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Alternative compounding methods for over-the-counter derivative transactions

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Summary

- There are two methods for compounding over-the-counter interest rate derivative cash flows in the 2006 ISDA Definitions, namely, Compounding and Flat Compounding.
- In addition, there exists a third compounding method that is not at present included in the Definitions.
- The three methods lead to different results when the Floating Amount includes a Spread component; if Spread is zero, there is no difference between the three methods.
- The method not included in the ISDA Definitions treats Floating Rate as compound interest but treats Spread as simple interest, and can be expressed as a simple formula as described below.

1. Introduction

Market participants have contacted ISDA regarding disputes over the calculation of interest rate swap cash flows that involve both a floating rate and a spread and are subject to Flat Compounding under the 2006 ISDA Definitions. Apparently there are two different understandings of flat compounding in the market, but only one is reflected in the ISDA Definitions. The following note seeks to explain and clarify the three types of compounding and how market participants use them in practice.

2. Overview of compounding methods

In over-the-counter derivatives transactions, the floating interest rate cash flow (hereafter referred to as the Floating Amount) is a function of two components. One the Floating Rate, Libor or Euribor, for example, which is reset periodically during the life of a transaction. The other is the Spread, which is a margin over the Floating Rate and normally does not change over the life of the transaction. The Spread can be positive, negative, or zero. Both Floating Rate and Spread are adjusted by the relevant Day Count Fractions.

The Floating Amount can be calculated in two ways. One is simple interest, which calculates interest in each period on the notional principal only. The other is compound interest, which calculates interest on the sum of notional principal and accumulated interest payments. In the

2006 ISDA Definitions (Section 6.1a), cash flows are calculated using simple interest unless the parties specify that Compounding or Flat Compounding is Applicable.

There are three known compound interest conventions, two of which are in the ISDA Definitions. Appendix A summarizes the main characteristics of the three conventions. All have in common that they in some way add interest back into the principal, allowing the instrument to earn interest on both initial principal and accumulated interest. But all differ according to how they treat the Spread component of interest. If Spread is zero, all three give the same result.

The most basic form of compound interest, known in the ISDA Definitions as Compounding, makes no distinction between Floating Rate and Spread. That is, Floating Rate plus Spread earned on the notional amount (hereafter Calculation Amount) at the end of each compounding period is added to the principal for the next compounding period, and so on.

A second form, known in the ISDA Definitions as Flat Compounding, treats Floating Rate and Spread differently in different periods. Current period interest is calculated using Floating Rate plus Spread. But in subsequent periods, accumulated interest is compounded using Floating Rate only.

A third form, not found in the ISDA Definitions, involves compounding the Floating Rate but treating the spread as simple interest. In other words, Floating Rate interest but not spread earned at the end of a period is added back into principal; Spread is then calculated on the principal for the entire calculation period without compounding. Some market participants apparently refer to this convention as "compounding flat," although usage varies (Appendix A). It has been suggested that this third method has become a new market standard, but as of this writing it has not been possible to substantiate these claims.

A common use of the various compounding conventions is in vanilla interest rate swaps that specify, say, a three-month floating rate versus a semiannual fixed rate; the three-month rate can be compounded over a six-month Calculation Period so the Floating Amount can be netted against the Fixed Amount. Another is in basis swaps between, for example, a one-month floating rate compounded over three months and a three-month floating rate; such swaps virtually always involve a Spread. A different use of compounding is in overnight indexed swaps, in which "self-compounding" Floating Rate Options—USD-Federal Funds-H.15-OIS-COMPOUND in the 2006 ISDA Definitions, for example —are calculated by means of a compounding formula but are then inserted into a simple interest formula to determine Floating Amount.

The following section will discuss each of the above in more detail, show how they can be calculated and, to the extent feasible, express them as formulas.

3. Alternative versions of Compounding

3.1 Compounding. The most basic form of compounding, which treats interest and spread the same, is defined by ISDA as "Compounding" in the following excerpted passages from the 2006 ISDA Definitions:

> **Section 6.1. Calculation of a Floating Amount.** Subject to the provisions of Section 6.4 (Negative Interest Rates), the Floating Amount payable by a party on a Payment Date will be:
>
> (…)
>
> (b) if "Compounding" is specified to be applicable to the Swap Transaction or that party and "Flat Compounding" is not specified, an amount equal to the sum of the Compounding Period Amounts for each of the Compounding Periods in the related Calculation Period…
>
> (…)
>
> **Section 6.3. Certain Definitions Relating to Compounding.** For purposes of the calculation of a Floating Amount where "Compounding" is specified to be applicable to a Swap Transaction:
>
> (…)
>
> (c) "Compounding Period Amount" means, for any Compounding Period, an amount calculated on a formula basis for that Compounding Period as follows:

$$\text{Compounding Period Amount} = \text{Adjusted Calculation Amount} \times \text{Floating Rate + Spread} \times \text{Floating Rate Day Count Fraction}$$

