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Improving efficiency is as important as radical change

Quants study ways to reduce noise in XVA Greeks calculations. By Nazneen Sherif

Price sensitivities to risk factors, otherwise known as Greeks, have recently been just as challenging to calculate as they are essential in efficient risk management and regulatory reporting.

This is especially true in the case of derivatives with early exercise features, such as Bermudan options, where exercise is allowed at certain specified dates before expiry.

It becomes even more complicated and time-consuming when one has to price valuation adjustments (XVAs) for these instruments, as this involves computation of the future values of the derivative at each exercise point.

Distributions of future values are computed using Monte Carlo simulations and regression. Finding Greeks, on the other hand, requires differentiation of these regressions at each future value, which can be computationally very intensive. It can also produce a lot of noise in the estimates, as the regressions themselves are occasionally noisy.

"To calculate the Greeks of prices or XVAs, we should go through these regressions and differentiate at each future value," says Alexandre Antonov, a director at Standard Chartered in London. "Regression techniques can be very complicated and painful. Sometimes you would like to avoid the differentiation of regressions."

In this month's technical, *Pathwise XVA Greeks for early-exercise products*, Antonov and his co-authors Serguei Mechkov, a senior vice-president at Numerix in Paris, Michael Konikov, an executive director, Andy McClelland, a director and Serguei Issakov, a senior vice-president within the same firm, show differentiation of regressions can in fact be avoided for some derivatives instruments.

Future values of products with early exercise are typically computed using a so-called backward Monte Carlo, a combination of Monte Carlo simulation and regression, carefully calculating the exercise conditions, and then running a forward Monte Carlo aggregating the final result.

Typically, the Greeks of prices are calculated by differentiating the regression function. Since prices are based on optimal exercise values in the future, differentiation at future exercise dates is not required for derivatives that are typically priced using backward Monte Carlo, because the differentials at these points are zero.

The quants show this conclusion can also be extended for XVAs to a wider class of products. For the calculation of credit valuation adjustment sensitivities of products with single exercise and simple collateral arrangements, such as one-sided credit support annexes, previous works have showed differentiation of regression can be avoided altogether by differentiating cashflows of the derivative instead and applying extra regressions.

The cashflows could then be combined in such a way that one can arrive at the Greeks.

The authors extend this to any instrument that can be priced using backward Monte Carlo, such as those with callable, multi-callable or combinations of callable and barrier options.

"If we manage to apply it to only cashflows, it is easy, because quite often we know all the cashflows analytically, so it's not a big deal. However, the derivative of the regression is complicated," says Antonov.

Numerix has been using the technique as part of its XVA pricing package for almost a year.

In recent years, techniques such as adjoint algorithmic differentiation (AAD) helped substantially speed up sensitivities calculation both for XVAs and regulatory reporting by anywhere between 10–500 times. AAD is already being used for a number of applications, such as risk management of complex derivatives, calculation of market risk sensitivities for regulatory requirements and computation of CVA Greeks.

Antonov says that for Greeks for certain instruments and under specific conditions, the results in their paper also extend to algorithmic techniques such as AAD, making it possible to avoid the inefficient step of differentiating regressions that is otherwise part of the algorithm.

> "Regression techniques can be very complicated and painful. Sometimes you would like to avoid the differentiation of regressions"
> Alexandre Antonov, Standard Chartered

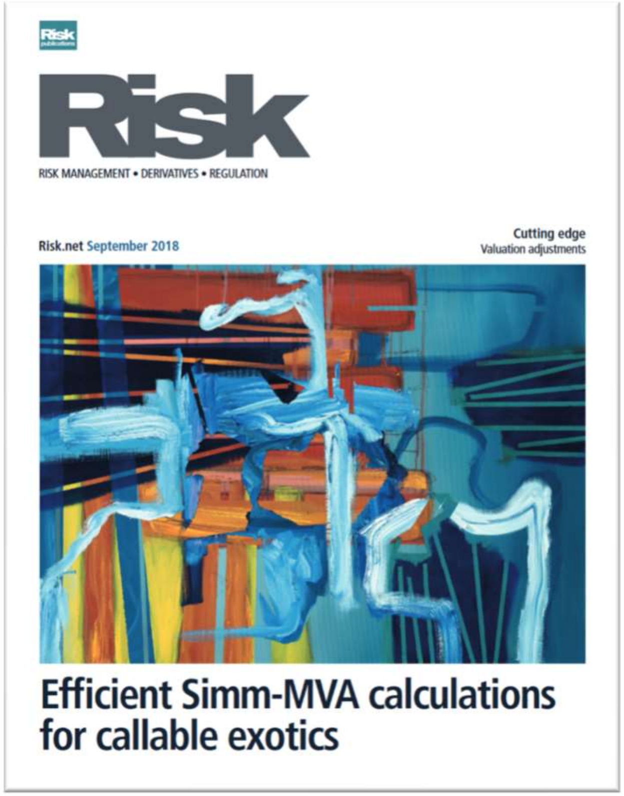
Avoiding differentiation has two advantages – speed and accuracy. Using this new technique, Greeks can be computed almost as quickly as the time it takes to price the derivatives. Cutting the step of differentiating regressions also helps reduce noise in the estimation.

