Advances in Tenor Basis Modeling: Boundedness, Specification & Calibration

Andy McClelland

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Numerix Webinar Series

October 10, 2019

High-Level Overview

1. Concerned with *e.g.* 3M IBOR-*vs*.-6M IBOR or OIS/SOFR-*vs*-.3M IBOR basis spreads
2. Tenor basis spreads drive CVA for tenor basis swaps
3. Model for IBORs & potential non-RFR IBOR alternatives, ICE's BBI *etc*.
4. Bound *e.g.* 3M IBOR-*vs*.-6M IBOR basis from below
5. Level-dependent volatility specification in multi-curve Cheyette model, akin to HW $\rightarrow$ CIR
6. No nonlinears for calibration, integrate historical data carefully

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Historical Basis Spread Behavior

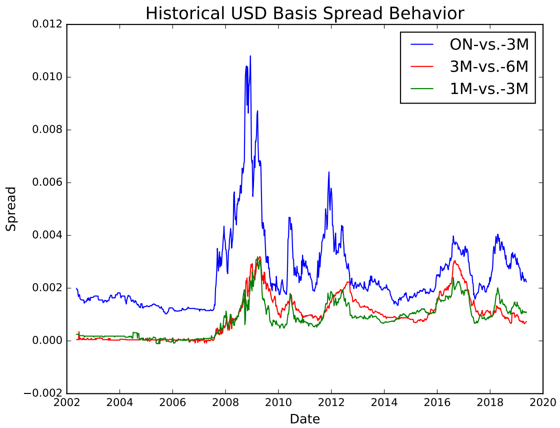

Figure: 2002-2019 USD Basis Spreads: 2Y Swap Tenor

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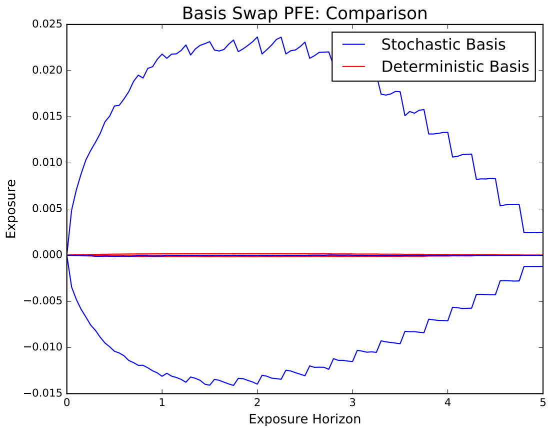

Figure: 5Y ON-3M Basis Swap PFEs: Stochastic *vs.* Deterministic Basis

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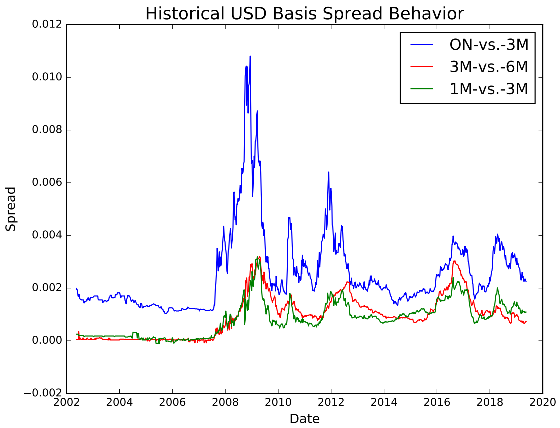

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Tenor Basis Spreads & CVA

Tenor Basis Spread Definitions

- IBORs are spot inter-bank unsecured rates with tenors τ_n

$$\text{Forward IBOR} = R(t, T)$$

- t is the calendar date & T is the fixing date
- $\tau_3 = 3\text{M} \implies R_3(t, T, \tau_3) = 3\text{M IBOR}$, $\tau_6 = 6\text{M} \implies R_6(t, T, \tau_6) = 6\text{M IBOR}$, *etc.*
- At fixing, $R_3(T, T, \tau_3)$ & $R_6(T, T, \tau_6)$ reflect different credit & liquidity fundamentals
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- Tenor basis swaps exchange *e.g.* 3M IBOR-plus-spread each quarter *vs.* 6M IBOR each half

$$\left(R_3(T_i, T_i, \tau_3) + \phi_{3,6}\right)\tau_3 \quad @\, T_{i+1} \quad vs. \quad R_6(\bar{T}_j, \bar{T}_j, \tau_6)\tau_6 \quad @\, \bar{T}_{j+1}$$

- $\phi_{3,6}$ is the spread, T_i, $\bar{T}_j$ are quarterly, semi dates

$$\text{Basis Swap Value}\,(t) = \sum_j P_0(t, \bar{T}_{j+1}) R_6(t, \bar{T}_j, \tau_6)\tau_6 - \sum_i P_0(t, T_{i+1})\left(R_3(t, T_i, \tau_3) + \phi_{3,6}\right)\tau_3$$