> (d) "Adjusted Calculation Amount" means (i) in respect of the first Compounding Period in any Calculation Period, the Calculation Amount for that Calculation Period and (ii) in respect of each succeeding Compounding Period in that Calculation Period, an amount equal to the sum of the Calculation Amount for that Calculation Period and the Compounding Period Amounts for each of the previous Compounding Periods in that Calculation Period.

The above definition can be expressed as a general formula. In order to do so, assume the following notation:

N = Notional amount (called Calculation Amount in the ISDA Definitions)

FA = Floating Amount for the relevant Calculation Period

CPA_t = Compounding Period Amount for Compounding Period t

ACA_t = Adjusted Calculation Amount for Compounding Period t (= N in Period 1)

R_t = Floating Rate for Compounding Period t

S = Spread

d_t = Day count fraction for Compounding Period t

Assume the Calculation Period contains three Compounding Periods. According to the above definition, the Compounding Period Amount is equal to:

$$CPA_1 = ACA_1 \text{ x } (R_1 + S) \text{ x } d_1 = N \times (R_1 + S) \times d_1$$

For the second period, Compounding Period Amount is:

$$CPA_2 = ACA_2 \text{ x } (R_2 + S) \text{ x } d_2 = (N + CPA_1) \times (R_2 + S) \times d_2$$

And for the third period,

$$CPA_3 = ACA_3 \text{ x } (R_3 + S) \text{ x } d_3 = (N + CPA_1 + CPA_2) \text{ x } (R_3 + S) \text{ x } d_3$$

So Floating Amount for the Calculation Period is

$$FA = CPA_1 + CPA_2 + CPA_3$$

The above definition can be expressed as the following well-known formula, derived in Appendix B:

$$FA = N \text{ x } [(1 + (R_1 + S) \times d_1) \text{ x } (1 + (R_2 + S) \times d_2) \text{ x } (1 + (R_3 + S) \times d_3) - 1]$$

A more generally applicable version for compounding over T periods is:

$$FA = N \times \left[\left(1 + (R_1 + S) \times d_1\right) \times \left(1 + (R_2 + S) \times d_2\right) \times \cdots \times \left(1 + (R_T + S) \times d_T\right) - 1\right]$$

$$= N \times \left[\prod_{t=1}^{T} (1 + (R_t + S) \times d_t) - 1\right]$$

Where the product operator $\prod_{i=1}^{n} a_i$ means the product $a_1 \times a_2 \times \ldots \times a_n$

3.2 Flat Compounding in the ISDA Definitions. The ISDA Definition of Flat Compounding is contained in the following passages from the 2006 ISDA Definitions.

Section 6.1. Calculation of a Floating Amount. Subject to the provisions of Section 6.4 (Negative Interest Rates), the Floating Amount payable by a party on a Payment Date will be:

(a) if Compounding is not specified for the Swap Transaction or that party, an amount calculated on a formula basis for that Payment Date or for the related Calculation Period as follows:

$$\text{Floating Amount} = \text{Calculation Amount} \times \text{Floating Rate + Spread} \times \text{Floating Rate Day Count Fraction}$$

(…)

(c) if "Flat Compounding" is specified to be applicable to the Swap Transaction or that party, an amount equal to the sum of the Basic Compounding Period Amounts for each of the Compounding Periods in the related Calculation Period plus the sum of the Additional Compounding Period Amounts for each such Compounding Period.

(…)

Section 6.3. Certain Definitions Relating to Compounding. For purposes of the calculation of a Floating Amount where "Compounding" is specified to be applicable to a Swap Transaction:

(…)

(e) "Basic Compounding Period Amount" means, for any Compounding Period, an amount calculated as if a Floating Amount were being calculated for that Compounding Period, using the formula set forth in Section 6.1(a).

(f) "Additional Compounding Period Amount" means, for any Compounding Period, an amount calculated on a formula basis for that Compounding Period as follows:

$$\text{Additional Compounding Period Amount} = \text{Flat Compounding Amount} \times \text{Floating Rate} \times \text{Floating Rate Day Count Fraction}$$

(g) "Flat Compounding Amount" means (i) in respect of the first Compounding Period in any Calculation Period, zero and (ii) in respect of each succeeding Compounding Period in that Calculation Period, an amount equal to the sum of the Basic Compounding Period Amounts and the Additional Compounding Period Amounts for each of the previous Compounding Periods in that Calculation Period.

