Lecture 4

Sketching with Path Analysis

Joint Treatments

Causal Inference Using Graphs
August 8, 2019

Adam Glynn
Department of Political Science and QTM
Emory University

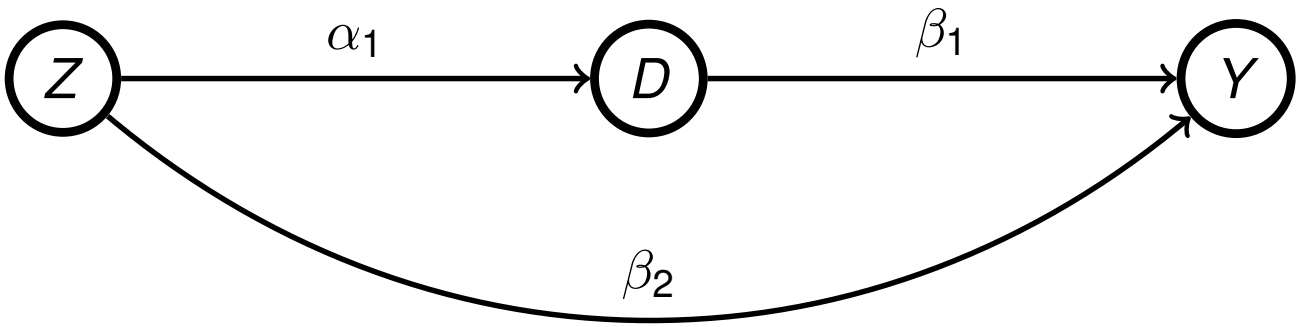
Acknowledgements

Daniel Arnon contributed to many of the slides from lectures 3 and 4 today.

Goals and Objectives for This Afternoon:

- Introduce the use of BDC with path analysis for joint treatments.
- Present TSCS examples.
- Discuss mediation with constant effects and preview mediation with non-constant effects.

Overview

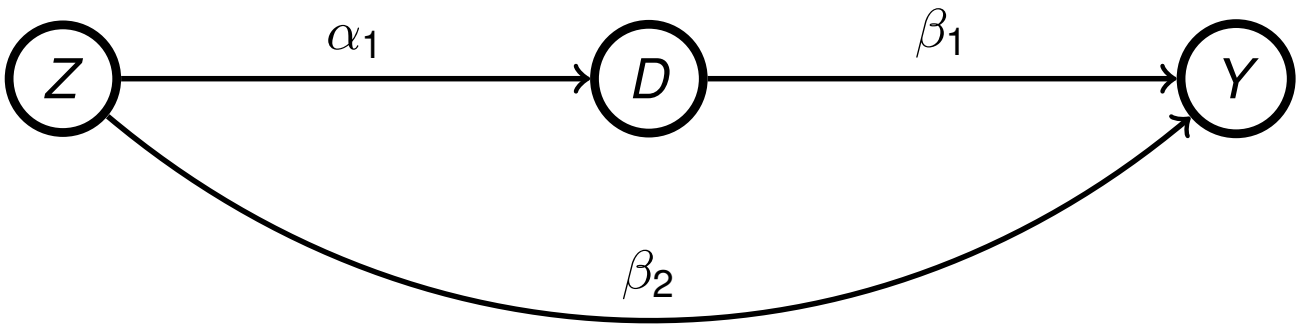


$$(1) D_i(z) = \alpha_0 + \alpha_1 z + \nu_i$$

$$(2) Y_i(d, z) = \beta_0 + \beta_1 d + \beta_2 z + \epsilon_i$$

$$Y_i(d_i(z), z) = Y_i(z) = \beta_0 + \beta_1(\alpha_0 + \alpha_1 Z + \nu_i) + \beta_2 Z + \epsilon_i$$

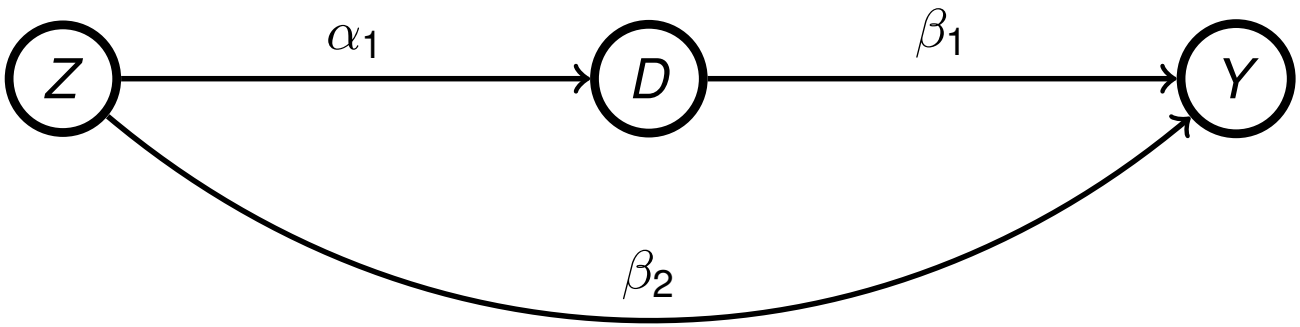
$$Y_i(z) = \underbrace{\beta_0 + \beta_1\alpha_0}_{\textit{Intercept}} + \underbrace{(\beta_1\alpha_1 + \beta_2)}_{\text{Effect}} z + \underbrace{\epsilon_i + \beta_1\nu_i}_{\text{Error}}$$



$$(1) D_i(z) = \alpha_0 + \alpha_1 z + \nu_i$$

$$(2) Y_i(d, z) = \beta_0 + \beta_1 d + \beta_2 z + \epsilon_i$$

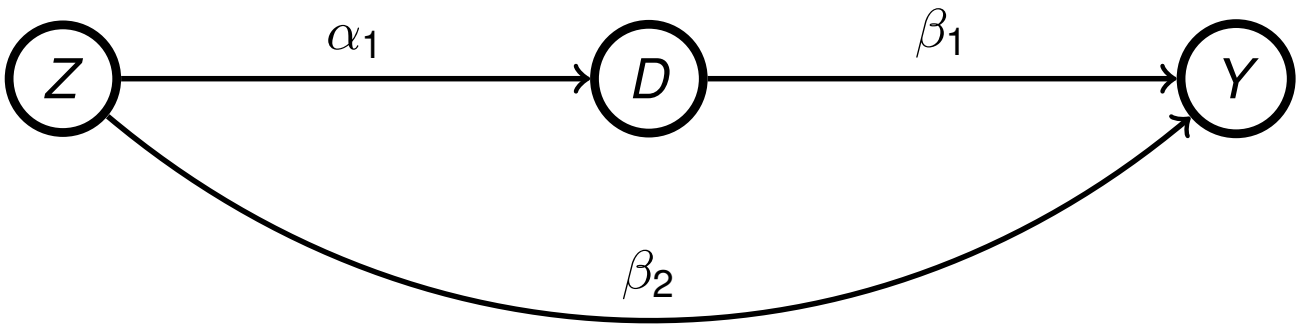
- Write joint effects in terms of the joint potential outcomes in (2) that correspond to β_1.