- Primary risk is difference in IBOR curves, value roughly $\propto \left(R_6^{av}(t) - R_3^{av}(t)\right) - \phi_{3,6}$
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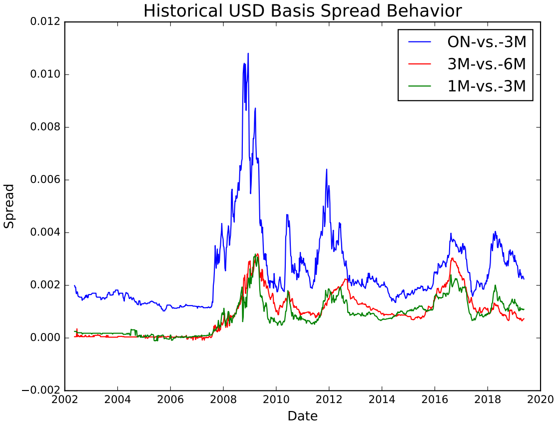

Figure: 2002-2019 USD Basis Spreads: 2Y Swap Tenor

CVA for Tenor Basis Swaps

- CVA is lifetime expected exposure, ignore credit, discounting *etc*.

$$\text{CVA} \approx \int_0^\infty \mathbb{E}_0\left[\left(V(t)\right)^+\right] dt$$

- Essentially have a "tenor basis swaption", which would be an ideal calibration instrument
- ... explains lack of pre-'08 tenor basis modeling efforts
- No tenor basis volatility $\implies$ low CVA, PFEs, CCR capital (& FVA, *some* MVA effect)

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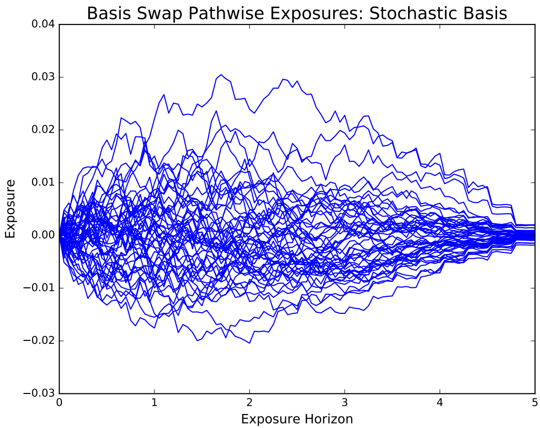

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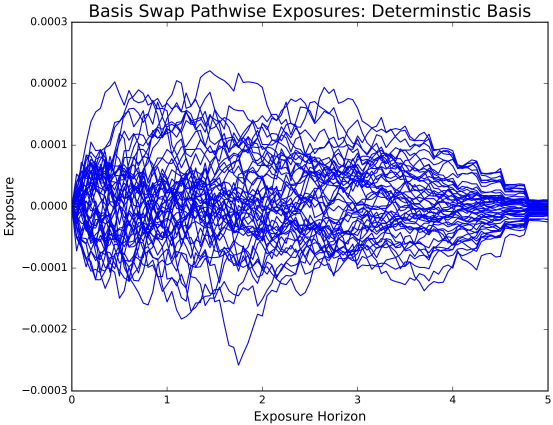

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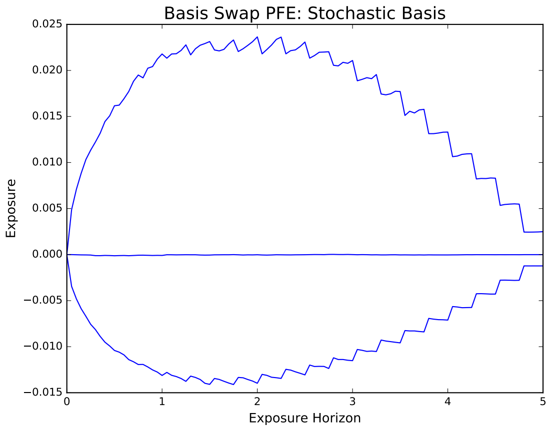

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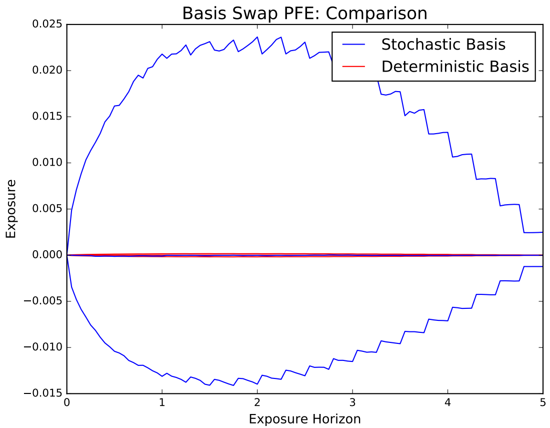