It is not apparent that the ISDA Definition of Flat Compounding can be expressed as a general formula. It can, however, be expressed in a more complex manner as a step-wise recursive process. Along with previous notation, define the following additional terms:

$BCPA_t$ = Basic Compounding Period Amount for compounding period t

$ACPA_t$ = Additional Compounding Period Amount for compounding period t

Assuming again a Calculation Period consisting of three Compounding Periods, the above provisions of the ISDA Definitions lead to the following formulation:

$$CPA_1 = BCPA_1 = N \times (R_1 + S) \times d_1$$

$$CPA_2 = BCPA_2 + ACPA_2 = N \times (R_2 + S) \times d_2 + (CPA_1 \times R_2 \times d_2)$$

$$CPA_3 = BCPA_3 + ACPA_3 = N \times (R_3 + S) \times d_3 + [(CPA_1 + CPA_2) \times R_3 \times d_3]$$

$$FA = CPA_1 + CPA_2 + CPA_3$$

Although ISDA Flat Compounding does not appear to reduce to a single formula, we can generalize the above procedure to the following recursive process for a Calculation Period consisting of T Compounding Periods:

For the first Compounding Period (t=1):

$$CPA_1 = N \times (R_1 + S) \times d_1$$

And for subsequent Compounding Periods (t>1):

$$CPA_t = N \times (R_t + S) \times d_t + \left(\sum_{i=1}^{t-1} CPA_i\right) \times R_t \times d_t$$

$$FA = \sum_{t=1}^{T} CPA_t$$

When Spread is equal to zero, these formulas collapse to the general formula for Compounding in Section 3.1.

3.3 Compounding treating Spread as simple interest. As mentioned above, there is a non-ISDA compounding convention in the market that treats Floating Rate interest as compound interest but Spread as simple interest. Using the same notation as the previous example but letting SA equal Spread Amount and d (= $d_1 + d_2 + d_3$) equal total days in the Calculation Period divided by the year basis, the convention would compute compound interest for a Calculation Period consisting of three Compounding Periods as follows:

$$CPA_1 = ACA_1 \times R_1 \times d_1 = N \times R_1 \times d_1$$

$$CPA_2 = ACA_2 \times R_2 \times d_2 = (N + CPA_1) \times R_2 \times d_2$$

$$CPA_3 = ACA_3 \times R_3 \times d_3 = (N + CPA_1 + CPA_2) \times R_3 \times d_3$$

$$SA = (N \times S \times d_1) + (N \times S \times d_2) + (N \times S \times d_3) = N \times S \times d$$

So Floating Amount for the Calculation Period is

$$FA = CPA_1 + CPA_2 + CPA_3 + SA$$

More generally, for T Compounding Periods in a Calculation Period:

$$FA = N \times [(1 + R_1 \times d_1) \times (1 + R_2 \times d_2) \times \cdots \times (1 + R_T \times d_T) - 1 + S \times d\,]$$

$$= N \times \left[\prod_{t=1}^{T}(1 + R_t \times d_t) - 1 + S \times d\right]$$

where:

$$d = \sum_{t=1}^{T} d_t$$

As with Flat Compounding, this formula collapses to the formula for Compounding when Spread is zero.

4. Examples

Assume an interest rate swap with a notional amount of $10 million. Assume further that one party agrees to pay one-month Libor compounded over a three-month Calculation Period, and that Spread is 0.10%. Finally, assume that the relevant one-month Libor fixings are:

Compounding Period 1	4.0%
Compounding Period 2	4.5%
Compounding Period 3	5.0%

For simplicity, let each month consist of 30 days so the Calculation Period is 90 days.