$$(1) D_i(z) = \alpha_0 + \alpha_1 z + \nu_i$$

$$(2) Y_i(d, z) = \beta_0 + \beta_1 d + \beta_2 z + \epsilon_i$$

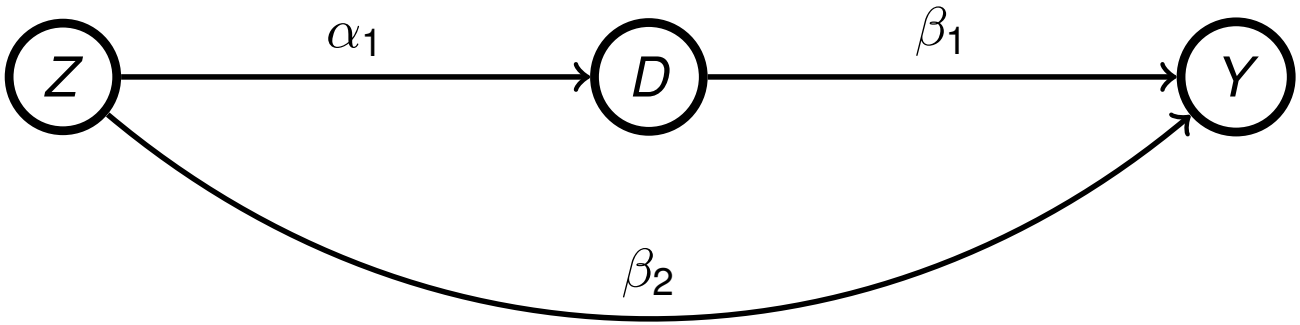
- Write joint effects in terms of the joint potential outcomes in (2) that correspond to β_1.
- Write joint effects in terms of the joint potential outcomes in (2) that correspond to β_2.



$$(1) D_i(z) = \alpha_0 + \alpha_1 z + \nu_i$$

$$(2) Y_i(d, z) = \beta_0 + \beta_1 d + \beta_2 z + \epsilon_i$$

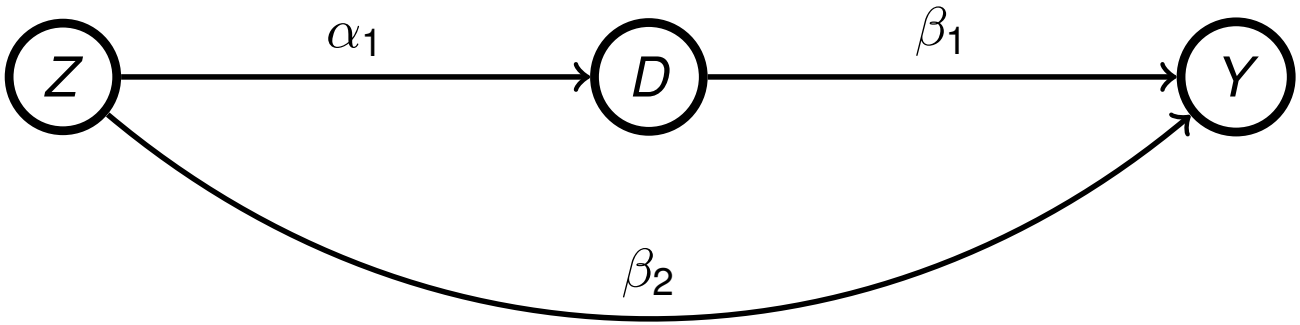
- Write joint effects in terms of the joint potential outcomes in (2) that correspond to β_1.
- Write joint effects in terms of the joint potential outcomes in (2) that correspond to β_2.
- Relate these to the effects of D on Y and Z on Y.



$$(1) D_i(z) = \alpha_0 + \alpha_1 z + \nu_i$$

$$(2) Y_i(d, z) = \beta_0 + \beta_1 d + \beta_2 z + \epsilon_i$$

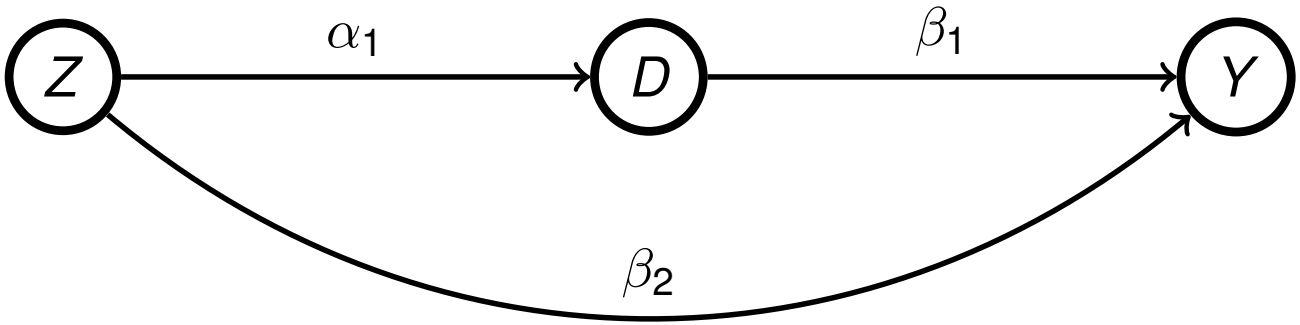
- Add an unmeasured variable to this graph so that the effect of D on Y is identified but the effect of Z on Y is not.



