{1}{3}$
5. 11/30
6. 5/9
7. $\frac{55}{221}$
8. 8/663

SOLVED OBJECTIVE PROBLEMS: HELPING HAND

1. A coin is tossed until a head appears or until the coin has been tossed five times. If a head does not occur on the first two tosses, then the probability that the coin will be tossed five times is *[CEE-1993]*
   (a) 1/2 (b) 3/5
   (c) 1/4 (d) 1/3

Solution

(c) P(tail in third) × P(tail in fourth)

$$=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$$

2. The sum of two positive numbers is 100. The probability that their product is greater than 1000 is *[RPET-1999]*
   (a) 7/9 (b) 7/10
   (c) 2/5 (d) None of these

Solution

(a) Required pair = (12, 88), (13, 87), ..., (88, 12)

Total number of such pairs are 77 and total number of pairs (for which sum is 100) = 99

∴ Required probability $=\frac{7}{9}$

3. If any four numbers are selected and they are multiplied, then the probability that the last digit will be 1, 3, 5 or 7 is

[RPET-2002]

(a) $\frac{4}{625}$ (b) $\frac{18}{625}$

(c) $\frac{16}{625}$ (d) None of these

Solution

(c) Total number of digits in any number at the unit place is 10.

$\therefore n(S) = 10$

To get the last digit in product is 1, 3, 5 or 7, it is necessary the last digit in each number must be 1, 3, 5 or 7.

$n(A) = 4,$

$\therefore P(A) = \frac{4}{10} = \frac{2}{5}$

$\therefore$ Required probability $= \left(\frac{2}{5}\right)^4 = \frac{16}{625}$

4. If Mohan has 3 tickets of a lottery containing 3 prizes and 9 blanks, then his chances of winning a prize are

(a) 34/55 (b) 21/55

(c) 17/55 (d) None of these

Solution

(a) Mohan can get 1, 2 or 3 prizes and his chance of failure means he gets no prize. Number of total ways $= {}^{12}C_3 = 220$. Favourable number of ways to be failure $= {}^9C_3 = 84$

Hence required probability

$$= 1 - \frac{84}{220} = \frac{34}{55}$$

5. A committee of 5 is to be chosen from a group of 9 people. The probability that a certain married couple will either serve together or not at all is *[CEE-1993]*

(a) 1/2 (b) 5/9

(c) 4/9 (d) 2/9

Solution

(c) Required probability

$$= \frac{{}^7C_3}{{}^9C_5} + \frac{{}^7C_5}{{}^9C_5} = \frac{56}{126} = \frac{4}{9}$$

6. n cadets have to stand in a row. If all possible permutations are equally likely, then the probability that two particular cadets stand side by side is

(a) $2/n$ (b) $1/n$

(c) $2/(n-1)!$ (d) None of these

Solution

(a) Total number of ways $= n!$

Favourable cases $= 2(n-1)!$

Hence required probability

$$= \frac{2(n-1)!}{n!} = \frac{2}{n}$$

7. Two numbers are selected at random from 1, 2, 3, …, 100 and are multiplied. The probability correct to two places of decimals that the product thus obtained is divisible by 3 is

[Kurukshetra CEE-1998]

(a) 0.55 (b) 0.44 (c) 0.22 (d) 0.33

Solution

(a) Total number of cases obtained by taking multiplication of only two numbers out of 100 $= {}^{100}C_2$.

Out of 100 (1, 2, …, 100) given numbers, there are the numbers 3, 6, 9, 12, ..., 99, which are 33 in number such that when any one of these is multiplied with any one of the remaining 67 numbers or any two of these 33 are multiplied, the resulting product is divisible by 3. Then the pair of number whose products is divisible by 3

$$3 = {}^{33}C_1 \times {}^{67}C_1 + {}^{33}C_2.$$

Hence the required probability

$$= \frac{{}^{33}C_1 \times {}^{67}C_1 + {}^{33}C_2}{{}^{100}C_2} = \frac{2739}{4950} = 0.55$$

8. Three ships A, B and C sail from England to India. If the ratio of their arriving safely are 2 : 5, 3 : 7 and 6 : 11, respectively, then the probability of all the ships for arriving safely is *[Pb. CET-2000]*

(a) $\frac{18}{595}$ (b) $\frac{6}{17}$

(c) $\frac{3}{10}$ (d) $\frac{2}{7}$

Solution

(a) We have ratio of the ships A, B and C for arriving safely are 2:5, 3:7 and 6:11, respectively. The probability of ship A for arriving safely

$$= \frac{2}{2+5} = \frac{2}{7}$$

Similarly, for $B = \frac{3}{3+7} = \frac{3}{10}$

and for $C = \frac{6}{6+11} = \frac{6}{17}$

$\therefore$ Probability of all the ships for arriving safely $= \frac{2}{7} \times \frac{3}{10} \times \frac{6}{17} = \frac{18}{595}$.

9. In a city, 20% persons read English newspaper, 40% read Hindi newspaper and 5% read both newspapers. The percentage of non-readers of either paper is

(a) 60% (b) 35% (c) 25% (d) 45%

Solution

(d) $P(A) = \frac{1}{5}$, $P(B) = \frac{2}{5}$ and $P(A \cap B) = \frac{1}{20}$

Then $P(\overline{A} \cap \overline{B}) = 1 - P(A \cup B)$

$= 1 - \left[\frac{1}{5} + \frac{2}{5} - \frac{1}{20}\right] = \frac{9}{20}$, i.e., 45%.

10. A, B, C are any three events. If $P(S)$ denotes the probability of S happening, then $P(A \cap (B \cup C)) =$ *[EAMCET-1994]*

(a) $P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C)$

(b) $P(A) + P(B) + P(C) - P(B)\, P(C)$

(c) $P(A \cap \mathrm{B}) + P(A \cap \mathrm{C}) - P(A \cap \mathrm{B} \cap \mathrm{C})$

(d) None of these

Solution

(c) $P[A \cap (B \cup C)] = P[(A \cap B) \cup (A \cap C)]$

$= P(A \cap B) + P(A \cap C) - P[(A \cap B) \cap (A \cap C)]$

$= P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)$

11. Let E_1, E_2, E_3 be three arbitrary events of a sample space S. Which of the following statements are correct? *[Pb. CET-2004]*

(a) P(only one of them occurs) $= P(\overline{E}_1 E_2 E_3 + E_1 \overline{E}_2 E_3 + E_1 E_2 \overline{E}_3)$

(b) P(none of them occurs) $= P(\overline{E}_1 + \overline{E}_2 + \overline{E}_3)$

(c) P(at least one of them occurs) $= P(E_1 + E_2 + E_3)$

(d) P(all the three occurs) $= P(E_1 + E_2 + E_3)$ where $P(E_1)$ denotes the probability of E_1 and $\overline{E}_1$ denotes complement of E_1.

Solution

(c) P(only one of them occurs)

$= P(E_1\overline{E}_2\overline{E}_3 + \overline{E}_1 E_2\overline{E}_3 + \overline{E}_1 \overline{E}_2 E_3)$

$\neq P(\overline{E}_1 E_2 E_3 + E_1 \overline{E}_2 E_3 + E_1 E_2 \overline{E}_3)$

$\therefore$ (a) is incorrect.

P (none of them occurs)

$= P(\overline{E}_1 \cap \overline{E}_2 \cap \overline{E}_3) \neq (\overline{E}_1 + \overline{E}_2 + \overline{E}_3)$

$\therefore$ (b) is not correct.

P (at least one of them occurs)

$= P(E_1 \cup E_2 \cup E_3) = P(E_1 + E_2 + E_3)$

$\therefore$ (c) is correct.

P (all the three occurs)

$= P(E_1 \cap E_2 \cap E_3) \neq P(E_1 + E_2 + E_3)$

$\therefore$ (d) is not correct.

12. In a certain population 10% of the people are rich, 5% are famous and 3% are rich and famous. The probability that a person picked at random from the population is either famous or rich but not both is equal to *[UPSEAT-2004]*

(a) 0.07 (b) 0.08

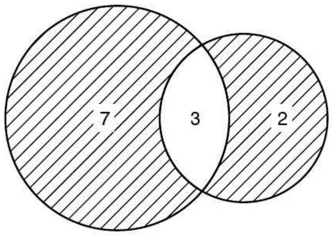
(c) 0.09 (d) 0.12

Solution

(c) Here, $P(R) = \dfrac{10}{100} = 0.1$

$P(F) = \dfrac{5}{100} = 0.05$

$P(F \cap R) = \dfrac{3}{100} = 0.03$



$\therefore$ Required probability

$= P(R) + P(F) - 2P(F \cap R)$

$= 0.1 + 0.05 - 2(0.03) = 0.09$

13. If E and F are events with $P(E) \leq P(F)$ and $P(E \cap F) > 0$, then ***[IIT-1998]***

(a) Occurrence of $E \Rightarrow$ Occurrence of F

(b) Occurrence of $F \Rightarrow$ Occurrence of E

(c) Non-occurrence of $E \Rightarrow$ Non-occurrence of F

(d) None of the above implications holds.

Solution

(d) $P(E) \leq P(F) \Rightarrow n(E) \leq n(F)$

$P(E \cap F) > 0 \Rightarrow E \cap F \neq \phi$

These do not mean that E is a sub-set of F or F is a sub-set of E, i.e., $E \subseteq F$ or $F \subseteq E$ or $\overline{E} \subseteq \overline{F}$.

14. An anti-aircraft gun take a maximum of four shots at an enemy plane moving away from it. The probability of hitting the plane at the first, second, third and fourth shot are 0.4, 0.3, 0.2 and 0.1, respectively. The probability that the gun hits the plane is ***[CEE-1993; IIT Screening]***

(a) 0.25 (b) 0.21
(c) 0.16 (d) 0.6976

Solution

(d) Let $p_1 = 0.4, p_2 = 0.3, p_3 = 0.2$ and $p_4 = 0.1$

P(the gun hits the plane) $= P$(the plane is hit atleast once)

$= 1 - P$(the plane is hit in none of the shots)

$= 1 - (1 - p_1)(1 - p_2)(1 - p_3)(1 - p_4) = 0.6976$

15. If $P(A) = 0.3$, $P(B) = 0.4$, $P(C) = 0.8$, $P(AB) = 0.08$, $P(AC) = 0.28$, $P(ABC) = 0.09$, $P(A + B + C) \geq 0.75$ and $P(BC) = x$, then ***[IIT-1983]***

(a) $0.23 \leq x \leq 0.48$ (b) $0.32 \leq x \leq 0.84$
(c) $0.25 \leq x \leq 0.73$ (d) None of these

Solution

(d) There will be no x because $P(AB)$ can never be less than $P(ABC)$.

16. A rifle man is firing at a distant target and has only 10% chance of hitting it. The minimum number of rounds he must fire in order to have 50% chance of hitting it at least once is ***[Kurukshetra CEE-1998]***

(a) 7 (b) 8 (c) 9 (d) 6

Solution

(a) The probability of hitting in one shot

$$= \frac{10}{100} = \frac{1}{10}$$

If he fires n shots, the probability of hitting at least once

$$= 1 - \left(1 - \frac{1}{10}\right)^n = 1 - \left(\frac{9}{10}\right)^n = \frac{1}{2}$$

(from the question)

$$\therefore \left(\frac{9}{10}\right)^n = \frac{1}{2}$$

$\therefore n\{2 \log_{10} 3 - 1\} = -\log_{10} 2$

$$n = \left\{\frac{\log_{10} 2}{1 - 12 \log_{10} 3} = \frac{0.3010}{1 - 2 \times 0.4771} = 6.5\right.$$

(nearly)

$\therefore$ For 6 shots, the probability is about 53%, while for 7 shots it is nearly 48%.

17. Probability that a student will succeed in the IIT entrance test is 0.2 and that he will succeed in the AIEEE is 0.5. If the probability that he will be successful at both the places is 0.3, then the probability that he does not succeed at both the places is

(a) 0.4 (b) 0.3 (c) 0.2 (d) 0.6

Solution

(d) Let A denotes the event that the student is selected in the IIT entrance test and B denotes the event that he is selected in the AIEEE. Then

$P(A) = 0.2$, $P(B) = 0.5$ and $P(A \cap B) = 0.3$

Required probability

$= P(\bar{A} \cap B) = 1 - P(A \cup B)$

$= 1 - (P(A) + P(B) - P(A \cap B))$

$= 1 - (0.2 + 0.5 - 0.3) = 0.6$

18. Consider two events A and B such that

$$P(A) = \frac{1}{4}, P\left(\frac{B}{A}\right) = \frac{1}{2}, P\left(\frac{A}{B}\right) = \frac{1}{4}.$$

For each of the following statements, which is true

I. $P\left(\frac{A^C}{B^C}\right) = \frac{3}{4}$

II. The events A and B are mutually exclusive

III. $P(A/B) + P(A/B^c) = 1$ ***[AMU-2000]***

(a) I only (b) I and II

(c) I and III (d) II and III

Solution

(a) $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} \Rightarrow \frac{1}{2} = \frac{P(A \cap B)}{1/4}$

$\Rightarrow \quad P(A \cap B) = \frac{1}{8}$

Hence events A and B are not mutually exclusive.

$\therefore$ Statement II is incorrect.

$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} \Rightarrow P(B) = \frac{1}{2}$

$\because P(A \cap B) = \frac{1}{8} = P(A).\ P(B)$

$\therefore$ Events A and B are independent events.

$$P\left(\frac{A^c}{B^c}\right) = \frac{P(A^c \cap B^c)}{P(B^c)} = \frac{P(A^c)P(B^c)}{P(B^c)}$$

$$= \frac{3}{4} \times \frac{1}{2} \times \frac{2}{1} = \frac{3}{4}$$

Hence statement I is correct.

Again, $P\left(\frac{A}{B}\right) + P\left(\frac{A}{B^c}\right) = \frac{1}{4} + \frac{P(A \cap B^c)}{P(B^c)}$

$$= \frac{1}{4} + \frac{P(A) - P(A \cap B)}{P(B^c)} = \frac{1}{4} + \frac{\frac{1}{4} - \frac{1}{8}}{\frac{1}{2}}$$

$$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

Hence statement III is incorrect.

19. The chance of an event happening is the square of the chance of a second event but the odds against the first are the cube of the odds against the second. The chances of the events are:

(a) $\frac{1}{9}, \frac{1}{3}$ (b) $\frac{1}{16}, \frac{1}{4}$

(c) $\frac{1}{4}, \frac{1}{2}$ (d) None of these

Solution

(a) Let p_1, p_2 be the chances of happening of the first and second event, respectively, then according to the given conditions, we have $p_1 = p_2^2$ and

$$\frac{1 - p_1}{p_1} = \left(\frac{1 - p_2}{p_2}\right)^3$$

$\Rightarrow \quad p_2 = \frac{1}{3}$ and so $p_1 = \frac{1}{9}$.

20. Three faces of a fair die are yellow, two faces red and one blue. The die is tossed three times. The probability that the colours yellow, red and blue appear in the first, second and third tosses, respectively, is ***[IIT-1992]***

(a) $\frac{1}{36}$ (b) $\frac{36}{1}$ (c) $\frac{2}{34}$ (d) $\frac{34}{2}$

Solution

(a) $P(Y)=\frac{3}{6};P(R)=\frac{2}{6};P(B)=\frac{1}{6}$

$\because$ Outcomes are independent in each toss.

$P(YRB) = P(Y) \times P(R) \times P(B)$

$$=\frac{3}{6}\times\frac{2}{6}\times\frac{1}{6}=\frac{1}{36}.$$

21. Three riflemen take one shot each at the same target. The probability of the first rifleman hitting the target is 0.4, the probability of the second rifleman hitting the target is 0.5 and the probability of the third rifleman hitting the target is 0.8. Find the probability that exactly two of them hit the target. ***[MNR-1997]***

(a) 0.4 (b) 0.54 (c) 0.44 (d) 0.14

Solution

(c) Let A, B, C be the three riflemen and we are given the probability of their hitting the right target as

$P(A) = 0.4 = 2/5$

$P(B) = 0.5 = 1/2$

$P(C) = 0.8 = 4/5$

$\therefore P(\bar{A})=\frac{3}{5},P(\bar{B})=\frac{1}{2},P(\bar{C})=\frac{1}{5}$

Now the probability that exactly two of them hit the target is $P(A \cap B \cap \bar{C})$ + $P(A \cap \bar{B} \cap C)$ + $P(\bar{A} \cap B \cap C)$

$$=\left(\frac{2}{5}\right)\left(\frac{1}{2}\right)\left(\frac{1}{5}\right)+\left(\frac{2}{5}\right)\left(\frac{1}{2}\right)\left(\frac{4}{5}\right)+\left(\frac{3}{5}\right)\left(\frac{1}{2}\right)\left(\frac{4}{5}\right)$$

$$=\frac{2+8+12}{50}=\frac{22}{50}=\frac{11}{25}=0.44$$

22. A person is assigned to 3 jobs A, B and C. The probabilities of his doing the jobs A, B, C, respectively, are p, q and 1/2. He gets the full payment only if he either does the jobs A and B or the jobs A and C. If the probability of his getting the full payment is 1/2, find the relation satisfied by p and q. ***[MNR-1996]***

(a) $p(1 + q) = 1$ (b) $q(1 + p) = 1$

(c) $p - q = 1$ (d) None of these

Solution

(a) The man will get full payment if he either does the jobs A and B or jobs A or C or does all the three jobs A, B and C. Given that the probability of getting full payment is

$$p(AB\bar{C})+p(AC\bar{B})+p(ABC)=\frac{1}{2}. \quad \ldots\ldots\text{(i)}$$

It is given that $P(A) = p$, $P(B) = q$

$\therefore p(\bar{B})=1-q,\ p(C)=\frac{1}{2}$

$\therefore p(\bar{C})=1-\frac{1}{2}=\frac{1}{2}.$

Hence from Eq. (i),

$$p\times q\times\frac{1}{2}+p\times\frac{1}{2}(1-q)+p\times q\times\frac{1}{2}=\frac{1}{2}$$

$\therefore pq + p - pq + pq = 1$

$\therefore p(1 + q) = 1$ is the required relation between p and q.

23. A and B are two independent events. The probability that both occur simultaneously is 1/6 and the probability that neither occurs is 1/3. Find the probabilities of occurrence of the events A and B separately. ***[Roorkee-2000]***

(a) $x=\frac{1}{3},\ y=\frac{1}{2}$ (b) $x=\frac{1}{2},\ y=\frac{1}{3}$

(c) $x=\frac{5}{6},\ y=\frac{1}{6}$ (d) None of these

Solution

(a) Let $P(A) = x$ and $P(B) = y$, where A and B are independent events

$\Rightarrow P(A)\times P(B)=\frac{1}{6}$

Now $P(A\cap B)=\frac{1}{6},\ P(A\cup B)'=\frac{1}{3}$

$\therefore 1-P(A\cup B)=\frac{1}{3}$

$\therefore P(A\cup B)=\frac{2}{3}$

or $P(A)+P(B)-P(A\cap B)=\frac{2}{3}$

or $x+y=\frac{2}{3}+\frac{1}{6}=\frac{5}{6}$

Also $xy=\frac{1}{6}$

Solving, we get

$x=\frac{1}{3}=P(A),\ \ y=\frac{1}{2}=P(B)$

24. A lot contains 50 defective and 50 non- defective bulbs. Two bulbs are drawn at random, one at a time, with replacement. The events A, B, C are defined as

$A=$ {The first bulb is defective.}

$B=$ {The second bulb is non-defective.}

$C=$ {The two bulbs are both defective or both non-defective.} ***[IIT-1992]***

Determine whether:

(a) A, B, C are pairwise independent.
(b) A, B, C are independent.
(c) A, B, C are pairwise dependent.
(d) A, B, C are dependent.

Solution

(i) Pairwise independent.
(ii) Not mutually independent.
We have

$$P(A)=\frac{50}{100}\times 1=\frac{1}{2};\ P(B)=1\times\frac{50}{100}=\frac{1}{2}$$

$$P(C)=\frac{50}{100}\times\frac{50}{100}+\frac{50}{100}\times\frac{50}{100}=\frac{1}{2}$$

$A\cap B$ is the event that the first bulb is defective and the second is non-defective.

$$\therefore P(A\cap B)=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$$

$A\cup B$ is the event that the first bulb is defective and the second is also defective.

$$\therefore P(A\cap C)=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$$

Similarly, $P(B\cap C)=\frac{1}{4}$

Thus, we have $P(A\cap B)=P(A)\times P(B)$; $P(A\cap C)=P(A)\times P(C)$; $P(B\cap C)=P(B)\times P(C)$;

$\therefore$ A, B and C are pairwise independent.

There is no element in $A\cap B\cap C$

$\therefore$ $P(A\cap B\cap C)=0$

$\therefore$ $P(A\cap B\cap C)\neq P(A)\times P(B)\times P(C)$

Hence A, B and C are not mutually independent.

25. If $\frac{1-3p}{2},\frac{1+4p}{3}$ and $\frac{1+p}{6}$ are the probabilities of three mutually exclusive, then the set of all values of p is ***[MNR-1992]***

(a) $(0, 1)$ (b) $\left(-\frac{1}{4},\frac{1}{3}\right)$

(c) $\left(0,\ \frac{1}{3}\right)$ (d) $(0,\infty)$

Solution

(b) Let A, B and C denote mutually exclusive and exhaustive events so that

$$P(A)=\frac{1-3p}{3},P(B)=\frac{1+4p}{3},P(C)=\frac{1+p}{6}$$

$\because$ Events are mutually exclusive.

Also $0\le P(A)\le 1, 0\le P(B)\le 1, 0\le P(C)\le 1$

$$\therefore 0\le\frac{1-3p}{2}\le 1$$

$$\Rightarrow\ -1\le -3p\le 1$$

$$\Rightarrow\ \frac{1}{3}\ge p\ge -\frac{1}{3}$$

$$\Rightarrow\ -\frac{1}{3}\le p\le\frac{1}{3}\qquad\ldots\ldots(1)$$

$$0\le\frac{1+4p}{3}\le 1$$

$$\Rightarrow\ -1\le 4p\le 2$$

$$\Rightarrow\ -\frac{1}{4}\le p\le\frac{1}{2}\qquad\ldots\ldots(2)$$

$$0 \le \frac{1+p}{6} \le 1 \Rightarrow 0 \le 1+p \le 6$$

$$\Rightarrow -1 \le p \le 5 \qquad \ldots\ldots(3)$$

Now set of values of p which satisfy all the above inequalities is $\left(-\frac{1}{4}, \frac{1}{3}\right)$.

26. For three events A, B and C, if
P (happening of exactly A or B) $= P$
P (happening of exactly B or C) $= P$
P (happening of exactly C or A) $= P$
P (happening of A, B, C together) $= P^2$
where $0 < p < 1/2$, then probability of happening of at least one of A, B, C is *[IIT-96]*

(a) $\frac{3p+2p^2}{2}$ (b) $\frac{p+3p^2}{4}$

(c) $\frac{p+3p^2}{2}$ (d) $\frac{3p+2p^2}{4}$

Solution

(a) P(happening of exactly A or B)
$= P(A) + P(B) - 2P(AB)$

$\Rightarrow \quad P = P(A) + P(B) - 2P(AB) \qquad \ldots\ldots(1)$

Similarly, $P = P(B) + P(C) - 2P(BC) \ldots\ldots(2)$

$p = P(C) + P(A) - 2P(CA) \qquad \ldots\ldots(3)$

$(1) + (2) + (3)$

$\Rightarrow \quad P(A) + P(B) + P(C) - P(AB) - P(BC) - P(CA) = 3p/2 \qquad \ldots\ldots(4)$

Also as given, $P(ABC) = P^2 \qquad \ldots\ldots(5)$

$\therefore$ Required probability $= P(A + B + C)$
$= P(A) + P(B) + P(C) + P(ABC) - P(AB) - P(BC) - P(CA)$

$$= 3p/2 + p^2 = \frac{3p+2p^2}{2}$$

27. In a throw of three dice, if they show different numbers then the probability that at least one will show 6 will be *[DCE-95]*

(a) 5/6 (b) 5/18
(c) 13/18 (d) 1/2

Solution

(d) Total cases of occurring different numbers on three dice $= 6 \times 5 \times 4 = 120$

In these cases, total number of cases in which no dice shows $6 = 5 \times 4 \times 3 = 60$

$\therefore$ Probability that no dice shows $6 = \frac{60}{120} = \frac{1}{2}$

$\Rightarrow$ Probability that at least one shows $6 = 1 - \frac{1}{2} = \frac{1}{2}$.

28. A bag contains 8 white and 6 red balls. Five balls are drawn from it at random. The probability that 3 or more balls are white will be: *[PET (Raj.)-96]*

(a) 317/1001 (b) 658/1001
(c) 210/1001 (d) None

Solution

(b) Favourable cases are: (3 white + 2 red), (4 white + 1 red), 5 white.

$\therefore$ Probability

$$= \frac{\left({}^8C_3 \times {}^6C_2\right) + \left({}^8C_4 \times {}^6C_1\right) \times \left({}^8C_5\right)}{{}^{14}C_5} = \frac{658}{1001}$$

29. If $0 < P(A) < 1$, $0 < P(B) < 1$ and $P(A \cup B) = P(A) + P(B) - P(A)P(B)$, then *[IIT, (Screening)-95; Haryana (CEE)-98]*

(a) $P(B/A) = P(B) - P(A)$
(b) $P(\overline{A} \cup \overline{B}) = P(\overline{A}) + P(\overline{B})$
(c) $P(A \cup B) = P(\overline{A})\, P(\overline{B})$
(d) $P(A/B) = P(A)$

Solution

(c) $\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore P(A \cap B) = P(A)\, P(B)$

$\Rightarrow \quad A, B$ are independent.

Now $P(A \cup B) = P(\overline{A} \cap \overline{B}) = P(\overline{A})\, P(\overline{B})$

30. A cricket team plays m number of matches in winter and wins x matches. Further, it plays n number of matches in summer and wins y matches. Its winning probability in both the seasons is *[MPPET-2007]*

(a) $\frac{x}{m} \times \frac{y}{n}$ (b) $\frac{x}{m} - \frac{y}{n}$

(c) $\frac{x+y}{m+n}$ (d) None of these

Solution

(c) $\dfrac{x+y}{m+n}$

31. The probabilities of three events A, B and C are $P(A) = 0.6$, $P(B) = 0.4$ and $P(C) = 0.5$, respectively.

If $P(A \cup B) = 0.8$, $P(A \cap C) = 0.3$,

$P(A \cap B \cap C) = 0.2$ and

$P(A \cup B \cup C) \geq 0.85$, find $P(B \cap C)$.

[Roorkee-96]

(a) $P(B \cap C) \geq 0.85$
(b) $P(B \cap C) \leq 0.2$
(c) $P(B \cap C) \geq 0.35$
(d) $0.2 \leq P(B \cap C) \leq 0.35$

Solution

(d) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.8 = 0.6 + 0.4 - P(A \cap B)$

$\therefore P(A \cap B) = 0.2$

Now $P(A \cup B \cup C) = S_1 - S_2 + S_3$

$= (0.6 + 0.4 + 0.5) - (0.2 + P(B \cap C) + 0.3) + 0.2$

$= 1.5 - 0.3 - P(B \cap C)$

We know $0.85 \leq P(A \cup B \cup C) \leq 1$ or

$0.85 \leq 1.2 - P(B \cap C) \leq 1$

$\therefore 0.2 \leq P(B \cap C) \leq 0.35$

32. A man alternatively tosses a coin and throws a die beginning with the coin. The probability that he gets a head on the coin before he gets a 5 or 6 on the dice is *[Aligarh-98]*

(a) 1/2 (b) 1/3 (c) 1/4 (d) 1/6

Solution

(d) Probability of coming $H = 1/2$

Probability of coming 5 or 6 $= \dfrac{2}{6} = \dfrac{1}{3}$

$\therefore$ Required probability $= \dfrac{1}{2} \times \dfrac{1}{3} = \dfrac{1}{6}$

33. For two events A and B, $P(A \cap B)$ is not

[UPSEAT-2005]

(a) less than $P(A) + P(B)$
(b) greater than $P(A) + P(B) - 1$
(c) equal to $P(A) + P(B) + P(A \cup B)$
(d) equal to $P(A) + P(B) - P(A \cup B)$

Solution

(c) $\because P(A \cap B) = P(A) + P(B) - P(A \cup B)$ ......(1)

$\therefore P(A \cap B) \neq P(A) + P(B) + P(A \cup B)$

unless $P(A) = 0 = P(B)$

Further from Eq. (1) $\Rightarrow$ $P(A \cap B) \leq P(A) + P(B)$

Also max. $P(A \cup B) = 1$, so

$P(A \cap B) \geq P(A) + P(B) - 1$

Hence only (c) is correct.

34. Four positive integers are taken at random and are multiplied together. The probability that the product ends in an odd digit other than 5 is

[DCE-95]

(a) 2/5 (b) 3/5
(c) 16/625 (d) 609/625

Solution

(c) The last digit in the product is an odd digit other than 5 may be any one of 1, 3, 7, 9. This shows that the product will not be divisible by 2 or 5. Now last digit in any number may be 0, 1, 2, ..., 8, 9.

The number will be divisible by 2 or 5. Its last digit is one of 0, 2, 4, 5, 6, 8. Hence the probability that a number is divisible by 2 or 5 is 6/10 = 3/5.

$\therefore$ The probability that the number is not divisible by 2 or 5 = 1 – 3/5 = 2/5.

Now the product of 4 numbers will not be divisible by 2 or 5 if none of the constituent numbers should be divisible by 2 or 5.

$\therefore$ Required probability $= \left(\dfrac{2}{5}\right)^4 = \dfrac{16}{625}$.

35. Odds 8 to 5 against a person who is 40 years old living till he is 70 and 4 to 3 against another person now 50 till he will be living 80. Probability that one of them will be alive next 30 years is *[MNR-1986]*

(a) $\dfrac{59}{91}$ (b) $\dfrac{44}{91}$

(c) $\dfrac{51}{91}$ (d) $\dfrac{32}{91}$

Solution

(b) Probability (Person A will die in 30 years) $= \frac{8}{8+5}$

$P(A) = \frac{8}{13} \quad P(\bar{A}) = \frac{5}{13}$

Similarly, $P(B) = \frac{4}{7}$

$\Rightarrow \quad P(\bar{B}) = \frac{3}{7}$

There are two ways in which one person is alive after 30 years. $\bar{A}B$ and $A\bar{B}$ and event are independent. So, required probability

$= P(\bar{A}) \times P(B) + P(A) \times P(\bar{B})$

$= \frac{5}{13} \times \frac{4}{7} + \frac{8}{13} \times \frac{3}{7} = \frac{44}{91}$

36. Three numbers are chosen at random without replacement from $\{1, 2, 3, ..., 10\}$. The probability that the minimum of the chosen numbers is 3 or their maximum is 7 is ***[IIT-97]***

(a) 7/40 (b) 3/10
(c) 11/40 (d) None of these

Solution

(c) Let A be the event that minimum number selected is 3 and B be the event that maximum number selected is 7 then

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ ……(1)

Total number of ways in which 3 numbers can be chosen from the given 10 numbers $= {}^{10}C_3 = 120$.

Now minimum of the chosen 3 numbers is 3 if one of them is 3 and the remaining 2 from 4, 5, 6, 7, 8, 9, 10. This can be obtained in ${}^{7}C_2 = 21$ ways.

$\therefore P(A) = 21/120$ ……(2)

Also, maximum of the chosen 3 numbers is 7 if one of them is 7 and the remaining 2 from 1, 2, 3, 4, 5, 6. This can be obtained in ${}^{6}C_2 = 15$ ways.

$\therefore P(B) = \frac{15}{120}$ ……(3)

Further favourable cases for $A \cap B = {}^{3}C_1 = 3$ because $A \cap B$ is possible if one number is chosen from 4, 5, 6 with 3 and 7.

$\therefore P(A \cap B) = \frac{3}{120} = \frac{1}{40}$ ……(4)

So from Eqs. (1) to (4), we have

$\therefore$ Required probability $= \frac{21}{120} + \frac{15}{120} - \frac{3}{120}$

$= \frac{33}{120} = \frac{11}{40}$.

37. The probabilities that a student passes in Mathematics, Physics and Chemistry are m, p and c, respectively. On these subjects, the student has a 75% chance of passing in at least one, a 50% chance of passing in at least two and a 40% chance of passing in exactly two. Which of the following relations are true? ***[IIT-1999]***

(a) $p + m + c = \frac{19}{20}$ (b) $p + m + c = \frac{27}{20}$

(c) $pmc = \frac{1}{10}$ (d) $pmc = \frac{1}{4}$

Solution

(b, c) Let M, P and C be the events of passing in mathematics, physics and chemistry respectively.