With growing regulatory requirements and the industry's rapid move towards clearing and electronic trading, optimisation is clearly the key to running a successful business, pushing quants to focus more on issues such as margin compression and price impact optimisation.

It is no surprise, therefore, that some would want to improve efficiency and reduce complexity wherever possible, even for techniques that already work well, such as AAD, for instance.

They may not seem like radical changes, but small improvements over a period of time could save someone a lot of pain in the long run. ■

Risk.net Cutting Edge Research on MVA



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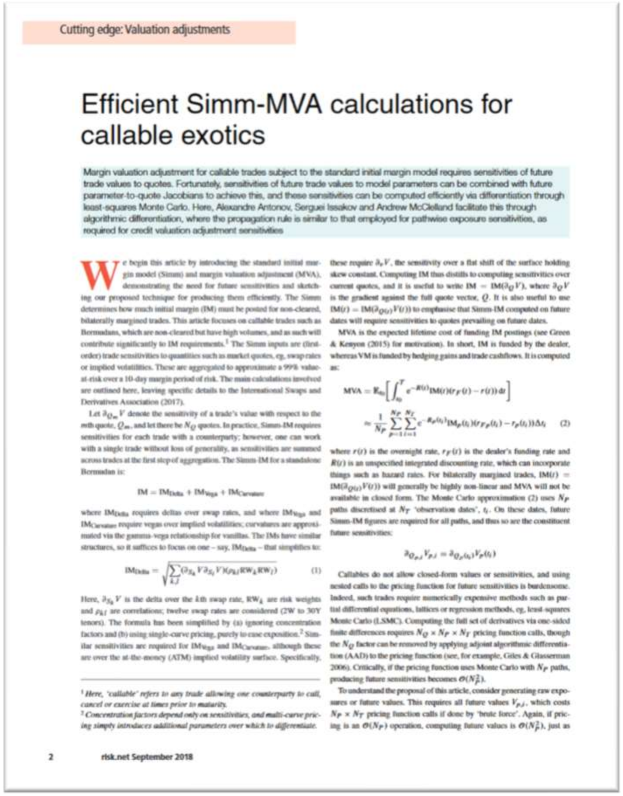
Efficient Simm-MVA calculations for callable exotics



Efficient Simm-MVA calculations for callable exotics

Margin valuation adjustment for callable trades subject to the standard initial margin model requires sensitivities of future trade values to quotes. Fortunately, sensitivities of future trade values to model parameters can be combined with future parameter-to-quote Jacobians to achieve this, and these sensitivities can be computed efficiently via differentiation through least-squares Monte Carlo. Here, Alexandre Antonov, Serguei Issakov and Andrew McClelland facilitate this through algorithmic differentiation, where the propagation rule is similar to that employed for pathwise exposure sensitivities, as required for credit valuation adjustment sensitivities

We begin this article by introducing the standard initial margin model (Simm) and margin valuation adjustment (MVA), demonstrating the need for future sensitivities and sketching our proposed technique for producing them efficiently. The Simm determines how much initial margin (IM) must be posted for non-cleared, bilaterally margined trades. This article focuses on callable trades such as Bermudans, which are non-cleared but have high volumes, and as such will contribute significantly to IM requirements.[1] The Simm inputs are (first-order) trade sensitivities to quantities such as market quotes, eg, swap rates or implied volatilities. These are aggregated to approximate a 99% value-at-risk over a 10-day margin period of risk. The main calculations involved are outlined here, leaving specific details to the International Swaps and Derivatives Association (2017).