$$(1) D_i(z) = \alpha_0 + \alpha_1 z + \nu_i$$

$$(2) Y_i(d, z) = \beta_0 + \beta_1 d + \beta_2 z + \epsilon_i$$

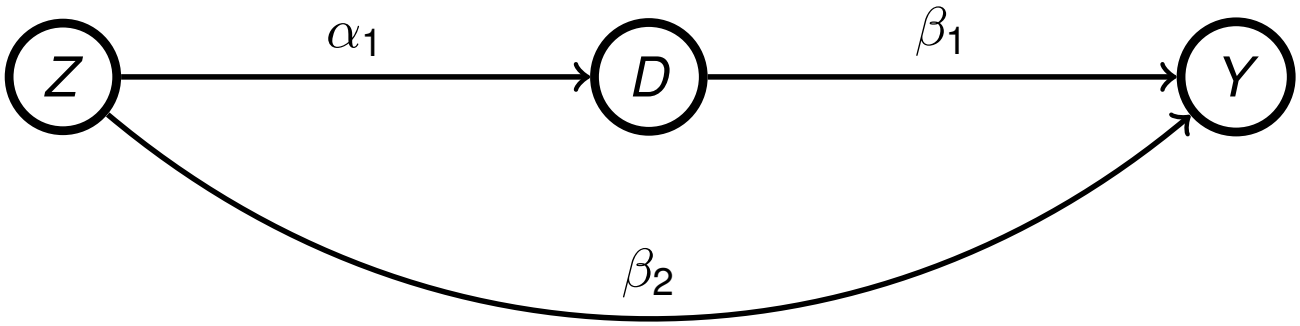
- Add an unmeasured variable to this graph so that the effect of D on Y is identified but the effect of Z on Y is not.
- Add an unmeasured variable to this graph so that the effect of Z on Y is identified but the effect of D on Y is not.



$$(1) D_i(z) = \alpha_0 + \alpha_1 z + \nu_i$$

$$(2) Y_i(d, z) = \beta_0 + \beta_1 d + \beta_2 z + \epsilon_i$$

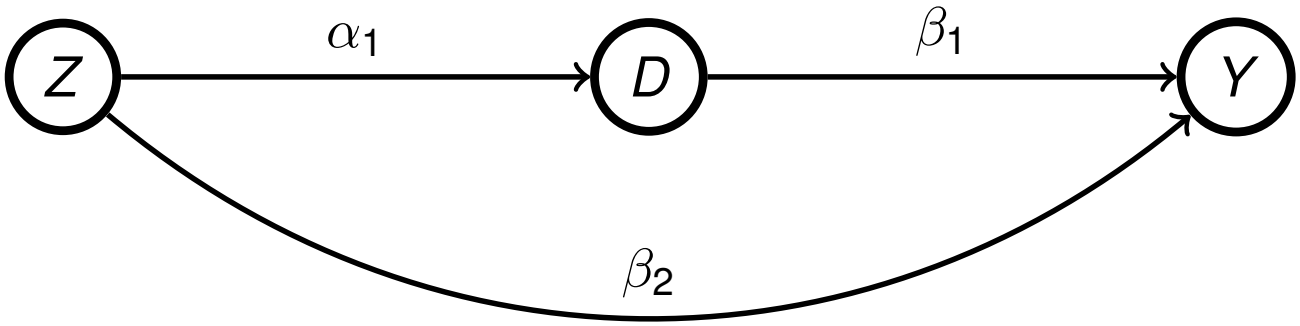
- Add an unmeasured variable to this graph so that the effect of D on Y is identified but the effect of Z on Y is not.
- Add an unmeasured variable to this graph so that the effect of Z on Y is identified but the effect of D on Y is not.
- For either of those two graphs, what does the BDC say about the joint effects of Z and D on Y.



$$(1) D_i(z) = \alpha_0 + \alpha_1 z + \nu_i$$

$$(2) Y_i(d, z) = \beta_0 + \beta_1 d + \beta_2 z + \epsilon_i$$

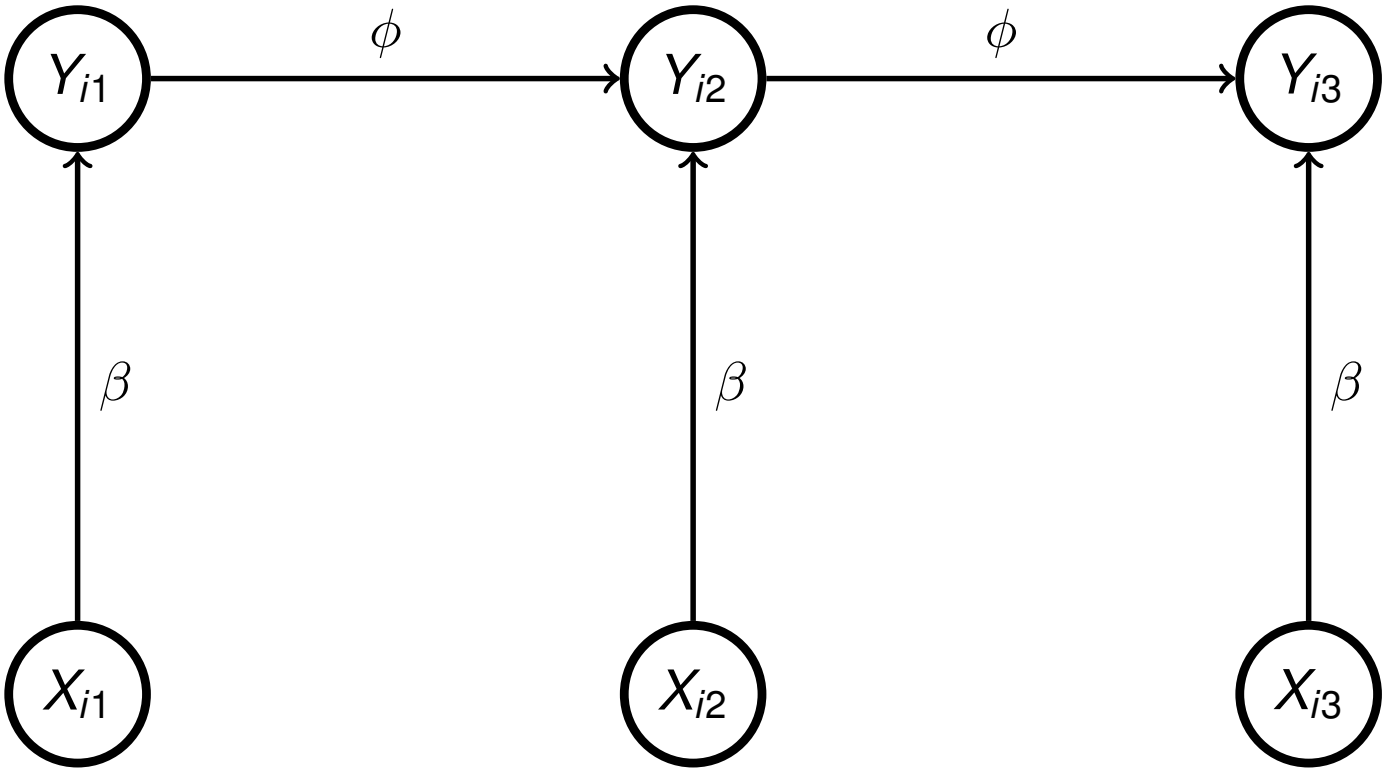
- Add a measured variable to this graph so that the effects of D on Y and Z on Y are identified but their joint effect is not.