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A Cheyette-Style Multi-Curve Model with Lower Bounds

- ON curve is "base" curve, modeled outright, used for $R_0(t, T, \tau_n)$
- $f_0(t, T)$ is instantaneous ON forward curve, used for discount bonds $P_0(t, T)$ & $R_0(t, T, \tau_3)$

$$P(t, T) = e^{-\int_t^T f_0(t,u)\, du}$$

$$R_0(t, T, \tau_3) = \frac{1}{\tau_3}\left(\frac{P(t, T)}{P(t, T + \tau_3)} - 1\right) = \frac{1}{\tau_3}\left(e^{\int_T^{T+\tau_3} f_0(t,u)\, du} - 1\right)$$

- Risk-neutral model with Cheyette ('96) dynamics, *a.k.a.* Markovian HJM dynamics

$$df_0(t, T) = \mu_0(t, T)\, dt + \psi_0(t)\, e^{-\kappa_0(T-t)}\, dW_0(t)$$

- Encounter "usual" HJM no-arbitrage drift condition

$$\int_t^T \mu_0(t, u)\, du = \frac{1}{2}\left(\int_t^T \psi_0(t) e^{-\kappa_0(u-t)}\, du\right)^2 \implies \mu_0(t, T) = \frac{1}{\kappa_0}\psi_0^2(t)\left(e^{-\kappa_0(T-t)} - e^{-2\kappa_0(T-t)}\right)$$

Modeling the Overnight Curve

- ON curve is "base" curve, modeled outright, used for $R_0(t, T, \tau_n)$
- $f_0(t, T)$ is instantaneous ON forward curve, used for discount bonds $P_0(t, T)$ & $R_0(t, T, \tau_3)$

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$$P(t, T) = e^{-\int_t^T f_0(t,u)\, du}$$

$$R_0(t, T, \tau_1) = \frac{1}{\tau_3}\left(\frac{P(t, T)}{P(t, T + \tau_3)} - 1\right) = \frac{1}{\tau_3}\left(e^{\int_T^{T+\tau_3} f_0(t,u)\, du} - 1\right)$$

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- Encounter "usual" HJM no-arbitrage drift condition

$$\int_t^{T} \mu_0(t, u)\,du = \frac{1}{2}\left(\int_t^{T} \psi_0(t)e^{-\kappa_0(u-t)}\,du\right)^2 \implies \mu_0(t, T) = \frac{1}{\kappa_0}\psi_0^2(t)\left(e^{-\kappa_0(T-t)} - e^{-2\kappa_0(T-t)}\right)$$

Modeling the Tenor Basis Spread Curves 1

- 3M IBOR curve, $R_3(t,T,\tau_3)$, built from $R_0(t,T,\tau_3)$ & "spread multiplier" $S_3(t,T,\tau)$

$$S_3(t,T,\tau) = e^{\int_T^{T+\tau} s_3(t,u)\,du}$$

$$R_0(t,T,\tau_3) = \frac{1}{\tau_3}\left(\frac{P(t,T)}{P(t,T+\tau_3)} - 1\right) = \frac{1}{\tau_3}\left(e^{\int_T^{T+\tau_3} f_0(t,u)\,du} - 1\right)$$

- $s_3(t,T)$ is instantaneous 3M-ON spread curve, capturing time-T credit & liquidity contributions[1]
- Introduce similar risk-neutral Cheyette model

$$ds_3(t,T) = \mu_3(t,T)\,dt + \psi_3(t)\,e^{-\kappa_3(T-t)}\,dW_3(t)$$

- We assume zero correlation without loss of generality $\implies$ Martingale $S_3(t,T)$

[1] $s_3(t,T)$: Martinez ('09), Grbac & Runggaldier ('15), $S_3(t,T)$: Henrard ('14), $S_3^+(t,T)$: Mercurio ('10)

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$$S_3(t,T,\tau) = e^{\int_T^{T+\tau} s_3(t,u)\,du}$$

$$R_0(t,T,\tau_3) = \frac{1}{\tau_3}\left(\frac{P(t,T)}{P(t,T+\tau_3)} - 1\right) = \frac{1}{\tau_3}\left(e^{\int_T^{T+\tau_3} f_0(t,u)\,du} - 1\right)$$

- $s_3(t,T)$ is instantaneous 3M-ON spread curve, capturing time-T credit & liquidity contributions[2]
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$$ds_3(t,T) = \mu_3(t,T)\,dt + \psi_3(t)\,e^{-\kappa_3(T-t)}\,dW_3(t)$$

- We assume zero correlation without loss of generality $\implies$ Martingale $S_3(t,T)$

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- We assume zero correlation without loss of generality $\implies$ Martingale $S_3(t,T)$

[3] $s_3(t,T)$: Martinez ('09), Grbac & Runggaldier ('15), $S_3(t,T)$: Henrard ('14), $S_3^+(t,T)$: Mercurio ('10)