Let $\partial_{Q_m} V$ denote the sensitivity of a trade's value with respect to the mth quote, Q_m, and let there be N_Q quotes. In practice, Simm-IM requires sensitivities for each trade with a counterparty; however, one can work with a single trade without loss of generality, as sensitivities are summed across trades at the first step of aggregation. The Simm-IM for a standalone Bermudan is:

$$\mathrm{IM} = \mathrm{IM}_{\mathrm{Delta}} + \mathrm{IM}_{\mathrm{Vega}} + \mathrm{IM}_{\mathrm{Curvature}}$$

where $\mathrm{IM}_{\mathrm{Delta}}$ requires deltas over swap rates, and where $\mathrm{IM}_{\mathrm{Vega}}$ and $\mathrm{IM}_{\mathrm{Curvature}}$ require vegas over implied volatilities; curvatures are approximated via the gamma-vega relationship for vanillas. The IMs have similar structures, so it suffices to focus on one – say, $\mathrm{IM}_{\mathrm{Delta}}$ – that simplifies to:

$$\mathrm{IM}_{\mathrm{Delta}} = \sqrt{\sum_{k,l} (\partial_{S_k} V \partial_{S_l} V)(\rho_{kl} \mathrm{RW}_k \mathrm{RW}_l)} \tag{1}$$

Here, $\partial_{S_k} V$ is the delta over the kth swap rate, RW_k are risk weights and ρ_{kl} are correlations; twelve swap rates are considered (2W to 30Y tenors). The formula has been simplified by (a) ignoring concentration factors and (b) using single-curve pricing, purely to ease exposition.[2] Similar sensitivities are required for $\mathrm{IM}_{\mathrm{Vega}}$ and $\mathrm{IM}_{\mathrm{Curvature}}$, although these are over the at-the-money (ATM) implied volatility surface. Specifically, these require $\partial_\sigma V$, the sensitivity over a flat shift of the surface holding skew constant. Computing IM thus distills to computing sensitivities over current quotes, and it is useful to write $\mathrm{IM} = \mathrm{IM}(\partial_Q V)$, where $\partial_Q V$ is the gradient against the full quote vector, Q. It is also useful to use $\mathrm{IM}(t) = \mathrm{IM}(\partial_{Q(t)} V(t))$ to emphasise that Simm-IM computed on future dates will require sensitivities to quotes prevailing on future dates.

MVA is the expected lifetime cost of funding IM postings (see Green & Kenyon (2015) for motivation). In short, IM is funded by the dealer, whereas VM is funded by hedging gains and trade cashflows. It is computed as:

$$\begin{aligned} \mathrm{MVA} &= \mathrm{E}_{t_0}\Big[\int_{t_0}^{T} e^{-R(t)} \mathrm{IM}(t)(r_F(t) - r(t))\,\mathrm{d}t\Big] \\ &\approx \frac{1}{N_P} \sum_{p=1}^{N_P} \sum_{i=1}^{N_T} e^{-R_p(t_i)} \mathrm{IM}_p(t_i)(r_{F,p}(t_i) - r_p(t_i))\Delta t_i \end{aligned} \tag{2}$$

where $r(t)$ is the overnight rate, $r_F(t)$ is the dealer's funding rate and $R(t)$ is an unspecified integrated discounting rate, which can incorporate things such as hazard rates. For bilaterally margined trades, $\mathrm{IM}(t) = \mathrm{IM}(\partial_{Q(t)} V(t))$ will generally be highly non-linear and MVA will not be available in closed form. The Monte Carlo approximation (2) uses N_P paths discretised at N_T 'observation dates', t_i. On these dates, future Simm-IM figures are required for all paths, and thus so are the constituent future sensitivities:

$$\partial_{Q_{p,i}} V_{p,i} = \partial_{Q_p(t_i)} V_p(t_i)$$

Callables do not allow closed-form values or sensitivities, and using nested calls to the pricing function for future sensitivities is burdensome. Indeed, such trades require numerically expensive methods such as partial differential equations, lattices or regression methods, eg, least-squares Monte Carlo (LSMC). Computing the full set of derivatives via one-sided finite differences requires $N_Q \times N_P \times N_T$ pricing function calls, though the N_Q factor can be removed by applying adjoint algorithmic differentiation (AAD) to the pricing function (see, for example, Giles & Glasserman 2006). Critically, if the pricing function uses Monte Carlo with N_P paths, producing future sensitivities becomes $\mathcal{O}(N_P^2)$.