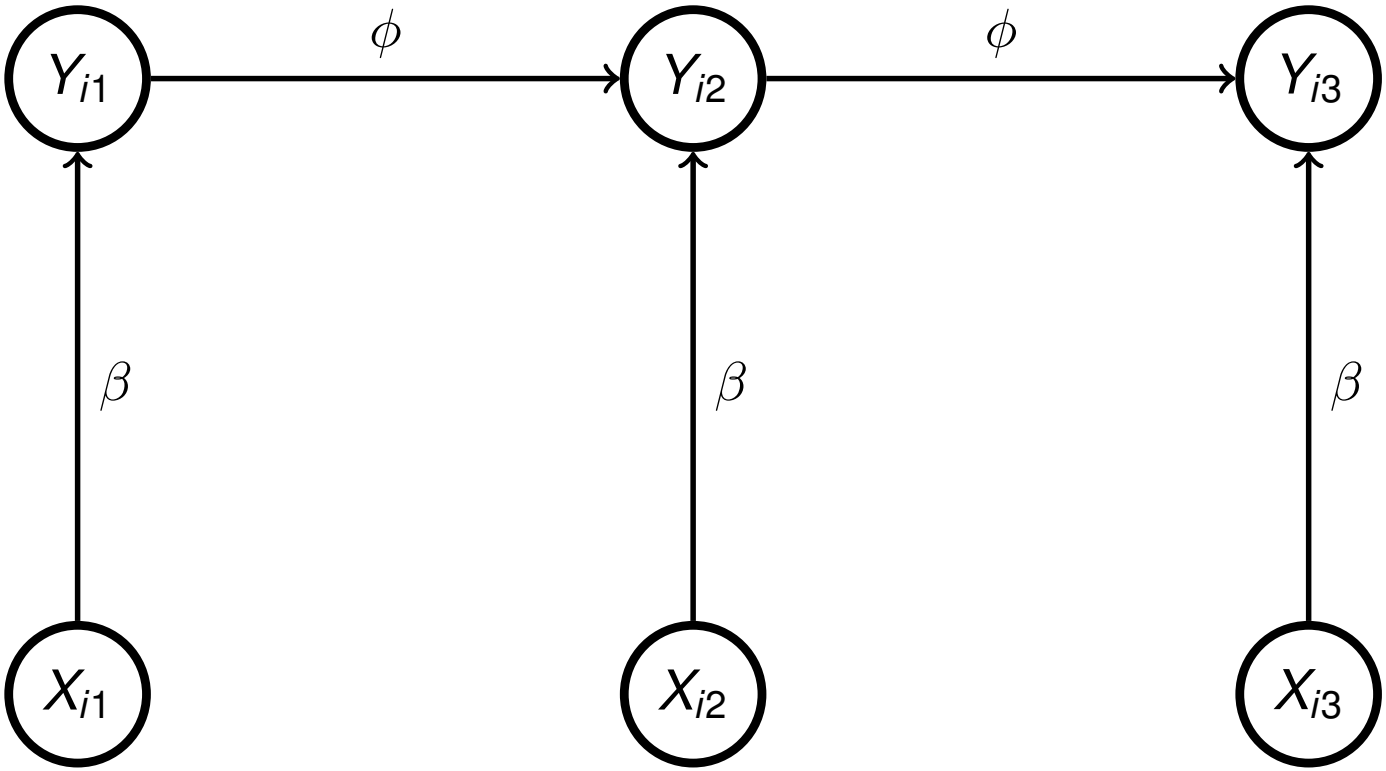
$$(1) D_i(z) = \alpha_0 + \alpha_1 z + \nu_i$$

$$(2) Y_i(d, z) = \beta_0 + \beta_1 d + \beta_2 z + \epsilon_i$$

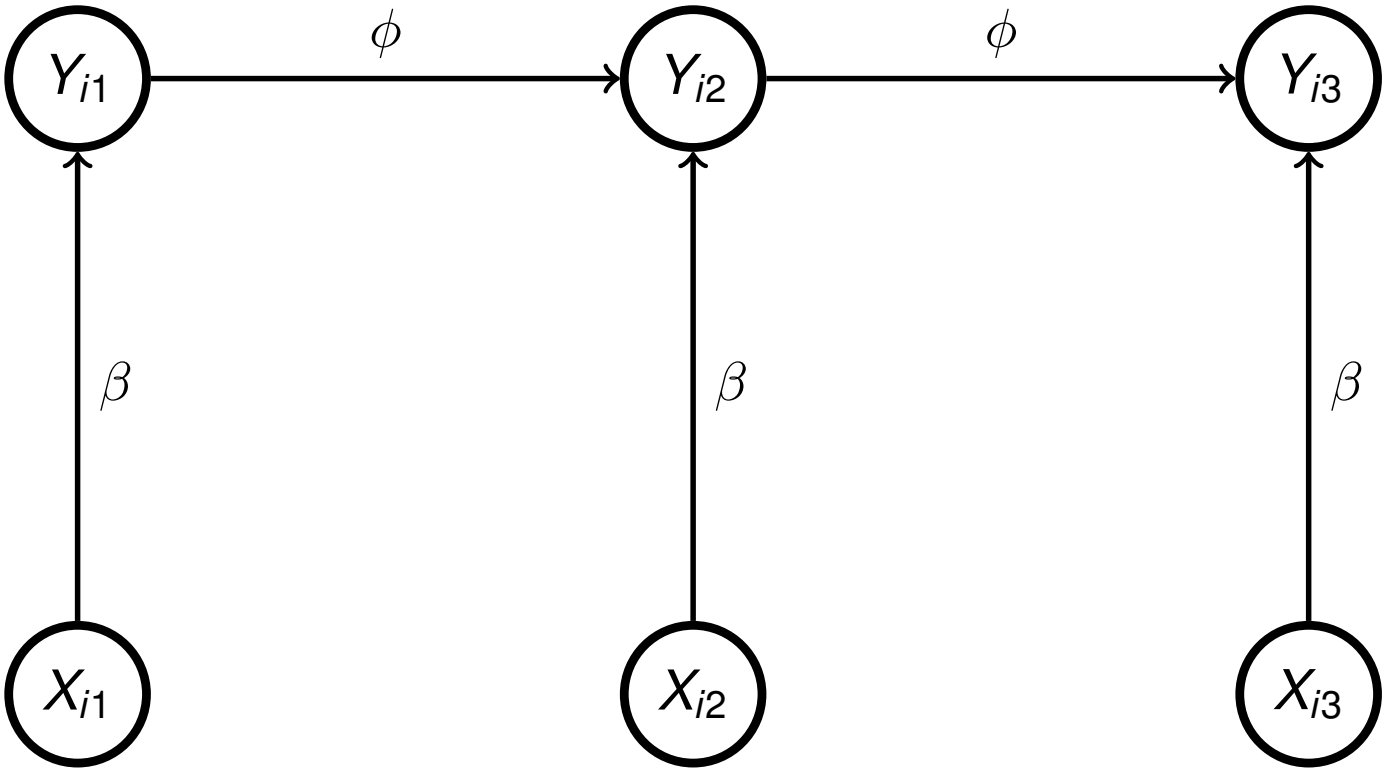
- Add a measured variable to this graph so that the effects of D on Y and Z on Y are identified but their joint effect is not.
- Standard methods will not work in this scenario. See Blackwell and Glynn 2018 for an explanation.