$P(M \cup P \cup C) = \frac{75}{100} = \frac{3}{4}$

$P(M \cap P) + P(P \cap C) + P(M \cap C)$
$- 2P(M \cap P \cap C) = \frac{50}{100} = \frac{1}{2}$

$P(M \cap P) + P(P \cap C) + P(M \cap C)$
$- 2P(M \cap P \cap C) = \frac{40}{100} = \frac{2}{5}$

$\therefore m(1-p)(1-c) + p(1-m)(1-c) + c(1-m)$
$(1-p) + mp(1-c) + mc(1-p) + pc(1-m)$
$+ mpc = \frac{3}{4}$

$\Rightarrow \quad m + p + c - mc - mp - pc + mpc = \frac{3}{4}$ ……(1)

Similarly, $mp(1-c)+pc(1-m)+mc(1-p)+mpc=\frac{1}{2}$

$\Rightarrow \quad mp+pc+mc-2mpc=\frac{1}{2}$ ......(2)

$mp\,(1-c)+pc\,(1-m)+mc\,(1-p)=\frac{2}{5}$

$\Rightarrow \quad mp+pc+mc-3mpc=\frac{2}{5}$ ......(3)

From Eqs. (2) to (3), $mpc=\frac{1}{2}-\frac{2}{5}=\frac{1}{10}$

From Eqs. (1) and (2), $m+p+c-mpc=\frac{3}{4}+\frac{1}{2}$

$\therefore\ m+p+c=\frac{3}{4}+\frac{1}{2}+\frac{1}{10}=\frac{15+10+2}{20}=\frac{27}{20}$

38. For any two events A and B in a sample space *[IIT-1991]*

(a) $P\left(\frac{A}{B}\right)\geq\frac{P(A)+P(B)-1}{P(B)}$, $P(B)\neq 0$ is always true.

(b) $P(A\cap\bar{B})=P(A)-P(A\cap B)$ does not hold.

(c) $P(A\cup B)=1-P(\bar{A})\,P(\bar{B})$, if A and B are disjoint.

(d) None of these.

Solution

(a) We know that

$P(A/B)=\frac{P(A\cap B)}{P(B)}$

Also we know that

$P(A\cup B)\leq 1$

$\Rightarrow \quad P(A)+P(B)-P(A\cap B)\leq 1$

$\Rightarrow \quad P(A\cap B)\geq P(A)+P(B)-1$

$\Rightarrow \quad \frac{P(A\cap B)}{P(B)}\geq\frac{P(A)+P(B)-1}{P(B)}$

$\Rightarrow \quad P(A/B)\geq\frac{P(A)+P(B)-1}{P(B)}$

39. Let E^C denote the complement of an event E. Let E, F, G be pairwise independent events with $P(G)>0$ and $P(E\cap F\cap G)=0$. Then $P(E^C\cap F^C\,|\,G)$ equals: *[IIT JEE-2007]*

(a) $P(E^C)+P(F^C)$ (b) $P(E^C)-P(F^C)$

(c) $P(E^C)-P(F)$ (d) $P(E)-P(F^C)$

Solution

(c) E, F, G are pairwise independent events.

$\therefore\ P(E\cap F)=P(E)\times P(F)$

$P(F\cap G)=P(F)\times P(G)$

$P(G\cap E)=P(G)\times P(E)$

$P\left(\frac{E^C\cap F^C}{G}\right)=\frac{P((E^C\cap F^C)\cap G)}{P(G)}$

$=\frac{P(G)-P(G\cap E)-P(G\cap F)}{P(G)}$

$=\frac{P(G)(1-P(E)-P(F))}{P(G)}$

$=1-P(E)-P(F)=P(E^C)-P(F)$

OBJECTIVE PROBLEMS: IMPORTANT QUESTIONS WITH SOLUTIONS

1. The probability that a leap year will have 53 Fridays or 53 Saturdays is *[MPPET-2002]*

(a) 2/7 (b) 3/7

(c) 4/7 (d) 1/7

2. A pair of dice is thrown, if 5 appears on at least one of the dice, then the probability that the sum is 10 or greater is *[MPPET-2001]*

(a) 11/36 (b) 2/9

(c) 3/11 (d) 1/12

3. Three coins are tossed together, then the probability of getting at least one head is *[RPET-01; MPPET-89]*

(a) 1/2 (b) 3/4

(c) 1/8 (d) 7/8

4. A fair coin is tossed repeatedly. If tail appears on the first four tosses, then the probability of head appearing on the fifth toss equals *[IIT-1998]*

(a) 1/2 (b) 1/32
(c) 31/32 (d) 1/5

5. Two dice are thrown simultaneously. The probability that sum is odd or less than 7 or both is
(a) 2/3 (b) 1/2
(c) 3/4 (d) 1/3

6. The probability of getting head and tail alternately in three throws of a coin (or a throw of three coins) is ***[RPET-1997]***
(a) 1/8 (b) 1/4
(c) 1/3 (d) 3/8

7. Let E and F be two independent events. The probability that both E and F happens is 1/12 and the probability that neither E nor F happens is 1/2, then ***[IIT-1993]***
(a) $P(E) = 1/3, P(F) = 1/4$
(b) $P(E) = 1/2, P(F) = 1/6$
(c) $P(E) = 1/6, P(F) = 1/2$
(d) None of these

8. A card is drawn at random from a pack of cards. What is the probability that the drawn card is neither a heart nor a king?
(a) 4/13 (b) 9/13
(c) 1/4 (d) 13/26

9. The probability that an ordinary or a non-leap year has 53 Sundays is ***[MPPET-1996]***
(a) 2/7 (b) 1/7
(c) 3/7 (d) None

10. Two dice are thrown simultaneously. What is the probability of obtaining sum of the numbers less than 11?
(a) 17/18 (b) 1/12
(c) 11/12 (d) None of these

11. The probabilities of three mutually exclusive events are 2/3, 1/4 and 1/6. The statement is
(a) True (b) False
(c) Could be either (d) Do not know

12. A coin is tossed three times. The probability of obtaining at least two heads is or Three coins are tossed all together. The probability of getting at least two heads is ***[MPPET-1995]***
(a) 1/8 (b) 3/8
(c) 1/2 (d) 2/3

13. The probability of happening an event A in one trial is 0.4. The probability that the event A happens at least once in three independent trials is
[IIT-1980; Kurukshetra CEE-1998; DCE-2001]
(a) 0.936 (b) 0.784
(c) 0.904 (d) 0.216

14. A bag contains 3 red, 7 white and 4 black balls. If 3 balls are drawn from the bag, then the probability that exactly 2 of them are of the same colour is
(a) 6/71 (b) 7/81
(c) 10/91 (d) None of these

15. A and B are tossing a coin alternatively, the first to show a head being the winner. If A starts the game, the chance of his winning is ***[MPPET-87]***
(a) 5/8 (b) 1/2
(c) 1/3 (d) 2/3

16. A five-digit number is formed by writing the digits 1, 2, 3, 4, 5 in a random order without repetitions. Then the probability that the number is divisible by 4 is ***[Orissa JEE-2003]***
(a) 3/5 (b) 18/5
(c) 1/5 (d) 6/5

17. Out of 30 consecutive numbers, 2 are chosen at random. The probability that their sum is odd is
(a) 14/29 (b) 16/29
(c) 15/29 (d) 10/29

18. Five coins whose faces are marked 2, 3 are tossed. The chance of obtaining a total of 12 is ***[MPPET-2001; Pb. CET-2000]***
(a) 1/32 (b) 1/16
(c) 3/16 (d) 5/16

19. If a committee of 3 is to be chosen from a group of 38 people; of which you are a member. What is the probability that you will be on the committee?
(a) $\left(\dfrac{38}{3}\right)$ (b) $\left(\dfrac{37}{2}\right)$
(c) $\left(\dfrac{37}{2}\right)\Big/\left(\dfrac{38}{3}\right)$ (d) $\dfrac{666}{8436}$

20. If $P(A) = 1/2, P(B) = 1/3$ and $P(A \cap B) = 7/12$, then the value of $P(A' \cap B')$ is

(a) 7/12 (b) 3/4
(c) 1/4 (d) 1/6

21. Two numbers are selected randomly from the set $S = \{1, 2, 3, 4, 5, 6\}$ without replacing one by one. The probability that minimum of the 2 numbers is less than 4 is *[IIT SC-2003]*

(a) 1/15 (b) 14/15
(c) 1/5 (d) 4/5

22. Let A and B be two events such that $\overline{P(A \cup B)} = \frac{1}{6}, P(A \cap B) = \frac{1}{4}$ and $P(\overline{A}) = \frac{1}{4}$, where $\overline{A}$ stands for complement of event A. Then events A and B are *[AIEEE-2005]*

(a) independent but not equally likely
(b) mutually exclusive and independent
(c) equally likely and mutually exclusive
(d) equally likely but not independent

23. A person has to go through three successive tests. The probability of his passing the first exam is P. The probability of passing successive tests is P or $P/2$ according as he passed the last test or not. He is selected if he passes at least two tests. Then the probability of his selection is *[IIT-2003]*

(a) $2P^2 - P^3$ (b) $P^2 - 2P^3$
(d) $2P^3 - P^2$ (d) None of these

24. If A and B are events such that $P(A \cup B) = 3/4$, $P(A \cap B) = 1/4$, $P(\overline{A}) = 2/3$, then $P(\overline{A} \cap B)$ is *[AIEEE-2002]*

(a) 5/12 (b) 3/8
(c) 5/8 (d) 1/4

25. Among 15 players, 8 are batsmen and 7 are bowlers. Find the probability that a team is chosen of 6 batsmen and 5 bowlers

(a) $\frac{{}^8C_6 \times {}^7C_5}{{}^{15}C_{11}}$ (b) $\frac{{}^8C_6 + {}^7C_5}{{}^{15}C_{11}}$
(c) 15/28 (d) None of these

26. If A speaks truth in 75% cases and B in 80% cases, then the probability that they contradict each other in starting the same statement is *[MPPET-1997, 2002]*

(a) 7/20 (b) 13/20 (c) 12/20 (d) 2/5

27. A six-faced dice is so biased that it is twice as likely to show an even number as an odd number when thrown. It is thrown twice. The probability that the sum of two numbers thrown is even is *[MPPET-1995]*

(a) 1/12 (b) 1/6
(c) 1/3 (d) 5/9

28. A die is thrown. Let A be the event that the number obtained is greater than 3. Let B be the event that the number obtained is less than 5. Then $P(A \cup B)$ is *[AIEEE-2008]*

(a) 2/5 (b) 3/5
(c) 0 (d) 1

29. If A and B are two mutually exclusive and exhaustive events with $P(B) = 3P(A)$, then what is the value of $P(\overline{B})$? *[NDA-2009]*

(a) 3/4 (b) 1/4
(c) 1/3 (d) 2/3

30. If A and B are two events such that $P(A \cup B) = \frac{5}{6}, P(A \cap B) = \frac{1}{3}$ and $P(\overline{B}) = \frac{1}{3}$, then the value of $P(A)$ is *[MPPET-2009]*

(a) 1/3 (b) 1/4
(c) 1/2 (d) 2/3

SOLUTIONS

1. (b) Total days in a leap year = 366

366 days = 7 × 52 + 2

(total possible cases for the remaining two days)

$n(s)$ = (Sun, Mon), (Mon, Tue), (Tue, Wed), (Wed, Thu), (Thu, Fri), (Fri, Sat), (Sat, Sun)

$n(s) = 7$

Favourable cases for friday or saturday $n(A) = 3$

$$\therefore P(A) = \frac{n(A)}{n(s)} = \frac{3}{7}$$

2. (d) Total outcomes = 6 × 6 = 36
Favourable outcomes = (5, 5), (5, 6), (6, 5)
$n(A) = 3$

$$\therefore P(A) = \frac{n(A)}{n(S)} \text{ or } P(A) = \frac{3}{36} = \frac{1}{12}$$

3. (d) Probability $= \dfrac{{}^3C_1 + {}^3C_2 + {}^3C_3}{2^3} = \dfrac{7}{8}$

4. (a) Appearance of head on the fifth toss does not depend on the outcomes of the first four tosses. Hence, $P(\text{head on the fifth toss}) = \dfrac{1}{2}$

5. (c) Requiredp robability

$= P(\text{less than } 7) + P(\text{odd}) - P(7 \cap \text{odd})$

$$P\,(\text{odd}) = \frac{18}{36} = \frac{1}{2}$$

$$P\,(\text{less than } 7) = \frac{15}{36} = \frac{5}{12},$$

$$P(\text{both}) = \frac{6}{36} = \frac{1}{6}$$

Hence required probability

$$= \frac{5}{12} + \frac{1}{2} - \frac{2}{12} = \frac{9}{12} = \frac{3}{4}$$

6. (b) Total probable ways = 8

Favourable number of ways = [HTH, THT]

Hence required probability $= \dfrac{2}{8} = \dfrac{1}{4}$

7. (a) Let $P(E) = x$ and $P(F) = y$

$ATQ\ P(E \cap F) = \dfrac{1}{12}$

As E and F are independent events,
$P(E \cap F) = P(E)\ P(F)$

$$\Rightarrow \quad \frac{1}{12} = xy$$

$$\Rightarrow \quad xy = \frac{1}{12} \qquad \ldots\ldots(1)$$

Also $P\overline{E} \cap \overline{F} = P(\overline{E \cup F}) = 1 - P(E \cap F)$

$$\Rightarrow \quad \frac{1}{2} = 1 - [P(E) + P(F) - P(E)\ P(F)]$$

$$\Rightarrow \quad x + y - xy = \frac{1}{2}$$

$$\Rightarrow \quad x + y = \frac{7}{12} \qquad \ldots\ldots(2)$$

Solving Eqs. (1) and (2), we get

either $x = \dfrac{1}{3}$ and $y = \dfrac{1}{4}$

or $x = \dfrac{1}{4}$ and $y = \dfrac{1}{3}$

8. (b) Required probability $= \dfrac{52-16}{52} = \dfrac{36}{52} = \dfrac{9}{13}$

9. (b) Total days in a non-leap year = 365

= 52 × 7 + 1

= 52 Sundays + 1 day

All possible days for 1 day = 7.

Favourable ways for Sunday = 1

$\therefore P(A) = 1/7$

10. (c) Favourable cases to get the sum not less than 11 are {(5, 6), (6, 6), (6, 5)} = 3

Hence favourable cases to get the sum less than 11 are (36 – 3) = 33.

So required probability $= \frac{33}{36} = \frac{11}{12}$.

11. (b) Since we have

$P(A + B + C) = P(A) + P(B) + P(C)$

$= \frac{2}{3} + \frac{1}{4} + \frac{1}{6} = \frac{13}{12}$

which is greater than 1.

Hence the statement is wrong.

12. (c) Sample space of 3 coins

(HHH), (HHT), (HTH); (THH)(HTT) (THT), (TTH), (TTT)

Total ways = 8

$2H$ or $3H$ may be taken in probability of at least two Hs.

(HHT), (HHT), (HTH), (THH)

Favourable ways = 4

P (at least two head) $= \frac{4}{8} = \frac{1}{2}$

13. (b) P(occurence of event atleast once)
= 1 – P(non-occurence of events thrice)

14. (c) Required probability $= \frac{{}^3C_3 + {}^7C_3 + {}^4C_3}{{}^{14}C_3}$

$$= \frac{1+35+4}{14 \times 13 \times 2} = \frac{40}{14 \times 26} = \frac{10}{91}$$

15. (d) Requiredpr obability

$$= \left\{ \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \ldots\ldots\ldots\ldots\ldots \right\}$$

Series are in G.P. So sum of infinite series

$$S_\infty = \frac{a}{1-r} = \frac{\frac{1}{2}}{1-\frac{1}{4}} = \frac{2}{3}$$

16. (c) To find favourable number of ways, we observe that a number is divisible by 4 if the last two digits are divisible by 4. Hence the last two digits can be {12, 24, 32, 52}

Corresponding to each of {12, 24, 32, 52} the remaining 3 places can be filled up in 3! ways.

So favourable ways = 3! × 4 = 24.

Then five-digit number is formed by taking all digits at a time 5! = 120

So, required probability $= \frac{24}{120} = \frac{1}{5}$.

17. (c) The total number of ways in which 2 integers can be chosen from the given 30 integers is ${}^{30}C_2$. The sum of the selected numbers is odd if exactly one of them is even and one is odd.

∴ Favourable number of outcomes

$$= {}^{15}C_1 \times {}^{15}C_1$$

∴ Required probability $= \frac{{}^{15}C_1 \times {}^{15}C_1}{{}^{30}C_2} = \frac{15}{29}$

18. (d) Suitable numbers for obtaining the total of 12 are (2, 2, 2, 3, 3).

Probability for obtaining 2 on coin is $P = \frac{1}{2}$

Probability for not obtaining 2 on coin is

$$q = \frac{1}{2}$$

Therefore, probability for obtaining 2 on 3 coins out of 5 coins is

$$P(A) = {}^5C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2 = \frac{10}{32} = \frac{5}{16}$$

19. (d) Required probability

$$= \frac{{}^{37}C_2}{{}^{38}C_3}$$

20. (b) $P(A' \cap B') = 1 - P(A \cup B)$

$$= 1 - \left(\frac{1}{2} + \frac{1}{3} - \frac{7}{12}\right)$$

$$= 1 - \frac{1}{4} = \frac{3}{4}$$

21. (d) Total number of selection of 2 out of 6 is

$${}^6C_2 = \frac{6 \times 5}{1 \times 2} = 15$$

Now for favourable ways, we have the following:

If smaller is chosen as 3, then greater can be 4, 5, 6, i.e., 6 choices. Similarly for 2 we have 4 choices, i.e., 3, 4, 5, 6 and for 1 we have 5 choices, i.e., 2, 3, 4, 5, 6.

$\therefore$ Total favourable choices is

$$3+4+5=12$$

Hence required probability is

$$\frac{12}{15}=\frac{4}{5}$$

22. (a) $P(\overline{A\cup B})=\frac{1}{6}$; $P(A\cap B)=\frac{1}{4}$

$$P(\bar{A})=\frac{1}{4}$$

$$\Rightarrow \quad P(A)=\frac{3}{4}$$

$$P(\overline{A\cup B})=1-P(A\cup B)$$
$$=1-P(A)-P(B)+P(A\cap B)$$

$$\Rightarrow \frac{1}{6}=\frac{1}{4}-P(B)+\frac{1}{4} \Rightarrow P(B)=\frac{1}{3}$$

Since $P(A \cap B) = P(A) \times P(B)$ and $P(A) \neq P(B)$, A and B are independent but not equally likely.

23. (a) Let E_i denote the event that the student will pass in the i^{th} exam, $i=1,2,3$, and E: denote the event that the student will qualify.

$$P(E) = P(E_1) \times P(E_2)$$
$$= P^2 + P \times (1-P)\frac{P}{2} + (1-P) \times \frac{P}{2} \times P$$
$$=\frac{2P^2+P^2-P^3+P^2-P^3}{2}$$
$$=2P^2-P^3$$

24. (a) $P(A\cup B)=\frac{3}{4}$, $\quad P(A\cap B)=\frac{1}{4}$

$$P(\bar{A})=\frac{2}{3} \qquad \Rightarrow \quad P(A)=\frac{1}{3}$$

$$\therefore\ P(A\cap B)=P(A)+P(B)-P(A\cup B)$$

$$\frac{1}{4}=\frac{1}{3}+P(B)-\frac{3}{4} \qquad \Rightarrow \quad P(B)=\frac{2}{3}$$

$$P(\bar{A}\cap B)=P(B)-P(A\cap B)$$
$$=\frac{2}{3}-\frac{1}{4}=\frac{8-3}{12}=\frac{5}{12}$$

25. (a) Total number of ways $= {}^{15}C_{11}$

Favourable cases $= {}^{8}C_6 \times {}^{7}C_5$

Required probability $=\frac{{}^{8}C_6 \times {}^{7}C_5}{{}^{15}C_{11}}$

26. (a) For the contradiction of A and B

A speaks true $\times$ B speaks false or A speaks false $\times$ B speaks true

$$\therefore\ P=\frac{75}{100}\times\frac{20}{100}+\frac{25}{100}\times\frac{80}{100}$$

or $P=\frac{35}{100}=\frac{7}{20}$

27. (d) Probability of an even number = 2/3 and probability of an odd number = 1/3. The sum will be even if either both times even numbers come up or both times odd numbers come up.

Required probability $=\frac{2}{3}\times\frac{2}{3}+\frac{1}{3}\times\frac{1}{3}=\frac{5}{9}$

28. (d) $A = \{4, 5, 6\}$, $B = \{1, 2, 3, 4\}$

$A \cap B = \{4\}$ where $A \cup B = \{1, 2, 3, 4, 5, 6\}$

$P(A \cup B) = 1$

29. (b) Since, A and B are mutually exclusive and exhaustive events,

$P(A \cap B) = 0$, $P(A \cup B) = 1$

we know that

$P(A \cup \mathrm{B}) = P(A) + P(B) - P(A \cap \mathrm{B})$

$\Rightarrow \quad 1 = P(A) + 3P(\mathrm{A})$

$$\Rightarrow \quad P(A)=\frac{1}{4}$$

$$\therefore\ P(B)=\frac{3}{4}$$

Hence, $P(\bar{B}) = 1 - P(B) = 1-\frac{3}{4}=\frac{1}{4}$

30. (c) $P(A) = P(A \cap B) + P(A \cup B) - P(B)$

$$=\frac{1}{3}+\frac{5}{6}-\frac{2}{3}=\frac{3}{6}=\frac{1}{2}$$

UNSOLVED OBJECTIVE PROBLEMS (IDENTICAL PROBLEMS FOR PRACTICE): FOR IMPROVING SPEED WITH ACCURACY

1. A coin is tossed four times. The probability that at least one head turns up is *[MPPET-2000]*
(a) 1/16 (b) 2/16
(c) 14/16 (d) 15/16

2. A man draws a card from a pack of 52 playing cards, replaces it and shuffles the pack. He continues this processe, until he gets a card of spade. The probability that he will fail the first two times is
(a) 9/16 (b) 1/16
(c) 9/64 (d) None of these

3. One card is drawn from each of two ordinary packs of 52 cards. The probability that at least one of them is an ace of heart is
(a) 103/2704 (b) 1/2704
(c) 2/52 (d) 2601/2704

4. The probability of happening an event A is 0.5 and that of B is 0.3. If A and B are mutually exclusive events, then the probability of happening neither A nor B is
(a) 0.6 (b) 0.2
(c) 0.21 (d) None of these

5. Two cards are drawn one by one at random from a pack of 52 cards. The probability that both of them are king is *[MPPET-1994]*
(a) 2/13 (b) 1/169
(c) 1/221 (d) 30/221

6. If a dice is thrown twice, then the probability of getting 1 in the first throw only is
(a) 1/36 (b) 3/36
(c) 5/36 (d) 1/6

7. If A and B are two events such that $P(A \cup B) = 5/6$, $P(A \cap B) = 1/3$ and $P(\overline{B}) = 1/3$, then $P(A) =$
(a) 1/4 (b) 1/3
(c) 1/2 (d) 2/3

8. A coin is tossed three times. What is the probability of getting head and tail (HTH) or tail and head (THT) alternately? *[NDA-2006]*
(a) 1/4 (b) 1/5
(c) 1/6 (d) 1/8

9. If a dice is thrown twice, the probability of occurrence of 4 at least once is *[UPSEAT-03]*
(a) 11/36 (b) 7/12
(c) 35/36 (d) None of these

10. If the probability of X to fail in the examination is 0.3 and that for Y is 0.2, then the probability that either X or Y fail in the examination is
(a) 0.5 (b) 0.44
(c) 0.6 (d) None of these

11. The chance of throwing at least 9 in a single throw with 2 dice is *[SCRA-1980]*
(a) 1/18 (b) 5/18
(c) 7/18 (d) 11/18

12. In a single throw of 2 dice what is the probability of obtaining a number greater than 7, if 4 appears on the first die?
(a) 1/3 (b) 1/2
(c) 1/12 (d) None of these

13. A coin is tossed successively three-times. The probability of getting exactly 1 head or 2 heads is
(a) 1/4 (b) 1/2
(c) 3/4 (d) None of these

14. Two cards are drawn from a pack of 52 cards. What is the probability that at least one of the cards drawn is an ace?
(a) 33/221 (b) 188/221
(c) 1/26 (d) 21/221

15. Suppose that A, B, C are events such that $P(A) = P(B) = P(C) = \frac{1}{4}, P(AB) = P(CB) = 0, P(AC) = \frac{1}{8}$, then $P(A + B) =$
(a) 0.125 (b) 0.25
(c) 0.375 (d) 0.5

16. Six cards are drawn simultaneously from a pack of playing cards. What is the probability that 3 will be red and 3 black?
(a) ${}^{26}C_6$ (b) ${}^{26}C_3/{}^{52}C_6$
(c) ${}^{26}C_3 \times {}^{26}C_3/{}^{52}C_6$ (d) 1/2

17. If $P(A) = P(B) = x$ and $P(A \cap B)$ $P(A9 \cup B9)$ 1/3, then $x =$
(a) 1/2 (b) 1/3
(c) 1/4 (d) 1/6

18. The two events A and B have probabilities 0.25 and 0.50, respectively. The probability that both A and B occur simultaneously is 0.14. Then the probability that neither A nor B occurs is ***[IIT-80; MPPET-94; UP-SEE-07]***
(a) 0.39 (b) 0.25
(c) 0.904 (d) None of these

19. The probability that a man will be alive in 20 years is 3/5 and the probability that his wife will be alive in 20 years is 2/3. Then the probability that at least one will be alive in 20 years is
(a) 13/15 (b) 7/15
(c) 4/15 (d) None of these

20. If odds against solving a question by 3 students are 2 : 1, 5 : 2 and 5 : 3, respectively, then probability that the question is solved only by one student is
(a) 31/56 (b)24/56
(c) 25/56 (d) None of these

21. The probabilities of occurrence of two events are 0.21 and 0.49, respectively. The probability that both occurs simultaneously is 0.16. Then the probability that none of the two occurs is ***[MPPET-1998]***
(a) 0.30 (b) 0.46
(c) 0.14 (d) None of these

22. In a box there are 2 red, 3 black and 4 white balls. Out of these 3 balls are drawn together. The probability of these being of the same colour is
(a) 1/84 (b) 1/21
(c) 5/84 (d) None of these

23. From a group of 7 men and 4 ladies a committee of 6 persons is formed, then the probability that the committee contains 2 ladies is
(a) 5/13 (b) 5/11
(c) 4/11 (d) 3/11

24. For any two independent events E_1 and E_2, $P\{(E_1 \cup E_2) \cap (E_1 \cap E_2)\}$ is
(a) ≤1/4 (b) >1/4
(c) ≥1/2 (d) None of these

25. A card is drawn at random from a pack of 100 cards numbered 1 to 100. The probability of drawing a number which is a square is
(a) 1/5 (b) 2/5
(c) 1/10 (d) None of these

26. In a non-leap year, the probability of getting 53 Sundays or 53 Tuesdays is
(a) 1/7 (b) 2/7
(c) 3/7 (d) 4/7 (e) 1/53

27. Let A and B be two events and $P(A') = 0.3$, $P(B) = 0.4$, $P(A \cap B') = 0.5$, then $P(A \cup B')$ is ***[Orissa JEE-2005]***
(a) 0.5 (b) 0.8
(c) 1 (d) 0.1

28. Three letters are to be sent to different persons and addresses on the three envelopes are also written. Without looking at the addresses, the probability that the letters go into the right envelope is equal to
[MNR-1972; MPPET-1990; Orissa JEE-2004]
(a) 1/27 (b) 1/9
(c) 4/27 (d) 1/6

29. Two dice are tossed. The probability that the total score is a prime number is
(a) 1/6 (b) 5/12
(c) 1/2 (d) None of these

WORKSHEET: TO CHECK THE PREPARATION LEVEL

Important Instructions

1. The answer sheet is immediately below the worksheet.
2. The test is of 19 minutes.
3. The worksheet consists of 19 questions. The maximum marks are 57.
4. Use Blue/Black Ball point pen only for writing particulars/marking responses. Use of pencil is strictly prohibited.

1. If two dice are thrown simultaneously, then probability that 1 comes on the first dice is *[RPET-2002]*

(a) 1/36 (b) 5/36
(c) 1/6 (d) None

2. Two dice are thrown together. The probability that at least one will show its digit 6 is *[RPET-96]*

(a) 11/36 (b) 36/11
(c) 5/11 (d) 1/6

3. The probability that a leap year selected randomly will have 53 Sundays is *[MPPET-1991,93,95; Pb. CET-2002;MP-2003]*

(a) 1/7 (b) 2/7
(c) 4/53 (d) 4/49

4. Three coins are tossed. If one of them shows tail, then the probability that all three coins show tail is

(a) 1/7 (b) 1/8
(c) 2/7 (d) 1/6

5. A bag contains 5 black, 4 white and 3 red balls. If a ball is selected randomwise, the probability that it is a black or red ball is *[EAMCET-2002]*

(a) 1/3 (b) 1/4
(c) 5/12 (d) 2/3

6. The probability that the same number appear on throwing three dice simultaneously is

(a) 1/6 (b) 1/36
(c) 5/36 (d) None of these

7. If A and B are any two events, then $P(\overline{A} \cap B) =$ *[MPPET-2001; AMU-99]*

(a) $P(\overline{A})\, P(\overline{B})$
(b) $1 - P(A) - P(B)$
(c) $P(A) + P(B) - P(A \cap B)$
(d) $P(B) - P(A \cap B)$

8. If $P(A) = 0.25$, $P(B) = 0.50$ and $P(A \cap B) = 0.14$, then $P(A \cap \overline{B})$ is equal to *[RPET-2001]*

(a) 0.61 (b) 0.39
(c) 0.48 (d) None of these

9. A bag contains 4 white, 5 black and 6 red balls. If a ball is drawn at random, then what is the probability that the drawn ball is either white or red

(a) 4/15 (b) 1/2
(c) 2/5 (d) 2/3

10. If $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{4}$, then $P(B/A) =$

(a) 1 (b) 0
(c) 1/2 (d) 1/3

11. If A and B are two events of a random experiment, $P(A) = 0.25$, $P(B) = 0.5$ and $P(A \cap B) = 0.15$, then $P(A \cap \overline{B}) =$

(a) 0.1 (b) 0.35
(c) 0.15 (d) 0.6

12. If $P(A) = 0.4$, $P(B) = x$, $P(A \cap B) = 0.7$ and the events A and B are independent, then $x =$

(a) 1/3 (b) 1/2
(c) 2/3 (d) None of these

13. A and B are 2 events such that $P(A) = 0.4$, $P(A + B) = 0.7$ and $P(AB) = 0.2$, then $P(B) =$

(a) 0.1 (b) 0.3
(c) 0.5 (d) None

14. The probability that a person will be alive in next 10 years is 1/4 and that of his wife is 1/3. The probability that none of them will be alive in next 10 years is *[Roorkee (Screening)-92]*

(a) 5/12 (b) 1/2
(c) 7/12 (d) 11/12

15. From the past experience it is known that an investor will invest in security A with a probability of 0.6, will invest in security B with a probability 0.3 and will invest in both A and B with probability 0.2. The probability that an investor will invest neither in A nor in B is *[NDA-2006]*

(a) 0.7 (b) 0.28
(c) 0.3 (d) 0.4

16. A bag contains 3 red and 5 black balls and a second bag contains 6 red and 4 black balls. A balls is drawn from each bag. The probability that one is red and other is black is

(a) 3/20 (b) 21/40
(c) 3/8 (d) None of these

17. If E_1 and E_2 are two events, then *[MP PET-2008]*

(a) $P(E_1 - E_2) = P(E_2) - P(E_1 \cap E_2)$
(b) $P(E_1 - E_2) = P(E_2) + P(E_1 \cap E_2)$
(c) $P(E_1 - E_2) = P(E_1) - P(E_1 \cap E_2)$
(d) $P(E_1 - E_2) = P(E_1) + P(E_1 \cap E_2)$

18. If A and B are two independent events such that $P(A) = 0.40$, $P(B) = 0.50$, find P(neither A nor B)

(a) 0.90 (b) 0.10
(c) 0. 2 (d) 0. 3

19. A lot consists of 12 good pencils, 6 with minor defects and 2 with major defects. A pencil is chosen at random. The probability that this pencil is not defective is

(a) 3/5 (b) 3/10
(c) 4/5 (d) 1/2

ANSWER SHEET

1. ⓐ ⓑ ⓒ ⓓ	8. ⓐ ⓑ ⓒ ⓓ	15. ⓐ ⓑ ⓒ ⓓ
2. ⓐ ⓑ ⓒ ⓓ	9. ⓐ ⓑ ⓒ ⓓ	16. ⓐ ⓑ ⓒ ⓓ
3. ⓐ ⓑ ⓒ ⓓ	10. ⓐ ⓑ ⓒ ⓓ	17. ⓐ ⓑ ⓒ ⓓ
4. ⓐ ⓑ ⓒ ⓓ	11. ⓐ ⓑ ⓒ ⓓ	18. ⓐ ⓑ ⓒ ⓓ
5. ⓐ ⓑ ⓒ ⓓ	12. ⓐ ⓑ ⓒ ⓓ	19. ⓐ ⓑ ⓒ ⓓ
6. ⓐ ⓑ ⓒ ⓓ	13. ⓐ ⓑ ⓒ ⓓ	
7. ⓐ ⓑ ⓒ ⓓ	14. ⓐ ⓑ ⓒ ⓓ	

HINTS AND EXPLANATIONS

1. (c) Probability that 1 comes on the first dice is $\frac{1}{6}$.