- 3M IBOR curve, $R_3(t,T,\tau_3)$, built from $R_0(t,T,\tau_3)$ & "spread multiplier" $S_3(t,T,\tau)$

$$S_3(t,T,\tau) = e^{\int_T^{T+\tau} s_3(t,u)\,du}$$

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- 3M IBOR curve, $R_3(t, T, \tau_3)$, built from $R_0(t, T, \tau_3)$ & "spread multiplier" $S_3(t, T, \tau)$

$$S_3(t, T, \tau) = S(0, T, \tau) e^{\cdots + \int_0^t \sigma_3(v,T)\, dW_3(t)}$$

$$R_3(t, T, \tau_3) = \frac{1}{\tau_3}\left(\frac{P(t, T)}{P(t, T+\tau_3)} S_3(t, T, \tau_3) - 1\right)$$

- $s_3(t, T)$ is instantaneous 3M-ON spread curve, capturing time-T credit & liquidity contributions[7]
- Introduce similar risk-neutral Cheyette model

$$ds_3(t, T) = \mu_3(t, T)\, dt + \psi_3(t)\, e^{-\kappa_3(T-t)}\, dW_3(t)$$

- We assume zero correlation without loss of generality $\implies$ Martingale $S_3(t, T)$

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$$R_3(t,T,\tau_3) = \frac{1}{\tau_3}\left(\frac{P(t,T)}{P(t,T+\tau_3)} S_3(t,T,\tau_3) - 1\right) \approx f_0(t,T) + s_3(t,T)$$

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$$ds_3(t,T) = \mu_3(t,T)\,dt + \psi_3(t)\,e^{-\kappa_3(T-t)}\,dW_3(t)$$

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[12] $s_3(t, T)$: Martinez ('09), Grbac & Runggaldier ('15), $S_3(t, T)$: Henrard ('14), $S_3^{+}(t, T)$: Mercurio ('10)

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$$R_3(t,T,\tau_3) = \mathbb{E}_t^{T+\tau_3}\left[R_3(T,T,\tau_3)\right] \quad \text{for} \quad P^*(T,T+\tau_3) = P(T,T+\tau_3)e^{-\int_T^{T+\tau_3}\lambda(u)\,du}$$

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- Introduce similar risk-neutral Cheyette model

$$ds_3(t,T) = \mu_3(t,T)\,dt + \psi_3(t)\,e^{-\kappa_3(T-t)}\,dW_3(t)$$

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$$ds_3(t,T) = \mu_3(t,T)\,dt + \psi_3(t)\,e^{-\kappa_3(T-t)}\,dW_3(t)$$

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[14] $s_3(t,T)$: Martinez ('09), Grbac & Runggaldier ('15), $S_3(t,T)$: Henrard ('14), $S_3^+(t,T)$: Mercurio ('10)

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$$S_3(t,T,\tau) = e^{\int_T^{T+\tau} s_3(t,u)\,du}$$

$$R_3(t,T,\tau_3) = \frac{1}{\tau_3}\mathbb{E}_t^{T+\tau_3}\left[\frac{1}{P^*(T,T+\tau_3)} - 1\right] \quad \text{for} \quad P^*(T,T+\tau_3) = P(T,T+\tau_3)e^{-\int_T^{T+\tau_3}\lambda(u)\,du}$$

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[15] $s_3(t,T)$: Martinez ('09), Grbac & Runggaldier ('15), $S_3(t,T)$: Henrard ('14), $S_3^+(t,T)$: Mercurio ('10)

- 3M IBOR curve, $R_3(t, T, \tau_3)$, built from $R_0(t, T, \tau_3)$ & "spread multiplier" $S_3(t, T, \tau)$

$$S_3(t, T, \tau) = e^{\int_T^{T+\tau} s_3(t,u)\, du}$$

$$R_3(t, T, \tau_3) = \frac{1}{\tau_3} \mathbb{E}_t^{T+\tau_3}\left[\frac{e^{\int_T^{T+\tau_3} \lambda(u)\, du}}{P(T, T+\tau_3)} - 1\right] \quad \text{for} \quad P^*(T, T+\tau_3) = P(T, T+\tau_3) e^{-\int_T^{T+\tau_3} \lambda(u)\, du}$$

- $s_3(t, T)$ is instantaneous 3M-ON spread curve, capturing time-T credit & liquidity contributions[16]
- Introduce similar risk-neutral Cheyette model

$$ds_3(t, T) = \mu_3(t, T)\, dt + \psi_3(t)\, e^{-\kappa_3 (T-t)}\, dW_3(t)$$

- We assume zero correlation without loss of generality $\implies$ Martingale $S_3(t, T)$

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Modeling the Tenor Basis Spread Curves 1