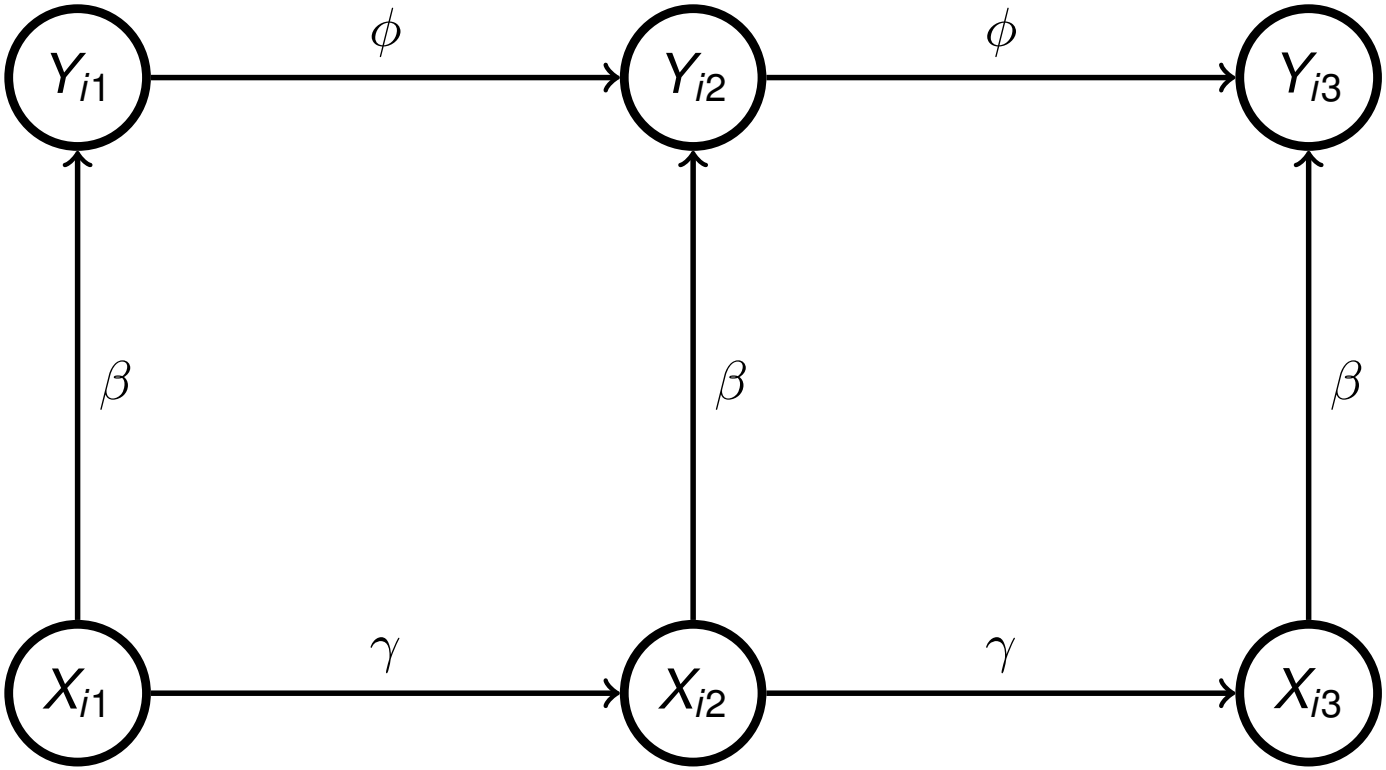
- Define the effect of X_3 on Y_3 in potential outcomes and coefficients.



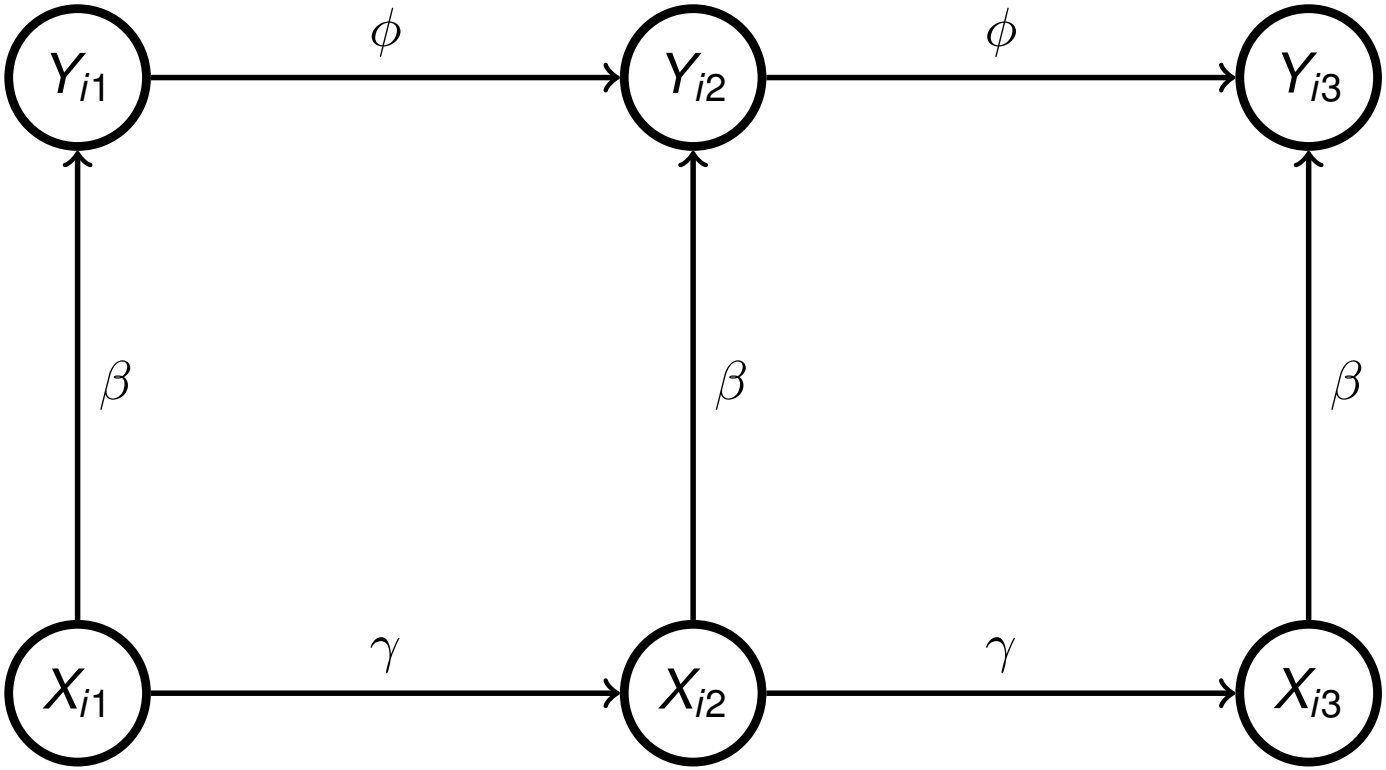
- Define the effect of X_3 on Y_3 in potential outcomes and coefficients.
- Define the effect of X_2 on Y_3 in potential outcomes and coefficients.