Total number of ways $= 6 \times 6$

Favourable number of ways

$= \{(1,1), (1, 2), (1, 3), (1, 4), (1, 5), (1,6)\}$

Required probability $= \frac{6}{36} = \frac{1}{6}$

2. (a) Sample space $= 6 \times 6 = 36$

Favourable number of ways

$$= \begin{Bmatrix} (6,1),(6,2),(6,3),(6,4), \\ (6,5),(1,6),(2,6),(3,6), \\ (4,6),(5,6),(6,6) \end{Bmatrix}$$

$\therefore$ Probability of getting at least one 6 is

$P(\text{one } 6) + P(\text{both } 6) = \frac{10}{36} + \frac{1}{36} = \frac{11}{36}$

9. (d) Probability for the white ball

$$P(W) = \frac{4}{15}$$

Probability for the red ball

$$P(R) = \frac{6}{15}$$

Probability (white or red ball)

$= P(W) + P(R)$

$= \frac{4}{15} + \frac{6}{15} = \frac{10}{15} = \frac{2}{3}$

14. (b) Probability that a person will not be alive = 3/4 and probability that his wife will not be alive = 2/3

So, required probability $= \frac{3}{4} \times \frac{2}{3} = \frac{1}{2}$

$\because$ Both events are mutually independent.

15. (c) Probability of investing in security A

$$P(A) = 0.6$$

Probability of investing in security B

$$P(B) = 0.4$$

$\because P(A \cap B) = 0.2$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 0.6 + 0.3 - 0.2$
$= 0.9 - 0.2 = 0.7$

$\therefore P(\text{neither in } A \text{ nor } B) = 1 - P(A \cup B)$
$= 1 - 0.7 = 0.3$

16. (b)

	Red(R)	Black(B)
Bag-1	3/8	5/8
Bag-2	6/10	4/10

Required probability $= \left(\frac{3}{8} \times \frac{4}{10} + \frac{5}{8} \times \frac{6}{10}\right)$

$= \frac{6}{40} + \frac{3}{8} = \frac{6+15}{40}$

$= \frac{21}{40}$

19. (a) Required probability $= \frac{{}^{12}C_1}{{}^{20}C_1} = \frac{3}{5}$

LECTURE

3

Conditional Probability and Binomial Distribution

BASIC CONCEPTS

1. **Conditional Probability** If A and B are dependent events, then the probability of B when A has already happened is called the conditional probability of B with respect to A, and it is denoted by $P(B/A)$. It may be seen that $P(B/A) = \dfrac{P(AB)}{P(A)}$.

   1.1 **Note:** It can be easily seen that favourable cases for B/A = favourable cases of AB and total cases for B/A = favourable cases of A

   1.2 $P(A/B) = P(A \cap B) \mid P(B)$

   $P(A \cap \bar{B}) = P(A - B) = P(A) - P(A \cap B)$

   $P(B \cap \bar{A}) = P(B) - P(A \cap B)$

   $P(\bar{A} \cap \bar{B}) = P(\bar{A})\, P(\bar{B})$, for two independent events.

   $P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$, for any two events.

   $$P(\bar{A}/B) = \frac{P(\bar{A} \cap B)}{P(B)} = \frac{P(B) - P(A \cap B)}{P(B)}$$

   $P(A/B) + P(\bar{A}/B) = 1$

   $$P(\bar{A}/\bar{B}) = \frac{P(\bar{A} \cap \bar{B})}{P(\bar{B})} = \frac{P\left(\overline{A \cup B}\right)}{P(\bar{B})}$$

   $$= \frac{1 - P(A \cup B)}{1 - P(B)}$$

   $P(A/A) = 1, P(A/\bar{A}) = 0$

   Example: For any two events A and B, $P\left(\dfrac{A}{A \cup B}\right)$ is equal to **[PET (Raj.), 95]**

   (a) $\dfrac{P(A)}{P(A \cup B)}$ (b) $\dfrac{P(B)}{P(A \cup B)}$

   (c) $\dfrac{P(A \cap B)}{P(A \cup B)}$ (d) None of these

2. **Binomial Distribution for Repeated Trials** Let n independent trials be repeated under identical conditions and there are only two M.E. outcomes: success or failure, for each trial. Also the probability of a success in each trial remains constant and does not change from trial to trial. Let P be the probability of success of an event and q be the probability of failure of the event in one trial, then probability of exactly r successes is

   i.e., $P(A) = (x = r) = {}^nC_r\, q^{n-r} p^r = P(r)$

   2.1 Probability of getting at most k successes is

   $$P(A)(0 \le r \le k) = \sum_{r=0}^{k} {}^nC_r q^{n-r} p^r$$

   2.2 Probability of getting at least k success is

   $$P(A)(r \ge k) = \sum_{r=k}^{n} {}^nC_r q^{n-r} p^r$$

   2.3 For the above binomial variate mean = np; standard deviation = $\sqrt{npq}$; variance = npq.

SOLVED SUBJECTIVE PROBLEMS (XII BOARD (C.B.S.E./STATE)): FOR BETTER UNDERSTANDING AND CONCEPT BUILDING OF THE TOPIC

1. There are n letters and n envelopes. Find the probability that none of the letters are kept in correct envelopes.

Solution

n letters can be kept in n envelopes in $n!$ ways. The probability that none letter is kept in the correct envelope:

$$=\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\ldots+\frac{(-1)^n}{n!}\right]$$

2. Six cards are drawn from a pack of cards with replacement. Find the probability that at the maximum 3 of them are spade.

Solution

Probability of drawing a spade

$$P=\frac{13}{52}=\frac{1}{4}$$

Probability of not drawing a spade

$$=1-\frac{1}{4}=\frac{3}{4}$$

Now, the probability of drawing 3 spades at the maximum = probability of drawing no spade + probability of drawing 1 spade + probability of drawing 2 spades + probability of drawing 3 spades

$$= {}^6C_0\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^6 + {}^6C_1\left(\frac{1}{4}\right)^1\left(\frac{3}{4}\right)^5$$

$$+ {}^6C_2\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^4 + {}^6C_3\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^3$$

$$=\left(\frac{3}{4}\right)^6+\frac{6}{4}\left(\frac{3}{4}\right)^5+\frac{15}{16}\left(\frac{3}{4}\right)^4+\frac{20}{64}\left(\frac{3}{4}\right)^3$$

$$=\left(\frac{3}{4}\right)^3\left[\left(\frac{3}{4}\right)^3+\frac{3}{2}\left(\frac{3}{4}\right)^2+\frac{15}{16}\left(\frac{3}{4}\right)+\frac{20}{6}\right]$$

$$=\frac{27}{64}\left[\frac{27}{64}+\frac{27}{32}+\frac{45}{64}+\frac{20}{64}\right]=\frac{27}{64}\left(\frac{146}{64}\right)$$

$$=\frac{27}{64}\times\frac{73}{32}=\frac{1971}{2048}$$

3. 10% of the bulbs manufactured by a factory, out of 1,000 samples 5 bulbs are drawn at random. Find the number of bulbs which are not defective.

Solution

$$p\,(\text{defective bulbs})=\frac{10}{100}=\frac{1}{10}$$

$$q\,(\text{undefective bulbs})=1-\frac{1}{10}=\frac{9}{10}$$

Probability that none of the 5 bulbs are defective $= {}^5C_0\left(\frac{1}{10}\right)^0\left(\frac{9}{10}\right)^5=\left(\frac{9}{10}\right)^5$

Number of undefective bulbs =

$$1000\times\left(\frac{9}{10}\right)^5=590$$

4. A coin is tossed three times: *[NCERT]*

(i) E: 'head on the third toss'
   F: 'heads on the first two tosses'

(ii) E: 'at least two heads'
   F: 'at most two heads'

(iii) E: 'at most two tails'
   F: 'at least one tail'

Find $P(E/F)$.

Solution

When a coin is tossed three times, the sample space S contains $2^3 = 8$ equally likely sample points.

In fact, $S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

(i) Here, E: 'head on the third toss'
   F: 'heads on the first two tosses'.
   i.e., $E = \{HHH, THH, THH, TTH\}$
   and $F = \{HHH, HHT\}$
   $\Rightarrow (E \cap F) = \{HHH\}$

Hence, $P(E)=\frac{4}{8}=\frac{1}{2}, P(F)=\frac{2}{8}=\frac{1}{4}$

and $P(E\cap F)=\frac{1}{8}$

$$\therefore\ P(E/F)=\frac{P(E\cap F)}{P(F)}=\frac{1/8}{2/8}=\frac{1}{2}$$

(ii) Here E: 'atleast two heads'
and F: 'atmost two heads'
i.e., $E = \{HHH, HHT, HTH, THH\}$
$F = \{TTT, THT, TTH, HTT, HHT, HTH, THH\}$
$\Rightarrow \quad E \cap F = \{HHT, HTH, THH\}$

Hence, $P(E) = \frac{4}{8} = \frac{1}{2}, P(F) = \frac{7}{8}$

and $P(E \cap F) = \frac{3}{8}$

$\therefore \ P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{3/8}{7/8} = \frac{3}{7}$

(iii) Here, E: 'at most two tails' and
F: 'at least one tail'.
i.e., $E = \{HHH, HHT, HTH, HTT, THH, THT, TTH\}$
and $F = \{HHT, HTH, HTT, THH, THT, TTH, TTT\}$
$\Rightarrow \quad E \cap F = \{HHT, HTH, HTT, THH, THT, TTH\}$

Hence, $P(E) = \frac{7}{8}, P(F) = \frac{7}{8}$ and

$P(E \cap F) = \frac{6}{8}$

$\therefore \ P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{6/8}{7/8} = \frac{6}{7}$

5. A die is thrown three times:
E: '4 appears on the third toss'
F: '6 and 5 appears, respectively, on the first two tosses'
Find $P(E/F)$. ***[NCERT]***

Solution

When a dice is thrown three times, the sample space contains $6 \times 6 \times 6 = 216$ equally likely simple events. The sample space is $S = \{(x, y, z): x, y, z \in \{1, 2, 3, 4, 5, 6\}\}$.
Here, E: 4 appears on the third toss and F: 6 and 5 appears, respectively, on the first two tosses
i.e., $E = \{(x, y, 4): x, y \in \{1, 2, 3, 4, 5, 6\}\}$
and $F = \{(6, 5, x): x \in \{1, 2, 3, 4, 5, 6\}\}$
$= \{(6, 5, 1), (6, 5, 2), (6, 5, 3), (6, 5, 4), (6, 5, 5), (6, 5, 6)\}$
$\Rightarrow \quad E \cap F = \{(6, 5, 4)\}$

It may be noted that E contains $6 \times 6 = 36$ simple events.
Required probability $P(E/F)$

$$= \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{216}}{\frac{6}{216}} = \frac{1}{6}$$

6. Consider the experiment of throwing a die, if a multiple of 3 comes up throw the die again and if any other number comes, toss a coin. Find the conditional probability of the event 'the coin shows a tail', given that 'atleast one die shows a 3'. ***[NCERT]***

Solution

Here, the sample space S is given by

$$S = \begin{Bmatrix} (3,1),(3,2),(3,3),(3,4),(3,5),(3,6), \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6), \\ (1,H),(1,T),(2,H),(2,T),(4,H),(4,T), \\ (5,H),(5,T) \end{Bmatrix}$$

The outcomes of S are not equally likely. First 12 outcomes are equally likely and are such that the sum of their probabilities is $\frac{2}{6} = \frac{1}{3}$.
So each of the first 12 outcomes has a probability equal to 1/36. Remaining eight outcomes are equally likely and are such that the sum of their probabilities is $\frac{4}{6} = \frac{2}{3}$
So, each of these has a probability equal to

$\frac{2}{3 \times 8} = \frac{1}{12}$.

Let E: the coin shows a tail and
F: at least one die shows up a 3.
i.e., $E = \{(1, T), (2, T), (4, T), (5, T)\}$
and $F = \{(3, 1), (3, 2), (3, 3), (3, 4), (3\ 5), (3, 6), (6, 3)\}$
$\Rightarrow \quad E \cap F = \phi$
Hence, the required probability $= P(E/F)$

$= \frac{P(E \cap F)}{P(F)} = \frac{0}{P(F)} = 0$

($\because E \cap F$ is an impossible event.)

7. A box of oranges is inspected by examining three randomly selected oranges drawn without replacement. If all the three oranges are good, the box is approved for sale otherwise it is rejected. Find the probability that a box containing 15 oranges, out of which 12 are good and 3 are bad ones, will be approved for sale. *[NCERT]*

Solution

Required probability = P(three good oranges are taken out one by one without replacement)

$$= \frac{12}{15} \times \frac{11}{14} \times \frac{10}{13}$$

(There are 12 good oranges in a total of 15.)

$$= \frac{44}{91}$$

8. If A and B are two events such that $A \subset B$ and $P(B) \neq 0$, then which of the following is correct: *[NCERT]*

(a) $P(A/B) = \dfrac{P(B)}{P(A)}$ (b) $P(A/B) < P(A)$

(c) $P(A/B) \geq P(A)$ (d) None of these

Solution

(c) When $A \subset B$, then $A \cap B = A$.

$$\therefore P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)} \geq P(A)$$

$$\left(\because 0 < P(B) \leq 1, \frac{1}{P(B)} \geq 1 \right)$$

9. A class has 15 students whose ages are 14, 17, 15, 14, 21, 17, 19, 20, 16, 18, 20, 17, 16, 19 and 20 years. One student is selected in such a manner that each has the same chance of being chosen and the age X of the selected student is recorded. What is the probability distribution of the random variable X? Find mean, variance and standard deviation of X. *[NCERT]*

Solution

We construct the table

X (age)	14	15	16	17	18	19	20	21
Number of students	2	1	2	3	1	2	3	1
$P(X)$	2/15	1/15	2/15	3/15	1/15	2/15	3/15	1/15

The third row gives the probability distribution of X.

Mean $X = \sum XP(X)$

$$= \frac{14 \times 2 + 15 \times 1 + 16 \times 2 + 17 \times 3 + 18 \times 1 + 19 \times 2 + 20 \times 3 + 21 \times 1}{15}$$

$$= \frac{28 + 15 + 32 + 51 + 18 + 38 + 60 + 21}{15}$$

$$= \frac{263}{15} = 17.53$$

Variance $X = \sum X^2 P(X) - (\text{mean})^2$

$$= \frac{(14)^2 \times 2 + (15)^2 \times 1 + (16)^2 \times 2 + (17)^2 \times 3 + (18)^2 \times 1 + (19)^2 \times 2 + (20)^2 \times 3 + (21)^2 \times 1}{15} - \left(\frac{263}{15} \right)^2$$

$$= \frac{392 + 225 + 512 + 867 + 324 + 722 + 1200 + 441}{15} - \left(\frac{263}{15} \right)^2$$

$$= \frac{4683}{15} - \left(\frac{263}{15} \right)^2$$

$$= 312.2 - 307.4 = 4.8$$

S.D. of $X = \sqrt{\text{Variance}} = \sqrt{4.8} = 2.1\,9$

10. Suppose that 90% of people are right-handed. What is the probability that at most 6 of a random sample of 10 people are right-handed? *[NCERT]*

Solution

It is a case of Bernoullian trials with $n = 10$, where success is 'a right-handed person'.

Here, $p = \dfrac{90}{100} = \dfrac{9}{10}$ and hence,

$$q = 1 - p = 1 - \frac{9}{10} = \frac{1}{10}$$

$\therefore$ Required probability

$$= P(X \leq 6) = P(0) + P(1) + P(2) + P(3) + P(4) + P(5) + P(6)$$

$$= {}^{10}C_0 p^0 q^{10} + {}^{10}C_1 p^1 q^9 + {}^{10}C_2 p^2 q^8 + {}^{10}C_3 p^3 q^7 + {}^{10}C_4 p^4 q^6 + {}^{10}C_5 p^5 q^5 + {}^{10}C_6 p^6 q^4$$

$$= q^{10} + 10pq^9 + \frac{10 \times 9}{1 \times 2} p^2 q^8 + \frac{10 \times 9 \times 8}{1 \times 2 \times 3} p^3 q^7$$

$$+\frac{10\times9\times8\times7}{1\times2\times3\times4}p^4q^6$$
$$+\frac{10\times9\times8\times7\times6}{1\times2\times3\times4\times5}p^5q^5$$
$$+\frac{10\times9\times8\times7}{1\times2\times3\times4}p^6q^4 \qquad (\because {}^{10}C_6={}^{10}C_4)$$
$$=q^{10}+10pq^9+45p^2q^8+120p^3q^7+210p^4q^6+252p^5q^5+210p^6q^4$$
$$=\left(\frac{1}{10}\right)^{10}+10\left(\frac{9}{10}\right)\left(\frac{1}{10}\right)^9+45\left(\frac{9}{10}\right)^2\left(\frac{1}{10}\right)^8$$
$$+120\left(\frac{9}{10}\right)^3\left(\frac{1}{10}\right)^7+210\left(\frac{9}{10}\right)^4\left(\frac{1}{10}\right)^6$$
$$+252\left(\frac{9}{10}\right)^5\left(\frac{1}{10}\right)^5+210\left(\frac{9}{10}\right)^6\left(\frac{1}{10}\right)^4$$
$$=\frac{1+90+45\times9^2+120\times9^3+210\times9^4+252\times9^5+210\times9^6}{10^{10}}$$

Alternatively, required probability

$$=P(X\le 6)=1-P(7\le X\le 10)$$
$$=1-\sum_{r=7}^{10}{}^{10}C_r\left(\frac{9}{10}\right)^r\left(\frac{1}{10}\right)^{10-r}$$

11. A die is thrown 6 times. If 'getting an odd number' is a success, what is the probability of:
(i) 5 successes?
(ii) at least 5 successes?
(iii) at most 5 successes? *[NCERT]*

Solution

It is a case of Bernoullian trials with $n = 6$. Here, success is 'getting an odd number' and $P = P(a \text{ success}) = P(\text{getting an odd number in a single throw of a die})$

$$=\frac{3}{6}=\frac{1}{2}$$

($\because$ 1, 3, 5 are only favourable outcomes.)

and $q = P(\text{failure}) = 1 - p = 1-\frac{1}{2}=\frac{1}{2}$

(i) $P(5 \text{ successes}) = {}^6C_5p^5q^1={}^6C_1(1/2)^5(1/2)^1$
($\because {}^nC_r={}^nC_{n-r}$)

$$=\frac{6}{2^6}=\frac{3}{32}$$

(ii) $P(\text{at least 5 successes})$

$$=P(5 \text{ successes})+P(6 \text{ successes})$$
$$={}^6C_5p^5q^1+{}^6C_6p^6q^0$$
$$=\frac{3}{32}+1\left(\frac{1}{2}\right)^6 \qquad \text{(see part (i))}$$
$$=\frac{3}{32}+\frac{1}{64}=\frac{7}{64}$$

(iii) $P(\text{at most 5 successes})$

$$=1-P(6 \text{ successes})$$
$$=1-{}^6C_6p^6q^0$$
$$=1-1\left(\frac{1}{2}\right)^6=\frac{64-1}{64}=\frac{63}{64}$$

12. Two coins are tossed once: *[NCERT]*
(i) E: 'tail appears on one coin'
F: 'one coin shows head'
(ii) E: 'no tail appears'
F: no head appears.
Find $P(E/F)$.

Solution

When two coins are tossed, the sample space S contains $2^2 = 4$ equally likely simple events. In fact $S = \{HH, HT, TH, TT\}$.

(i) Here, E: 'tail appears on one coin'
and F: 'one coin shows head'
i.e., $E = \{HT, TH\}$ and $F = \{TH, HT\}$
$\Rightarrow \quad E\cap F = \{HT, TH\} = F$

$$\therefore P(E/F)=\frac{P(E\cap F)}{P(F)}=\frac{P(F)}{P(F)}=1$$

(ii) Here, E: = 'no tail appears'
and F: 'no head appears'
i.e., $E = \{HH\}$ and $F = \{TT\}$
$\Rightarrow \quad E\cap F = \{\ \} = \phi$
Hence, $P(E\cap F) = 0$ and $P(F) = 1/4$.

$$\therefore P(E/F)=\frac{P(E\cap F)}{P(F)}=\frac{P(\phi)}{P(F)}=\frac{0}{1/4}=0$$

13. An instructor has a question bank consisting of 300 easy true/false questions, 200 difficult true/false questions, 500 easy multiple choice questions and 400 difficult multiple choice questions. If a question is selected at random from the test bank, what is the probability that it will be an easy question, given that it is a multiple choice question? *[NCERT]*

Solution

Let E: 'it is an easy question' and
F: 'it is a multiple choice question', then
$E \cap F$: 'it is an easy multiple choice question'.

Total number of questions
$= 300 + 200 + 500 + 400 = 1400$.

$$\therefore P(E \cap F) = \frac{500}{1400} = \frac{5}{14} \text{ and}$$

$$P(F) = \frac{500+400}{1400} = \frac{9}{14}$$

Hence, required probability $= P(E/F)$

$$= \frac{P(E \cap F)}{P(F)} = \frac{5/14}{9/14} = \frac{5}{9}$$

14. An urn contains 5 red and 5 black balls. A ball is drawn at random; its colour is noted and is returned to the urn. Moreover, 2 additional balls of the colour drawn are put in the urn and then a ball is drawn at random. What is the probability that the second ball is red?

[NCERT]

Solution

Required probability $= P$(second ball is red) $= P$(a red ball is drawn and returned along with 2 red balls and then a red ball is drawn) $+ P$(a black ball is drawn and returned along with 2 black balls and then a red ball is drawn)

$$= \frac{5}{10} \times \frac{7}{12} + \frac{5}{10} \times \frac{5}{12} = \frac{35+25}{120} = \frac{60}{120} = \frac{1}{2}$$

15. In a hurdle race, a player has to cross 10 hurdles. The probability that he will clear each hurdle is 5/6. What is the probability that he will knock down fewer than 2 hurdles?

[NCERT]

Solution

It is a case of Bernoullian trials where success is 'crossing a hardle successfully without knocking it down' and $n = 10$.

$$p = P(\text{a success}) = \frac{5}{6} \Rightarrow \quad q = \frac{1}{6}$$

Required probability

$$P(9) + P(10) = {}^{10}C_9 p^9 q + {}^{10}C_{10} p^{10} q^0.$$

16. If the mean and the variance of a binomial variate X are 2 and 1, respectively, then the probability that X takes a value greater than one is equal to …… *[IIT-91]*

Solution

The binomial distribution of X is given by:

$$(q+p)^n = \Sigma\, {}^nC_x q^{n-x} P_x,$$

where $X = 0, 1, 2, \ldots n$ and $p + q = 1$.
The mean of this distribution $= np$ and its variance $= npq$.

As given in this question, we have $np = 2$ and $npq = 1$, where $q = 1/2$, Then $p = 1 - q = 1/2$ and $n = 4$.

So in this case, the distribution is

$$1 = \left(\frac{1}{2} + \frac{1}{2}\right)^4 = {}^4c_0\left(\frac{1}{2}\right)^4 + {}^4c_1\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)$$
$$+ {}^4c_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2 + {}^4c_3\frac{1}{2}\left(\frac{1}{2}\right)^3 + {}^4c_4\left(\frac{1}{2}\right)^4 \quad \ldots\ldots \text{(i)}$$

Now the probability that X takes a value greater than 1 is the sum of the last three terms on the L.H.S of Eq. (i).

Hence the required probability

$$= 6\left(\frac{1}{2}\right)^4 + 4\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4 = \frac{11}{16}$$

UNSOLVED SUBJECTIVE PROBLEMS (XII BOARD (C.B.S.E./STATE)): SOLVE THESE PROBLEMS TO GRASP THE TOPIC

EXERCISE 1

1. A dice is thrown twice and the sum of the numbers appearing is observed to be 6. What is the conditional probability that the number 4 has appeared at least once? *[CBSE-91, 2000]*
2. A speaks untruth in 30% cases and B speaks truth in 60% cases. Find the probability when they contradict each other. *[MP-2000]*
3. Three coins are thrown simultaneously. Find:
   (i) probability of getting at least two heads. *[MP-2000]*
   (ii) probability of getting at most two heads. *[MP-2000; CBSE -2005]*
4. A question of mathematics is given to three students to solve. Probabilities of solving the question by them are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$, respectively. If they try to solve it, what is the probability that the problem will be solved? *[MP-93, 98, 2002]*
5. Two dice are thrown simultaneously. Find the probability that the first die shows an even number or both the dice show the sum 8.
   [MP-89, 93(S), 96, 98, 2000, 2005(A), CBSE-2004]
6. If $P(A) = \frac{1}{2}, P(B) = \frac{1}{4}$ and $P(A \cap B) = \frac{1}{4}$, then find the value of the following: *[MP-2007]*
   (i) $P\left(\frac{A}{B}\right)$ (ii) $P\left(\frac{B}{A}\right)$ (iii) $P(A \cup B)$
7. The probability of student A passing an examination is 3/7 and of student B passing is 5/7. Assuming the two events 'A passes', 'B passes', as independent, find the probability of: *[CBSE-95(C)]*
   (i) only a passing the examination.
   (ii) only one of them passing the examination.
8. In a college 25% students fail in maths, 15% fail in chemistry and 10% students fail in maths and chemistry both. A student is selected at random, then
   (i) What is the probability that he fails in Maths, if he is failed in Chemistry?
   (ii) What is the probability that he fails in Chemistry, if he is failed in Maths?
   (iii) What is the probability that he is failed in Maths or Chemistry?
9. A box contains 16 bulbs; out of which 4 bulbs are defective. Three bulbs are drawn one by one from the box without replacement. Find the probability distribution of the number of defective bulbs drawn.
   [CBSE-Practice Sample Paper-X]
10. In a multiple choice examination with three possible answers (out of which only one is correct) for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing? *[CBSE-2009]*
11. Coloured balls are distributed in three bags, as shown in the following table:

	Colour of the ball		
Bag	**Black**	**White**	**Red**
I	2	1	3
II	4	2	1
III	5	4	3

A bag is selected at random and then two balls are randomly drawn from the selected bag. They happen to be white and red. What is the probability that they came from bag II? *[CBSE-2009]*

EXERCISE 2

1. A group of children contains 6 boys and 4 girls. Three children are chosen at random from this group. Find the probability that this group chosen:
   (i) contains only a particular girl.
   (ii) contains at least one girl.
2. A player draws a playing card from a set of playing cards. What will be the probability of not being a diamond card? *[MP-2004(A)]*
3. In three groups of children there are 3 girls and 1 boy, 2 girls and 2 boys, 1 girl and 3 boys, respectively. One child is chosen at random from each group. Prove that if there is 1 girl and 2 boys among the chosen children then the probability is 13/32.
   [MP-99; Roorkee-85; CBSE-Sample Paper-III]

4. A bag contains 50 bolts and 150 nuts. Half of the bolts and half of the nuts are rusted. If one item is taken at random. Find the probability that it is rusted or is a bolt. *[MP-2000]*

5. Mohan tells the truth in 75% cases while Sohan in 80% cases. Find the probability that Mohan tells the truth and Sohan tells lie to narrate an incident. *[MP-2001]*

6. Two cards are drawn successively with replacement from a well-shuffled pack of 52 cards. Find the probability distribution of the number of jacks.
[CBSE-2006 (outside-Delhi)-I, II and III]

7. A can solve 90% of the problem given in a book and B can solve 70%. What is the probability that at least one of them will solve a problem selected at random from the book.
[MP-99, 2003; CBSE-92(C)]

8. Out of 9 outstanding students in a college, there are 4 boys and 5 girls. A team of four students is to be selected for a quiz programme. Find the probability that 2 are boys and 2 are girls. *[CBSE-94]*

9. A fair die is tossed twice. If the number appearing on the top is less than 3, it is a success. Find the probability distribution of the number of successes. *[CBSE-2004]*

10. A man is known to speak the truth 3 out of 5 times. He throws a die and reports that it is a number greater than 4. Find the probability that it is actually a number greater than 4.
[CBSE-2009]

ANSWERS

EXERCISE 1

1. 2/5
2. 46%
3. (i) 1/2
(ii) 7/8
4. 3/4
5. 5/9
7. (i) 6/49
(ii) 26/49
8. (i) 2/3
(ii) 2/5
(iii) 0.3

9.

X	0	1	2	3
$P(X)$	11/28	33/70	9/70	1/140

10. $\frac{11}{243}$
11. $\frac{2}{147}$

EXERCISE 2

1. (i) 1/8
(ii) 5/6
2. $\frac{39}{52}$
4. 5/8
5. 15%

6.

X	0	1	2
$P(X)$	144/169	24/169	1/169

7. 0.97
8. 10/21

9.

X	0	1	2
$P(X)$	4/9	4/9	1/9

10. $\frac{3}{7}$

SOLVED OBJECTIVE PROBLEMS: HELPING HAND

1. Seven chits are numbered 1 to 7. Three are drawn one by one with replacement. The probability that the least number on any selected chit is 5 is *[EAMCET-1991]*

(a) $1-\left(\frac{2}{7}\right)^4$ (b) $4\left(\frac{2}{7}\right)^4$

(c) $\left(\frac{3}{7}\right)^3$ (d) None of these

Solution

(c) $P(5 \text{ or } 6 \text{ or } 7)$ in one draw $= \frac{3}{7}$

$\therefore$ Probability that in each of 3 draws, the chits bear 5, 6 or 7 $= \left(\frac{3}{7}\right)^3$

2. A box contains 2 black, 4 white and 3 red balls. One ball is drawn at random from the box and kept aside. From the remaining balls in the box, another ball is drawn at random and kept aside the first. This process is repeated till all the balls are drawn from the box. The probability that the balls drawn are in the sequence of 2 black, 4 white and 3 red is

(a) $\frac{1}{1260}$ (b) $\frac{1}{7560}$

(c) $\frac{1}{126}$ (d) None of these

Solution

(a) The required probability

$$=\frac{2}{9}\times\frac{1}{8}\times\frac{4}{7}\times\frac{3}{6}\times\frac{2}{5}\times\frac{1}{4}\times1\times1\times1=\frac{1}{1260}$$

3. The probability that a teacher will give an unannounced test during any class meeting is 1/5. If a student is absent twice, then the probability that the student will miss at least one test is *[Aligarh-97]*

(a) 4/5 (b) 2/5 (c) 7/5 (d) 9/25

Solution

(d) The probability that one test is held

$$=2\times\frac{1}{5}\times\frac{4}{5}=\frac{8}{25}$$

The probability that one test is held on both days

$$=\frac{1}{5}\times\frac{1}{5}=\frac{1}{25}$$

Thus, the probability that the student misses at least one test $=\frac{8}{25}+\frac{1}{25}=\frac{9}{25}$

4. Suppose that a die (with faces marked 1 to 6) is loaded in such a manner that for $K = 1, 2, 3, \ldots, 6$, the probability of the face marked K turning up when die is tossed is proportional to K. The probability of the event that the outcome of a toss of the die will be an even number is equal to *[AMU-2000]*

(a) 1/2 (b) 4/7

(c) 2/5 (d) 1/21

Solution

(a) Required probability $=\frac{3}{6}=\frac{1}{2}$

5. A bag contains 3 red and 7 black balls, 2 balls are taken out at random, without replacement. If the first ball taken out is red, then what is the probability that the second taken out ball is also red? *[Pb. CET-2000]*

(a) 1/10 (b) 1/15 (c) 3/10 (d) 2/21

Solution

(b) We have total number of balls = 10

$\therefore$ Number of red balls = 3, number of black balls = 7 and number of balls in the bag = $3 + 7 = 10$

$\therefore$ The probability for taking out 1 red ball out of 10 balls $=\frac{3}{10}$ and the probability for taking out 1 red ball out of remaining 9 balls $=\frac{2}{9}$

$\therefore$ Probability for both balls to be red, i.e.,

$$p=\frac{3}{10}\times\frac{2}{9}=\frac{1}{15}$$

6. A bag contains 3 white and 2 black balls and another bag contains 2 white and 4 black balls. A ball is picked up randomly. The probability of its being black is *[MP PET-1989]*

(a) 2/5 (b) 8/15

(c) 6/11 (d) 2/3

Solution

(b) Required probability $=\frac{1}{2}\times\frac{2}{5}+\frac{1}{2}\times\frac{4}{6}=\frac{8}{15}$

7. A box containing 4 white and 2 black pens. Another box contains 3 white and 5 black pens. If 1 pen is selected from each box, then the probability that both the pens are white is equal to *[Pb. CET-2002]*

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{5}$

Solution

(c) Total number of pens in the first bag = $4 + 2 = 6$ and total number of pens in second bag $= 3 + 5 = 8$. The probability of selecting a white pen from the first bag $=\frac{4}{6}=\frac{2}{3}$ and the probability of selecting a white pen from the second bag $=\frac{3}{8}$.

$\therefore$ Required probability that both the pens are white $= \frac{2}{3} \times \frac{3}{8} = \frac{1}{4}$.