4.1 *Compounding*. Using the general formula in Section 3.1, Floating Amount is:

$$FA = N \times \left[\prod_{t=1}^{T} (1 + (R_t + S) \times d_t) - 1 \right]$$

$$= \$10{,}000{,}000 \times \left[(1 + 4.1\% \times \tfrac{30}{360}) \times \left(1 + 4.6\% \times \tfrac{30}{360}\right) \times \left(1 + 5.1\% \times \tfrac{30}{360}\right) - 1 \right]$$

$$= \$10{,}000{,}000 \times [(1.003417) \times (1.003833) \times (1.004250) - 1]$$

$$= \$115{,}439.65$$

Alternatively, one can calculate each of the Compounding Period Amounts separately as described in Section 3.1 and then sum them to obtain the Floating Amount:

$$CPA_1 = N \times (R_1 + S) \times d_1 = \$10{,}000{,}000 \times 0.041 \times \frac{30}{360} = \$34{,}166.67$$

$$CPA_2 = (N + CPA_1) \times (R_2 + S) \times d_2 = (\$10{,}000{,}000 + \$34{,}166.67) \times 0.046 \times \frac{30}{360} = \$38{,}464.31$$

$$CPA_3 = (N + CPA_1 + CPA_2) \text{ x } (R_3 + S) \text{ x } d_3$$

$$= (\$10{,}000{,}000 + \$34{,}166.67 + \$38{,}464.31) \times 0.051 \times \frac{30}{360} = \$42{,}802.62$$

So Floating Amount for the Calculation Period is

$$FA = CPA_1 + CPA_2 + CPA_3 = \$34{,}166.67 + \$38{,}464.31 + \$42{,}808.68 = \$115{,}439.65$$

which is the same quantity as with the single formula.

4.2 *Flat Compounding (ISDA)*. As mentioned above in Section 3.2, ISDA Flat Compounding does not reduce to a single formula so must be calculated period-by-period as follows:

$$CPA_1 = N \times (R_1 + S) \times d_1 = \$10{,}000{,}000 \times 0.041 \times \frac{30}{360} = \$34{,}166.67$$

$$CPA_2 = N \times (R_2 + S) \times d_2 + (CPA_1 \times R_2 \times d_2) = \$10{,}000{,}000 \times 0.046 \times \frac{30}{360} + (\$34{,}166.67 \times 0.045 \times \frac{30}{360})$$

$$= \$38{,}461.46$$

$$CPA_3 = N \times (R_3 + S) \times d_3 + [(CPA_1 + CPA_2) \times R_3 \times d_3]$$

$$= \$10{,}000{,}000 \times 0.051 \times \frac{30}{360} + (\$34{,}166.67 + \$38{,}461.46) \times 0.050 \times \frac{30}{360} = \$42{,}802.62$$

$$FA = CPA_1 + CPA_2 + CPA_3 = \$34{,}166.67 + \$38{,}461.46 + \$42{,}802.62 = \$115{,}430.74$$

4.3 *Compounding treating Spread as simple interest*. The non-ISDA compounding convention and formula described in Section 3.3 leads to the following result:

$$FA = N \times \left[\prod_{t=1}^{T}(1 + R_t \times d_t) - 1 + (S \times d)\right]$$

$$= \$10{,}000{,}000 \times \left[\left(1 + 4.0\% \times \frac{30}{360}\right) \times \left(1 + 4.5\% \times \frac{30}{360}\right) \times \left(1 + 5.0\% \times \frac{30}{360}\right) - 1 + 0.01\% \times \frac{90}{360}\right]$$

$$= \$10{,}000{,}000 \times [(1.003333) \times (1.003750) \times (1.004167) - 1 + 0.00025]$$

$$= \$115{,}420.66$$

Again, one can calculate each of the Compounding Period Amounts separately as described in Section 3.3 and then sum them to obtain the Floating Amount:

$$CPA_1 = N \times R_1 \times d_1 = \$10{,}000{,}000 \times 0.040 \times \frac{30}{360} = \$33{,}333.33$$

$$CPA_2 = (N + CPA_1) \text{ x } R_2 \text{ x } d_2 = (\$10{,}000{,}000 + \$33{,}333.33) \times 0.045 \times \frac{30}{360} = \$37{,}625.00$$

$$CPA_3 = (N + CPA_1 + CPA_2) \times R_3 \times d_3 = (\$10{,}000{,}000 + \$33{,}333.33 + \$37{,}625.00) \times 0.050 \times \frac{30}{360}$$
$$= \$41{,}962.33$$

$$SA = N \times S \times d = \$10{,}000{,}000 \times 0.01\% \times \frac{90}{360} = \$2{,}500.00$$

$$FA = CPA_1 + CPA_2 + CPA_3 + SA = \$33{,}333.33 + \$37{,}625.00 + \$41{,}962.33 + \$2{,}500.00$$

$$= \$115{,}420.66$$

which is the same result as with the single formula.