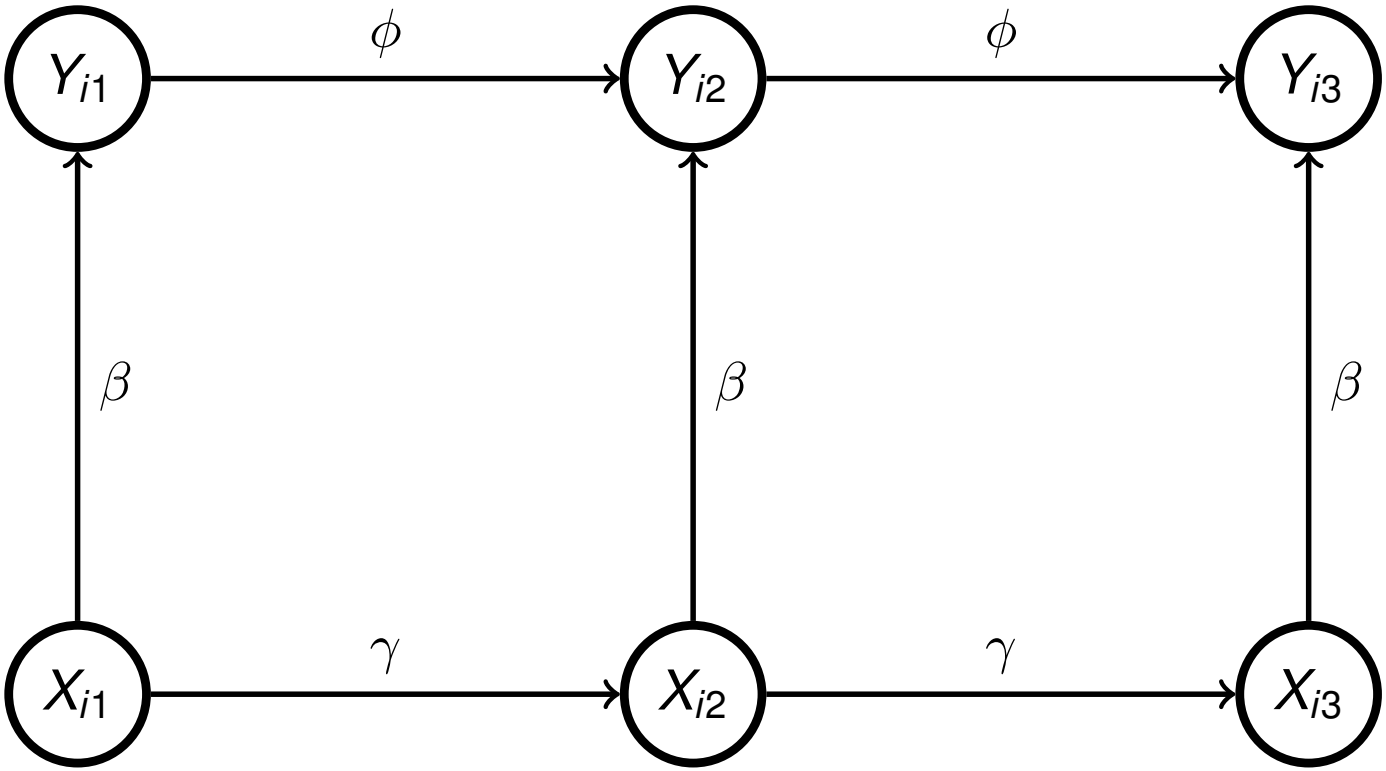
- Define the effect of X_3 on Y_3 in potential outcomes and coefficients.
- Define the effect of X_2 on Y_3 in potential outcomes and coefficients.
- Define the effect of X_1 on Y_3 in potential outcomes and coefficients.