8. A box contains 25 tickets numbered 1, 2, ..., 25. If 2 tickets are drawn at random, then the probability that the product of their numbers is even is
(a) 11/50 (b) 13/50
(c) 37/50 (d) None of these

Solution

(c) Required probability is $1 - P$.
(Both odd numbers are chosen.)

$$= 1 - \frac{^{13}C_2}{^{25}C_2} = 1 - \frac{13 \times 12}{25 \times 24} = \frac{37}{50}$$

9. A box contains 10 mangoes; out of which 4 are rotten. Two mangoes are taken out together. If one of them is found to be good, the probability that the other is also good is **[EAMCET-1992]**
(a) 1/3 (b) 8/15
(c) 5/13 (d) 2/3

Solution

(c) A = event that first mango is good
B = event that second mango is good

Then required probability $= P\left(\frac{B}{A}\right) = \frac{P(AB)}{P(A)}$

But $P(AB) = \frac{^6C_2}{^{10}C_2} = \frac{1}{3}$

and $P(A) = \frac{^6C_2}{^{10}C_2} + \frac{^6C_1 \times {^4C_1}}{^{10}C_2} = \frac{13}{15}$

$\therefore$ Required probability $= \frac{1/3}{13/15} = \frac{5}{13}$.

10. A party of 23 persons take their seats at a round table. The odds against two persons sitting together are **[RPET-1999]**
(a) 10:1 (b) 1:11
(c) 9:10 (d) None of these

Solution

(a) Required probability

$$= \frac{(21)!2!}{(22)!} = \frac{1}{11} = \frac{1}{1+10}$$

$\therefore$ Odds against = 10 : 1.

11. If E and F are independent events such that $0 < P(E) < 1$ and $0 < P(F) < 1$, then **[IIT-1989]**
(a) E and F^c (the complement of the event F) are independent.
(b) E^c and F^c are independent.
(c) $P\left(\frac{E}{F}\right) + P\left(\frac{E^c}{F^c}\right) = 1$.
(d) All of the above.

Solution

(d) $P(E \cap F) = P(E).\,P(F)$
Now, $P(E \cap F^c) = P(E) - P(E \cap F)$
$= P(E)\,[1 - P(F)] = P(E).P(F^c)$
and $P(E^c \cup F^c)$
$= 1 - P(E \cup F) = 1 - [P(\text{E}) + P(F)$
$- P(E \cap F)]$
$= [1 - P(E)]\,[1 - P(F)\,] = P(E^c)\,P(F^c)$
Also $P(E/F) = P(E)$ and $P(E^c/F^c) = P(E^c)$
$\Rightarrow \quad P(E/F) + P(E^c/F^c) = 1$

12. If two events A and B are such that $P(A^c) = 0.3$, $P(B) = 0.4$ and $P(AB^c) = 0.5$, then $P[B/(A \cup B^c)]$ is equal to **[IIT-1994]**
(a) 1/2 (b) 1/3
(c) 1/4 (d) None of these

Solution

(c) $P[B/(A \cup B^c)] = \frac{P(B \cap (A \cup B^c))}{P(A \cup B^c)}$

$$= \frac{P(A \cap B)}{P(A) + P(B^c) - P(A \cap B^c)}$$

$$= \frac{P(A) - P(A \cap B^c)}{P(A) + P(B^c) - P(A \cap B^c)}$$

$$= \frac{0.7 - 0.5}{0.8} = \frac{1}{4}$$

13. Two cards are drawn one by one from a pack of cards. The probability of getting first card an ace and second a coloured one is (before drawing the second card the first card is not placed again in the pack)

[UPSEAT-1999; 2003]

(a) $\frac{1}{26}$ (b) $\frac{5}{52}$

(c) $\frac{5}{221}$ (d) $\frac{4}{13}$

Solution

(c) $P(E_1) = \frac{4}{52} = \frac{1}{13}, P\left(\frac{E_2}{E_1}\right) = \frac{15}{51} = \frac{5}{17}$

$$P(E_1 \cap E_2) = P(E_1) \times P\left(\frac{E_2}{E_1}\right) = \frac{1}{13} \times \frac{5}{17} = \frac{5}{221}$$

14. A bag X contains 2 white and 3 black balls and another bag Y contains 4 white and 2 black balls. One bag is selected at random and a ball is drawn from it. Then the probability for the ball chosen be white is ***[EAMCET-2003]***

(a) $\frac{2}{15}$ (b) $\frac{7}{15}$

(c) $\frac{8}{15}$ (d) $\frac{14}{15}$

Solution

(c) Let A be the event of selecting bag X, B be the event of selecting bag Y and E be the event of drawing a white ball, then $P(A) = 1/2$, $P(B) = 1/2$, $P(E/A) = 2/5$, $P(E/B) = 4/6 = 2/3$.
$P(E) = P(A)\,P(E/A) + P(B)\,P(E/B)$

$$= \frac{1}{2} \times \frac{2}{5} + \frac{1}{2} \times \frac{2}{3} = \frac{8}{15}$$

15. In a box containing 100 eggs, 10 eggs are rotten. The probability that out of a sample of 5 eggs none is rotten if the sampling is with replacement is

[MPPET-1991; MNR-1986; RPET-1995; UPSEAT-2000]

(a) $\left(\frac{1}{10}\right)^5$ (b) $\left(\frac{1}{5}\right)^5$

(c) $\left(\frac{9}{5}\right)^5$ (d) $\left(\frac{9}{10}\right)^5$

Solution

(d) Let $P(\text{fresh egg}) = \frac{90}{100} = \frac{9}{10} = p$

$P\,(\text{rotten egg}) = \frac{10}{100} = \frac{1}{10} = q;\ n = 5, r = 5$

So the probability that none egg is rotten

$$= {}^5C_5 \left(\frac{9}{10}\right)^5 \times \left(\frac{1}{10}\right)^0 = \left(\frac{9}{10}\right)^5$$

16. A bag contains 2 white and 4 black balls. A ball is drawn five times with replacement. The probability that at least 4 of the balls drawn are white is ***[AMU-2001]***

(a) $\frac{8}{141}$ (b) $\frac{10}{243}$

(c) $\frac{11}{243}$ (d) $\frac{8}{41}$

Solution

(c) Probability for white ball $= \frac{2}{6} = \frac{1}{3}$

Probability for black ball $= \frac{4}{6} = \frac{2}{3}$

$\therefore$ Required probability

$$= {}^5C_5 \left(\frac{1}{3}\right)^5 \left(\frac{2}{3}\right)^0 + {}^5C_4 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)$$

$$= \left(\frac{1}{3}\right)^4 \left[\frac{1}{3} + 5 \cdot \frac{2}{3}\right] = \frac{11}{3^5} = \frac{11}{243}$$

17. An unbiased coin is tossed. If the result is a head, a pair of unbiased dice is rolled and the number obtained by adding the numbers on the two faces is noted. If the result is a tail, a card from a well-shuffled pack of 11 cards numbered 2, 3, 4, …, 12 is picked and the number on the card is noted. The probability that the noted number is either 7 or 8 is

[IIT-1994]

(a) 0.24 (b) 0.244

(c) 0.024 (d) None of these

Solution

(b) Required probability = probability that either the number is 7 or 8,

i.e., Required probability $= P_7 + P_8$

Now $P_7 = \frac{1}{2}\times\frac{1}{11}+\frac{1}{2}\times\frac{6}{36}=\frac{1}{2}\left(\frac{1}{11}+\frac{1}{6}\right)$

$P_8 = \frac{1}{2}\times\frac{1}{11}+\frac{1}{2}\times\frac{5}{36}=\frac{1}{2}\left(\frac{1}{11}+\frac{5}{36}\right)$

$\therefore\ P=\frac{1}{2}\left(\frac{2}{11}+\frac{11}{36}\right)=0.244$

18. One bag contains 5 white and 4 black balls. Another bag contains 7 white and 9 black balls. A ball is transferred from the first bag to the second bag and then a ball is drawn from the second bag. The probability that the ball is white is ***[DSSE-1987]***

(a) $\frac{8}{17}$ (b) $\frac{40}{153}$ (c) $\frac{5}{9}$ (d) $\frac{4}{9}$

Solution

(d) Let a white ball be transferred from the first bag to the second. The probability of selecting a white ball from the first bag $=\frac{5}{9}$.

Now the second bag has 8 white and 9 black balls. The probability of selecting white ball from the second bag $=\frac{8}{17}$.

Hence, required probability $=\frac{5}{9}\times\frac{8}{17}=\frac{40}{153}$.

If a black ball is transferred from the first bag to the second, then the probability

$=\frac{4}{9}\times\frac{7}{17}=\frac{28}{153}$

Therefore, required probability

$=\frac{40}{153}+\frac{28}{153}=\frac{4}{9}$

19. Three groups A, B, C are competing for positions on the Board of Directors of a company. The probabilities of their winning are 0.5, 0.3, 0.2, respectively. If the group A wins, the probability of introducing a new product is 0.7 and the corresponding probabilities for group B and C are 0.6 and 0.5, respectively. The probability that the new product will be introduced is ***[Roorkee-1994]***

(a) 0.18 (b) 0.35 (c) 0.10 (d) 0.63

Solution

(d) Let E be the event that a new product is introduced. Then $P(A) = 0.5$, $P(B) = 0.3$, $P(C) = 0.2$ and $P(E/A) = 0.7$, $P(E/B) = 0.6$, $P(E/C) = 0.5$.

$\therefore$ A, B and C are mutually exclusive and exhaustive events.

$P(E) = P(A)\times P(E/A) + P(B)\times P(E/B)$
$\quad + P(C)\times P(E/C)$
$\quad 0.5\times 0.7 + 0.3\times 0.6 + 0.2\times 0.5$
$\Rightarrow\ 0.35 + 0.18 + 0.10 = 0.63$

20. An unbiased die with faces marked 1, 2, 3, 4, 5 and 6 is rolled four times. Out of four face values obtained the probability that the minimum face value is not less than 2 and the maximum face value is not greater than 5 is ***[IIT-1993; DCE-2000; Roorkee-2000]***

(a) 16/81 (b) 1/81 (c) 80/81 (d) 65/81

Solution

(a) P(minimum face value not less than 2 and maximum face value not greater than 5)

$=P(2 \text{ or } 3 \text{ or } 4 \text{ or } 5)=\frac{4}{6}=\frac{2}{3}$

Hence required probability $= {}^4C_4$

$\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^0=\frac{16}{81}$

21. India plays two matches each with West Indies and Australia. In any match the probabilities of India getting point 0, 1 and 2 are 0.45, 0.05 and 0.50, respectively. Assuming that the outcomes are independents, the probability of India getting at least 7 points is ***[IIT-1992; Orissa JEE-2004]***

(a) 0.8750 (b) 0.0875
(c) 0.0625 (d) 0.0250

Solution

(b) Matches played by India are 4. Maximum points in any match are 2.

$\therefore$ Maximum points in 4 matches can be 8 only. Therefore probability

$P = p(7) + p(8)$

$p(7) = {}^4C_1\,(0.05)(0.5)^3 = 0.0250$

$p(8) = (0.5)^4 = 0.0625$

$\Rightarrow\ P = 0.0875$

22. A box contains 24 identical balls; of which 12 are white and 12 are black. The balls are drawn at random from the box, one at a time with replacement. The probability that a white ball is drawn for the fourth time on the seventh draw is *[IIT Screening-1994]*

(a) $\frac{5}{64}$ (b) $\frac{27}{32}$ (c) $\frac{5}{32}$ (d) $\frac{1}{2}$

Solution

(c) To draw fourth white ball in seventh draw, 3 white balls have to be drawn from first six draws

$$P = {}^6C_3 \times \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) \Rightarrow P = \frac{5}{32}$$

23. A man takes a step forwards with probability 0.4 and backwards with probability 0.6. Find the probability that at the end of 11 steps he is one step away from the starting point.

(a) ${}^{11}C_6(0.4)^6(0.6)^5$ (b) ${}^{11}C_6(0.6)^6(0.5)^5$

(c) $462(0.24)^5$ (d) None of these

Solution

(c) The man will be one step away from the starting point if:

(i) either he is one step ahead or

(ii) one step behind the starting point.

Now if at the end of 11 steps the man is one step ahead the starting point, he must take 6 steps forwards and 5 steps backwards. The probability of this event

$= {}^{11}C_6 \times (0.4)^6 \times (0.6)^5$

$= 462 \times (0.4)^6 \times (0.6)^5$

Again if at the end of 11 steps, the man is 1 step behind the starting point, then out of 11 steps he must have taken 6 steps backwards and five steps forwards. The probability of this event

$= {}^{11}C_6 \times (0.6)^6 \times (0.4)^5$

$= 462 \times (0.6)^6 \times (0.4)^5$

Since the events (i) and (ii) are mutually exclusive the probability that one of these events happens

$= 462 \times (0.4)^6 \times (0.6)^5 + 462 \times (0.6)^6 \times (0.4)^5$

$= 462 \times (0.4)^5 \times (0.6)^5[0.4 + 0.6]$

$= 462 \times (0.4 \times 0.6)^5 \times 1 = 462 \times (0.24)^5$

24. A cricket team has 15 members; out of which only 5 can bowl. If the names of the 15 members are put into a hat and 11 drawn random, then the chance of obtaining an 11 containing at least 3 bowlers is

(a) 7/13 (b) 11/15

(c) 12/13 (d) None of these

Solution

(c) Requiredpr obability

$$= \frac{{}^5C_3 \times {}^{10}C_8}{{}^{15}C_{11}} + \frac{{}^5C_4 \times {}^{10}C_7}{{}^{15}C_{11}} + \frac{{}^5C_5 \times {}^{10}C_6}{{}^{15}C_{11}}$$

$$= \frac{1}{{}^{15}C_{11}}(10 \times 45 + 5 \times 120 + 1 \times 210)$$

$$= \frac{1260 \times 1 \times 2 \times 3 \times 4}{15 \times 14 \times 13 \times 12} = \frac{12}{13}$$

25. If two events A and B are such that $P(\bar{A}) = 0.3$, $P(B) = 0.4$ and $P(A\bar{B}) = 0.5$, then $P\left(\frac{B}{A \cup \bar{B}}\right)$ equals *[IIT-1994]*

(a) 1/2 (b) 1/4

(c) 1/3 (d) None of these

Solution

(b) $P(\bar{A}) = 0.3 \Rightarrow P(A) = 0.7$

$P(B) = 0.4 \Rightarrow \quad P(\bar{B}) = 0.6$

Now $P(A \cup \bar{B}) = P(A) + P(\bar{B}) - P(A\bar{B})$

$= 0.7 + 0.6 - 0.5 = 0.8$ ……(1)

and $P(A \cap \bar{B}) = P(A) - P(A \cap B)$

$\Rightarrow \quad P(A \cap B) = P(A) - P(A \cap \bar{B})$

$= 0.7 - 0.5 = 0.2$ ……(2)

$\therefore$ Required probability

$$p\left(\frac{B}{A \cup \bar{B}}\right) = \frac{P\left[B \cap (A \cup \bar{B})\right]}{P(A \cup \bar{B})}$$

$$= \frac{P[(B \cap A) \cup (B \cap \bar{B})]}{P(A \cup \bar{B})} = \frac{P(A \cap B)}{P(A \cup \bar{B})} = \frac{0.2}{0.8} = 1/4$$

26. Two dice are thrown together four times. The probability that both dice will show the same number twice is *[EAMCET-91]*

(a) 1/3 (b) 25/36

(c) 25/216 (d) None

Solution

(c) The probability of showing the same number by both dice

$p = 6/36 = 1/6$

In binomial distribution

$n = 4, r = 2, p = 1/6, q = 5/6$

$\therefore$ Required probability

$= {}^nC_r q^{n-r} p^r = {}^4C_2 (5/6)^2 (1/6)^2$

$$= 6\left(\frac{25}{36}\right)\left(\frac{1}{36}\right) = \frac{25}{216}$$

27. Two dice are thrown thrice. The probability of getting at most twice equal numbers on dice is **[DCE-94]**

(a) 1/6 (b) 5/72

(c) 215/216 (d) None

Solution

p = probability of occurring the same number on two dice

$$= \frac{6}{36} = \frac{1}{6} \qquad q = 1 - \frac{1}{6} = \frac{5}{6}$$

$\therefore$ Using binomial distribution, probability

$$= \sum_{r=0}^{2} {}^3C_r q^{3-r} p^r = {}^3C_0 q^3 + {}^3C_1 q^2 p + {}^3C_2 q p^2$$

$$= \frac{215}{216}$$

28. One card is drawn from the pack of cards. It is now replace in the pack and a card is again drawn. If it is done six times, then the probability of coming 2 cards of heart, 2 of diamond and 2 red cards in order is **[Ranchi-05]**

(a) $(1/4)^4$ (b) $(1/4)^5$

(c) $(1/4)^6$ (d) None of these

Solution

(b) Probability for the first 2 cards of heart = $(1/4)^2$ and probability for two cards of red colour = $(1/2)^2$.

$\therefore$ Required probability

$$= \left(\frac{1}{4}\right)^2 \left(\frac{1}{4}\right)^2 \left(\frac{1}{2}\right)^2 = \left(\frac{1}{4}\right)^5.$$

29. Six positive and 8 negative numbers are given. If 4 numbers are chosen and multiplied, then the probability of getting a positive product is **[Kurukshetra, (CEE)-97]**

(a) 15/1001 (b) 70/1001

(c) 420/1001 (d) 505/1001

Solution

(d) The number of selections of 4 numbers from 6 + 8 = 14 numbers = ${}^{14}C_4 = 1001$. The selected four numbers product will be positive in the following cases

(i) All numbers are positive.

(ii) All numbers are negative.

(iii) Two number are positive and two negative.

$\therefore$ Favourable cases = ${}^6C_4 + {}^8C_4 + ({}^6C_2 + {}^8C_2)$ = 505

$$\therefore \text{Probability} = \frac{505}{1001}$$

30. Two events A and B are such that $P(A) = 1/4$, $P(B/A) = 1/2$, $P(A/B) = 1/4$, then $P(\overline{A} / \overline{B})$ is equal to **[NDA-2005]**

(a) 1/4 (b) 3/4 (c) 1/2 (d) 2/3

Solution

(b) $P(B/A) = \frac{1}{2}$

$$\Rightarrow \frac{P(BA)}{P(A)} = \frac{1}{2}$$

$$\Rightarrow P(BA) = \frac{1}{2} P(A) = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$$

$$\Rightarrow P(AB) = 1/8 \qquad \ldots\ldots(1)$$

Also $P(A/B) = \frac{1}{4}$

$$\Rightarrow \frac{P(AB)}{P(B)} = \frac{1}{4}$$

$$\Rightarrow P(B) = 4\left(\frac{1}{8}\right) = \frac{1}{2} \qquad \ldots\ldots(2)$$

Further

$P(\overline{A} \cap \overline{B}) = P(\overline{A \cup B}) = 1 - P(A + B)$

$= 1 - [P(A) + P(B) - P(AB)]$

$$= 1 - \left(\frac{1}{4} + \frac{1}{2} - \frac{1}{8}\right) = \frac{3}{8} \qquad \ldots\ldots(3)$$

$$= P\left(\frac{\bar{A}}{\bar{B}}\right) = \frac{P(\bar{A}\cap\bar{B})}{P(\bar{B})} = \frac{(3/8)}{(1/2)} =$$

31. Team A has probability 2/3 of winning whenever it plays. Suppose A plays 4 games; then the probability that A wins more than half of its games is ***[VIT-2006]***

(a) 16/27 (b) 19/27
(c) 19/81 (d) 32/81

Solution

(a) In binomial probability distribution

$p = 2/3$, $q = 1/3$, $n = 4$.

So, required probability $= P(x = 3, 4)$

$$= {}^4C_3\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^3 + {}^4C_4\left(\frac{2}{3}\right)^4$$

$$\therefore 4\times\frac{8}{81}+\frac{16}{81}=\frac{16}{27}$$

32. The mean and variance of a binomial variates X are 2 and 1, respectively. The probability that X takes a values greater than 1 is

(a) 1/16 (b) 5/16
(c) 11/16 (d) 15/16

Solution

(c) Required probability $= 1 - P(x \le 1)$

$$= 1-\left[{}^4C_0\left(\frac{1}{2}\right)^4 + {}^4C_1\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)\right] = 1-\frac{5}{16}=\frac{11}{16}$$

33. A dice is thrown $(2n + 1)$ times. The probability of getting 1, 3 or 4 at most n times is

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) None

Solution

(a) Let X be the number of times 1, 3 or 4 occur on the die. Then X follows a binomial distribution with probability $p = \frac{3}{6} = \frac{1}{2}$.

We have P(1, 3 or 4 occur at most n times on the die)

$$= P(0 \le X \le n) = P(X = 0) + P(X = 1) + \ldots + P(X = n)$$

$$= {}^{2n+1}C_0\left(\frac{1}{2}\right)^{2n+1} + {}^{2n+1}C_1\left(\frac{1}{2}\right)^{2n+1} + \ldots + {}^{2n+1}C_n\left(\frac{1}{2}\right)^{2n+1}$$

$$= \left[{}^{2n+1}C_0 + {}^{2n+1}C_1 + \ldots + {}^{2n+1}C_n\right]\left(\frac{1}{2}\right)^{2n+1}$$

Let $S = {}^{2n+1}C_0 + {}^{2n+1}C_1 + \ldots + {}^{2n+1}C_n$

$$\Rightarrow\quad 2S = 2.{}^{2n+1}C_0 + 2.{}^{2n+1}C_1 + \ldots + 2.{}^{2n+1}C_n$$

$$= ({}^{2n+1}C_0 + {}^{2n+1}C_{2n+1}) + ({}^{2n+1}C_1 + {}^{2n+1}C_{2n}) + \ldots + ({}^{2n+1}C_n + {}^{2n+1}C_{n+1})$$

$$\Rightarrow\quad S = 2^{2n}.$$

Hence, required probability $= 2^{2n}\left(\frac{1}{2}\right)^{2n+1} = \frac{1}{2}$

34. In binomial probability distribution, mean is 3 and standard deviation is 3/2. Then the probability distribution is ***[AISSE-1979; Pb. CET-2003]***

(a) $\left(\frac{3}{4}+\frac{1}{4}\right)^{12}$ (b) $\left(\frac{1}{4}+\frac{3}{4}\right)^{12}$

(c) $\left(\frac{1}{4}+\frac{3}{4}\right)^{9}$ (d) $\left(\frac{3}{4}+\frac{1}{4}\right)^{9}$

Solution

(a) Mean $= np = 3$, S.D. $= \sqrt{npq} = \frac{3}{2}$

$$\Rightarrow\quad q = \frac{npq}{np} = \frac{9}{4\times 3} = \frac{3}{4}$$

$$\Rightarrow\quad p = 1-\frac{3}{4} = \frac{1}{4}$$

Hence binomial distribution is

$$(q+p)^n = \left(\frac{3}{4}+\frac{1}{4}\right)^{12}$$

35. A box contains 3 white and 2 red balls. If the first drawn ball is not replaced, then the probability that the second drawn ball will be red is ***[Roorkee-95]***

(a) 8/25 (b) 2/5 (c) 3/5 (d) 21/25

Solution

(b) Let A = event that drawing ball is white
B = event that drawing ball is red

There are two mutually exclusive cases of the required event: WR and RR.

Now $P(WR) = P(W)\,P(R/W) = \frac{3}{5} \times \frac{2}{4} = \frac{6}{20}$

$P(RR) = P(R)\,P(R/R) = \frac{2}{5} \times \frac{1}{4} = \frac{2}{20}$

$\therefore$ Required probability

$= P(WR + RR)$

$= P(WR) + P(RR) = \frac{6}{20} + \frac{2}{20} = \frac{8}{20} = \frac{2}{5}$

36. There are four machines and it is known that exactly two of them are faulty. They are tested, one by one, in a random order till both the faulty machines are identified. Then the probability that only two tests are needed is *[IIT-1998]*

(a) 1/3 (b) 1/6 (c) 1/2 (d) 1/4

Solution

(a) Two test are needed in two ways i.e. if both tested machines are faulty or both are without faulty.

$\therefore$ faulty machines or machines without fault can be tested with probability $= \frac{1}{{}^4C_2} = \frac{1}{6}$

$\therefore$ Required probability $= 2 \times \frac{1}{6} = \frac{1}{3}$

37. There are n urns each containing $(n + 1)$ balls such that the i^{th} urn contains i white balls and $(n + 1 - i)$ red balls. Let u_i be the event of selecting the i^{th} urn, $i = 1, 2, 3 \ldots, n$, and W denote the event of getting a white ball.

(i) If $P(u_i) \alpha\, i$, where $i = 1, 2, 3, \ldots, n$, then $\lim\limits_{n\to\infty} P(W)$ is equal to *[IIT-JEE-2006]*

(a) 1 (b) 1/4 (c) 2/3 (d) 3/4

Solution

(c) Let $P(u_i) = ki$,

Since u_i, $i = 1, 2, \ldots, n$ are mutually exclusive total events, $\Sigma P(ui) = 1$.

$\Rightarrow \quad k + 2k + 3k + \ldots + nk = 1$

$\Rightarrow \quad \frac{kn(n+1)}{2} = 1 \Rightarrow \quad k = 2/n(n + 1)$

Now

$$P(W) = P(u_1)P\left(\frac{W}{u_1}\right) + P(u_2)P\left(\frac{W}{u_2}\right) + \ldots + P(u_n)P\left(\frac{W}{u_n}\right)$$

$$= \frac{2}{n(n+1)} \times \frac{1}{n+1} + \frac{2.2}{n(n+1)} \times \frac{2}{n+1} + \ldots + \frac{2n}{n(n+1)} \times \frac{n}{n+1}$$

$$= \frac{2}{n(n+1)^2}[1 + 2^2 + 3^2 + \ldots + n^2]$$

$$= \frac{2}{n(n+1)^2} = \frac{n(n+1)(2n+1)}{6} = \frac{2n+1}{3(n+1)}$$

$\therefore \lim\limits_{n\to\infty} P(W) = \frac{2}{3}$

(ii) If in the above question $P(u_i) = c$, where c is a constant, then $P(u_n/W)$ is equal to *[IIT-JEE-2006]*

(a) $\frac{1}{2}$ (b) $\frac{1}{n+1}$

(c) $\frac{2}{n+1}$ (d) $\frac{n}{n+1}$

Solution

(c) $P\left(\frac{u_n}{W}\right) = \frac{P(u_n \cap W)}{P(W)} = \frac{P(u_n)P(W/u_n)}{\sum\limits_{i=1}^{n} P(u_i)P(W/u_i)}$

$$= \frac{c\left(\frac{n}{n+1}\right)}{c\sum\limits_{i=1}^{n}\frac{i}{n+1}} = \frac{n}{1+2+3+\ldots+n}$$

$\Rightarrow \quad \frac{n}{\frac{n(n+1)}{2}} = \frac{2}{n+1}$

(iii) If in the above question $P(u_i) = 1/n$, then the value of $P(W/E)$ is equal to *[IIT-JEE-2006]*

(a) $\frac{1}{n+1}$ (b) $\frac{n}{n+1}$

(c) $\frac{n+2}{2n+1}$ (d) $\frac{n+2}{2(n+1)}$

Solution

Here, $E = u_2 \cup u_4 \cup u_6 \cup \ldots \cup u_n$

$\Rightarrow \quad P(E) = P(u_2) + P(u_4) \ldots P(u_n)$

$$= \frac{1}{n} + \frac{1}{n} + \ldots + \frac{1}{n} = \frac{n}{2}\left(\frac{1}{n}\right) = \frac{1}{2} \qquad \ldots\ldots(1)$$

Now $P\left(\frac{W}{E}\right) = \frac{P(W \cap E)}{P(E)}$

$$= 2\left[\frac{1}{n} \times \frac{2}{n+1} + \frac{1}{n} \times \frac{4}{n+1} + \ldots + \frac{1}{n} \times \frac{n}{n+1}\right]$$

$$= \frac{4}{n(n+1)}\left[1 + 2 + 3 + \ldots + \frac{n}{2}\right]$$

$$= \frac{4}{n(n+1)} \frac{\frac{n}{2}\left(\frac{n}{2}+1\right)}{2} = \frac{n+2}{2(n+1)}$$

OBJECTIVE PROBLEMS: IMPORTANT QUESTIONS WITH SOLUTIONS

1. If $4P(A) = 6P(B) = 10(A \cap B) = 1$, then $P(B/A) =$ *[MPPET-2003]*
(a) 2/5 (b) 3/5
(c) 7/10 (d) 19/60

2. A fair coin is tossed n times. If the probability that head occurs six times is equal to the probability that head occurs eight times, then n is equal to *[Kurukshetra CEE-1998; AMU-2000]*
(a) 15 (b) 14
(c) 12 (d) 7

3. A coin is tossed $m + n$ times, where $m \geq n$. The probability of getting at least m consecutive heads is
(a) $\frac{n+1}{2^{m+1}}$ (b) $\frac{n+2}{2^{m+1}}$
(c) $\frac{m+2}{2^{n+1}}$ (d) None of these

4. If the probability that a student is not a swimmer is 1/5, then the probability that out of 5 students 1 is swimmer is
(a) ${}^5C_1\left(\frac{4}{5}\right)^4\left(\frac{1}{5}\right)$ (b) ${}^5C_1\frac{4}{5}\left(\frac{1}{5}\right)^4$
(c) $\frac{4}{5}\left(\frac{1}{5}\right)^4$ (d) None of these

5. A coin is tossed n times. The probability of getting head at least once is greater than 0.8, then the least value of n is *[EAMCET-2003]*
(a) 2 (b) 3
(c) 4 (d) 5

6. A die is tossed five times. Getting an odd number is considered a success. Then the variance of distribution of the success is *[AIEEE-2002]*
(a) 8/3 (b) 3/8
(c) 4/5 (d) 5/4

7. The mean and variance of a binomial distribution are 4 and 3, respectively, then the probability of getting exactly 6 successes in this distribution is *[MP PET-2002]*
(a) ${}^{16}C_6\left(\frac{1}{4}\right)^{10}\left(\frac{3}{4}\right)^6$ (b) ${}^{16}C_6\left(\frac{1}{4}\right)^6\left(\frac{3}{4}\right)^{10}$
(c) ${}^{12}C_6\left(\frac{1}{4}\right)^{10}\left(\frac{3}{4}\right)^6$ (d) ${}^{12}C_6\left(\frac{1}{4}\right)^6\left(\frac{3}{4}\right)^6$

8. The mean and variance of a binomial distribution are 6 and 4. The parameter n is *[MPPET-2000]*
(a) 18 (b) 12
(c) 10 (d) 9

9. A coin is tossed three times by 2 persons. What is the probability that both get equal number of heads? *[DCE-1999]*
(a) 3/8 (b) 1/9
(c) 5/16 (d) None of these

10. If $\overline{E}$ and $\overline{F}$ are the complementary events of events E and F, respectively, and if $0 < P(F) < 1$, then?
(a) $P(E/F) + P(\overline{E}/F) = 1$
(b) $P(E/F) + P(E/\overline{F}) = 1$
(c) $P(\overline{E}/F) + P(E/\overline{F}) = 1$
(d) $P(E/\overline{F}) + P(\overline{E}/\overline{F}) = 1$

11. Seven white balls and 3 black balls are randomly placed in a row. The probability that no 2 black balls are placed adjacently equals

(a) 1/2 (b) 7/15
(c) 2/15 (d) 1/3

12. A bag contains 30 balls numbered from 1 to 30; 1 ball is drawn randomly. The probability that the number of the ball is multiple of 5 or 7 is *[RPET-1997]*

(a) 1/2 (b) 1/3
(c) 2/3 (d) 1/4

13. The probabilities of a student getting I, II and III division in an examination are 1/10, 3/5 and 1/4, respectively. The probability that the student fails in the examination is *[MPPET-1997]*

(a) 197/200 (b) 27/100
(c) 83/100 (d) None of these

14. In tossing 10 coins, the probability of getting exactly 5 heads is *[MP PET-1996]*

(a) 9/128 (b) 63/256
(c) 1/2 (d) 193/256

15. A coin is tossed three times in succession. If E is the event that there are at least 2 heads and F is the event in which the first throw is a head, then $P(E/F) =$

(a) 3/4 (b) 3/8
(c) 1/2 (d) 1/8

16. If a party of n persons sits at a round table, then the odds against two specified individuals sitting next to each other are

(a) $2 : (n - 3)$ (b) $(n - 3) : 2$
(c) $(n - 2) : 2$ (d) $2 : (n - 2)$

17. The chance of an event happening is the square of the chance of a second event but the odds against the first are the cube of the odds against the second. The chances of the events are

(a) 1/9, 1/3 (b) 1/16, 1/4
(c) 1/4, 1/2 (d) None of these

18. In order to get at least once a head with probability ≥ 0.9, the number of times a coin needs to be tossed is

(a) 3 (b) 4
(c) 5 (d) None of these

19. The probability of India winning a test match against West Indies is 1/2. Assuming independence from match to match, the probability that in a 5 match series India's second win occurs at the third test is

(a) 2/3 (b) 1/2
(c) 1/4 (d) 1/8

20. A die is thrown three times. Getting a 3 or a 6 is considered a success. Then the probability of at least 2 successes is

(a) 2/9 (b) 7/27
(c) 1/27 (d) None of these

21. A man and his wife appear for an interview for two posts. The probability of the husband's selection is 1/7 and that of the wife's selection is 1/5. What is the probability that only one of them be selected?