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$$R_3(t,T,\tau_3) = \frac{1}{\tau_3}\left(\frac{P(t,T)}{P(t,T+\tau_3)} - 1\right) + S_3^+(t,T,\tau_3) = R_0(t,T,\tau_3) + S_3^+(t,T,\tau_3)$$

- $s_3(t,T)$ is instantaneous 3M-ON spread curve, capturing time-T credit & liquidity contributions[20]
- Introduce similar risk-neutral Cheyette model

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$$S_3(t, T, \tau) = e^{\int_T^{T+\tau} s_3(t,u)\, du}$$

$$R_3(t, T, \tau_3) = \frac{1}{\tau_3}\left(e^{\int_T^{T+\tau} f_0(t,u)+s_3(t,u)\, du} - 1\right)$$

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- Introduce similar risk-neutral Cheyette model

$$ds_3(t, T) = \mu_3(t, T)\, dt + \psi_3(t)\, e^{-\kappa_3(T-t)}\, dW_3(t)$$

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- $s_3(t, T)$ is instantaneous 3M-ON spread curve, capturing time-T credit & liquidity contributions[26]

- Introduce similar risk-neutral Cheyette model

$$ds_3(t, T) = \mu_3(t, T)\, dt + \nu_3(t)\sqrt{s_3(t, t)}\, e^{-\kappa_3(T-t)}\, dW_3(t)$$

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Modeling the Tenor Basis Spread Curves 2

- Recall $S_3(t,T) = e^{\int_T^{T+\tau_3} s_3(t,u)\,du}$ where $ds_3(t,T) = \mu_3(t,T)\,dt + \psi_3(t)\,e^{-\kappa_3(T-t)}\,dW_3(t)$
- Enforcing $S_3(t,T)$ Martingality reduces to $\mathbb{E}_t[dS_3(t,T)] = 0$

$$\mathbb{E}_t[dS_3(t,T)] = S_3(t,T)\left(\left(\int_T^{T+\tau_3} \mu_3(t,u)\,du\right) + \frac{1}{2}\left(\int_T^{T+\tau_3} \psi_3(t)e^{-\kappa_3(u-t)}\,du\right)^2\right)dt = 0$$

- As such, no-arbitrage condition on $\mu_3(t,T)$ can be inferred from integral condition

$$\implies \int_T^{T+\tau_3} \mu_3(t,u)\,du = -\frac{1}{2}\left(\int_T^{T+\tau_3} \psi_3(t)e^{-\kappa_3(u-t)}\,du\right)^2$$

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Modeling the Tenor Basis Spread Curves 2

- Recall $S_3(t,T) = e^{\int_T^{T+\tau_3} s_3(t,u)\,du}$ where $ds_3(t,T) = \mu_3(t,T)\,dt + \psi_3(t)\,e^{-\kappa_3(T-t)}\,dW_3(t)$
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- As such, no-arbitrage condition on $\mu_3(t, T)$ can be inferred from integral condition

$$\Longrightarrow \int_{\color{red}{T}}^{\color{red}{T}+\tau_3} \mu_3(t, u)\, du = -\frac{1}{2}\left(\int_{\color{red}{T}}^{\color{red}{T}+\tau_3} \psi_3(t) e^{-\kappa_3(u-t)}\, du\right)^2$$

Modeling the Tenor Basis Spread Curves 3

- Differentiate over T to ensure agreement for all T, *i.e.* all coverage periods $[T, T+\tau_3)$

$$\mu_3(t, T+\tau_3) - \mu_3(t, T) = -\left(\int_T^{T+\tau_3} \psi_3(t) e^{-\kappa_3(u-t)}\, du\right)\left(\psi_3(t)\, e^{-\kappa_3(T+\tau_3-t)} - \psi_3(t)\, e^{-\kappa_3(T-t)}\right)$$

$$= \frac{\psi_3(t)^2}{\kappa_3} e^{-2\kappa_3(T-t)}\left(1 - e^{-\kappa_3\tau_3}\right)^2$$

- Unusual... involves a *difference* of drifts at T & $T+\tau_3$

$$S_3(t, T) = e^{\int_T^{T+\tau_3} s_3(t,u)\, du} \implies T \to T+\epsilon \text{ pulls in } s_3(t, T+\tau_3+\epsilon) \text{ but drops } s_3(t, T)$$

- $\mu_3(t, T+\tau_3)$ compensates for the loss of convexity offset from $\mu_3(t, T)$

Modeling the Tenor Basis Spread Curves 3

- Differentiate over T to ensure agreement for all T, *i.e.* all coverage periods $[T, T + \tau_3)$

$$
\begin{aligned}
\mu_3(t, T+\tau_3) - \mu_3(t, T) &= -\left(\int_T^{T+\tau_3} \psi_3(t) e^{-\kappa_3(u-t)}\, du\right)\left(\psi_3(t)\, e^{-\kappa_3(T+\tau_3-t)} - \psi_3(t)\, e^{-\kappa_3(T-t)}\right) \\
&= \frac{\psi_3(t)^2}{\kappa_3} e^{-2\kappa_3(T-t)} \left(1 - e^{-\kappa_3\tau_3}\right)^2
\end{aligned}
$$