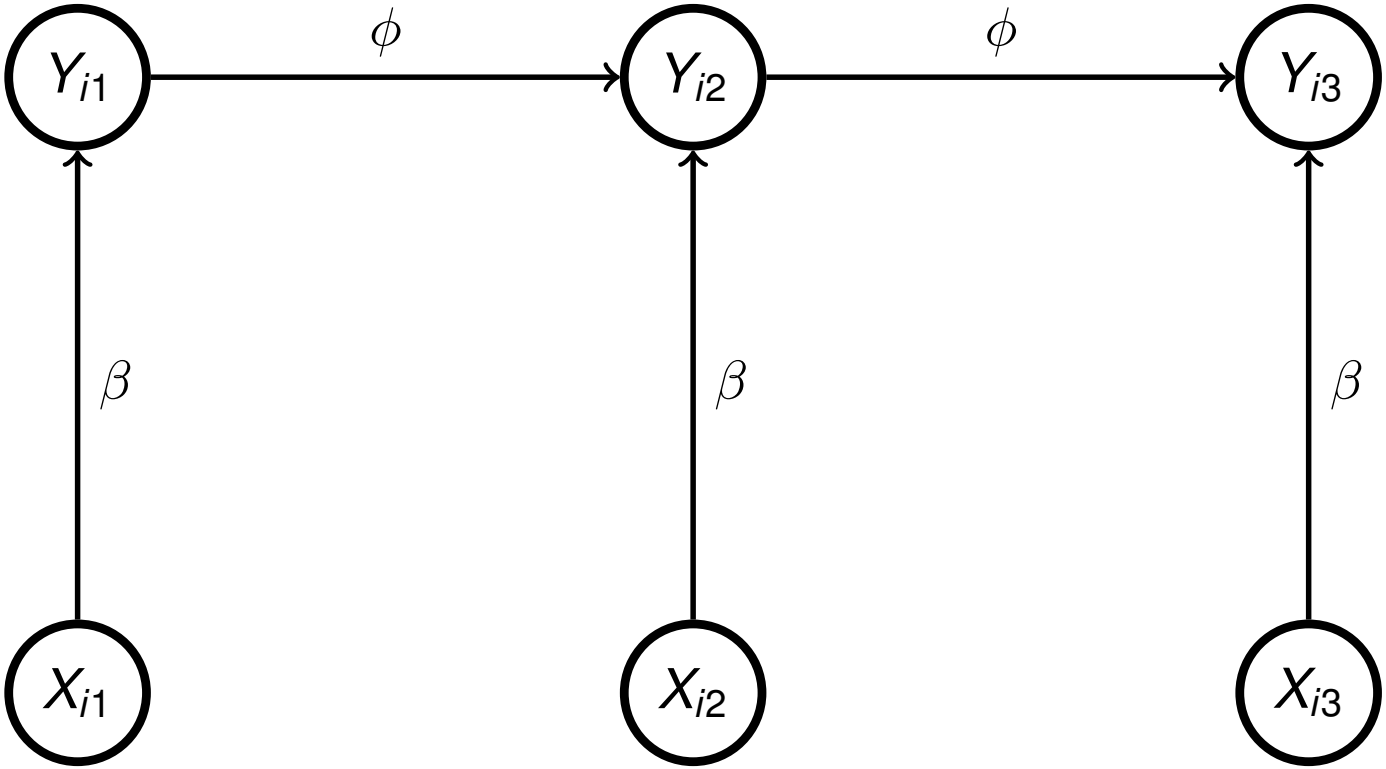
- Define the effect of X_3 on Y_3 in potential outcomes and coefficients.



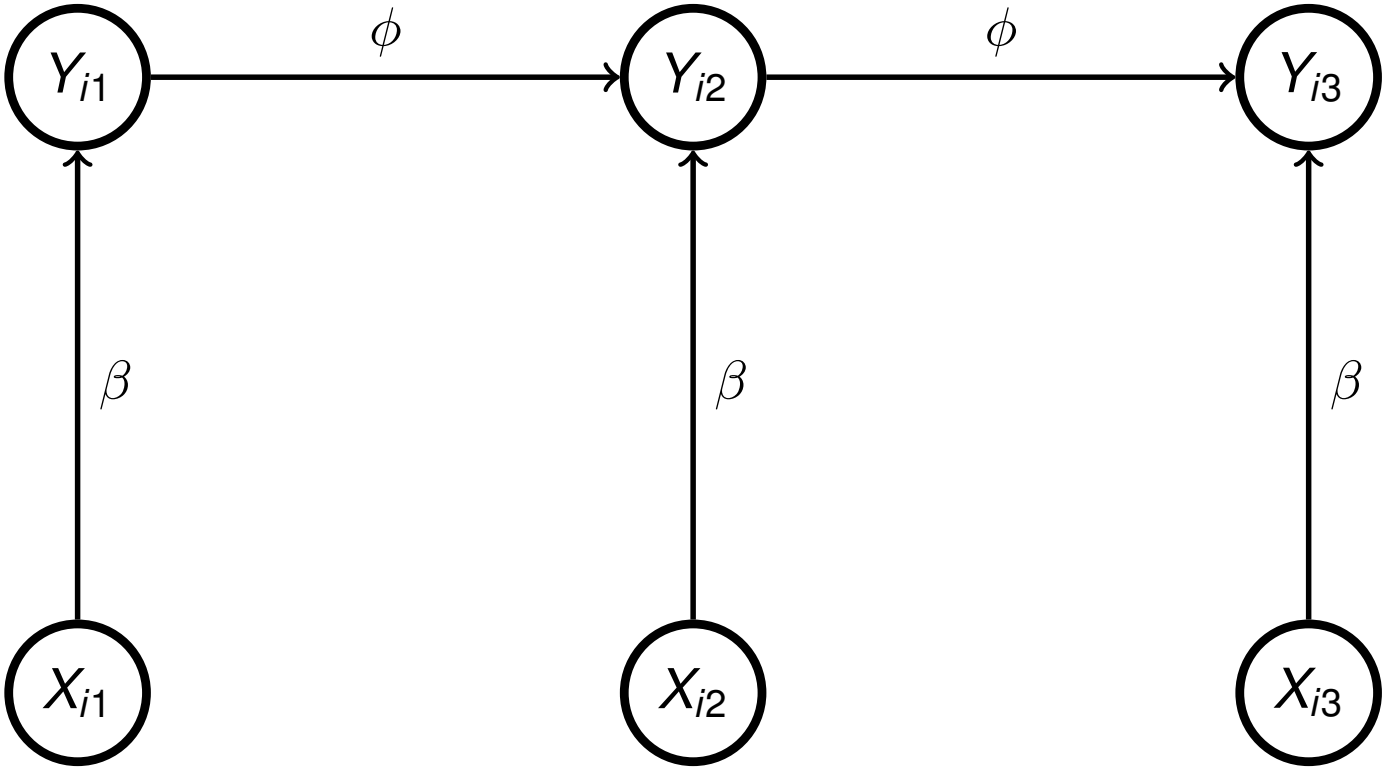
- Define the effect of X_3 on Y_3 in potential outcomes and coefficients.
- Define the effect of X_2 on Y_3 in potential outcomes and coefficients.