(a) 1/7 (b) 2/7
(c) 3/7 (d) None of these

22. A purse contains 4 copper coins and 3 silver coins, the second purse contains 6 copper coins and 2 silver coins. If a coin is drawn out of any purse, then the probability that it is a copper coin is

(a) 4/7 (b) 3/4
(c) 37/56 (d) None of these

23. If out of 20 consecutive whole numbers 2 are chosen at random, then the probability that their sum is odd is

(a) 5/19 (b) 10/19
(c) 9/19 (d) None of these

24. In a box of 10 electric bulbs, 2 are defective. Two bulbs are selected at random one after the other from the box. The first bulb after selection being put back in the box before making the second selection. The probability that both the bulbs are without defect is *[MPPET-1987]*

(a) 9/25 (b) 16/25
(c) 4/5 (d) 8/25

25. A box contains 6 nails and 10 nuts. Half of the nails and half of the nuts are rusted. If one item is chosen at random, what is the probability that it is rusted or is a nail? *[MPPET-1992; 2000]*

(a) 3/16 (b) 5/16
(c) 11/16 (d) 14/16

26. For two events A and B, if $P(A) = P\left(\frac{A}{B}\right) = \frac{1}{4}$ and $P\left(\frac{B}{A}\right) = \frac{1}{2}$, then

(a) A and B are independent
(b) $P\left(\frac{A'}{B}\right) = \frac{3}{4}$
(c) $P\left(\frac{B'}{A'}\right) = \frac{1}{2}$
(d) All of these

27. If from each of the three boxes containing 3 white and 1 black, 2 white and 2 black, 1 white and 3 black balls, 1 ball is drawn at random from each box, then the probability that 2 white and 1 black ball will be drawn is

(a) 13/32 (b) 1/4
(c) 1/32 (d) 3/16

28. A bag contains 4 white, 5 red and 6 black balls. If 2 balls are drawn at random, then the probability that one of them is white is

(a) 44/105 (b) 11/105
(c) 11/21 (d) None of these

29. If a coin be tossed n times, then probability that the head comes odd number of times is *[RPET-2002]*

(a) 1/2 (b) $1/2^n$
(c) $1/2^{n-1}$ (d) None of these

30. Five horses are in a race. Mr A selects two of the horses at random and bets on them. The probability that Mr A selected the winning horse is

(a) 4/5 (b) 3/5
(c) 1/5 (d) 2/5

31. A die is thrown four times. The probability of getting at most two 6 is

(a) 0.984 (b) 0.802
(c) 0.621 (d) 0.721

32. Given that $P(A) = 1/3$, $P(B) = 1/4$, $P(A|B) = 1/6$, then $P(B|A)$ equal to *[NDA-2009]*

(a) 1/4 (b) 1/8
(c) 3/4 (d) 1/2

33. For a binomial distribution $b(n, p)$, $np = 4$ and variance = 4/3. What is the probability $P(x \geq 5)$ equal to? *[NDA-2009]*

(a) $(2/3)^6$ (b) $2^5/3^6$
(c) $(1/3)^6$ (d) $2^8/3^6$

34. If bag A contains 2 white and 3 red balls and bag B contains 4 white and 5 red balls. A ball is selected randomly from a randomly selected bag and is found to be red. Then the probability that it is selected from bag B is *[MPPET-2009]*

(a) 25/52 (b) 5/18
(c) 21/52 (d) 13/18

35. The probability that A speaks truth is 4/5 and the probability that B speaks truth is 3/4. The probability that they contradict each other when asked to speak on a fact is *[MPPET-2009]*

(a) 3/10 (b) 7/20
(c) 1/4 (d) 2/5

36. The mean and the variance of a binomial distribution are 4 and 2, respectively, then the probability of 2 successes is *[MPPET-2009]*

(a) 28/256 (b) 42/256
(c) 56/256 (d) 72/256

37. If three students A, B, C can solve a problem with probabilities 1/3, 1/4 and 1/5 respectively, then the probability that the problem will be solved is *[MPPET-2009]*

(a) 3/5 (b) 4/5
(c) 2/5 (d) 47/60

SOLUTIONS

1. (a) $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{\left(\frac{1}{10}\right)}{\left(\frac{1}{4}\right)} = \frac{2}{5}$

2. (b) Here $P(\text{without defected}) = \frac{8}{10} = \frac{4}{5} = p$

$P\text{ (defected)} = \frac{2}{10} = \frac{1}{5} = q$ and $n = 2, r = 2$

Hence required probability $= {}^nC_r p^r \times q^{n-r}$

$= {}^2C_2 \left(\frac{4}{5}\right)^2 \times \left(\frac{1}{5}\right)^0 = \frac{16}{25}$

3. (a) Starting with the first or second or third or, … or n^{th} or $(n + 1)^{th}$ toss, we must get head m times and head or tail at later draws. Required probability

$= P$(getting head at first m tosses)

$+ P$(getting a tail at second toss and head on next m tosses)

$+ P$(getting a tail at second toss and head on next m tosses) …

$+ P$(getting a tail at n^{th} toss and head on next m tosses)

$$= \left(\frac{1}{2}\right)^m$$

$$+ \left\{\frac{1}{2} \times \left(\frac{1}{2}\right)^m + \frac{1}{2} \times \left(\frac{1}{2}\right)^m + \frac{1}{2} \times \left(\frac{1}{2}\right)^m + \ldots \text{ up to } n \text{ terms}\right\}$$

$$= \frac{1}{2^m} + \frac{n}{2^{m+1}} = \frac{2+n}{2^{m+1}} = \frac{n+2}{2^{m+1}}$$

4. (b) Required probability $= {}^5C_1 \left(\frac{4}{5}\right)\left(\frac{1}{5}\right)^4$

5. (b) $1 - \left(\frac{1}{2}\right)^n \geq 0.8$

$\left(\frac{1}{2}\right)^n \leq 0.2$

$\left(\frac{1}{2}\right)^n \leq \frac{1}{5}$

The least value of n is 3.

6. (d) Probability of getting odd number

$p = \frac{3}{6} = \frac{1}{2}$

Probability of getting others even number q

$= \frac{3}{6} = \frac{1}{2}$

$\therefore$ Variance $= npq = 5 \times \frac{1}{2} \times \frac{1}{2} = \frac{5}{4}$

7. (b) For the binomial distribution of n trials, p = successes q = failures, mean = np and variance = npq.

$np = 4$ and $npq = 3 \quad \Rightarrow \quad q = \frac{3}{4}$

$\Rightarrow \quad p = 1 - q = \frac{1}{4}$

8. (a) By binomial distribution

Mean = $npq = 4$ Variance = $npq = 4$

$\therefore \frac{npq}{np} = \frac{4}{6}$,

$q = \frac{2}{3}$ from $p + q$ as $q = \frac{2}{3}$

$\therefore p = \frac{1}{3}$

Put $p = \frac{1}{3}$ in $np = 6$ or $n = 18$.

9. (c) The condition will be satisfied, if both get 0, 1, 2 or 3 heads.

$\therefore$ Either 0 head by A and 0 head by B

or 1 head by A and 1 head by B

or 2 head by A and 2 head by B

or 3 head by A and 3 head by B

$\therefore$ Required probability

$= \left[\frac{1}{8} \times \frac{1}{8} + \frac{3}{8} \times \frac{3}{8} + \frac{3}{8} \times \frac{3}{8} + \frac{1}{8} \times \frac{1}{8}\right] = \frac{5}{16}$

10. (a, d)

$P\left(\frac{E}{F}\right) + P\left(\frac{\bar{E}}{F}\right) = \frac{P(E \cap F) + P(\bar{E} + F)}{P(F)}$

$= \frac{P\{(E \cap F) \cup (\bar{E} \cap \bar{F})\}}{P(F)}$

$[\because E \cap F$ and $\bar{E} \cap F$ are disjoint$]$

$$= \frac{P\{(E \cup \bar{E}) \cap F\}}{P(F)} = \frac{P(F)}{P(F)} = 1$$

Similarly, we can show that (b) and (c) are not true while (d) is true.

$$P\left(\frac{E}{F}\right) + P\left(\frac{\bar{E}}{F}\right) = \frac{P(E \cup \bar{F})}{P(\bar{F})} + \frac{P(\bar{E} \cap \bar{F})}{P(\bar{F})}$$

$$= \frac{P(\bar{F})}{P(\bar{F})} = 1$$

11. (b) The white balls can be kept in 7! ways. Now there are 8 places for black balls between white balls. Hence the black balls can be placed n^8P_3 ways. The total number of arrangements is (10)!. Required probability

$$= \frac{(7!)\,{}^8P_3}{(10)!}$$

$$= \frac{8 \times 7 \times 6}{10 \times 9 \times 8} = \frac{7}{15}$$

12. (b) Numbers multiple of 5 = (5, 10, 15, 20, 25, 30)

Numbers multiple of 7 = (7, 14, 21, 28)

Multiple of both = (0); $P(5 \text{ or } 7) = \frac{10}{30} = \frac{1}{3}$

13. (b) Given, probability to get I class

$$P(A) = \frac{1}{10}$$

Probability to get II class

$$P(B) = \frac{3}{5}$$

Probability to get III class

$$P(C) = \frac{1}{4}$$

If the student fails, it means he does not get I, II, III class. Probability that the student fails

$$P = P(\bar{A})P(\bar{B})P(\bar{C})$$

Required probability

$$P = \{1 - P(A)\}\ \{1 - P(B)\}\{1 - P(C)\}$$

$$P = \left(1 + \frac{1}{10}\right)\left(1 - \frac{3}{5}\right)\left(1 - \frac{1}{4}\right)$$

$$P = \frac{9}{10} \times \frac{2}{5} \times \frac{3}{4} = \frac{27}{100}$$

14. (b) Probability of getting head

$$p = \frac{1}{2}$$

By $p + q = 1$, $q = \frac{1}{2}$

given $n = 10$, $r = 5$

By binomial distribution formula probability of r successes

$$P(r) = {}^n\mathrm{C}_r\, \mathrm{p}^r\, q^{\,n-r}$$

$$\therefore\ P(5) = {}^{10}C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^{10-5} = {}^{10}C_5 \left(\frac{1}{2}\right)^{10} = \frac{63}{256}$$

15. (a) Formula $P\left(\frac{E}{F}\right) = \frac{P(E \cap F)}{PF}$ ......(1)

sample space of 3 coins.

(HHH) (HHT) (HTH) (HTT) (THH) (THT) (TTH) (TTT)

$\therefore\ n(s) = 8$

Event E: at least 2 heads

(HHH) (HTH) (HTH) (THH)

and Event F: first throw is a head

(HHH) (HHT) (HTH) (HTH)

$\therefore\ n(f) = 4$

E and F : at least 2 heads and the first throw is a head

(HHH) (HHT) (HTH)

$\therefore\ n\ (E \text{ and } F) = 3$

$$\therefore\ P(F) = \frac{4}{8}, P(E \cap F) = \frac{3}{8}$$

Using $F\left(\frac{E}{F}\right) = \frac{3/8}{4/8} = \frac{3}{4}$

16. (d) Total cases, for n persons to sit on a round table = n! Total cases when two specified persons sit together = $2!(n - 2)!$.

Probability of two persons sitting together

$$P=\frac{2!(n-2)!}{(n-1)!}$$

$$P=\frac{2\times 1}{(n-1)}=\frac{a}{a+b}$$

On comparing we get $a=2$

$a+b=(n-1)$

$\therefore\ b=(n-1)-2=n-3$

Odd against two specified persons sitting together $=\frac{b}{a}=\frac{n-3}{2}$.

17. (a) Let p_1, p_2 be the chances of happening of the first and second event, respectively, thus by the given condition, we have

$$p_1=p_2^2 \text{ and } \frac{1-p_1}{p_1}=\left(\frac{1-p_2}{p_2}\right)^3$$

$$\Rightarrow\ \frac{1-p_2^2}{p_2^2}=\left(\frac{1-p_2}{p_2}\right)^3$$

$$\Rightarrow\ p_2(1+p_2)=(1-p_2)^2$$

$$\Rightarrow\ p_2+p_2^2=1+p_2^2-2p_2$$

$$\Rightarrow\ 3p_2=1$$

$$\Rightarrow\ p_2=\frac{1}{3}$$

$$\Rightarrow\ p_1=\left(\frac{1}{3}\right)^2=\frac{1}{9}$$

18. (b) Probability of getting at least 1 head

$$=1-\left(\frac{1}{2}\right)^n\geq 0.9$$

$$\Rightarrow\ \left(\frac{1}{2}\right)^n\leq 0.1\ \Rightarrow\ -n\log 2\leq -1$$

or, $n\geq\frac{1}{\log 2}=\frac{1}{0.30103}=3.32\Rightarrow\quad n=4$

19. (c) The sample space is $[LWW, WLW]$.

$\therefore\ P(LWW)+P(WLW)$

Probability that in 3 match series, it is India's second win

$=P(L)\,P(W)\,P(W)+P(W)\,P(L)\,P(W)$

$$=\frac{1}{8}+\frac{1}{8}=\frac{2}{8}=\frac{1}{4}$$

20. (b) Required Probability $=P(\text{exactly }2)+P(\text{exactly }3)$

$$={}^3C_2\times\left(\frac{2}{6}\right)^2\left(\frac{4}{6}\right)+{}^3C_3\times\left(\frac{2}{6}\right)^3$$

$$=\frac{2}{9}+\frac{1}{27}=\frac{7}{27}$$

21. (b) The probability that the husband is not selected $=1-\frac{1}{7}=\frac{6}{7}$

The probability that the wife is not selected

$$=1-\frac{1}{5}=\frac{4}{5}$$

The probability that only husband is selected

$$=\frac{1}{7}\times\frac{4}{5}=\frac{4}{35}$$

The probability that only wife is selected

$$=\frac{1}{5}\times\frac{6}{7}=\frac{6}{35}$$

Hence required probability

$$=\frac{6}{35}+\frac{4}{35}=\frac{10}{35}=\frac{2}{7}$$

22. (d) Let

$A\equiv$ event of selecting the first purse

$B\equiv$ event of selecting the second purse

$C\equiv$ event of drawing a copper coin from the first purse

$D\equiv$ event of drawing a copper coin from the second purse

Then the given event has two disjoint cases: AC and BD

$\therefore$ Required probability

$=P(AC+BD)=P(AC)+P(BD)$

$=P(A)\,P(C)+P(B)\,P(D)$

$$=\frac{1}{2}\times\frac{4}{7}+\frac{1}{2}\times\frac{6}{8}=\frac{37}{56}$$

23. (b) The sum of two positive integers is odd and the other is even.

$\therefore$ The probability of the required event is

$$\frac{{}^{10}C_1\times{}^{10}C_1}{{}^{20}C_2}=\frac{2\times 10\times 10}{20\times 19}=\frac{10}{19}$$

24. (b) Here $P(\text{without defected}) = \frac{8}{10} = \frac{4}{5} = p$

$P(\text{defected}) = \frac{2}{10} = \frac{1}{5} = q$ and $n = 2, r = 2$

Hence required probability $= {}^nC_r p^r \times q^{n-r}$

$$= {}^2C_2\left(\frac{4}{5}\right)^2 \times \left(\frac{1}{5}\right)^0 = \frac{16}{25}$$

25. (c) Probability that the item is rusted

$$P(A) = \frac{8}{16}$$

Probability that the item is nail

$$P(B) = \frac{6}{16}$$

Probability (rusted and nail)

$$P(A \cap B) = \frac{3}{16}$$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\therefore P(A \cup B) = \frac{8}{16} + \frac{6}{16} - \frac{3}{16} = \frac{11}{16}$$

26. (c) $P\left(\frac{B}{A}\right) = \frac{1}{2} P(B \cap A) = \frac{1}{8}$

$$P\left(\frac{A}{B}\right) = \frac{1}{4} P(B) = \frac{1}{2}$$

$$P(A \cap B) = \frac{1}{8} = P(A)P(B)$$

So, events A and B are independent.

$$P\left(\frac{A'}{B}\right) = \frac{P(A' \cap B)}{P(B)} = \frac{3}{4}$$

$$P\left(\frac{B'}{A'}\right) = \frac{1}{2}$$

27. (a) The selections can be (ω, ω, b), (ω, b, ω), (b, ω, ω)

The probabilities are

$$\frac{3}{4} \times \frac{2}{4} \times \frac{3}{4} + \frac{3}{4} \times \frac{2}{4} \times \frac{1}{4} + \frac{1}{4} \times \frac{2}{4} \times \frac{1}{4}$$

$$= \frac{18 + 6 + 2}{64} = \frac{13}{32}$$

28. (a) Required probability

$$= \frac{{}^4C_1 \times {}^5C_1}{{}^{15}C_2} + \frac{{}^4C_1 \times {}^6C_1}{{}^{15}C_2} = \frac{44}{105}$$

29. (a) Let x denote the number of heads in n trials then,

$$P(x = r) = {}^nC_r \left(\frac{1}{2}\right)^r \cdot \left(\frac{1}{2}\right)^{n-r} = {}^nC_r \left(\frac{1}{2}\right)^n$$

$\therefore$ Required probability

$= P(x = 1) + P(x = 3) + P(x = 5) + \ldots$

$$= {}^nC_1\left(\frac{1}{2}\right)^n + {}^nC_3\left(\frac{1}{2}\right)^n + {}^nC_5\left(\frac{1}{2}\right)^5 + \ldots$$

$$= \left(\frac{1}{2}\right)^n \{{}^nC_1 + {}^nC_3 + {}^nC_5 + \ldots\}$$

$$= \left(\frac{1}{2}\right)^n (2^{n-1})^n = \frac{1}{2}$$

30. (d) Out of 5 horses only 1 is the winning horse. The probability that Mr A selected the losing horse $= \frac{4}{5} \times \frac{3}{4}$

$\therefore$ The probability that Mr A selected the winning horse $= 1 - \frac{4}{5} \times \frac{3}{4} = \frac{2}{5}$

31. (a) Probability of getting $6 = \frac{1}{6} = p$,

$q = 5/6$, $n = 4$, $r = 2$

Probability of getting at most 2 six

$$= {}^4C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^2 + {}^4C_1\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^3 + {}^4C_0\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^4$$

$= 0.984$

32. (b) $\because P(A) = \frac{1}{3}, P(B) = \frac{1}{4}, P\left(\frac{A}{B}\right) = \frac{1}{6}$

But $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$$\Rightarrow \quad \frac{1}{6} = \frac{P(A \cap B)}{\frac{1}{4}}$$

$$\Rightarrow \quad P(A \cap B) = \frac{1}{24}$$

$$\therefore P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{1/24}{1/3} = \frac{1}{8}$$

33. (d) $\because np = 4$ and $npq = \frac{4}{3}$

$\therefore\ 4q = \frac{4}{3} \quad q = \frac{1}{3}$

$\therefore\ p = 1 - \frac{1}{3} = \frac{2}{3}$

$\Rightarrow \quad n = \frac{4\times 3}{2} = 6$

Now, $P(X \geq 5) = {}^6C_5\,(p)^5\,(q)^1 + {}^6C_6\,p^6\,q^0$

$= {}^6C_5\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right) + {}^6C_6\left(\frac{2}{3}\right)^6$

$= \frac{6\times 32}{3^6} + \frac{64}{3^6} = \frac{256}{3^6} = \frac{2^8}{3^6}$

34. (a) Let E be the event of selecting a red ball.

$\therefore\ P\left(\frac{E}{A}\right) = \frac{3}{5}, P\left(\frac{E}{B}\right) = \frac{5}{9}, P(A) = P(B) = \frac{1}{2}$

$\therefore\ P\left(\frac{B}{E}\right) = \dfrac{P(B)\times P\left(\frac{E}{B}\right)}{P(A)\times P\left(\frac{E}{A}\right) + P(B)\times P\left(\frac{E}{B}\right)}$

$= \dfrac{\frac{1}{2}\times\frac{5}{9}}{\frac{1}{2}\times\frac{3}{5} + \frac{1}{2}\times\frac{5}{9}} = \dfrac{\frac{5}{9}}{\frac{52}{45}} = \frac{25}{52}$

35. (b) Given, $P(A) = \frac{4}{5}, P(B) = \frac{3}{4}$

$\Rightarrow \quad P(\bar{A}) = \frac{1}{5}, P(\bar{B}) = \frac{1}{4}$

$\therefore$ Required probability $= P(A\cap\bar{B}) + P(\bar{A}\cap B)$

$= P(A)P(\bar{B}) + P(\bar{A})P(B)$

$= \frac{4}{5}\times\frac{1}{4} + \frac{1}{5}\times\frac{3}{4} = \frac{7}{20}$

36. (a) Given mean $np = 4$ and variance $npq = 2$

$\Rightarrow \quad \frac{npq}{np} = \frac{2}{4}$

$\Rightarrow \quad q = \frac{1}{2} = p$

$\therefore \quad n = \frac{4}{1/2} = 8$

$\therefore$ Probability of 2 successes $= {}^8C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^6$

$= 28\times\left(\frac{1}{2}\right)^8 = \frac{28}{256}$

37. (a) Given, the probabilities of solving the problem are 1⁄3, 1⁄4 and 1⁄5, respectively, and corresponding probabilities of not solving the problem are 2⁄3, 3⁄4 and 4⁄5, respectively.

$\therefore$ Required probability = 1 – P (not solving the problem)

$= 1 - \left(\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}\right)$

$= 1 - \frac{2}{5} = \frac{3}{5}$

UNSOLVED OBJECTIVE PROBLEMS (IDENTICAL PROBLEMS FOR PRACTICE): FOR IMPROVING SPEED WITH ACCURACY

1. Three mangoes and 3 apples are in a box. If 2 fruits are chosen at random, the probability that 1 is a mango and the other is an apple is *[EAMCET-1990]*

(a) 2/3 (b) 3/5
(c) 1/3 (d) None of these

2. A die is tossed thrice. If getting a 4 is considered a success, then the mean and variance of the probability distribution of the number of successes are

(a) 1/2, 1/12 (b) 1/6, 5/12
(c) 5/6, 1/2 (d) None of these

3. $P(B) = \frac{3}{4}, P(\bar{A}\cap B\cap\bar{C}) = \frac{1}{3},$

$P(A\cap B\cap\bar{C}) = \frac{1}{3}$ then $P(B\cap C)$: *[IIT Screening 2003]*

(a) 1/12 (b) 3/4
(c) 5/12 (d) 23

4. Fifteen coupons are numbered 1, 2, 3, …, 15, respectively. Seven coupons are selected at random one at a time with replacement. The probability that the largest number appearing on a selected coupon is 9 is *[IIT-83; Ranchi-90]*

(a) $(9/16)^6$ (b) $(8/15)^7$
(c) $(3/5)^7$ (d) None of these

5. Five boys and 5 girls are sitting in a row randomly. The probability that boys and girls sit alternatively is *[Kerala (CEE)-2005]*

(a) 1/63 (b) 1/126
(c) 5/126 (d) 6/126

6. An aircraft has 3 engines A, B and C. The aircraft crashes if all the 3 engines fail. The probabilities of failure are 0.03, 0.02 and 0.05 for engines A, B and C, respectively, that the aircraft will not crash? *[NDA-2006]*

(a) 0.00003 (b) 0.90
(c) 0.99997 (d) 0.90307

7. If A and B are two events such that $P(A \cup B) = P(A \cap B)$, then the true relation is

(a) $P(A) + P(B) = 0$
(b) $P(A) + P(B) = P(A)\, P(B/A)$
(c) $P(A) + P(B) = 2P(A)\, P(B/A)$
(d) None of these

8. If 2 coins are tossed 5 times, then the probability of getting 5 heads and 5 tails is *[AMU-2002]*

(a) 63/256 (b) 1/1024
(c) 2/205 (d) 9/64

9. If A and B are 2 events such that $P(A) = 3/8$, $P(B) = 5/8$ and $P(A \cup B) = 3/4$, then $P(A/B) =$

(a) 2/5 (b) 2/3
(c) 3/5 (d) None of these

10. Eight coins are tossed simultaneously. The probability of getting at least 6 heads is

(a) 57/64 (b) 229/256
(c) 7/64 (d) 37/256

11. The mean and variance of a random variable X having a binomial distribution are 4 and 2, respectively, then $P(X = 1)$ is

(a) 1/32 (b) 1/16
(c) 1/8 (d) 1/4

12. In a binomial distribution the probability of getting a success is 1/4 and standard deviation is 3, then the its mean is *[EAMCET-2002]*

(a) 6 (b) 8
(c) 12 (d) 10

13. A person can kill a bird with probability 3/4. He tries 5 times. What is the probability that he may not kill the bird? *[RPET-97]*

(a) 243/1024 (b) 781/1024
(c) 1/1024 (d) 1023/1024

14. The records of a hospital show that 10% of the cases of a certain disease are fatal. If 6 patients are suffering from the disease, then the probability that only 3 will die is

(a) 1458×10^{-5} (b) 1458×10^{-6}
(c) 41×10^{-6} (d) 8748×10^{-5}

15. A drawer contains 5 brown and 4 blue socks well mixed. A man reaches the drawer and pulls out 2 socks at random. What is the probability that they match?

(a) 4/9 (b) 5/8
(c) 5/9 (d) 7/12

16. A draws 2 cards with replacement from a pack of 52 card and B throws a pair of dice what is the chance that A gets both cards of the same suit and B gets total of 6?

(a) 1/144 (b) 1/4
(c) 5/144 (d) 7/144

17. The items produced by a firm are supposed to contain 5% defective items. The probability that a sample of 8 items will contain less than 2 defective items is

(a) $\frac{27}{20}\left(\frac{19}{20}\right)^7$ (b) $\frac{533}{400}\left(\frac{19}{20}\right)^6$
(c) $\frac{153}{20}\left(\frac{1}{20}\right)^7$ (d) $\frac{35}{16}\left(\frac{1}{20}\right)^6$

18. A die is tossed twice. Getting a number greater than 4 is considered a success. Then the variance of the probability distribution of the number of successes is *[DSSE-87]*

(a) 2/9 (b) 4/9
(c) 1/3 (d) None of these

19. If E_1 denotes the event of coming sum 6 in throwing of two dice and E_2 is the event of coming 2 in any one of the two, then $P(E_2/E_1)$ is

(a) 1/5 (b) 4/5
(c) 3/5 (d) 2/5

20. A party of 10 sit round a table. What are the odds against two specified persons A, B sitting together?

(a) 7 : 2 (b) 7 : 3
(c) 1 : 2 (d) 7 : 1

21. If the probabilities of a boy and a girl to be born are the same, then in a 4 children family the probability of being at least 1 girl is

(a) 14/16 (b) 15/16
(c) 1/8 (d) 3/8

22. For a. B.D. the parameters n and p are 16 and 1/2, respectively. Then its S.D. σ is equal to

(a) 2 (b) $\sqrt{2}$
(c) $2\sqrt{2}$ (d) 4

23. A drawer contains 50 bolts and 150 nuts. Half of the bolts and half of the nuts are rusted. If 1 item is chosen at random. What the probability that it is rusted or is a bolt?

(a) 1/8 (b) 3/8
(c) 5/8 (d) 7/8

24. It is given that the events A and B are such that $P(A) = 1/4$, $P(A/B) = 1/2$ and $P(B/A) = 2/3$. Then $P(B) =$ ***[AIEEE-2008]***

(a) 1/2 (b) 1/6
(c) 1/3 (d) 2/3

25. The probability that a certain kind of component will survive a given shock test is 3/4. The probability that exactly 2 of the next 4 components tested survive is

[VITEEE-2008]

(a) 9/41 (b) 25/128
(c) 1/5 (d) 27/128

26. What is the probability that in a family of 4 children there will be at least 1 boy?

[NDA-2008]

(a) 15/16 (b) 3/8
(c) 1/16 (d) 7/8

27. A purse contains 4 nickel and 9 copper coins while another purse contains 6 nickel and 7 copper coins. A purse is chosen at random and a coin is drawn from it. The probability that it is a nickel coin is ***[PET (Raj.), 93]***

(a) 10/13 (b) 6/13
(c) 5/13 (d) 4/13

WORKSHEET: TO CHECK THE PREPARATION LEVEL

Important Instructions

1. The answer sheet is immediately below the worksheet.
2. The test is of 24 minutes.
3. The worksheet consists of 24 questions. The maximum marks are 72.
4. Use Blue/Black Ball point pen only for writing particulars/marking responses. Use of pencil is strictly prohibited.

1. A coin is tossed 10 times. The probability of getting exactly 6 heads is
(a) 512/513 (b) 105/512
(c) 100/153 (d) ${}^{10}C_6$

2. The probability of solving a question by 3 students are 1/2, 1/4, 1/6, respectively. The probability of question being solved will be:
(a) 33/48 (b) 35/48
(c) 31/48 (d) 37/48

3. Two persons A and B stand in a row with 10 other persons. What is the probability that there are exactly 2 persons between A and B?
(a) 3/22 (b) 2/22
(c) 1/22 (d) None of these

4. Each of A and B tosses two coins. What is the probability that they get equal number of heads? *[NDA-2007]*
(a) 3/16 (b) 5/16 (c) 4/16 (d) 6/16

5. If A and B are two events such that $P(A) = 1/3$, $P(B) = 1/4$ and $P(A \cap B) = 1/5$, then $P(\overline{B}/\overline{A}) =$
(a) 37/40 (b) 37/45
(c) 23/40 (d) None of these

6. Assuming that for a husband-wife couple the chances of their child being a boy or a girl are the same, the probability of their two children being a boy and a girl is
(a) 1/4 (b) 1 (c) 1/2 (d) 1/8

7. Out of 40 consecutive natural numbers, 2 are chosen at random. Probability that the sum of the numbers is odd is
(a) 14/29 (b) 20/39
(c) 1/2 (d) None of these

8. A and B are two events such that $P(A) = 0.8$, $P(B) = 0.6$ and $P(A \cap B) = 0.5$, then the value of $P(A/B)$ is
(a) 5/6 (b) 5/8
(c) 9/10 (d) None of these

9. Five coins are tossed simultaneously. The probability of at least one head turning up is
(a) 1/32 (b) 1/8
(c) 15/16 (d) 31/32

10. The probability that a man can hit a target is 3/4. He tries five times The probability that he will hit the target at least three times is *[MNR-1994]*
(a) 291/364 (b) 371/264
(c) 471/502 (d) 459/512

11. If A and B are mutually exclusive events with $P(B) \neq 1$, then $P(A/\overline{B}) =$ (Here $\overline{B}$ is the complement of the event B.) *[EAMCET-2007]*
(a) $\dfrac{1}{P(B)}$ (b) $\dfrac{1}{1-P(B)}$
(c) $\dfrac{P(A)}{P(B)}$ (d) $\dfrac{P(A)}{1-P(B)}$

12. A bag contains 6 white and 4 black balls. Two balls are drawn at random. The probability that they are of the same colour is *[EAMCET-2007]*
(a) 1/15 (b) 2/5
(c) 4/15 (d) 7/15

13. The mean and standard deviation of a binomial variate X are 4 and $\sqrt{3}$, respectively. Then $P(X \geq 1) =$ *[EAMCET-2007]*
(a) $1-\left(\dfrac{1}{4}\right)^{16}$ (b) $1-\left(\dfrac{3}{4}\right)^{16}$
(c) $1-\left(\dfrac{2}{3}\right)^{16}$ (d) $1-\left(\dfrac{1}{3}\right)^{16}$

14. If A and B are two events of a random experiment, $P(A) = 0.25$, $P(B) = 0.5$ and $P(A \cap B) = 0.15$, then $P(A \cap \overline{B}) =$ *[MPPET-1987]*
(a) 0.1 (b) 0.35
(c) 0.15 (d) 0.6

15. In a binomial distribution, the mean is 4 and variance is 3. Then its mode is *[MPPET-2006]*

(a) 5 (b) 6
(c) 4 (d) None of these

16. A fair die is tossed eight times. The probability that a third 6 is observed on the eighth throw is *[AIEEE-2002]*

(a) $\frac{{}^7C_2 \times 5^5}{6^6}$ (b) $\frac{{}^7C_2 \times 5^5}{6^2}$
(c) $\frac{{}^7C_2 \times 5^5}{6^8}$ (d) None of these

17. The mean and variance of a binomial variable X are 2 and 1, respectively, then $P(X \geq 1)$ is

(a) 2/3 (b) 4/5
(c) 7/8 (d) 15/16

18. The probability that England wins a cricket match against India is 2/3. If India and England play 3 test matches, the probability that England will win exactly 2 test matches is

(a) 1/3 (b) 4/9
(c) 5/9 (d) None of these

19. An experiment succeeds twice as often as it fails. Find the probability that in 4 trials there will be at least 3 successes. *[AMU-1999]*

(a) 4/27 (b) 8/27
(c) 16/27 (d) 24/27

20. In a simultaneous toss of 4 coins, what is the probability of getting exactly 3 heads?

(a) 1/2 (b) 1/3
(c) 1/4 (d) None of these

21. A bag contains 5 white, 7 red and 8 black balls. If 4 balls are drawn one by one without replacement, what is the probability that all are white?