- Unusual... involves a *difference* of drifts at T & $T + \tau_3$

$$S_3(t, T) = e^{\int_T^{T+\tau_3} s_3(t,u)\, du} \implies T \to T + \epsilon \text{ pulls in } s_3(t, T + \tau_3 + \epsilon) \text{ but drops } s_3(t, T)$$

- $\mu_3(t, T + \tau_3)$ compensates for the loss of convexity offset from $\mu_3(t, T)$

- ON curve is "base" curve, modeled outright, used for $R_0(t, T, \tau_n)$
- $f_0(t, T)$ is instantaneous ON forward curve, used for discount bonds $P_0(t, T)$ & $R_0(t, T, \tau_3)$

$$P(t, T) = e^{-\int_t^T f_0(t,u)\, du}$$

$$R_0(t, T, \tau_1) = \frac{1}{\tau_1}\left(\frac{P(t, T)}{P(t, T+\tau_1)} - 1\right) = \frac{1}{\tau_1}\left(e^{\int_T^{T+\tau_1} f_0(t,u)\, du} - 1\right)$$

- Risk-neutral model with Cheyette ('96) dynamics, *a.k.a.* Markovian HJM dynamics

$$df_0(t, T) = \mu_0(t, T)\, dt + \psi_0(t)\, e^{-\kappa_0(T-t)}\, dW_0(t)$$

- Encounter "usual" HJM no-arbitrage drift condition

$$\int_t^T \mu_0(t, u)\, du = \frac{1}{2}\left(\int_t^T \psi_0(t) e^{-\kappa_0(u-t)}\, du\right)^2 \implies \mu_0(t, T) = \frac{1}{\kappa_0}\psi_0^2(t)\left(e^{-\kappa_0(T-t)} - e^{-2\kappa_0(T-t)}\right)$$

Modeling the Tenor Basis Spread Curves 3

- Differentiate over T to ensure agreement for all T, *i.e.* all coverage periods $[T, T+\tau_3)$

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\begin{aligned}
\mu_3(t, T+\tau_3) - \mu_3(t, T) &= -\left(\int_T^{T+\tau_3} \psi_3(t) e^{-\kappa_3(u-t)}\, du\right)\left(\psi_3(t)\, e^{-\kappa_3(T+\tau_3-t)} - \psi_3(t)\, e^{-\kappa_3(T-t)}\right) \\
&= \frac{\psi_3(t)^2}{\kappa_3} e^{-2\kappa_3(T-t)}\left(1 - e^{-\kappa_3\tau_3}\right)^2
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- Unusual... involves a *difference* of drifts at T & $T+\tau_3$

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S_3(t, T) = e^{\int_T^{T+\tau_3} s_3(t,u)\, du} \implies T \to T+\epsilon \text{ pulls in } s_3(t, T+\tau_3+\epsilon) \text{ but drops } s_3(t, T)
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- See Martinez ('09), & jointly by Grbac & Runggaldier ('15) & Miglietta ('15)
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- Recall $S_3(t,T) = e^{\int_T^{T+\tau_3} s_3(t,u)\,du}$ where $ds_3(t,T) = \mu_3(t,T)\,dt + \psi_3(t)\,e^{-\kappa_3(T-t)}\,dW_3(t)$
- Enforcing $S_3(t,T)$ Martingality reduces to $\mathbb{E}_t[dS_3(t,T)] = 0$

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- As such, no-arbitrage condition on $\mu_3(t,T)$ can be inferred from integral condition

$$\Longrightarrow \int_T^{T+\tau_3} \mu_3(t,u)\,du = -\frac{1}{2}\left(\int_T^{T+\tau_3} \psi_3(t)e^{-\kappa_3(u-t)}\,du\right)^2$$

Modeling the Tenor Basis Spread Curves 2

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numerix

- Must solve a difference equation for $\mu_3(t, T)$ in the variable T

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- As a function of T we basically just have $\mu_3(T + \tau_3) - \mu_3(T) = C(t; \kappa_3, \tau_3) e^{-2\kappa_3 T}$
- Postulate a functional form $\mu_3(t, T) = A(t; \kappa_3, \tau_3) e^{-2\kappa_3 T}$ and substitute, solve for $A(t; \kappa_3, \tau_3)$

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Modeling the Tenor Basis Spread Curves 5

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Modeling the Tenor Basis Spread Curves 6