To understand the proposal of this article, consider generating raw exposures or future values. This requires all future values $V_{p,i}$, which costs $N_P \times N_T$ pricing function calls if done by 'brute force'. Again, if pricing is an $\mathcal{O}(N_P)$ operation, computing future values is $\mathcal{O}(N_P^2)$, just as

[1] *Here, 'callable' refers to any trade allowing one counterparty to call, cancel or exercise at times prior to maturity.*

[2] *Concentration factors depend only on sensitivities, and multi-curve pricing simply introduces additional parameters over which to differentiate.*

Tenor Basis and Commodity Models

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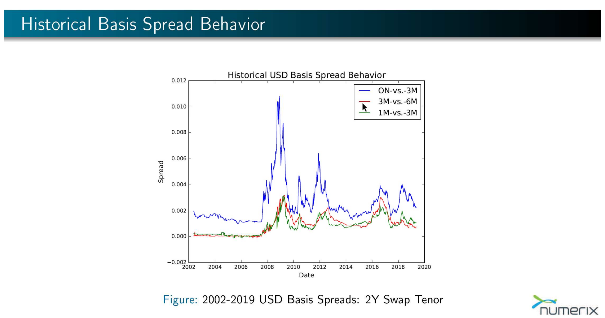

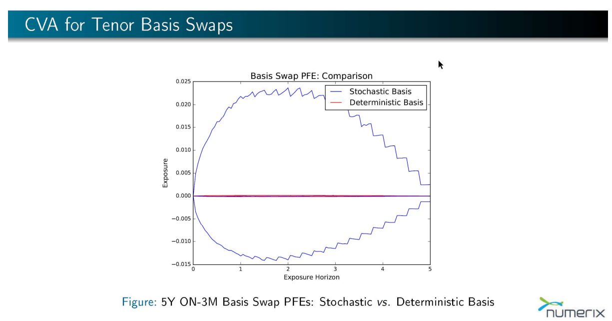

Risk.net Cutting Research: Volatility Modelling

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Risk.net August 2020

Cutting edge
Volatility modelling

A new arbitrage-free parametric volatility surface

A new arbitrage-free parametric volatility surface

Alexandre Antonov, Michael Spector and Michael Konikov describe a new parametric volatility surface that is arbitrage free, is extremely rich and flexible, and has closed-form expressions for both European option values and local volatilities. The volatility surface is based on previous work by Carr and Pelts, for which the present authors provide a simple derivation and a concrete implementation

Parametric volatility surfaces are used in finance and financial modelling for many purposes, such as:

- vanilla option pricing, risk, hedging and market making;
- data cleansing, information reduction and producing derived data;
- arbitrage detection and prevention, trade signals;
- scenario generation and volatility surface dynamics; and
- as an input for stochastic models, ie, local volatility (LV) and local stochastic volatility (LSV) models.

A particularly important use of such surfaces is in Dupire's formulation of the LV model, where the parameterisation is employed to obtain derivatives of the implied volatility (or option price) that appear in Dupire's formula. The local volatility itself usually is not available analytically, but implied volatility (or option price) usually is. It is very important for the volatility surface not only to be free of any sort of arbitrage but also to be sufficiently smooth if we want to produce a sensible local volatility function using Dupire's formula. More generally, some desirable properties of a volatility surface are:

- absence of static arbitrage, or, equivalently, positivity of call spreads, butterfly spreads and calendar spreads (see, for example, Cousot 2004);
- parsimonious formulation, ie, the ability to describe the continuum of option prices with just a few parameters;
- flexibility, ie, the ability to fit as rich an options data set as needed (eg, the Standard & Poor's 500 (SPX));
- smoothness, which is an important feature to facilitate the use of Dupire's formula (see Dupire 1994);
- fast option valuation, preferably in closed form; and
- fast local volatility valuation, preferably in closed form.

It is usually very hard to guarantee the total absence of arbitrage working directly in the implied volatility space, since arbitrage conditions are difficult to translate into volatility terms. If we parameterise in the price space, however, we can ensure our surface is free of arbitrage, but then it is hard to achieve a plausible shape for its implied volatility.

One of the first parametric volatility surfaces that guaranteed the absence of arbitrage under some conditions was the stochastic volatility inspired (SVI) volatility surface introduced in Gatheral (2004) and Gatheral & Jacquier (2012, 2014). This has many desirable properties (although not all of those listed above) and is still very popular in the industry. Its shortcomings include a lack of flexibility to fit rich options data when arbitrage conditions are enforced as well as time-direction interpolation in price terms; strike interpolation at input maturities is, however, done in volatility terms. The latter may lead to some unintuitive shapes for the interpolated volatilities and may introduce arbitrage in the strike dimension.