(a) 1/969 (b) 1/380
(c) 5/20 (d) None of these

22. The odds against A solving a certain problem are 4 to 3 and the odds in favour of B solving the same problem are 7 to 5, then the chance that the problem will be solved if they both try is

(a) 5/21 (b) 16/21
(c) 4. 5/3 .7 (d) None of these

23. There are n letters and n addressed envelopes. The probability that all the letters are not kept in the right envelope is

(a) $\frac{1}{n!}$ (b) $1-\frac{1}{n!}$
(c) $1-\frac{1}{n}$ (d) None of these

24. A box contains 10 identical electronic components of which 4 are defective. If 3 components are selected at random from the box in succession, without replacing the units already drawn, what is the probability that two of the selected components are defective? *[NDA-2007]*

(a) 3/10 (b) 1/40
(c) 1/5 (d) 5/24

ANSWER SHEET

1. (a) (b) (c) (d)	9. (a) (b) (c) (d)	17. (a) (b) (c) (d)
2. (a) (b) (c) (d)	10. (a) (b) (c) (d)	18. (a) (b) (c) (d)
3. (a) (b) (c) (d)	11. (a) (b) (c) (d)	19. (a) (b) (c) (d)
4. (a) (b) (c) (d)	12. (a) (b) (c) (d)	20. (a) (b) (c) (d)
5. (a) (b) (c) (d)	13. (a) (b) (c) (d)	21. (a) (b) (c) (d)
6. (a) (b) (c) (d)	14. (a) (b) (c) (d)	22. (a) (b) (c) (d)
7. (a) (b) (c) (d)	15. (a) (b) (c) (d)	23. (a) (b) (c) (d)
8. (a) (b) (c) (d)	16. (a) (b) (c) (d)	24. (a) (b) (c) (d)

HINTS AND EXPLANATIONS

1. (b) Probability of getting head $= \frac{1}{2}$

Probability of getting tail $= \frac{1}{2}$

By binomial distribution,

probability of getting exactly 6 heads

$= {}^{10}C_6\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^6$

$= \frac{105}{512}$

2. (a) Probability of question not being solved

$= \frac{1}{2}\times\frac{3}{4}\times\frac{5}{6} = \frac{15}{48}$

So, required probability $= 1 - \frac{15}{48} = \frac{33}{48}$

4. (b) Required probability

$= \frac{1}{4}\times\frac{1}{4}+\frac{1}{2}\times\frac{1}{2} = \frac{5}{16}$

7. (b) Case1: Sum even = first number even + second number odd.

Let event A.

Case 2: Sum even = odd + even.

Let event B.

There are two mutually exclusive event.

$\therefore P(\text{sum even}) = P(A \cup B) = P(A \cup B)$

$= P(A) + P(B)$

$= \frac{20}{40}\times\frac{20}{39}+\frac{20}{40}\times\frac{20}{39} = \frac{20}{39}$

NOTE

There are 20 even and odd numbers present in 40 consecutive natural numbers.

10. (d) Probability that a man hits a target

$p = \frac{3}{4}$

miss the target

$q = \frac{1}{4}$ and $n = 5$

Probability of hitting target at least three times

$= P(3) + P(4) + P(5)$

$= {}^5C_3\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right)^2 + {}^5C_4\left(\frac{3}{4}\right)^4\left(\frac{1}{4}\right) + {}^5C_5\left(\frac{3}{4}\right)^5$

$= \frac{10\times 27}{64\times 16}+\frac{5\times 81}{64\times 16}+\frac{243}{16\times 16}$

$= \frac{270+405+243}{1024} = \frac{459}{512}$

15. (c) Given that $np = 4$ and $npq = 3$

$q = \frac{npq}{np} = \frac{3}{4}$

Also, $p = 1 - q = 1 - \frac{3}{4} = \frac{1}{4}$

$\therefore n = 16$

We know that in binomial distribution, the value of r for which $p(x = r)$ is maximum is known as mode of binomial distribution. $(n + 1)p - 1 \le r \le (n + 1)p$

$\frac{17}{4} - 1 \le r \le \frac{17}{4}$

$\frac{13}{4} \le 1 \le \frac{17}{4}$

$3.25 \le r \le 4.25$

$r = 4$

16. (c) Probability of getting 6 exactly two times in 7 throws

$= {}^7C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^5$

So, required probability

$= {}^7C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^5\left(\frac{1}{6}\right) = \frac{{}^7C_2\times 5^5}{6^3}$

22. (b) The odds against A are 4: 3

$\therefore P(A) = \frac{3}{7}$

The odds in favour of B are 7: 5

$\therefore P(B) = \frac{7}{12}$

The required probability $= 1 - P(\bar{A})P(\bar{B})$

$$= 1 - \left(1 - \frac{3}{7}\right)\left(1 - \frac{7}{12}\right)$$

$$= \frac{64}{84} = \frac{16}{21}$$

24. (a) Required probability $= \dfrac{{}^6C_1 \times {}^4C_2}{{}^{10}C_3}$

$$= \frac{6 \times 6 \times 6}{10 \times 9 \times 8} = \frac{3}{10}$$

LECTURE

4

Probability Distribution and Baye's Theorem

BASIC CONCEPTS

1. Probability Distribution

Let the corresponding probabilities of the values $x_1, x_2, \ldots, x_n$ of a variate X be $p_1, p_2, \ldots p_n$, respectively. If these are such that

(i) $0 \le p_i \le 1, i = 1, 2, \ldots, n,$

and (ii) $p_1 + p_2 + \ldots + p_n$, i.e., $\sum_{i=1}^{n} p_i = 1$

then we say that variate X satisfy probability distribution. In tabular form it is expressed as

X:	x_1	x_2	x_3	……	x_n
$P(X)$:	p_1	p_2	p_3	……	p_n

Further in a probability distribution, for its variate X

Mean $= \mu = \sum_{i=1}^{n} p_i x_i$

variance $= \sum (x_i - \mu)^2 p_i$ (when μ is different)

Variance $= \sum_{i=1}^{n} p_i x_i^2 -$ (mean)^2

2. Poisson Probability Distribution

When an experiment is repeated n time where n is very large, i.e., $n \to \infty$, and the probability p of success is very small, i.e., $p \to 0$, then in such a condition poisson distribution is used which is a limiting case of binomial distribution. Under poisson distribution, for a variate X, the probability of r successes is given by

$$P(X = r) = e^{-m} \times \frac{m^r}{r!}, \ e = 2.7183$$

where m is a parameter and r can take any non-negative integral value. Further, for the variate of a poisson distribution

Mean $= m = \underset{n\to\infty}{Lt}\, np$, SD $= \sqrt{m}$ variance $= m$

NOTE

The mean and the variance of the poisson distribution both are equal to the parameter m.

Example 1: A random variate x has Poisson distribution with mean 2. Then $P(X > 1.5)$ equals: *[AIEEE-2005]*

(a) $3/e^2$ (b) $1 - 3/e^2$

(c) 0 (d) $2/e^2$

Solution

(b) $\because P(X = r) = \dfrac{e^{-m} m^r}{r!}$, where m is the mean.

$\therefore P(X = r) = \dfrac{e^{-2} 2^r}{r!}$

Now $P(X = r > 1.5) = P(2) + P(3) + \ldots$ up to ∞

$= e^{-2}\left[\dfrac{2^2}{2!} + \dfrac{2^3}{3!} + \dfrac{2^4}{4!} + \ldots\right]$

$= e^{-2}\left[e^2 - \left(1 + \dfrac{2}{1!}\right)\right]$

$= 1 - 3e^2$

Example 2: At a telephone enquiry system the number of phone calls regarding relevant enquiry follow Poisson distribution with an average of 5 phone calls during 10-minute time intervals. The probability that there is at the most 1 phone call during a 10-minute time period is *[AIEEE 2006]*

(a) 6/55 (b) 5/6
(c) $6/e^5$ (d) $6/5^e$

Solution

(c) Here $m = 5$, so $P(x = 0,1) = e^{-5}\dfrac{5^0}{0!} + e^{-5}\dfrac{5^1}{1!}$

Example 3: In a Poisson distribution if $2P(X = 1) = (X = 2)$, then the variance is *[VIT, 2005]*

(a) 0 (b) −1
(c) 4 (d) 2

Solution

(a) $2\dfrac{e^{-m}m^1}{1!} = \dfrac{e^{-m}m^2}{2!} \Rightarrow m^2 - 4m = 0 \Rightarrow m = 4$ $[\because m \neq 0]$

$\therefore$ variance $= m = 4$

3. **Bayes Theorem (Inverse Probability)**
If event B can happen (occur) with any of the following mutually exclusive and collectively exclusive events A_1, A_2, A_3 then for given $P(B/A_1)$ $P(B/A_2)$, $P(B/A_3)$ process of finding any one of $P(A_i/B)$, $i = 1, 2, 3$ is called **Bayes' thorem or inverse probability,** which is as follows (for example):

$$P\left(\frac{A_1}{B}\right) = \frac{P(A_1)P(B/A_1)}{P(A_1)P(B/A_1) + P(A_2)P(B/A_2) + P(A_3)P(B/A_3)} = \frac{P(A_1)P(B/A_1)}{\sum_{i=1}^{3} P(A_i)P(B/A_i)}$$

NOTE

The event B can happen with 2 or more of mutually exclusive and collectively exclusive events.

Example 1: A letter is known to have come either from LONDON or CLIFTON; on the postmark only the two consecutive letters ON are legible. The probability that it came from LONDON is

(a) 5/17 (b) 12/17
(c) 17/30 (d) 3/5

Solution

(b) We define the following events:
A_1: 'Selecting a pair of consecutive letter from the word LONDON'
A_2: 'Selecting a pair of consecutive letters from the word CLIFTON'
E: 'Selecting a pair of letters ON'

Then $P(A_1 \cap E) = \dfrac{2}{5}$; as there are 5 pairs of consecutive letters out of which 2 are ON.

$P(A_2 \cap E) = \dfrac{1}{6}$; as there are 6 pairs of consecutive letters of which one is ON.

$\therefore$ The required probability is

$$P\left(\frac{A_1}{E}\right) = \frac{P(A_1 \cap E)}{P(A_1 \cap E) + P(A_2 \cap E)} = \frac{\frac{2}{5}}{\frac{2}{5} + \frac{1}{6}} = \frac{12}{17}$$

Example 2: In an entrance test there are multiple choice questions. There are four possible answers to each question; out of which one is correct. The probability that a student knows the answer to a question is 90%. If he gets the correct answer to a question, then the probability that he was guessing is

(a) 37/40 (b) 1/37
(c) 36/37 (d) 1/9

Solution

(b) We define the following events:
A_1: 'He knows the answer'
A_2: 'He does not know the answer'
E: 'He gets the correct answer'

Then $P(A_1) = \dfrac{9}{10}$, $P(A_2) = 1 - \dfrac{9}{10} = \dfrac{1}{10}$,

$P\left(\dfrac{E}{A_1}\right) = 1$, $P\left(\dfrac{E}{A_2}\right) = \dfrac{1}{4}$

$\therefore$ Required probability

$$= P\left(\frac{A_2}{E}\right) = \frac{P(A_2)P(E/A_2)}{P(A_1)P(E/A_1) + P(A_2)P(E/A_2)} = \frac{1}{3}$$

Example 3: A bag A contains 2 white and 3 red balls and bag B contains 4 white and 5 red balls. One ball is drawn at random from a randomly chosen bag and is found to be red. The probability that it was drawn from bag B was: ***[BIT Ranchi 1988; IIT -76]***

(a) $\frac{5}{14}$ (b) $\frac{5}{16}$ (c) $\frac{5}{18}$ (d) $\frac{25}{52}$

Solution

(d) Let E_1 be the event that the ball is drawn from bag A, E_2 the event that it is drawn from bag B and E that the ball is red. We have to find $P(E_2/E)$. Since both the bags are equally likely to be selected, we have $P(E_1) = P(E_2) = \frac{1}{2}$

Also $P(E/E_1) = 3/5$ and $P(E/E_2) = 5/9$

Hence by Bayes theorem, we have

$$P(E_2/E) = \frac{P(E_2)P(E/E_2)}{P(E_1)P(E/E_1) + P(E_2)P(E/E_2)}$$

$$= \frac{\frac{1}{2}\times\frac{5}{9}}{\frac{1}{2}\times\frac{3}{5}+\frac{1}{2}\times\frac{5}{9}} = \frac{25}{52}$$

4. **Mathematical Expectation:**

Let X be discrete random variable which assumes the values $x_1, x_2, \ldots, x_n$ with the corresponding probability $p_1, p_2, p_3, \ldots, p_n$. The expected value of X or the mathematical expectation of X denoted by $E(X)$ is defined as

$$E(X) = \sum_{i=1}^{n} x_i p_i \text{ where } \sum_{i=1}^{n} p_i = 1$$

$$= x_1p_1 + x_2p_2 + \ldots + x_n p_n \text{ where } p_1 + p_2 + \ldots + p_n = 1$$

NOTE

Mean is nothing else but the expected value of the variable in the BINOMIAL DISTRIBUTION.

5. **Use of Multinomial Theorem:**

Suppose a die has m faces marked 1, 2, 3, ... m and n such dices are thrown. Then the probability that the sum of the numbers shown on the faces equal to S is given by:

$$P(S) = \frac{\text{Coefficient of } x^s \text{ in the expansion of } (x + x^2 + x^3 + \ldots + x^m)^n}{m^n}$$

SOLVED SUBJECTIVE PROBLEMS (XII BOARD (C.B.S.E./STATE)): FOR BETTER UNDERSTANDING AND CONCEPT BUILDING OF THE TOPIC

1. Of the students in a college, it is known that 60% reside in hostel and 40% are day scholars (not residing in hostel). Previous year results report that 30% of all students who reside in hostel attain A grade and 20% of day scholars attain A grade in their annual examination. At the end of the year, one student is chosen at random from the college and he has an A-grade, what is the probability that the student is a hostler? ***[NCERT]***

Solution

Let E_1: 'a student is residing in hostel'
and E_2: 'a student is not residing in the hostel'
then E_1 and E_2 are mutually exclusive and exhaustive.

Moreover, $P(E_1) = \frac{60}{100} = \frac{3}{5}$ and $P(E_2) = \frac{40}{100} = \frac{2}{5}$

Let E: 'a student attains A grade'

then $P(E/E_1) = \frac{30}{100} = \frac{3}{10}$ and $P(E/E_2) = \frac{20}{100} = \frac{2}{10}$

Required probability $= P(E_1/E)$

$$= \frac{P(E/E_1)P(E_1)}{P(E/E_1)P(E_1) + P(E/E_2)P(E_2)}$$

(by Bayes' theorem)

$$= \frac{\frac{3}{10}\times\frac{3}{5}}{\frac{3}{10}\times\frac{3}{5}+\frac{2}{10}\times\frac{2}{5}} = \frac{9}{9+4} = \frac{9}{13}$$

2. In answering a question on a multiple choice test, a student either knows the answer or guesses. Let 3/4 be the probability that he knows the answer and 1/4 be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability 1/4. What is the probability that a student knows the answer given that he answered it correctly? *[NCERT]*

Solution

Let E_1: 'the student knows the answer' and E_2: 'the student guesses the answer' then E_1 and E_2 are mutually exclusive and exhaustive. Moreover, $P(E_1) = 3/4$ and $P(E_2) = 1/4$

Let E: 'the answer is correct', then $P(E/E_1) = 1$ and $P(E/E_2) = 1/4$

($\because$ When the student knows the answer, it is a sure event that the answer is correct)

$\therefore$ Required probability

$$= P(E_1/E) = \frac{P(E/E_1)P(E_1)}{P(E/E_1)P(E_1) + P(E/E_2)P(E_2)}$$

(by Bayes' theorem)

$$= \frac{1 \times \frac{3}{4}}{1 \times \frac{3}{4} + \frac{1}{4} \times \frac{1}{4}} = \frac{\frac{3}{4}}{\frac{3}{4} + \frac{1}{16}} = \frac{\frac{3}{4}}{\frac{12+1}{16}} = \frac{3}{4} \times \frac{16}{13} = \frac{12}{13}$$

3. Suppose a girl throws a die. If she gets a 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3 or 4 she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head, what is the probability that she threw 1, 2, 3 or 4 with the die? *[NCERT]*

Solution

Let E_1: '1, 2, 3 or 4 is shown on die', E_2: '5 or 6 is shown on die' then E_1 and E_2 are mutually exclusive and exhaustive. Moreover,

$$P(E_1) = \frac{4}{6} = \frac{2}{3} \text{ and } P(E_2) = \frac{2}{6} = \frac{1}{3}$$

Let E: 'exactly one head shows up', then $P(E/E_1) = P$(head shows up when coin is tossed once) $= \frac{1}{2}$

and $P(E/E_2) = P$(exactly one head shows up when coin is tossed thrice)

$$= P(\{HTT, THT, TTH\}) = \frac{3}{8}$$

$\therefore$ Required probability $= P(E_1/E)$

$$= \frac{P(E/E_1)P(E_1)}{P(E/E_1)P(E_1) + P(E/E_2)P(E_2)}$$

(using Bayes theorem)

$$= \frac{\frac{1}{2} \times \frac{2}{3}}{\frac{1}{2} \times \frac{2}{3} + \frac{3}{8} \times \frac{1}{3}} = \frac{\frac{1}{3}}{\frac{1}{3} + \frac{1}{8}} = \frac{8}{8+3} = \frac{8}{11}$$

4. Probability that A speaks truth is $\frac{4}{5}$. A coin is tossed. A reports that a head appears. The probability that actually there was head is *[NCERT]*

(a) $\frac{4}{5}$ (b) $\frac{1}{2}$ (c) $\frac{1}{5}$ (d) $\frac{2}{5}$

Solution

Let E_1: 'coin comes up with a head', E_2: 'coin comes up with a tail' then E_1 and E_2 are mutually exclusive and exhaustive. Moreover,

$P(E_1) = P(E_2) = 1/2$

Let E: 'A reports that a head appears', then $P(E/E_1) = P$(head comes up and A speaks truth) $= 4/5$ and $P(E/E_2) = P$(tail comes up and A tells a lie) $= 1/5$

Required probability $= P(E_1/E)$

$$= \frac{P(E/E_1)P(E_1)}{P(E/E_1)P(E_1) + P(E/E_2)P(E_2)}$$

$$= \frac{\frac{4}{5} \times \frac{1}{2}}{\frac{4}{5} \times \frac{1}{2} + \frac{1}{5} \times \frac{1}{2}} = \frac{4}{4+1} = \frac{4}{5}$$

Hence, (a) is the correct option.

5. A random variable X has the following probability distribution. *[NCERT]*

X	0	1	2	3	4	5	6	7
$P(X)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

Determine:

(i) k (ii) $P(X<3)$

(iii) $P(X>6)$ (iv) $P(0<X<3)$

Solution

(i) Since $\Sigma P(X)=1$, therefore

$P(0)+P(1)+P(2)+P(3)+P(4)+P(5)+P(6)+P(7)=1$

$\Rightarrow 0+k+2k+2k+3k+k^2+2k^2+7k^2+k=1$

$\Rightarrow 10k^2+9k-1=0$

$$\Rightarrow k=\frac{-9\pm\sqrt{81+40}}{2\times 10}$$

$$\Rightarrow k=\frac{-9\pm 11}{20}=\frac{1}{10},-1$$

But k cannot be negative $(\because P(1)=k \nless 0)$

therefore, $k=\dfrac{1}{10}$

Hence, probability distribution of X is

X	0	1	2	3	4	5	6	7
$P(X)$	0	1/10	2/10	2/10	3/10	1/100	2/100	(7/100) + (1/10)

(Substituting $k=1/10$ in the given distribution)

(ii) $P(X<3)=P(0)+P(1)+P(2)=0+\dfrac{1}{10}+\dfrac{2}{10}=\dfrac{3}{10}$

(iii) $P(X>6)=P(7)=\dfrac{7}{100}+\dfrac{1}{10}=\dfrac{17}{100}$

(iv) $P(0<X<3)=P(1)+P(2)=\dfrac{1}{10}+\dfrac{2}{10}=\dfrac{3}{10}$

6. Suppose that 5% of men and 0.25% of women have grey hair. A grey haired person is selected at random. What is the probability of this person being male? Assume that there are equal number of males and females. ***[NCERT]***

Solution

Let E_1: 'selected person is a male'

E_2: 'selected person is a female'.

then E_1 and E_2 are mutually exclusive and exhaustive

Also, $P(E_1)=P(E_2)=1/2$

Let E: 'selected person is grey haired'

then $P(E/E_1)=\dfrac{5}{100}=\dfrac{1}{20}$

and $P(E/E_2)=\dfrac{0.25}{100}=\dfrac{1}{400}$

Required probability $=P(E_1/E)$

$$=\frac{P(E/E_1)P(E_1)}{P(E/E_1)P(E_1)+P(E/E_2)P(E_2)}$$

(using Bayes theorem)

$$=\frac{\frac{1}{20}\times\frac{1}{2}}{\frac{1}{20}\times\frac{1}{2}+\frac{1}{400}\times\frac{1}{2}}=\frac{\frac{1}{20}}{\frac{1}{20}+\frac{1}{400}}=\frac{\frac{1}{20}}{\frac{21}{400}}=\frac{20}{21}$$

7. Suppose we have four boxes A, B, C and D containing coloured marbles as given below: ***[NCERT]***

Box	Marble colour		
	Red	White	Black
A	1	6	3
B	6	2	2
C	8	1	1
D	0	6	4

One of the boxes has been selected at random and a single marble is drawn from it. If the marble is red, what is the probability that it was drawn from box A? Box B? Box C?

Solution

Let E_1: 'box A is selected',

E_2: 'box B is selected',

E_3: 'box C is selected' and

E_4: 'box D is selected',

then E_1, E_2, E_3, E_4 are mutually exclusive and exhaustive. Moreover,

$P(E_1)=P(E_2)=P(E_3)=P(E_4)=1/4$

Let E: 'marble drawn is red',

then $P(E/E_1)=\dfrac{1}{1+6+3}=\dfrac{1}{10}$

$P(E/E_2)=\dfrac{6}{6+2+2}=\dfrac{6}{10}$

$P(E/E_3)=\dfrac{8}{8+1+1}=\dfrac{8}{10}$

and $P(E/E_4)=\dfrac{0}{6+6+4}=0$

P (marble is drawn from box A) $= P(E_1/E)$

$$= \frac{P(E/E_1)P(E_1)}{\sum_{i=1}^{4} P(E/E_i)P(E_i)} \quad \text{(by Bayes theorem)}$$

$$= \frac{\frac{1}{10}\times\frac{1}{4}}{\frac{1}{10}\times\frac{1}{4}+\frac{6}{10}\times\frac{1}{4}+\frac{8}{10}\times\frac{1}{4}+0\times\frac{1}{4}}$$

$$= \frac{1}{1+6+8} = \frac{1}{15}$$

P(marble is drawn from box B) $= P(E_2/E)$

$$= \frac{P(E/E_2)P(E_2)}{\sum_{i=1}^{4} P(E/E_i)P(E_i)}$$

$$= \frac{\frac{6}{10}\times\frac{1}{4}}{\frac{1}{10}\times\frac{1}{4}+\frac{6}{10}\times\frac{1}{4}+\frac{8}{10}\times\frac{1}{4}+0\times\frac{1}{4}}$$

$$= \frac{6}{1+6+8} = \frac{6}{15}$$

and P (marble is drawn from box C)
$= P(E_3/E)$

$$= \frac{P(E/E_3)P(E_3)}{\sum_{i=1}^{4} P(E/E_i)P(E_i)}$$

$$= \frac{\frac{8}{10}\times\frac{1}{4}}{\frac{1}{10}\times\frac{1}{4}+\frac{6}{10}\times\frac{1}{4}+\frac{8}{10}\times\frac{1}{4}+0\times\frac{1}{4}}$$

$$= \frac{8}{1+6+8} = \frac{8}{15}$$

8. Find the probability distribution of ***[NCERT]***

(i) Number of heads in two tosses of a coin.
(ii) Number of tails in the simultaneous tosses of three coins.
(iii) Number of heads in four tosses of a coin.

Solution

(i) When a coin is tossed twice, then the sample space is

$S = \{HH, HT, TH, TT\}$, which contains four equally likely sample points.
Let X denote the number of heads in any outcome in S, then $X(HH) = 2$, $X(HT) = 1$, $X(TH) = 1$ and $X(TT) = 0$

$\therefore P(X = 0) = P$(tail occurs on both tosses)

$$= P(\{TT\}) = \frac{1}{4}$$

$P(X = 1) = P$(one head and one tail occurs)

$$= P(\{TH, HT\}) = \frac{2}{4} = \frac{1}{2}$$

and $P(X = 2) = P$(head occurs on both tosses)

$$= P(\{HH\}) = \frac{1}{4}$$

$\therefore$ Probability distribution of X is

X	0	1	2
$P(X)$	1/4	1/2	1/4

(ii) When a coin is tossed thrice, the sample space is

$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$,

which contains eight equally likely sample points.
Let X denote the number of tails in any outcome $\omega \in S$, then X can take values 0, 1, 2 and 3.

$P(X = 0) = P(\text{no tail}) = P(\{HHH\}) = \frac{1}{8}$

$P(X = 1) = P$(one tail and two heads show up)

$$= P(\{HHT, HTH, THH\}) = \frac{3}{8}$$

$P(X = 2) = P$(two tails and one head show up)

$$= P(\{HTT, THT, TTH\}) = \frac{3}{8} \text{ and}$$

$P(X = 3) = P(\text{three tails show up}) = P(\{TTT\})$

$$= \frac{1}{8}$$

$\therefore$ Probability distribution of X is

X	0	1	2	3
$P(X)$	1/8	3/8	3/8	1/8

(iii) When a coin is tossed four times, the sample space is

$$S = \left\{\begin{matrix} HHHH, HHHT, HHTH, HHTT, HTHH, HTHT, \\ HTTH, HTTT, THHH, THHT, THTH, THTT, \\ TTHH, TTHT, TTTH, TTTT \end{matrix}\right\}$$

which contains 16 equally likely sample points. Let X denote the number of heads in any outcome $\omega \sigma S$, then X can take values 0, 1, 2, 3 and 4.

$P(X = 0) = P(\text{no head shows up}) = P(\{TTTT\})$

$= \frac{1}{16}$

$P(X = 1) = P(\text{one head and three tails show up})$

$= P(\{HTTT, THTT, TTHT, TTTH\}) = \frac{4}{16} = \frac{1}{4}$

$P(X = 2) = P(\text{two heads and two tails show up})$

$= P(\{HHTT, HTHT, HTTH, THHT, THTH, TTHH\})$

$= \frac{6}{16} = \frac{3}{8}$

$P(X = 3) = P(\text{three heads and one tail shows up})$

$= (\{HHHT, HHTH, HTHH, THHH\})$

$= \frac{4}{16} = \frac{1}{4}$

$P(X = 4) = P(\text{four heads show up})$

$= P(\{HHHH\}) = \frac{1}{16}$

$\therefore$ Probability distribution of X is

X	0	1	2	3	4
$P(X)$	1/16	1/4	3/8	1/4	1/16

9. Suppose that two cards are drawn at random from a deck of cards. Let X be the number of aces obtained. What is the value of $E(X)$? ***[NCERT]***

(a) $\frac{37}{221}$ (b) $\frac{5}{13}$

(c) $\frac{1}{13}$ (d) $\frac{2}{13}$

Solution

Here, X can take values 0, 1, 2

$P(X = 0) = P(\text{one ace is drawn})$

$= \frac{^{48}C_2}{^{52}C_2} = \frac{48 \times 47}{52 \times 51} = \frac{188}{221}$

$P(X = 1) = P(\text{one ace and one non-ace})$

$= \frac{^{4}C_1 \times {}^{48}C_1}{^{52}C_2} = \frac{4 \times 48}{\frac{52 \times 51}{2!}} = \frac{32}{221}$

and $P(X = 2) = P(\text{two aces are drawn})$

$= \frac{^{4}C_2}{^{52}C_2} = \frac{4 \times 3}{52 \times 51} = \frac{1}{221}$

$\therefore$ Probability distribution of X is

X	0	1	2
$P(X)$	188/221	32/221	1/221

$\therefore E(X) = \Sigma X\, P(X)$

$= 0 \times \frac{188}{221} + 1 \times \frac{32}{221} + 2 \times \frac{1}{221}$

$= \frac{34}{221} = \frac{2}{13}$

$\therefore$ (d) is the correct option.

UNSOLVED SUBJECTIVE PROBLEMS (XII BOARD (C.B.S.E./STATE)): SOLVE THESE PROBLEMS TO GRASP THE TOPIC

EXERCISE 1

1. A coin is tossed twice. Find the probability distribution of the number of heads.
2. Two cards are drawn successively with replacement from well-shuffled pack of 52 cards. Find the probability distribution of the number of aces. ***[CBSE-1995, 2001]***
3. Tickets are numbered from 1 to 10. Two tickets are drawn one after the other at random. Find the probability that the number on one of the tickets is a multiple of 5 and on the other a multiple of 4. ***[CBSE-94]***
4. A bag contains 5 red and 7 black balls. Second bag contains 4 blue and 3 green balls. One ball is drawn from each bag. Find the probability for: ***[MP-2005 (B)]***
   (i) 1 red and 1 blue ball
   (ii) 1 green and 1 black ball
5. In a bulb factory, machines A, B and C manufacture 60%, 30% and 10% bulbs, respectively. 1%, 2% and 3% of the bulbs produced, respectively, by A, B and C are found to be defective. A bulb is picked up at random from the total production and found to be defective. Find the probability that this bulb was produced by the machine A. ***[CBSE-2008]***
6. A, B and C are three horses participating in a race. The chance of A's win is double of B and chance of B's win is double of C. Find out the probability for winning of each of them. Also find the probability that which horse wins the race, B or C. ***[MP-2008]***
7. Twelve cards, numbered 1 to 12, are placed in a box, mixed up throughly. Then a card is drawn at random from the box. If it is known that the number on the drawn card is more than 3, then find the probability that it is an even number. ***[CBSE-2008]***
8. Tickets are marked from 1 to 12 and mixed up. One ticket is taken out at random. Find the probability of its being a multiple of 2 or 3. ***[MP-91, 94, 2000, 2009]***

EXERCISE 2

1. A die is thrown twice. In the throw getting odd numbers is taken as a success. Find the probability distribution of the success. ***[MP-2001]***
2. There are three urns A, B and C. Urn A contains 4 white balls and 5 blue balls. Urn B contains 4 white balls and 3 blue balls. Urn C contains 2 white balls and 4 blue balls. One ball is drawn from each of these urns. What is the probability that out of these three balls drawn, two are white balls and one is a blue ball. ***[CBSE-96]***
3. A bag contains 4 white and 5 black balls and another bag contains 3 white and 4 black balls. A ball is taken out from the first bag and without seeing its colour is put in the second bag. A ball is taken out from another bag. Find the probability that the ball drawn is white. ***[CBSE-94]***
4. In a class 30% students fail in physics, 25% fail in maths and 10% fail in both. A student is chosen at random. Find the probability that
   (a) He fails in maths if fail in physics.
   (b) He fails in physics if he has been failed in maths.
   (c) He is fail in maths or physics.
5. To form a committee of 4 persons from 5 women and 7 men, find the probability when the committee contains:
   (i) 3 women and 1 men
   (ii) 2 women and 2 men
   (iii) 4 women
6. In a group of students, there are 3 boys and 3 girls. Four students are to be selected at random from the group. Find the probability that either 3 boys and 1 girl or 3 girls and 1 boy are selected. ***[CBSE-Practice Sample paper - V and VII]***
7. There are 3 red and 5 black balls in bag. A and 2 red and 3 black balls in bag B. One ball is drawn from bag A and two from bag B. Find the probability that out of the 3 balls drawn, 1 is red and 2 are black. ***[CBSE-96 (C)]***
8. Two cards are drawn at random from a well-shuffled pack of 52 cards. What is the probability that either both are red or both are kings? ***[MP-2008]***

ANSWERS

EXERCISE 1

1.

X_i	0	1	2
P_i	1/4	1/2	1/4

2.

X	0	1	2
$P(X)$	144/169	24/169	1/169

3. 4/45
4. (i) $\frac{5}{21}$ (ii) $\frac{1}{4}$
5. $\frac{2}{5}$
6. $\frac{3}{7}$
7. $\frac{5}{9}$
8. $\frac{2}{3}$

EXERCISE 2

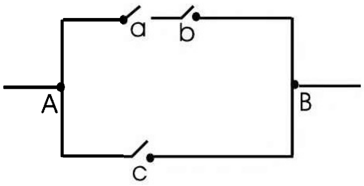
1.

x_i	0	1	2
p_i	1/4	1/2	1/4

2. 64/189
3. 31/32
4. (a) 1/3 (b) 2/5 (c) 9/20
5. (i) 14/99 (ii) 14/33 (iii) 1/99
6. 2/5
7. 39/80
8. $\frac{55}{221}$

SOLVED OBJECTIVE PROBLEMS: HELPING HAND

1. Consider the circuit,



If the probability that each switch is closed is p, then find the probability of current flowing through AB: ***[DCE-2005]***

(a) $p^2 + p$ (b) $p^3 + p - 1$
(c) $p^3 + p$ (d) $p^2 + p + 1$

Solution

(a) Current in the upper part will flow only if both the switches a and b are closed.

$\therefore$ Their probability $= p \times p = p^2$

Now current will flow in lower part of c, if c is closed, its probability is p. Thus, current will flow from A to B if current flows either in upper part or flow in lower part.

$\therefore$ Required probability $= p^2 + p$.