- $\mu_3(t,T)$ not uniquely determined by no-arbitrage conditions, pointed out by G&R & Miglietta
- Add function $k(t,T)$ integrating to zero over any $[T, T+\tau_3)$, *e.g.* $a(t)\cos\big((2\pi/\tau_3)T\big)$

$$\mu_3'(t,T) = \mu_3(t,T) + k(t,T) \implies \int_T^{T+\tau_1} \mu_3'(t,u)\,du = \int_T^{T+\tau_1} \mu_3(t,u)\,du$$

- Miglietta points out rightly that the definition of $s_3(t,T)$ in $S_3(t,T)$ is not unique

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Modeling the Tenor Basis Spread Curves 2

- Recall $S_3(t,T) = e^{\int_T^{T+\tau_3} s_3(t,u)\,du}$ where $ds_3(t,T) = \mu_3(t,T)\,dt + \psi_3(t)\,e^{-\kappa_3(T-t)}\,dW_3(t)$
- Enforcing $S_3(t,T)$ Martingality reduces to $\mathbb{E}_t[dS_3(t,T)] = 0$

$$\mathbb{E}_t[dS_3(t,T)] = S_3(t,T)\left(\left(\int_T^{T+\tau_3} \mu_3(t,u)\,du\right) + \frac{1}{2}\left(\int_T^{T+\tau_3} \psi_3(t)e^{-\kappa_3(u-t)}\,du\right)^2\right)dt = 0$$

- As such, no-arbitrage condition on $\mu_3(t,T)$ can be inferred from integral condition

$$\Longrightarrow \int_T^{T+\tau_3} \mu_3(t,u)\,du = -\frac{1}{2}\left(\int_T^{T+\tau_3} \psi_3(t)e^{-\kappa_3(u-t)}\,du\right)^2$$

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Modeling the Tenor Basis Spread Curves 6

- $\mu_3(t,T)$ not uniquely determined by no-arbitrage conditions, pointed out by G&R & Miglietta
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Modeling the Tenor Basis Spread Curves 6

- $s_3(t,T)$ in general has nonzero lower bound & CEV dependence, square-root for affine machinery

$$ds_3(t,T) = \mu_3(t,T)\,dt + \nu_3(t)\sqrt{s_3(t,t)}\,e^{-\kappa_3(T-t)}\,dW_3(t)$$

- Cheyette-style representation similar to that for $f_0(t,T)$

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Modeling the Tenor Basis Spread Curves 6

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Approximate Swaprate Dynamics

- Can use frozen coefficients[33] approach to determine approximate swaprate dynamics
- Let $\omega_3(t)$ denote IRS rate and $\phi_{0,3}(t)$ denote basis swap rate

$$\begin{aligned} d\omega_3(t) &= \gamma_0\, e^{-\kappa_0(T-t)}\nu_0(t)\, d\overline{W}_0(t) + \gamma_3\, e^{-\kappa_3(T-t)}\nu_3(t)\sqrt{\lambda_3(t)}\, d\overline{W}_3(t) \\ d\phi_{0,3}(t) &= \gamma_0'\, e^{-\kappa_0(T-t)}\nu_0(t)\, d\overline{W}_0(t) + \gamma_3'\, e^{-\kappa_3(T-t)}\nu_3(t)\sqrt{\lambda_3(t)}\, d\overline{W}_3(t) \end{aligned}$$

- For affine setup compute swaption prices[34] & (co-)moments via Fourier methods

$$\mathbb{E}_t\left[\omega_3(T)^m \phi_{0,3}(T)^n\right] = \partial^m_{\theta_\omega}\partial^n_{\theta_\phi}\mathbb{E}_t\left[e^{\theta_\omega \omega_3(T)+\theta_\phi \phi_{0,3}(T)}\right]\Big|_{\theta_\omega=\theta_\phi=0}$$

[33] Schrager and Pelsser ('06) techniques for affine short-rate models can be adapted
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Joint Calibration to Historical Data & Implied Data

Calibration Approach 1

- Formally swaption volatilities $\upsilon(k, T^{ex})$ reflect $\nu_0(t)$ and $\nu_3(t)$
- Skew does respond to $\nu_3(t)$, but $\nu_3(t)$ is present for other purposes
- Could require extreme $\nu_3(t)$ to mimic skew if $f_0(t, T)$ skew is flat, & skew varies over time
- The base model $f_0(t, T)$ should be augmented to hit skew before turning to $s_3(t, T)$ model
- Stochastic $s_3(t, T)$ to produce sensible basis dynamics, as compared with history[37]

[37] Hull & White ('16), Henrard ('16) also note the use of historical data

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- Could require extreme $\nu_3(t)$ to mimic skew if $f_0(t, T)$ skew is flat, & skew varies over time
- The base model $f_0(t, T)$ should be augmented to hit skew before turning to $s_3(t, T)$ model
- Stochastic $s_3(t, T)$ to produce sensible basis dynamics, as compared with history[38]