The methodology first described in Carr & Pelts (2015) is somewhat unique in that it allows both a guarantee of no arbitrage and a reasonable shape of the implied volatility surface at the same time. Moreover, we show that both implied and local volatilities are known essentially in closed form. The only drawback of this surface is its restricted flexibility, similar to the SVI construction. The obvious reason for this is that the two-dimensional volatility surface is parameterised with two one-dimensional functions.

The contribution of the present article is to provide a simplified explanation of how the Carr-Pelts (CP) surface can be implemented in practice as well as a generalisation allowing for a much more flexible (possibly arbitrarily flexible) surface able to fit very rich sets of options data. We also show how to incorporate discrete dividends into the construction of our volatility surface. Going forward, we refer to our new volatility surface (first introduced in Antonov *et al* (2019)) as the ensemble Carr-Pelts (ECP) surface.

The CP surface

In this section, we provide some intuition behind the CP construction. Take a stochastic variable X with cumulative distribution $\Psi(k) = \mathbb{E}[1_{X<k}]$ such that $\mathbb{E}[e^X]$ exists and equals 1. Define a function $F(\cdot)$ mapping the distribution into its measure change version (an interval $[0,1]$ to $[0,1]$) as follows:

$$F(\mathbb{E}[1_{X<k}]) := \mathbb{E}[e^X 1_{X<k}]$$

or, explicitly:

$$F(u) := \mathbb{E}[e^X 1_{X<\Psi^{-1}(u)}]$$

implying:

$$F'(u) = e^{\Psi^{-1}(u)} \quad \text{and} \quad F''(u) = \frac{e^{\Psi^{-1}(u)}}{\Psi'(\Psi^{-1}(u))}$$

Note that knowing $F(u)$ allows us to find the distribution $\Psi(k)$ and that $F(1) = 1$ is equivalent to $\mathbb{E}[e^X] = 1$. Also, both $F'(u)$ and $F''(u)$ are positive, which means $F(u)$ is increasing and convex.

Now, consider an exponential martingale process X_t with $\mathbb{E}[e^{X_t}] = 1$ and cumulative distribution $\Psi(t,k)$. We extend the previous considerations from the stochastic variable to the process by adding the time argument:

$$F(t,u) := \mathbb{E}[e^{X_t} 1_{X_t<\Psi_u^{-1}(t,u)}]$$

where the inverse of u is taken for a fixed t, and, like before, we have:

$$\partial_u F(t,u) = e^{\Psi_u^{-1}(t,u)} \tag{1}$$

$$\partial_u^2 F(t,u) = \frac{e^{\Psi_u^{-1}(t,u)}}{\partial_u \Psi(\Psi_u^{-1}(t,u))}$$

Again, the function $F(t,u)$ is increasing and convex in u. One can prove that $F(t,u)$ is decreasing in time. Indeed, a 'European put option' value can

Key Takeaways

- **Fast AD (Algorithmic Differentiation) XVA Greeks** – speed up your XVA Greeks by a factor of 10.
  - Available for IR, FX, EQ, CMDTY, INFL, CR Deterministic – for Gaussian Multi-Currency Hybrid models (IR Hull-White and FX/EQ/CMDTY Black-Scholes).
  - Roadmap: add for stochastic CR model to complete coverage of asset classes; support collateral.
- **MVA for non-cleared derivatives based on SIMM margin rules** – practical algorithm for general trades – estimate your future SIMM margin requirements for the entire life of your portfolio.
  - Available for IR, FX, EQ, CMDTY, CR Deterministic – for Gaussian Cross-Currency Hybrid models (IR Hull-White and FX/EQ/CMDTY Black-Scholes) with a single EQ, FX, and CMDTY.
  - Roadmap: add for INFL and stochastic CR model to complete coverage of asset classes; add support for general model configurations with multiple FX, EQ, and CMDTY.
- **Tenor Basis models** – evaluate your Tenor Basis risk in XVA by simulating stochastic basis spreads, also for municipal SIFMA rates.
- **Commodity Andersen model** – evaluate your commodity basis risks in XVA. Use the **Bachelier** model as a special limit of the Andersen model to manage negative oil prices.
- **New Arbitrage-Free Equity Volatility Surface** – the first arbitrage-free vol surface by construction based on open publication. Can fit 3,300 SPX options at once. Can fit the challenging W-shaped volatility curves.
- **Focus on Jacobian Greeks** – speed up Greeks for complex models (not covered by Algorithmic Differentiation) by using Jacobians.

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