4.4 *Zero Spread*. All three of the above methods yield the same result when Spread is zero. Assuming the same Floating Rates as in the preceding examples but that spread is zero, Compounding with no spread will lead to the following:

$$FA = N \times \left[\prod_{t=1}^{T}(1 + R_t \times d_t) - 1\right]$$

$$= \$10{,}000{,}000 \times \left[\left(1 + 4.0\% \times \frac{30}{360}\right) \times \left(1 + 4.5\% \times \frac{30}{360}\right) \times \left(1 + 5.0\% \times \frac{30}{360}\right) - 1\right]$$

$$= \$10{,}000{,}000 \times [(1.003333) \times (1.003750) \times (1.004167) - 1]$$

$$= \$112{,}920.66$$

Second, inserting a zero spread into the non-ISDA compounding formula described in Section 3.3 (compounding treating Spread as simple interest) leads immediately to the simple Compounding formula from Section 3.1, so the result is the same as with Compounding.

Finally, setting Spread to zero in ISDA Flat Compounding leads to the following changes to Section 4.3:

$$CPA_1 = \$10{,}000{,}000 \times 0.040 \times \frac{30}{360} = \$33{,}333.33$$

$$CPA_2 = \$10{,}000{,}000 \times 0.045 \times \frac{30}{360} + \$33{,}333.33 \times 0.045 \times \frac{30}{360} = \$37{,}625.00$$

$$CPA_3 = \$10{,}000{,}000 \times 0.050 \times \frac{30}{360} + (\$33{,}333.33 + \$37{,}625.00) \times 0.050 \times \frac{30}{360} = \$41{,}962.33$$

$$FA = CPA_1 + CPA_2 + CPA_3 = \$33{,}333.33 + \$37{,}625.00 + \$41{,}962.33 = \$112{,}920.66$$

which is the same result as with the other two methods.

Appendix A: Comparison of compounding conventions

Type	Defined by ISDA?	Treatment of interest:		Other names (source)
		Floating Rate	*Spread*	
Compounding	Yes	Compound	Compound	Compounding with spread included (Summit); Basic (RBS)
Flat Compounding	Yes	Compound	Mixed	Flat compounding (ISDA method) (Summit); Index flat (RBS)
Compounding with simple spread	No	Compound	Simple	Compounding flat (not verified); Compounding with spread excluded (Summit); Spread exclusive compounding (CS); Spread on profile (RBS); New formula (TD)

Appendix B: Derivation of general formula for Compounding in 2006 ISDA Definitions

Assume the following notation:

N = Notional amount (called Calculation Amount in the Definitions)

FA = Floating Amount for the relevant Calculation Period

CPA_t = Compounding Period Amount for Compounding Period t

ACA_t = Adjusted Calculation Amount for Compounding Period t (= N in Period 1)

R_t = Floating Rate for period t

S = Spread

d_t = Day count fraction for period t

Let $X_t = (R_t + S)d_t$ and assume that the Calculation Period contains three Compounding Periods. Following Sections 6.1(b) and 6.3(a-d) of the 2006 ISDA Definitions, the three Compounding Period Amounts are:

$$CPA_1 = ACA_1(R_1 + S)d_1 = NX_1$$

For the second period, Compounding Period Amount is:

$$CPA_2 = ACA_2(R_2 + S)d_2 = (N + CPA_1)X_2 = (N + NX_1)X_2 = N(1 + X_1)X_2$$

And for the third period,

$$CPA_3 = ACA_3(R_3 + S)d_3 = (N + CPA_1 + CPA_2)X_3 = [N + NX_1 + N(1 + X_1)X_2]X_3 = N[1 + X_1 + (1 + X_1)X_2]X_3$$

So Floating Amount for this period is

$$\begin{aligned} FA &= CPA_1 + CPA_2 + CPA_3 = NX_1 + N(1 + X_1)X_2 + N[1 + X_1 + (1 + X_1)X_2]X_3 \\ &= N\{X_1 + (1+X_1)X_2 + [(1+X_1) + (1+X_1)X_2]\}X_3 = N\{1 + X_1 + (1+X_1)X_2 + [(1+X_1) + (1+X_1)X_2]X_3 - 1\} \\ &= N\{(1+X_1) + (1+X_1)X_2 + [(1+X_1) + (1+X_1)X_2]X_3 - 1\} = N[(1+X_1)(1+X_2) + (1+X_1)(1+X_2)X_3 - 1] \\ &= N[(1+X_1)(1+X_2)(1+X_3) - 1] \end{aligned}$$

More generally, the above definition can be expressed as the following formula for any Calculation Period consisting of T Compounding Periods:

$$FA = N\left[\prod_{t=1}^{T}(1+X_t) - 1\right] = N\left[\prod_{t=1}^{T}(1 + (R_t+S)d_t) - 1\right]$$