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Calibration Approach 2

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- Multi-curve Cheyette model which enforces desired lower bounds through level dependence
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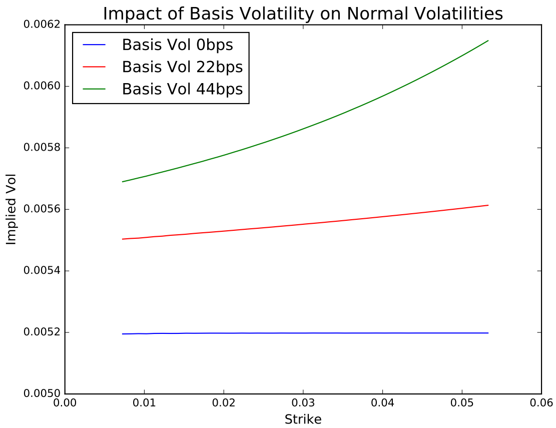
Impact of Basis Volatility on Implied Volatilities



Impact of Benchmark Rate Reforms

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- LIBOR for USD, GBP, EUR, *etc.*, being phased out by 2021

  EUR EURIBOR, JPY TIBOR being reformed, CAD CORA, AUD BBSW, *etc.* to remain[40]
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  Bank funding strategies under RFR-linked assets, basis risks, corporate preferences

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[40] https://www.fsb.org/2018/11/reforming-major-interest-rate-benchmarks-progress-report/
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Appendix A – Mapping of major interest rate benchmarks to alternative reference rates

Currency	Interest rate benchmark	Alternative reference rate (candidates)	Type of alternative reference rate	Remarks	Key milestones
AUD	BBSW	RBA Cash Rate	Unsecured	Multiple-rate approach has been adopted	
BRL	DI rate	Selic	Secured	Multiple-rate approach has been adopted	
CAD	CDOR	Enhanced CORRA	Secured	Multiple-rate approach has been adopted Term RFR to be developed in 2019	White paper on enhanced CORRA is to be published in Q1 2019.
CHF	LIBOR	SARON	Secured	Transition is necessary. Compounded SARON is recommended. A forward-looking term rate seems not feasible.[101]	The FCA has said it will not use its powers to maintain LIBOR beyond end-2021.
EUR	LIBOR	ESTER or Euribor	Unsecured	EUR LIBOR is not in scope of the working group on euro RFR owing to its limited market usage as compared to Euribor. Possible alternatives could be ESTER or the reformed Euribor.	The FCA has said it will not use its powers to maintain LIBOR beyond end-2021.
EUR	Euribor	ESTER	Unsecured	Term RFR under consideration, meanwhile Euribor is being reformed to meet BMR requirements.	A phase-in to a new hybrid methodology for Euribor is expected in the course of 2019.
EUR	EONIA	ESTER	Unsecured	A possible recommendation on a specific path for transition is planned around end-2018.	Usage of EONIA is to be prohibited for new contracts at the end of the BMR's transitional period, i.e. from 1 January 2020.

[101] See minutes of the 20th NWG meeting, available at https://www.snb.ch/en/ifor/finmkt/fnmkt_benchm/id/finmkt_NWG_documents.

Figure: 5Y ON-3M Basis Swap PFEs: Stochastic *vs.* Deterministic Basis

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Calibration of Overnight (Discount) Curve Volatilities

- Swaptions over IBOR-referencing swaps: assume 3M floating leg $R_3(T_i, T_i, \tau_3)$
- Decompose 3M IBOR into equivalent 3M IBOR off ON curve plus hypothetical 3M-ON spread

$$R_3(t, T, \tau_3) = R_0(t, T, \tau_3) + S^+_{0,3}(t, T)$$

- 3M IBOR vols thus composition of ON & 3M-ON volatilities (& correlation)

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Historical IRS Component Behavior

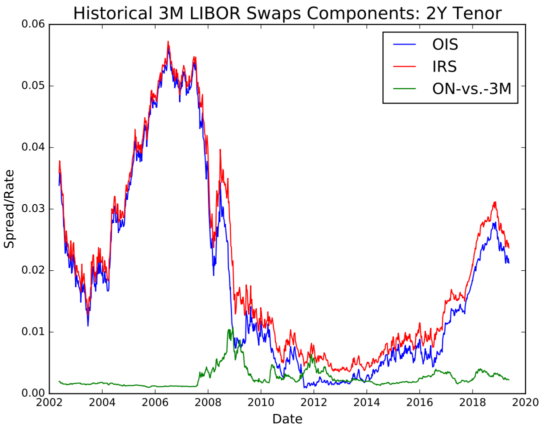

Figure: 2002-2019 Historical Swap and Basis Swap Volatilities: 2Y Tenor

Historical Volatility Components

Historical OIS Volatility	59.7 bps
Historical ON-*vs*.-3M Volatility	22.2 bps
Historical OIS, ON-*vs*.-3M Correlation	-0.015
Historical IRS Volatility	63.0 bps
"Theoretical" IRS Volatility	63.4 bps

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