2. Word UNIVERSITY is arranged randomly. Then the probability that both I does not come together is: ***[UPSEAT-2001]***

(a) 3/5 (b) 2/5
(c) 4/5 (d) 1/5

Solution

(c) Total number of ways $= \frac{10!}{2!}$

Favourable number of ways for I come together is 9!

Thus, probability that I come together

$$= \frac{9! \times 2!}{10!} = \frac{2}{10} = \frac{1}{5}$$

Hence, required probability $= 1 - \frac{1}{5} = \frac{4}{5}$

3. Let S be a set containing n elements. If we select two subsets A and B of S at random then the probability that $A \cup B = S$ and $A \cap B = \phi$ is: ***[Orissa JEE 2005]***

(a) 2^n (b) n^2
(c) $1/n$ (d) $1/2^n$

Solution

(d) Ways of selecting two subsets of $A = (2^n)^2$

Ways of selecting $A \cup B$ and $A \cap B$ are 2^n

$\therefore$ Required probability =

$$\frac{\text{Favourable cases}}{\text{Total cases}} = \frac{2^n}{(2^n)^2} = \frac{1}{2^n}$$

4. A die is tossed thrice. A success is getting 1 or 6 on a toss. The mean and the variance of number of successes are: ***[AI CBSE-1985]***

(a) $\mu = 1, \sigma^2 = 2/3$ (b) $\mu = 2/3, \sigma^2 = 1$
(c) $\mu = 2, \sigma^2 = 2/3$ (d) None of these

Solution

(a) For binomial distribution, mean = np and variance = npq

$$n = 3, p = \frac{2}{6} = \frac{1}{3}, q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$$

So, mean (μ) = $3 \times \frac{1}{3} = 1$

Variance(σ^2) = $3 \times \frac{1}{3} \times \frac{2}{3}$

$= \frac{2}{3}$

5. The value of C for which $P(X = k) = Ck^2$ can serve as the probability function of a random variable X that takes 0, 1, 2, 3, 4 is ***[EAMCET-1994]***

(a) 1/30 (b) 1/10
(c) 1/3 (d) 1/15

Solution

(a) $\sum_{k=0}^{4} P(X = k) = 1 \Rightarrow \sum_{k=0}^{4} C_k^2 = 1$

$\Rightarrow C\,(1^2 + 2^2 + 3^2 + 4^2) = 1$

$\Rightarrow C = \frac{1}{30}$

6. If X has binomial distribution with mean np and variance npq, then $\frac{P(X=k)}{P(X=k-1)}$ is ***[Pb. CET 2004]***

(a) $\frac{n-k}{k-1} \times \frac{p}{q}$ (b) $\frac{n-k+1}{k} \times \frac{p}{q}$

(c) $\frac{n+1}{k} \times \frac{q}{p}$ (d) $\frac{n-1}{k+1} \times \frac{q}{p}$

Solution

(b) Here mean = np and variance = npq

$$\therefore \frac{P(X=k)}{P(X=k-1)} = \frac{{}^nC_k (p)^k (q)^{n-k}}{{}^nC_{k-1} (p)^{k-1} (q)^{n-k-1}} = \frac{{}^nC_k}{{}^nC_{k-1}} \times \frac{p}{q}$$

$$\therefore \frac{P(X=k)}{P(X=k-1)} = \frac{n-k+1}{k} \times \frac{p}{q}$$

7. Two cards are drawn successively with replacement from a well-shuffled deck of 52 cards then the mean of the number of aces is ***[J & K 2005]***

(a) 1/13 (b) 3/13
(c) 2/13 (d) None of these

Solution

(c) Let X denote a random variable which is the number of aces. Clearly, X takes values 1, 2.

$$\therefore p = \frac{4}{52} = \frac{1}{13}, q = 1 - \frac{1}{13} = \frac{12}{13}$$

$$P(X = 1) = 2 \times \left(\frac{1}{13}\right)\left(\frac{12}{13}\right) = \frac{24}{169}$$

$$P(X = 2) = 2 \times \left(\frac{1}{13}\right)^2 \left(\frac{12}{13}\right)^0 = \frac{1}{169}$$

$$\text{Mean} = \sum P_i X_i = \frac{24}{169} + \frac{2}{169} = \frac{26}{169} = \frac{2}{13}$$

8. A sample of 4 items is drawn at a random without replacement from a lot of 10 items. Containing 3 defective. If X denotes the number of defective items in the sample then $P(0 < x < 3)$ is equal to ***[J&K-2005]***

(a) 3/10 (b) 4/5
(c) 1/2 (d) 1/6

Solution

(b) Since items are chosen without replacement.

$$P(X = x) = \frac{{}^3C_x \times {}^7C_{4-x}}{{}^{10}C_4}$$

Putting $x = 1, 2$, we have

$$P\,(0 < x < 3) = \frac{{}^3C_1 \times {}^7C_3}{210} + \frac{{}^3C_2 \times {}^7C_2}{210}$$

$$= \frac{3 \times 35 + 3 \times 21}{210} = \frac{105 + 63}{210} = \frac{168}{210} = \frac{4}{5}$$

9. Two persons A and B take turns in throwing a pair of dice. The first person to throw 9 from both dice will be awarded the prize. If A throws first then the probability that B wins the game is ***[Orissa JEE-2003]***

(a) 9/17 (b) 8/17
(c) 8/9 (d) 1/9

Solution

(b) The probability of throwing 9 with two dice $= \frac{4}{36} = \frac{1}{9}$

$\therefore$ The probability of not throwing 9 with two dice = 8/9

If A is to win he should throw 9 in 1st or 3rd or 5th attempt

If B is to win, he should throw, 9 in 2nd, 4th attempt

B's chances

$$= \left(\frac{8}{9}\right) \times \frac{1}{9} + \left(\frac{8}{9}\right)^3 \times \frac{1}{9} + \ldots = \frac{\frac{8}{9} \times \frac{1}{9}}{1 - \left(\frac{8}{9}\right)^2} = \frac{8}{17}$$

10. Two numbers a and b are chosen at random from the set of first 30 natural numbers. The probability that $a^2 - b^2$ is divisible by 3 is

(a) 9/87 (b) 12/87
(c) 15/87 (d) 47/87

Solution

(d) The total number of ways of choosing two numbers out of 1, 2, 3, …, 30 is ${}^{30}C_2 = 435$

Since $a^2 - b^2$ is divisible by 3 if either a and b both are divisible by 3 or none of a and b is divisible by 3.

Thus, the favourable number of cases

$$= {}^{10}C_2 + {}^{20}C_2 = 235.$$

Hence, the required probability

$$= \frac{235}{435} = \frac{47}{87}$$

11. A has 3 shares in a lottery containing 3 prizes and 9 blanks. B has two shares in a lottery containing 2 prizes and 6 blanks; Find the ratio of A's chance of success to B's chance of success.

(a) 927:715 (b) 972:751
(c) 925:715 (d) 715:972

Solution

(c) Since A has 3 shares in a lottery, his chance of success means that he gets at least 1 prize, that is, he gets either 1 prize or 2 prizes or 3 prizes and his chance of failure means that he gets no prize. It is certain that either he succeeds or fails.

If p denotes his chance of success and q the chance of his failure, then $p + q = 1$ or $p = 1 - q$

We now find $q \times n$ = total number of ways

$$= {}^{12}C_3 = \frac{12 \times 11 \times 10}{1 \times 2 \times 3} = 220$$

Since out of 12 tickets in the lottery, he can draw any 3 tickets by virtue of his having 3 shares in the lottery and m = favourable number of ways

$$= {}^{9}C_3 = \frac{9 \times 8 \times 7}{1 \times 2 \times 3} = 84$$

Since he will fail to draw a prize if all the tickets drawn by him are blanks.

$$\therefore q = \frac{m}{n} = \frac{84}{220} = \frac{21}{55}.$$

$$\therefore p = A\text{'s chance of success} = 1 - \frac{21}{55} = \frac{34}{55}$$

Similarly B's chance of success

$$p' = 1 - q' = 1 - \frac{{}^{6}C_2}{{}^{8}C_2} = 1 - \frac{6 \times 5}{8 \times 7}$$

$$= 1 - \frac{15}{28} = \frac{13}{28}.$$

$\therefore$ A's chance of success: B's chance of success

$$= p : p' = \frac{34}{35} : \frac{13}{28} = \frac{952}{1540} : \frac{715}{1540} = 952 : 715$$

12. A and B throw a pair of dice. A wins if he throws 6 before B throws 7 and B wins if he throws 7 before A throws 6. If A begins, what is his chance of winning?

[MNR 1995]

(a) 30/61 (b) 31/61
(c) 61/30 (d) 61/31

Solution

(a) Let E_1 denote the event of A's throwing 6 and E_2 the event of B's throwing 7 with a pair of dice.

Then $\bar{E}_1$, $\bar{E}_2$ are the complementary events. There are five ways of obtaining 6, namely, (1, 5), (2, 4), (3, 3), (4, 2), (5, 1) and similarly there are six ways of getting 7, namely, (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)

$$\therefore P(E_1) = \frac{5}{36}$$

$$\text{and } P(\bar{E}_1) = 1 - \frac{5}{36} = \frac{31}{36}$$

$$P(E_2) = \frac{6}{36} = \frac{1}{6}$$

$$\text{and } P(\bar{E}_2) = 1 - \frac{1}{6} = \frac{5}{6}$$

It is given that A starts the game and he will win in the following mutually exclusive ways.

(i) E_1 happens, i.e., A wins at the first draw.

(ii) $\bar{E}_1 \cap \bar{E}_2 \cap E_1$ happens, i.e., A wins at the third draw when both A and B fail at 1st and 2nd draw.

(iii) $\bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_1 \cap \bar{E}_2 \cap E_1$ happens, i.e., A wins at the 5th draw when both A and B fail at 1st, 2nd, 3rd and 4th draw and so on …

Hence the required probability of A winning say $P(A)$ is given by $P(A) = P(\text{i}) + P(\text{ii}) + P(\text{iii}) + \ldots$

$$= P(E_1) + P(\bar{E}_1 \cap \bar{E}_2 \cap E_1) + P(\bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_1 \cap \bar{E}_2 \cap E) + \ldots$$

$$= P(E_1) + P(\bar{E}_1)P(\bar{E}_2) + P(\bar{E}_1)P(\bar{E}_2)P(\bar{E}_1)P(\bar{E}_2)P(E_1)$$

$$= \frac{5}{36} + \frac{31}{36} \times \frac{5}{6} \times \frac{5}{36} + \frac{31}{36} \times \frac{5}{6} \times \frac{31}{36} \times \frac{5}{6} \times \frac{5}{36} + \ldots$$

$$= \frac{5}{36} + \left(\frac{31}{36} \times \frac{5}{6}\right) \times \frac{5}{36} + \left(\frac{31}{36} \times \frac{5}{6}\right)^2 \times \frac{5}{36} + \ldots$$

$$= \frac{5}{36} \times \frac{1}{1 - \left(\frac{31}{36} \times \frac{5}{6}\right)} = \frac{5}{36} \times \frac{216}{61} = \frac{30}{61}$$

13. Out of $(2n + 1)$ consecutively numbered tickets, three are drawn at random. The chance that the numbers on them are in A.P. is

(a) $4n^2 - 1/3n$ (b) $4n^2 + 1/3n$
(c) $3n/4n^2 - 1$ (d) $3n/4n^2 + 1$

Solution

(b) If the smallest number is 1, the groups of three numbers in A.P. are as 1, 2, 3; 1, 3, 5; 1, 4, 7; …; 1, $n + 1$, $2n + 1$; and they are n in number. If the smallest number selected is 2, the possible groupings are 2, 3, 4; 2, 4, 6; 2, 5, 8; …; 2, $n + 1$, $2n$; and their number is $n - 1$.

If the lowest number is 3, the groupings are 3, 4, 5; 3, 5, 7; 3, 6, 9; …; 3, $n + 2$, $2n + 1$; their number being $n - 1$.

Similarly, it can be seen that if the lowest numbers selected are 4, 5, 6, $2n - 2$, $2n - 1$, the numbers of selections, respectively, are $n - 2$, $n - 2$, $n - 3$, $n - 3$, …, 2, 2, 1, 1. Thus, the favourable ways for 2, 3 are the same and similarly they are the same for 4, 5 and so on. Hence, number of favourably ways

$$M = 2\,(1 + 2 + 3 + \ldots\ n - 1) + n$$

$$= 2 \times \frac{(n-1)n}{2} + n = n^2 - n + n = n^2$$

Also the total number of ways

$$N = {}^{2n+1}C_3 = \frac{(2n+1) \times 2n \times (2n-1)}{1 \times 2 \times 3}$$

$$= \frac{n(4n^2 - 1)}{3}$$

Hence, the required probability

$$= \frac{M}{N} = \frac{3n^2}{n(4n^2 - 1)} = \frac{3n}{4n^2 - 1}$$

14. In a test an examinee either guesses or copies or knows the answer to a multiple choice question with four choices. The probability that he makes a guess is 1/3 and the probability that he copies the answer is 1/6. The probability that his answer is correct given that he copied it, is 1/8. The probability that he knew the answer to the question given that he correctly answered it is *[IIT-1991]*

(a) 24/29 (b) 25/24
(c) 29/24 (d) 24/25

Solution

(a) Let A_1 be the event that the examinee guesses the answer; A_2 the event that he copies the answer and A_3 the event that he knows the answer. Also let A be the event that he answers correctly. Then as given, we have

$$P(A_1)=\frac{1}{3}, P(A_2)=\frac{1}{6}, P(A_3)=1-\frac{1}{3}-\frac{1}{6}=\frac{1}{2}.$$

(We have assumed here that the events A_1, A_2 and A_3 are mutually exclusive and totally exhaustive.)

Now $P(A/A_1)=\frac{1}{4}, P(A/A_2)=\frac{1}{8}$ (as given)

Again it is reasonable to take the probability of answering correctly given that he knows the answer as 1, that is,

$P(A/A_3) = 1$ We have to find $P(A_3/A)$

By Bayes' theorem, we have

$$P(A_3/A)=\frac{P(A_3)P(A/A_3)}{P(A_1)P(A/A_1)+P(A_2)P(A/A_2)+P(A_3)P(A/A_3)}$$

$$=\frac{(1/2)\times 1}{(1/3)(1/4)+(1/6)(1/8)+(1/2)\times 1}=\frac{24}{29}$$

15. In a combat between A, B and C, A tries to hit B and C, and B and C try to hit A. Probability of A, B and C hitting the targets are 2/3, 1/2 and 1/3, respectively. If A is hit, find the probability that B hits A and C does not. *[IIT-2003]*

(a) 1/1 (b) 1/2
(c) 2/2 (d) 2/1

Solution

(c) We have to find the probability of A being hit by B but not by C, i.e.,

$$P(BC'/A)=\frac{P(A/BC')P(BC')}{P(A/BC')P(BC')+P(A\,BC')P(B'C)+P(A/BC)P(BC)+P(A/B'C')P(R'C')}$$

Now putting the values from the given data, we have

$$P(BC'/A)=\frac{1\times\frac{1}{2}\times\frac{2}{3}}{1\times\frac{1}{2}\times\frac{2}{3}+1\times\frac{1}{2}\times\frac{1}{3}+0\times\frac{1}{2}\times\frac{2}{3}}$$

$$=\frac{\frac{1}{3}}{\frac{1}{3}+\frac{1}{6}+\frac{1}{6}}=\frac{1}{2}$$

16. Two persons A and B throw a die alternately till one of them gets 3 and wins the game. If A begins, then their respective probabilities of winning will be *[Kerala (CEE)-05]*

(a) 6/11, 5/11 (b) 5/11, 4/11
(c) 7/11, 6/11 (d) 1/2, 1/2

Solution

(a) A will win either in first or in third or in fifth throw. So

$$P(A)=\frac{1}{6}+\left(\frac{5}{6}\right)^2\frac{1}{6}+\left(\frac{5}{6}\right)^4\frac{1}{6}+\ldots$$

$$=\frac{1}{6}\left[1+\left(\frac{5}{6}\right)^2+\left(\frac{5}{6}\right)^4\right]+\ldots$$

$$=\frac{1}{6}\left[\frac{1}{1-\frac{25}{36}}\right]=\frac{6}{11}$$

B will win in second or in fourth or in sixth throw. So

$$P(B)=\left(\frac{5}{6}\right)\frac{1}{6}+\left(\frac{5}{6}\right)^3\frac{1}{6}+\left(\frac{5}{6}\right)^5\frac{1}{6}+\ldots$$

$$=\left(\frac{5}{6}\right)\frac{1}{6}\left[1+\left(\frac{5}{6}\right)^2+\left(\frac{5}{6}\right)^4+\ldots\right]$$

$$=\frac{5}{36}\times\frac{36}{11}=\frac{5}{11}$$

Hence, required respective probabilities = 6/11, 5/11.

17. A letter is known to have come from either TATANAGAR or CALCUTTA. On the envelope, just two consecutive letters. TA are visible. The probability that the letters have come from CALCUTTA is
(a) 1/3 (b) 4/11
(c) 5/12 (d) None of these

Solution

(b) Let A: 'the event that letters came from TATANAGAR'
B: 'the event that letters came from CALCUTTA'
C: 'the event that two consecutive letters visible by TA'
Then $P(A) = 1/2$, $P(B) = 1/2$, $P(C/B) = 1/7$.
Hence, by Bayes' theorem

$$P(B/C) = \frac{P(B) \times P(C/B)}{P(A)P(C/A) + P(B)P(C/B)} = \frac{4}{11}$$

18. A pair of unbiased dice is rolled together till a sum of either 5 or 7 is obtained. The probability that 5 comes before 7 is
(a) 1/5 (b) 2/5
(c) 3/5 (d) 4/5

Solution

(b) Let A: 'event that sum is 5'
B: 'event that sum is 7'
C: 'event that sum is neither 5 nor 7'

Then $P(A) = \frac{4}{36} = \frac{1}{9}, P(B) = \frac{6}{36} = \frac{1}{6},$

$$P(C) = \frac{26}{36} = \frac{13}{18}$$

Now probability that A occurs before B
$= P(A + CA + CCA + \ldots)$
$= P(A) + P(CA) + P(CCA) + \ldots$
$= P(A) + P(C)\,P(A) + P(C)\,P(C)\,P(A) + \ldots$

$$= \frac{1}{9} + \left(\frac{13}{18}\right)\frac{1}{9} + \left(\frac{13}{18}\right)^2 \frac{1}{9} + \ldots$$

$$= \frac{1/9}{1 - 13/18} = \frac{2}{5}$$

19. The probability distribution of a discrete random variable X is given by:

X	-1	0	1	2
$P(X)$	1/3	1/6	1/6	1/3

Then the value of $6E(X^2)$ – variance (X) is
(a) 12/113
(b) 113/12
(c) 19/12
(d) 1/2

Solution

(c) Mean $= \mu = \sum xi\, P(xi)$

$$= (-1)\left(\frac{1}{3}\right) + (0)\left(\frac{1}{6}\right) + (1)\left(\frac{1}{6}\right) + (2)\left(\frac{1}{3}\right) = \frac{1}{2}$$

Variance $= \sum x^2\, P(x) = (\text{Mean})^2$

$$= (-1)^2\left(\frac{1}{3}\right) + (0)(1(1)^2\left(\frac{1}{6}\right) + \frac{(2)^2}{3} - \left(\frac{1}{2}\right)^2 = \frac{19}{12}$$

Hence $6\,E(x^2)$ – variance(x)
$= 6\sum x^2\, P(x) - \text{variance}\,(x)$

$$= 6\left(\frac{1}{3} + 0 + \frac{1}{6} + \frac{3}{4}\right) - \frac{19}{12} = 11 - \frac{19}{12}$$

20. For a binomial variate X if $n = 5$ and $P(X = 1) = 8P(X = 3)$, then P is
(a) 4/5 (b) 1/5
(c) 1/3 (d) 2/3

Solution

(b) ${}^5C_1\, q^4 p^1 = 8 \times {}^5C_3\, q^2 p^3$
$\Rightarrow q = 4P$
$\Rightarrow 1 - p = 4P$
$\Rightarrow P = 1/5$

21. A random variable X is specified by the following distribution law:

X:	2	3	4
$P(X = x)$:	0.3	0.4	0.3

Then the variance of this distribution is
(a) 0.6 (b) 0.7
(c) 0.77 (d) 1.55

Solution

(a) Mean = (2)(0.3) + (3)(0.4) + (4)(0.3) = 3

$$\sigma x^2 = \text{Variance}\,(x) \sum (x - \overline{x})^2 P$$
$$= (2-3)^2(0.3) + (3.3)^2(0.4) + (4-3)^2 \times 0.3$$
$$= 0.6$$

22. The probability distribution of a random variable X is given by:

$X = x$	0	1	2	3	4
$P(X = x)$	0.4	0.3	0.1	0.1	0.1

The variance of X is: ***[EAMCET-2007]***

(a) 1.76 (b) 2.45 (c) 3.2 (d) 4.8

Solution

(a)

$X = x$	0	1	2	3	4
$P(X = x)$	0.4	0.3	0.1	0.1	0.1
$xP(X = x)$	0	0.3	0.2	0.3	0.4

$$\text{Mean} = \sum x_i P(X = x_1) = 1.2 = \bar{x}$$

$$\text{Variance} = \sum x_i^2 P(X = x_i) - \bar{x}^2$$

$$= 3.20 - 1.44 = 1.76$$

23. In a bag there are three tickets numbered 1, 2, 3. A ticket is drawn at random and put back. This is done four times. The probability that the sum of the numbers is even is

(a) $\frac{41}{81}$ (b) $\frac{39}{81}$

(c) $\frac{40}{81}$ (d) None of these

Solution

(a) The total number of ways of selecting 4 tickets $= 3^4 = 81$.

The favourable number of ways
$=$ sum of coefficients of $x^2, x^4, \ldots$ in $(x + x^2 + x^3)^4$
$=$ sum of coefficients of $x^2, x^4, \ldots$ in $x^4(1 + x + x^2)^4$

Let $(1 + x + x^2)^4 = 1 + a_1 x + a_2 x^2 + \ldots + a_8 x^8$

Then $3^4 = 1 + a_1 + a_2 + a_3 + \ldots + a_8$, (On putting $x = 1$)

and $1 = 1 - a_1 + a_2 - a_3 + \ldots + a_8$, (On putting $x = -1$)

$\therefore 3^4 + 1 = 2(1 + a_2 + a_4 + a_6 + a_8)$

$\Rightarrow a_2 + a_4 + a_6 + a_8 = 41$

Thus sum of the coefficients of $x^2, x^4, \ldots = 41$

Hence, required probability $= 41/81$.

24. If the integers m and n are chosen at random between 1 and 100, then the probability that a number of the form $7^m + 7^n$ is divisible by 5 equals ***[IIT-1999]***

(a) 1/4 (b) 1/7 (c) 1/8 (d) 1/49

Solution

(a) Since m and n are selected between 1 and 100, hence sample space $= 100 \times 100$.

Also $7^1 = 7, 7^2 = 49, 7^3 = 343, 7^4 = 2401$, $7^5 = 16807$ etc.

Hence 1, 3, 7 and 9 will be the last digits in the powers of 7. Hence, for favourable cases

$nm \rightarrow$
$\downarrow$

1, 1	1, 2	1, 3 …..	1, 100
2, 1	2, 2	2, 3 …..	2, 100
…………			
100, 1	100, 2	100, 3….	100, 100

For $m = 1$; $n = 3, 7, 11 \ldots 97$

$\therefore$ Favourable cases $= 25$

For $m = 2$; $n = 4, 8, 12 \ldots\ldots 100$

$\therefore$ Favourable cases $= 25$

Similarly for every m, favourable n are 25.

$\therefore$ Total favourable cases $= 100 \times 25$

Hence, required probability $= \frac{100 \times 25}{100 \times 100} = \frac{1}{4}$

25. Four tickets marked 00, 01, 10, 11, respectively, are placed in a bag. A ticket is drawn at random five times, being replaced each time. The probability that the sum of the numbers on tickets thus drawn is 23 will be ***[DCE-99]***

(a) 25/256 (b) 100/256

(c) 231/256 (d) None of these

Solution

(a) Total number of ways in which 4 tickets can be drawn 5 times $= 4^5$. Favourable cases of getting a sum of 23

$=$ Coefficients of x^{23} in $(x^{00} + x^{01} + x^{10} + x^{11})^5$

$=$ Coefficients of x^{23} in $(1 + x)^5 (1 + x^{10})^5$

$=$ Coefficients of x^{23} in $(1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5)(1 + 5x^{10} + 10x^{20} + 10x^{30} + \ldots)$

$= 100$

$\therefore$ Required probability $= \frac{100}{4^5} = \frac{100}{1024} = \frac{25}{256}$

26. A ten digit number is formed using the digits from zero to nine, every digit being used exactly once. The probability that the number is divisible by four is ***[Roorkee-1991]***

(a) 20/81 (b) 18/20
(c) 81/20 (d) 20/18

Solution

n = Total number of ways = $10! - 9!$
To find the favourable number of ways, we observe that a number is divisible by 4 if the last two digits are divisible by 4. Hence, the last two digits can be 20, 40, 60, 80, 12, 32, 52, 72, 92, 04, 24, 64, 84, 16, 36, 56, 76, 96, 08, 28, 48, 68 corresponding to each of 20, 40, 60, 80, 04, 08. The remaining 8 places can be filled up in 8! ways so that the number of ways in this case = 6.8!.
And corresponding to remaining 16 possibilities the number of ways = $16(8! - 7!)$
Hence m = favourable number of ways
$= 22.8! - 16.7!$
$\therefore$ The required probability = m/n

$$= \frac{22.8! - 16.7!}{10! - 9!}$$

$$= \frac{22.8 - 16}{1\,09.8 - 9.8} = \frac{160}{648} = \frac{20}{81}$$

27. Urn A contains 6 red and 4 black balls and urn B contains 4 red and 6 black balls. One ball is drawn at random from urn A and placed in urn B. Then 1 ball is drawn at random from urn B and placed in urn A. If 1 ball is now drawn at random from urn A, the probability that it is found to be red is ***[IIT-1988]***

(a) $\frac{32}{55}$ (b) $\frac{21}{55}$

(c) $\frac{19}{55}$ (d) None of these

Solution

(a) Let the events are
R_1: 'a red ball is drawn from urn A and placed in B'
B_1: 'a black ball is drawn from urn A and placed in B'
R_2: 'a red ball is drawn from urn B and placed in A'
B_2: 'a black ball is drawn from urn B and placed in A'
R: 'a red ball is drawn in the second attempt from A'

Then the required probability

$= P(R_1 R_2 R) + P(R_1 B_2 R) + P(B_1 R_2 R) + P(B_1 B_2 R)$

$= P(R_1)\,P(R_2)\,P(R) + P(R_1)\,P(B_2)\,P(R) + P(B_1)\,P(R_2)\,P(R) + P(B_1)\,P(B_2)\,P(R)$

$$= \frac{6}{10} \times \frac{5}{11} \times \frac{6}{10} + \frac{6}{10} \times \frac{6}{11} \times \frac{5}{10} + \frac{4}{10} \times \frac{4}{11} \times \frac{7}{10} + \frac{4}{10} \times \frac{7}{11} \times \frac{6}{10}$$

$$= \frac{32}{55}$$

28. Each coefficient in the equation $ax^2 + bx + c = 0$ is determined by throwing an ordinary die. The probability that the equation will have equal roots is ***[Roorkee-1998]***

(a) 216/5 (b) 261/5
(c) 5/261 (d) 5/216

Solution

Roots equal $\Rightarrow b^2 - 4ac = 0$

$$\therefore \left(\frac{b}{2}\right)^2 = ac \qquad \ldots\ldots(1)$$

Each coefficient is an integer, so we consider the following cases: $b = 1 \quad \therefore \frac{1}{4} = ac$

No integral values of a and c
$b = 2 \quad 1 = ac \quad \therefore (1, 1)$
$b = 3 \quad 9/2 = ac,$
No integral values of a and c
$b = 4 \quad 4 = ac$
$\therefore (1, 4), (2, 2), (4, 1)$
$b = 5 \quad 25/2 = ac,$
No integral values of a and c
$b = 6 \quad 9 = ac \quad \therefore 3, 3$
Thus we have 5 favourable ways for $b = 2, 4, 6$
Total number of equations is 6.6.6 = 216
$\therefore$ Required probability is 5/216.

Paragraph for Questions 29 to 31

A fair die is tossed repeatedly until a 6 is obtained. Let X denote the number of tosses required

29. The probability that $X = 3$ equals *[IIT-2009]*

(a) $\frac{25}{216}$ (b) $\frac{25}{36}$

(c) $\frac{5}{36}$ (d) $\frac{125}{216}$

Solution

(a) $P(X=3)=\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\frac{1}{6}=\frac{25}{216}$

30. The probability that $X \geq 3$ equals *[IIT-2009]*

(a) $\frac{125}{216}$ (b) $\frac{25}{36}$

(c) $\frac{5}{36}$ (d) $\frac{25}{216}$

Solution

(b) $P(X\leq 2)=\frac{1}{6}+\frac{5}{6}\times\frac{1}{6}=\frac{11}{36}$

$\therefore$ Required probability $= 1-\frac{11}{36}=\frac{25}{36}$

31. The conditional probability that $X \geq 6$ given $X > 3$ equals *[IIT-2009]*

(a) $\frac{125}{216}$ (b) $\frac{25}{216}$

(c) $\frac{5}{36}$ (d) $\frac{25}{36}$

Solution

(d) For $X \geq 6$, the probability is

$$\frac{5^5}{6^6}+\frac{5^6}{6^7}+\ldots\infty=\frac{5^5}{6^6}\left(\frac{1}{1-5/6}\right)=\left(\frac{5}{6}\right)^5$$

for $X > 3$, $\frac{5^3}{6^4}+\frac{5^4}{6^5}+\frac{5^5}{6^6}+\ldots\infty=\left(\frac{5}{6}\right)^3$

Hence the conditional probability $\frac{(5/6)^6}{(5/6)^3}=\frac{25}{36}$

32. One ticket is selected at random from 50 tickets numbered 00, 01, 02,..., 49. The probability that the sum of the digits on the selected ticket is 8, given that the product of these digits is zero, equals *[AIEEE-2009]*

(a) $\frac{1}{14}$ (b) $\frac{1}{7}$

(c) $\frac{5}{14}$ (d) $\frac{1}{50}$

Solution

(a) $S = (00, 01, 02, \ldots, 49)$

Let A be the even that sum of the digits on the selected ticket is 8 then

$A = \{08, 17, 26, 35, 44\}$

Let B be the event that the product of the digits is zero.

$B = \{00, 01, 02, 03, \ldots, 09, 10, 20, 30, 40\}$

$A \cap B = \{08\}$

$$P(A/B)=\frac{P(A\cap B)}{P(B)}=\frac{n(A\cap B)}{n(B)}=\frac{1}{14}$$

OBJECTIVE PROBLEMS: IMPORTANT QUESTIONS WITH SOLUTIONS

1. For a biased die, the probabilities for different faces to turn up are

Face	1	2	3	4	5	6
Probability	0.2	0.22	0.11	0.25	0.05	0.17

The die is tossed and you are told that either face 4 or face 5 has turned up. The probability that it is face 4 is

(a) 1/6 (b) 1/4

(c) 5/6 (d) None

2. Four boys and three girls stand in a queue for an interview, probability that they will stand in alternate position is

(a) 1/34 (b) 1/35

(c) 1/17 (d) 1/68

3. If four vertices of a regular octagon are chosen at random, then the probability that the quadrilateral formed by them is a rectangle is

(a) 1/8 (b) 2/21

(c) 1/32 (d) 1/35

4. There are three bags which are known to contain 2 white and 3 black; 4 white and 1 black and 3 white and 7 black balls, respectively. A ball is drawn at random from one of the bags and found to be a black ball. The probability that it was drawn from the bag containing the most black balls is
(a) 7/15 (b) 5/19
(c) 3/4 (d) None of these

5. In a certain town, 40% of the people have brown hair, 25% have brown eyes and 15% have both brown hair and brown eyes. If a person selected at random from the town has brown hair, the probability that he also has brown eyes is
(a) 1/5 (b) 3/8
(c) 1/3 (d) 2/3

6. A man alternately tosses a coin and throws a dice beginning with the coin. The probability that he gets a head in the coin before he gets a 5 or 6 in the dice is ***[Roorkee-1988]***
(a) 3/4 (b) 1/2
(c) 1/3 (d) None

7. A bag contains 3 white, 3 black and 2 red balls. One by one three balls are drawn without replacing them. The probability that the third ball is red is ***[MNR-1994]***
(a) 1/2 (b) 1/3
(c) 2/3 (d) 1/4

8. A determinant is chosen at random. The set of all determinants of order 2 with elements 0 or 1 only. The probability that value of the determinant chosen is positive is ***[IIT-1982]***
(a) 3/16 (b) 3/8
(c) 1/4 (d) None

9. Out of 21 tickets marked with numbers from 1 to 21, three are drawn at random. The chance that the numbers on them are in A.P., is ***[Roorkee-1988; DCE-1999]***
(a) 10/133 (b) 9/133
(c) 9/1330 (d) None of these

10. A bag x contains 3 white balls and 2 black balls and another bag y contains 2 white balls and 4 black balls. A bag and a ball out of it are picked at random. The probability that the ball is white is ***[IIT-71]***
(a) 3/5 (b) 7/15
(c) 1/2 (d) None

11. A biased die is tossed and the respective probabilities for various faces to turn up are given below: ***[MNR-1982]***

Face	1	2	3	4	5	6
Probability	0.1	0.24	0.19	0.18	0.15	0.14

If an even face has turned up, then the probability that it is face 2 or face 4 is
(a) 0.25 (b) 0.42
(c) 0.75 (d) 0. 9

12. Two squares are chosen at random on a chessboard. The probability that they have a side in common is
(a) 1/9 (b) 2/7
(c) 1/18 (d) None of these

13. There are 4 envelopes with addresses and 4 concerning letters. The probability that letter does not go into concerning proper envelope is
(a) 19/24 (b) 21/23
(c) 23/24 (d) 1/24

14. The probability distribution of a random variable X is given below:

$X = x_i$	2	3	4
$P : (X = x_i)$	1/4	1/8	5/8

Then its mean is
(a) 27/8 (b) 5/4
(c) 1 (d) 4/5

15. The probability for a randomly chosen month to have its 10th day as Sunday is
(a) 1/84 (b) 10/12
(c) 10/84 (d) 1/7 (e) 1/12

16. Three letters are drawn from the alphabet of 26 letters without replacement. The probability that they appear in alphabetical order is
(a) ${}^{23}C_1/{}^{26}C_3$ (b) $24/{}^{26}C_3$
(c) 1/6 (d) 1/3

17. A random variable X has the following probability distribution

x:	0	1	2	3	4	5	6	7	8
$P(X = x)$:	a	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$17a$

Then value of a is ***[DCE-98, AMU-90]***
(a) 1/81 (b) 2/81
(c) 5/81 (d) 7/81

18. In 324 throws of 4 dice, the expected number of times three sixes occur is
(a) 81 (b) 5
(c) 9 (d) 31

19. If x denotes the number of sixes in four consecutive throws of a dice, then $P(x = 4)$ is
(a) 1/1296 (b) 4/6
(c) 1 (d) 1295/1296

20. A and B are two independent events. The probability that both A and B occur is 1/6 and the probability that none of them occurs is 1/3. The minimum value of probability of occurrence of A is ***[RPET-2000]***
(a) 1/2 (b) 1/3
(c) 1/4 (d) None of these

21. Two players A and B play a game in which their chance of winning are in the ratio 3:2 A's chance of winning at least 2 games out of 3 is
(a) 73/125 (b) 81/125
(c) 67/625 (d) 71/625

22. A bag contains 9 white balls and 5 black balls. Another bag contains 8 white balls and 6 black balls. One ball is transferred from the first bag into the second, and then a ball is drawn from the latter. The probability that it will be a white ball is
(a) 1/2 (b) 1/5
(c) 1/21 (d) 121/210

23. If the letters of the word REGULATIONS be arranged at random. What is the chance that there will be exactly 4 letters between the R and the E?
(a) 6/55 (b) 8/55
(c) 10/55 (d) 12/55

24. Eight prizes are distributed by a lottery. The first participant takes 5 tickets from the box containing 50 tickets. What is probability of extracting exactly two winning tickets?

(a) $\dfrac{{}^{8}C_2 \times {}^{42}C_3}{{}^{50}C_5}$ (b) $\dfrac{{}^{9}C_2 \times {}^{42}C_3}{{}^{50}C_4}$

(c) $\dfrac{{}^{8}C_2 \times {}^{42}C_2}{{}^{50}C_4}$ (d) $\dfrac{{}^{8}C_2 \times {}^{42}C_3}{{}^{50}C_3}$

25. The first twelve letters of the alphabet are written at random. Find the probability that there are exactly four letters between A and B.
(a) 7/66 (b) 8/66
(c) 7/56 (d) None of these

26. If the letters of the word ATTEMPT are written down at random. Find the probability if
(i) all Ts are together
(ii) no two Ts are together
(a) (i) 1/7 (ii) 2/7 (b) (i) 2/7 (ii) 1/7
(c) (i) 2/7 (ii) 3/7 (d) (i) 3/7 (ii) 1/7

27. If four people are chosen at random, find the probability that no two of them were born on the same day of the week?
(a) 120/310 (b) 120/245
(c) 120/240 (d) 120/343

28. Two letters are taken at random from the word HOME. Find the probability that both the letters are vowels.
(a) 1/6 (b) 1/12
(c) 3/8 (d) None of these

29. The mean and the variance of a binomial distribution are 4 and 2, respectively. Then the probability of at most 2 successes is ***[AIEEE-2004]***
(a) 128/256 (b) 219/256
(c) 37/256 (d) 28/256

30. Three houses are available in a locality. Three persons apply for the houses. Each applies for one house without consulting others. The probability that all the three apply for the same house is ***[AIEEE-2005]***
(a) 8/9 (b) 7/9
(c) 2/9 (d) 1/9

31. A six faced fair die is thrown until 1 comes. The probability that 1 comes in even number of trials is ***[IIT (Screening)-2005]***
(a) 5/11 (b) 6/11
(c) 5/6 (d) 1/6

32. By Bayes' theorem, which one of the following probabilities is calculated? ***[NDA-2009]***
(a) Priorp robability
(b) Likelihood probability
(c) Posterior probability
(d) Conditional probability

33. A random variable X has the probability distribution:

X	1	2	3	4	5	6	7	8
$P(X)$	0.15	0.23	0.12	0.10	0.20	0.08	0.07	0.05

For the events $E = \{X \text{ is a prime number}\}$ and $F = \{X < 4\}$, then $P(E \cup F)$ is: ***[MPPET-2009]***

(a) 0.77 (b) 0.87
(c) 0.35 (d) 0.50

34. A pair of fair dice is thrown independently, 4 times. The probability of getting a sum of exactly 7 twice is: ***[MPPET-2009]***

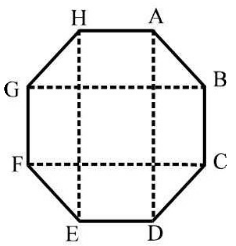
(a) $\frac{5}{81}$ (b) $\frac{25}{243}$
(c) $\frac{25}{216}$ (d) $\frac{125}{648}$

SOLUTIONS

1. (c) Let A be the event that face 4 turns up and B be the event that face 5 turns up. Then $P(A) = 0.25$, $P(B) = 0.05$.
Since A and B are mutually exclusive, so
$P(A \cup B) = P(A) + P(B)$
$= 0.25 + 0.05 = 0.30$
We have to find $P\left(\frac{A}{A \cup B}\right)$, which is equal to
$$P\frac{[A \cap (A \cup B)]}{P(A \cup B)} = \frac{P(A)}{P(A \cup B)} = \frac{0.25}{0.30} = \frac{5}{6}$$

2. (b) Boys can be arranged in 4! ways, the girls in the space between the boys in 3! ways. Total arrangement is 7! ways.
$$\text{Required probability} = \frac{(4!)(3!)}{7!} = \frac{1}{35}$$

3. (d) Here only two rectangles are formed $ADEH$, $GFCB$.



$\therefore$ Number of favourable cases = 2 and total number of cases = ${}^8C_4$
$$\therefore \text{Required probability} = \frac{2}{{}^8C_4} = \frac{1}{35}$$

4. (a) Consider the following events:
$A \rightarrow$ bag drawn is black;
$E_1 \rightarrow$ bag I is chosen;
$E_2 \rightarrow$ bag II is chosen and
$E_3 \rightarrow$ bag III is chosen.

Then $P(E_1) = (E_2) = P(E_3) = \frac{1}{3}, P\left(\frac{A}{E_1}\right) = \frac{3}{5}$
$$P\left(\frac{A}{E_2}\right) = \frac{1}{5}, P\left(\frac{A}{E_3}\right) = \frac{7}{10}$$
$$\text{Required probability} = P\left(\frac{E_3}{A}\right)$$
$$= \frac{P(E_3)P(A/E_3)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3)}$$
$$= \frac{7}{15}$$

5. (b) Required probability $= \frac{15\%}{40\%} = \frac{3}{8}$
(If E_1: 'a person with brown hair is selected' E_2: 'a person with brown eyes is selected' then required probability
$$= P\left(\frac{E_2}{E_1}\right) = \frac{P(E_2 \cap E_1)}{P(E_1)}.$$

6. (a) Probability of getting head $= \frac{1}{2}$ and probability of throwing 5 or 6 with dice
$= \frac{2}{6} = \frac{1}{3}$
He starts with a coin and alternately tosses the coin and throws the dice and he will win if he get a head before he get 5 or 6.
$\therefore$ Required probability
$$= \frac{1}{2} + \left(\frac{1}{2} \times \frac{2}{3}\right) \times \frac{1}{2} + \left(\frac{1}{2} \times \frac{2}{3}\right) \times \left(\frac{1}{2} \times \frac{2}{3}\right) \times \frac{1}{2} + \ldots$$
$$= \frac{1}{2}\left[1 + \frac{1}{3} + \left(\frac{1}{3}\right)^2 + \ldots\right] = \frac{1}{2} \times \frac{1}{1-(1/3)} = \frac{3}{4}$$

7. (d) Required probability $= P(RNR) + P(NRR) + P(NNR)$
where R stands for red ball and N for non-red.
There are six non-red balls and two red balls.
$\therefore$ Required probability

$$= \frac{2}{8} \times \frac{6}{7} \times \frac{1}{6} + \frac{6}{8} \times \frac{2}{7} \times \frac{1}{6} + \frac{6}{8} \times \frac{5}{7} \times \frac{2}{6}$$

$$= \frac{2}{56} + \frac{2}{56} + \frac{10}{56} = \frac{14}{56} = \frac{1}{4}$$

8. (a) The positive determinants are

$$\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

Required probability $= \dfrac{3}{16}$

[$\because$ total number of determinants $= 2^4 = 16$]

9. (a) Suppose 3 no's are a, b, c
$\therefore$ $2b = a + c$ or $a + c =$ even
$\Rightarrow$ a and c are both even or both odd
$\therefore$ favourable number of ways
$= {}^{10}C_2 + {}^{11}C_2 = 100$
($\because$ there are 10 even and 11 odd number)
sample space $= {}^{21}C_3 = 1330$

probability $= \dfrac{100}{1330} = \dfrac{10}{133}$

10. (b) Requiredpr obability

$$= \frac{1}{2}\left(\frac{3}{5} + \frac{2}{6}\right) = \frac{9+5}{30} = \frac{7}{15}$$

11. (c) If A = event of occurring even face
B = event of showing 2 or 4 on the face, then
$P(A) = 0.24 + 0.18 + 0.14 = 0.56$
$P(B) = 0.24 + 0.18 = 0.42$
$\therefore$ Required probability $= P(B/A)$

$$= \frac{P(AB)}{P(A)} = \frac{P(B)}{P(A)}$$

$$= \frac{0.42}{0.56} = \frac{3}{4} = 0.75$$

12. (c) The number of ways of choosing the first square is 64 and that for the second square is 63. Therefore, the number of ways of choosing the first and second square is $64 \times 63 = 4032$. Now we proceed to find the number of favourable ways. If the first happens to be any of the four squares in the corner, the second square can be chosen in two ways. If the first square happens to be any of the 24 square on either side of the chess board, the second square can be chosen in three ways. If the first square happens o be any of the 36 remaining squares, the second square can be chosen in four ways.

Therefore, the number of favourable ways is

$(4) \times (2) + (24)(3) + (36)(4) = 224$

Hence, the required probability

$$= \frac{224}{4032} = \frac{1}{18}$$

13. (c) Total ways to despatch 4 letters in four envelopes $= 4!$
Number of ways to despatch in right envelope $= 1$
Probability to despatch correctly

$$P(A) = \frac{1}{4!} \text{ and } P(\bar{A}) = 1 - \frac{1}{4!} = \frac{23}{24}$$

14. (a)

$x = x_i$	2	3	4
$P_i(x = x_i)$	1/4	1/8	5/8
$x \cdot p(x = x_i)$	1/2	3/8	5/8

$$\text{Mean} = \sum x_i p(x - x_i) = \frac{1}{2} + \frac{3}{8} + \frac{5}{2} = \frac{27}{8}$$

15. (d) Any day of a month can fall on any one day of the week.

$\therefore$ Required probability $= \dfrac{1}{7}$

16. (c) Probability $= \dfrac{{}^{26}C_3}{{}^{26}P_3} = \dfrac{1}{6}$

17. (a) We must have
$a + 3a + 5a + 7a + 9a + 11a + 13a + 15a + 17a = 1$

$\therefore \frac{9}{2}(a+17a) = 1$ [$\because$ sum of the probability = 1]

or $\frac{9\times 18a}{2} = 1$

$\therefore a = \frac{2}{9\times 18} = \frac{1}{81}$

18. (b) Probability of getting 3 sixes

$= {}^4C_3\left(\frac{1}{6}\right)^3 \times \frac{5}{6} = \frac{5}{324}$

$\therefore$ expected number of times of getting 3 sixes $= 324\times\frac{5}{324} = 5$

19. (a) $P(x=4) = {}^4C_4\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^0 = \frac{1}{6^4} = \frac{1}{1296}$

20. (a, b) $P(A\cap B) = P(A)\,P(B) = \frac{1}{6}$,

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$= 1 - \frac{1}{3} = \frac{2}{3}$

as $P(A\cup B)' = \frac{1}{3}$

Let $P(A) = x$, $P(B) = y$ then

$x + y = \frac{2}{3} + \frac{1}{6} = \frac{5}{6}, xy = \frac{1}{6}$

$\Rightarrow \quad y = \frac{1}{3}$ or $\frac{1}{2}, y = \frac{1}{2}$ or $\frac{1}{3}$

21. (b) Probability of winning of player $A = \frac{3}{5}$

Probability of winning of player $A = \frac{2}{5}$

$P(A$ winning at least 2 games)

$= {}^3C_2\left(\frac{3}{5}\right)^2\left(\frac{2}{5}\right) + {}^3C_3\left(\frac{3}{5}\right)^3 = \frac{81}{125}$

22. (d) Bag I: 9 white + 5 black; Bag II: 8 white + 6 black

Case 1: White ball is transferred

Probability $= P(W_I)\times P(W_{II}) = \frac{9}{14}\times\frac{9}{15}$

Case 2: Black ball is transferred

Probability $= P(B_I)\times P(W_{II}) = \frac{5}{14}\times\frac{8}{15}$

Totalpr obability

$= \frac{9}{14}\times\frac{9}{15} + \frac{5}{14}\times\frac{8}{15} = \frac{121}{210}$

23. (a) There are eleven places to be occupied by the letters of the word REGULATIONS. Two places for G and T can be chosen in ${}^{11}P_2 = 11\times 10 = 110$ ways. Since we want exactly four letters between G and T, therefore, G and T may occupy

(i) 1st and 6th places
(ii) 2nd and 7th places
(iii) 3rd and 8th places
(iv) 4th and 9th places
(v) 5th and 10th places
(vi) 6th and 11th places

Moreover, G and T can interchange their places in 2 ways.

So, favourable number of ways $= 6\times 2 = 12$

$\therefore$ Required probability $= \frac{12}{110} = \frac{6}{55}$

24. (a) Eight prizes are distributed by a lottery So, probability of exactly two winning tickets $= \frac{{}^8C_2\times{}^{42}C_3}{{}^{50}C_5}$

25. (a) ……… A …………. B ………

x $\quad$ 4 $\quad$ y

Let number of letters at left of A and right of B be x and y, respectively.

$\therefore x + y + 4 = 10 \quad \Rightarrow \quad x + y = 6$

No. of ways of selection

$= {}^{6+2-1}C_{2-1} = {}^7C_1$

$\therefore$ Required Probability

$= \frac{7\times 10!\times 2!}{12!} = \frac{7\times 2}{12\times 11} = \frac{7}{66}$

26. (a) $n(S) = \frac{7!}{3!}, n(E) = 5!$

So, $P(E) = \frac{5!}{\frac{7!}{3!}} = \frac{1}{7}$

27. (d) Requiredpr obability

$$=\frac{{}^7C_4}{7\times7\times7\times7}=\frac{7\times6\times5\times4}{7^4}=\frac{120}{343}$$

28. (a) Required Probability $=\frac{{}^2C_2}{{}^4C_2}=\frac{1}{6}$

29. (d) $\left.\begin{array}{r} np=4 \\ npq=2 \end{array}\right\}=q=\frac{1}{2}, p=\frac{1}{2}, n=8$

$$P(x=2)={}^8C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^6=28\times\frac{1}{2^8}=\frac{28}{256}$$

30. (d) For a particular house being selected, probability $=\frac{1}{3}$

Probability (all the persons apply for the same house)

$$=\left(\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}\right)3=\frac{1}{9}$$

31. (a) In single throw of dice, probability of getting 1 is $=\frac{1}{6}$ and probability of not getting 1 is $\frac{5}{6}$. Then getting 1 in even number of chances = getting 1 in 2nd in 9th chance or 6th chance and so on

$\therefore$ Required probability

$$=\frac{1}{5}\times\frac{1}{6}+\left(\frac{5}{6}\right)^3\times\frac{1}{6}+\left(\frac{5}{6}\right)^5\times\frac{1}{6}+\ldots$$

$$=\frac{5}{36}\left\{\frac{1}{1-\frac{25}{36}}\right\}=\frac{5}{36}\times\frac{36}{11}=\frac{5}{11}$$

32. (d) We know that, by Bayes' theorem conditional probability is calculated.

33. (a) $P(E)=P(X=2)+P(X=3)+P(X=5)+P(X=7)$

$$=0.23+0.12+0.20+0.07=0.62$$

$$P(F)=P(X=1)+P(X=2)+P(X=3)$$

$$=0.15+0.23+0.12=0.5$$

$$P(E\cap F)=P(X=2)+P(X=3)$$

$$=0.23+0.12=0.35$$

$$\therefore P(E\cup F)=P(E)+P(F)-P(E\cap F)$$

$$=0.62+0.5-0.35=0.77$$

34. (c) Let E = Event of getting sum of 7 in two dice = {(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)}

Now, $P(E)=\frac{6}{36}=\frac{1}{6}=P$ (say)

$\therefore$ $q=1-p=\frac{5}{6}$

Required probability $={}^4C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^2$

$$=6\times\frac{5^2}{6^4}=\frac{25}{216}$$

UNSOLVED OBJECTIVE PROBLEMS (IDENTICAL PROBLEMS FOR PRACTICE): FOR IMPROVING SPEED WITH ACCURACY

1. The probability that a number selected at random from the set of no. {1, 2, 3, …, 100} is a cube is:

(a) 1/25 (b) 2/25
(c) 3/25 (d) 4/25

2. The following table represents a probability distribution for a random variable X:

X:	1	2	3	4	5	6
$P(X=x)$:	0.1	$2k$	k	0.2	$3k$	0.1

Then, the value of k is

(a) 0.1 (b) 0.2
(c) 0.3 (d) 0.4

3. If the letters of the word ASSASSIN are written down at random in a row. Find the probability that no two Ss occur together.

(a) 9/14 (b) 3/14
(c) 5/14 (d) 1/14

4. An unbiased die is tossed until a number greater than 4 appears. The probability that an even number of tosses is needed is ***[IIT-1994]***

(a) 1/2 (b) 2/5
(c) 1/5 (d) 2/3

5. A biased coin with probability p, $0 < p < 1$ of heads is tossed until a head appears for the first time. If the probability that the number of tosses required is even is 2/5, then p is equal to *[AIEEE-2002]*
(a) 1/3 (b) 2/3
(c) 2/5 (d) 3/5

6. Ram and Shyam throw a coin turn by turn. One who gets head first wins the game. If Ram starts the game then the probability of winning by Ram is *[MP-87; PET (Raj.)-2003]*
(a) 2/3 (b) 1/3
(c) 1/2 (d) None of these

7. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. The probability that it is actually a six is
(a) 3/8 (b) 5/8
(c) 7/8 (d) 3/4

8. Two aeroplanes I and II bomb a target in succession. The probabilities of I and II scoring a hit correctly are 0.3 and 0.2, respectively. The second plane will bomb only if the first misses the target. The probability that the target is hit by the second plane is *[AIEEE-2007]*
(a) 0.2 (b) 0.7
(c) 0.06 (d) 0.14

9. *A* pair of fair dice is thrown independently three times. The probability of getting a score of exactly 9 twice is *[AIEEE-2007]*
(a) 8/729 (b) 8/243
(c) 1/729 (d) 8/9

10. Four numbers are chosen at random from $\{1, 2, 3, \dots, 40\}$. The probability that they are not consecutive is *[EAMCET-2007]*
(a) $\frac{1}{2470}$ (b) $\frac{4}{7969}$
(c) $\frac{2469}{2470}$ (d) $\frac{7965}{7969}$

11. One Indian and four American men and their wives are to be seated randomly around a circular table. Then the conditional probability that the Indian man is seated adjacent to his wife given that each American man is seated adjacent to his wife is *[IIT JEE-2007]*
(a) 1/2 (b) 1/3
(c) 2/5 (d) 1/5

12. Two boys b_1, b_2 and three girls g_1, g_2, g_3 play a tournament. Those of the same sex have equal probabilities of winning but each boy is twice as likely to win as any girl. The probability of winning the tournament by a girl is *[MP PET-2007]*
(a) 2/7 (b) 3/7
(c) 1/7 (d) None of these

13. A couple has three children. The probability of having two sons and a daughter, if the eldest child is a son is *[MPPET-2007]*
(a) 2/3 (b) 1/2
(c) 3/4 (d) None of these

14. Let the probability that a batter gets a hit is 1/4. If he plays 4 bats the probability that he gets at least one hit is *[MPPET-2007]*
(a) 175/256 (b) 1
(c) 1/16 (d) None of these

15. Three groups of children contain 3 girls and 1 boy, 2 girls and 2 boys, one girl and 3 boys, respectively. One child is selected at random from each group. The chance that three selected consisting of 1 girl and 2 boys is
(a) 9/32 (b) 3/32
(c) 13/32 (d) None of these

16. The probability of hitting a target by three marksmen are 1/2, 1/3 and 1/4, respectively. The probability that one and only one of them will hit the target when they fire simultaneously is
(a) 11/24 (b) 1/12
(c) 1/8 (d) None of these

17. A bag contains tickets numbered from 1 to 20. Two tickets are drawn. The probability that both the numbers are prime is
(a) 14/95 (b) 7/95
(c) 1/95 (d) None of these

18. A bag contains 4 white ball and 2 black balls. Another contains 3 white balls and 5 black balls. If one ball is drawn from each bag, then the probability that both are white is
(a) 0.25 (b) 0.2
(c) 0.3 (d) None of these

19. The number of cadets standing in a line is 5, all possible permutations being equally likely. Find the probability of two particular cadets being together.
(a) 1/5 (b) 2/5
(c) 3/5 (d) None of these

20. What is the probability that four *S*s appear consecutively in the word MISSISSIPPI? Assume that the letters are arranged at random.
(a) $\frac{1}{8}$ (b) $\frac{8!\,4!}{11!}$
(c) $\frac{4}{165}$ (d) None of these

21. A bag contains 5 white and 3 red balls. Balls are drawn in succession and are not replaced. Show that the chance that the first red ball will appear on the fifth draw is
(a) 1/56 (b) 3/56
(c) 7/56 (d) 5/56

22. Let A and B be two events such that
$P(\overline{A \cup B}) = \frac{1}{6}, P(A \cap B) = \frac{1}{4}$ and $P(\overline{A}) = \frac{1}{4}$
where $\overline{A}$ stands for complement of event A. Then events A and B are
(a) equally likely but not independent
(b) equally likely and mutually exclusive
(c) mutually exclusive and independent
(d) independent but not equally likely

23. A manufacturer of cotter pins knows that 5% of his product is defective. He sells pins in boxes of 100 and guarantees that not more than one pin will be defective in a box. In order to find the probability that a box will fail to meet the guaranteed quality, the probability distribution one has to employ is *[VITEEE-2008]*
(a) Binomial (b) Poisson
(c) Normal (d) Exponential

24. Which of the following numbers is nearest to the probability that three randomly selected persons are born on three different days of the week? *[NDA-2008]*
(a) 0.7 (b) 0.6
(c) 0.5 (d) 0.4

25. In a school there are 40% Science students and the remaining 60% are Arts students. It is known that 5% of the Science students are girls and 10% of the Arts students are girls. One student selected at random is a girl. What is the probability that she is an Arts student? *[NDA-2008]*
(a) 1/3 (b) 3/4
(c) 1/5 (d) 3/5

26. A purse contains 4 copper and 3 silver coins. Another purse contains 6 copper and 2 silver coins. A coin is taken out from any purse, the probability that it is a silver coin is *[MPPET-2008]*
(a) 37/56 (b) 19/56
(c) 4/7 (d) 2/3

WORKSHEET: TO CHECK THE PREPARATION LEVEL

Important Instructions

1. The answer sheet is immediately below the worksheet.
2. The test is of 18 minutes.
3. The worksheet consists of 18 questions. The maximum marks are 54.
4. Use blue/black ball point pen only for writing particulars/marking responses. Use of pencil is strictly prohibited.

1. A purse contains 2 silver and 4 copper coins. A second purse contains 4 silver and 3 copper coins. If a coin is pulled out at random from one of the two purses, what is the probability that it is a silver coin?

(a) 1/6 (b) 2/7
(c) 19/42 (d) None of these

2. For a biased die the probabilities for different faces to turn up are given below:

Face:	1	2	3	4	5	6
Probability:	0.1	0.32	0.21	0.15	0.05	0.17

The die is tossed and you are told that either face 1 or 2 has turned up. Then the probability that it is face 1 is *[IIT-1981]*

(a) 5/21 (b) 5/22
(c) 4/21 (d) None

3. A, B are two events and $\overline{A}$ denotes the complement of A. Consider the following statements: *[NDA-2007]*

1. $P(A \cup B) \le P(B) + P(A)$
2. $P(A) + P(\overline{A} \cup B) \le 1 + P(B)$

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

4. Six text books numbered 1, 2, 3, 4, 5 and 6 are arranged at random. What is the probability that the text books 2 and 3 will occupy consecutive places? *[NDA-2007]*

(a) 1/2 (b) 1/3
(c) 1/4 (d) 1/6

5. What is the probability of getting 5 heads and 7 tails in 12 flips of a balanced coin? *[NDA-2007]*

(a) $C(12, 5)/(2^{12})$ (b) $C(12, 7)/(2^6)$
(c) $C(12, 5)/2^5$ (d) $C(12, 5)/(2^7)$

6. In a lottery, 16 tickets are sold and 4 prizes are awarded. If a person buys 4 tickets, then what is the probability of his winning a prize? *[NDA-2007]*

(a) 1/4 (b) 81/256
(c) $4/16^4$ (d) 175/256

7. If A and B are any two events such that $P(A \cup B) = \frac{3}{4}$, $P(A \cap B) = \frac{1}{4}$ and $P(\overline{A}) = \frac{2}{3}$ where $\overline{A}$ stands for the complementary event of A, then what is $P(B)$? *[NDA-2007]*

(a) 1/9 (b) 2/9
(c) 1/3 (d) 2/3

8. A can hit a target 4 times in 5 shots;
B can hit a target 3 times in 4 shots;
C can hit a target 2 times in 3 shots;
All the three fire a shot each. What is the probability that two shots are at least hit? *[NDA-2007]*

(a) 5/6 (b) 1/3
(c) 1/6 (d) 3/5

9. A random variable X takes values 0, 1, 2, 3, ... with probability $P(X = x) = k\,(x + 1)\left(\frac{1}{5}\right)^x$ where k is constant, then $P(X = 0)$ is:

(a) 7/25 (b) 18/25
(c) 13/25 (d) 16/25

10. Out of 15 persons 10 can speak Hindi and 8 can speak English. If two persons are chosen at random, then the probability that one person speaks Hindi only and the other speaks both Hindi and English is *[Kerala PET-2007]*

(a) 3/5 (b) 7/12
(c) 1/5 (d) 2/5

11. A random variable X has the following probability distribution

$X = x_i$	1	2	3	4
$P(X = x_i)$	0.1	0.2	0.3	0.4

The mean and the standard deviation are, respectively: *[KeralaPET-2007]*

(a) 3 and 2 (b) 3 and 1
(c) 3an d $\sqrt{3}$ (d) 2 and 1

12. The last three digits of a telephone number beginning with 135 … have been erased. The probability that the erased digits will be all identical is *[AMU Engg.-2007]*
(a) 1/50 (b) 1/100
(c) 3/100 (d) None of these

13. Bag A contains 4 green and 3 red balls and bag B contains 4 red and 3 green balls. One bag is taken at random and a ball is drawn and noted it is green. The probability that it comes from bag B is *[DCE-2005]*
(a) 2/7 (b) 2/3
(c) 3/7 (d) 1/3

14. A die is tossed 5 times. Getting an odd number is considered a success. Then the variance of distribution of success is
(a) 8/3 (b) 3/8
(c) 4/5 (d) 5/4

15. The probability that a student passes in Mathematics is 4/9 and that he passes in Physics is 2/5. Assuming that passing in Mathematics and Physics are independent of each other. What is the probability that he passes in Mathematics but fails in Physics? *[NDA-2006]*
(a) 4/15 (b) 8/45
(c) 26/45 (d) 19/45

16. The corners of regular tetrahedrons are numbered 1, 2, 3, 4. Three tetrahedrons are tossed. The probability that the sum of upward corner will be 5 is
(a) 5/24 (b) 5/64
(c) 3/32 (d) 3/16

17. One bag contains 5 white balls and 3 black balls, and a second bag contains 2 white balls and 4 black balls. One ball is drawn from the first bag and placed unseen in the second bag. What is the probability that a ball now drawn from the second bag is black? *[NDA-2008]*
(a) $\frac{15}{56}$ (b) $\frac{35}{56}$
(c) $\frac{37}{56}$ (d) $\frac{25}{48}$

18. One bag contains 3 white and 2 black balls, another contains 5 white and 3 black balls. If a bag is chosen at random and a ball is drawn from it then what is the chance that it is white?
(a) 49/80 (b) 49/160
(c) 3/8 (d) 31/80

ANSWER SHEET

1. (a) (b) (c) (d)
2. (a) (b) (c) (d)
3. (a) (b) (c) (d)
4. (a) (b) (c) (d)
5. (a) (b) (c) (d)
6. (a) (b) (c) (d)
7. (a) (b) (c) (d)
8. (a) (b) (c) (d)
9. (a) (b) (c) (d)
10. (a) (b) (c) (d)
11. (a) (b) (c) (d)
12. (a) (b) (c) (d)
13. (a) (b) (c) (d)
14. (a) (b) (c) (d)
15. (a) (b) (c) (d)
16. (a) (b) (c) (d)
17. (a) (b) (c) (d)
18. (a) (b) (c) (d)

HINTS AND EXPLANATIONS

1. (c) $P(S) = P(S/\text{I})\,P(\text{I}) + P(S/\text{II})\,P(\text{I})$

$$= \frac{2}{6} \times \frac{1}{2} + \frac{4}{7} \times \frac{1}{2} = \frac{19}{42}$$

2. (a)

$$P(E) = \frac{P(\text{face 1})}{P(\text{face 1 or face 2})} = \frac{0.1}{0.1 + 0.3\ 2} = \frac{10}{42} = \frac{5}{2}$$

4. (b) $P(E)=\dfrac{n(E)}{n(S)}$; $n(E)=5!\times 2!$; $n(S)=5!$

$$P(E)=\frac{5!\times 2!}{6!}=\frac{1}{3}$$

8. (a) $P(A)=\dfrac{4}{5}=P$ (probability of A hitting a target)

$P(B)=\dfrac{3}{4}$, $P(C)=\dfrac{2}{3}$

$P(E) = P$(2 shots are hit) + P(3 shots are hit)

$= P(A)\,P(B)\,P(\bar{C}) + P(A)\,P(\bar{B})\,P(C) + P(\bar{A})\,P(B)\,P(C) + P(A)\,P(B)\,P(C)$

$$=\frac{4}{5}\times\frac{3}{4}\times\frac{1}{3}+\frac{4}{5}\times\frac{1}{4}\times\frac{2}{3}+\frac{1}{5}\times\frac{3}{4}\times\frac{2}{3}+\frac{4}{5}\times\frac{3}{4}\times\frac{2}{3}$$

$$=\frac{50}{60}=\frac{5}{6}$$

10. (c) No. of person who speak English and Hindi both

$= n(H \cap E) = n(H) + n(E) - n(H \cup E)$

$= 10 + 8 - 15 = 3$, n(persons peaking Hindi only)

$= 10 - 3 = 7$

$$= P(E)=\frac{{}^{3}C_1\times {}^{7}C_1}{{}^{15}C_2}=\frac{21}{105}=\frac{1}{5}$$

12. (b) $P(E)=\dfrac{10}{10\times 10\times 10}=\dfrac{1}{100}$

($\because$ since favourable cases are 000, 111, 222, …, 999)

13. (c) $P(B/G)=\dfrac{P(G/B)\times P(B)}{P(G/B)\,P(B)+P(G/A)\;P(A)}$

$$=\frac{3/7\times 1/2}{3/7\times 1/2+4/7+1/2}=\frac{3}{7}$$

14. (d) Variance = $npq = 5\times\dfrac{3}{6}\times\dfrac{3}{6}=\dfrac{5}{4}$