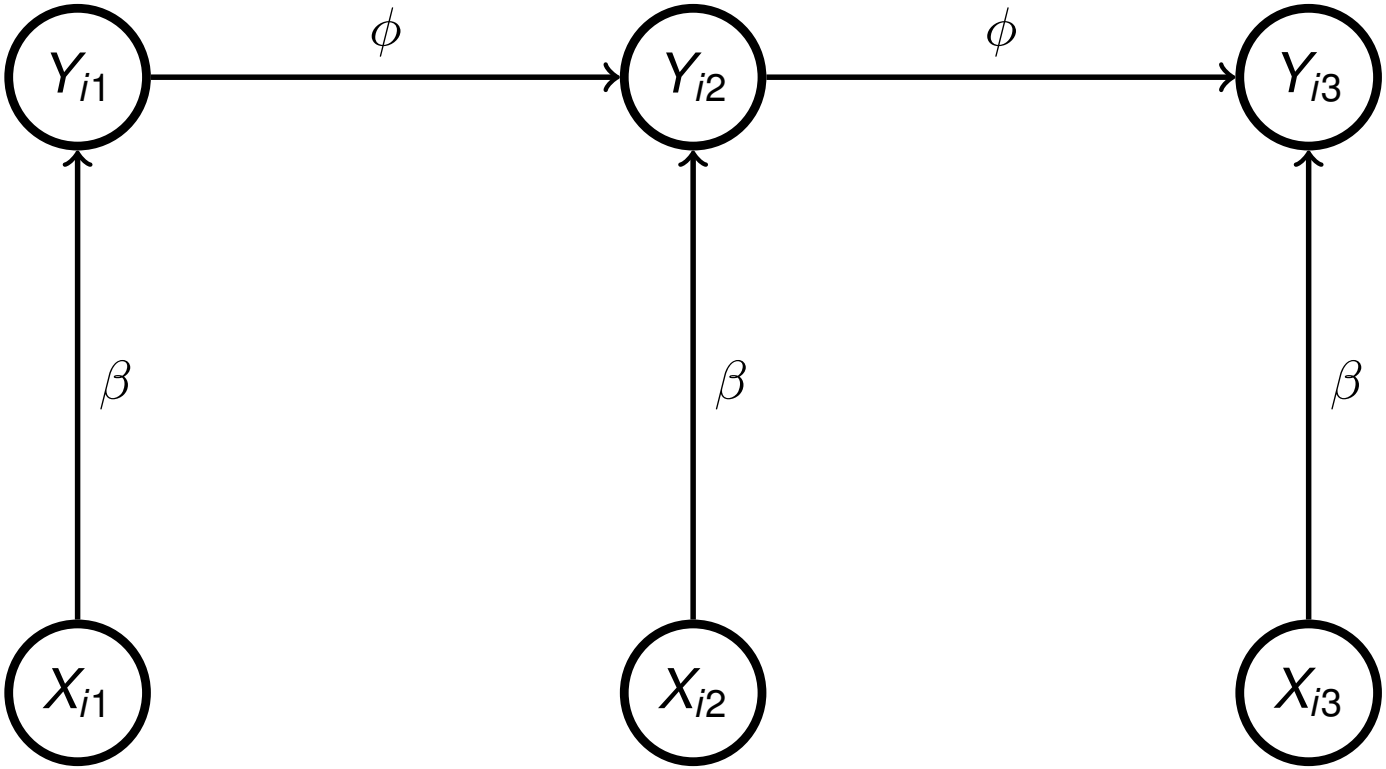
- Define the effect of X_3 on Y_3 in potential outcomes and coefficients.
- Define the effect of X_2 on Y_3 in potential outcomes and coefficients.
- Define the effect of X_1 on Y_3 in potential outcomes and coefficients.



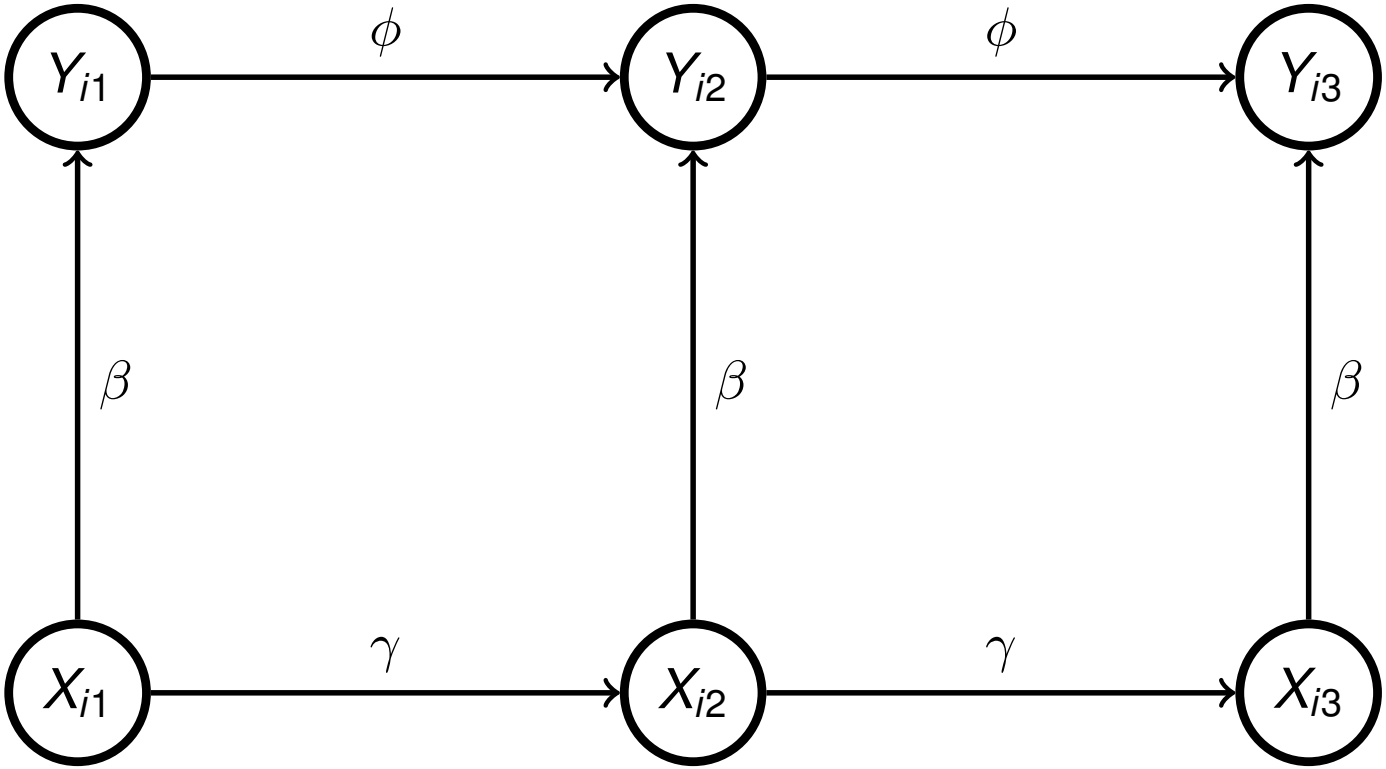
- Define some joint effects of X_2 and X_3 on Y_3 in potential outcomes and coefficients.



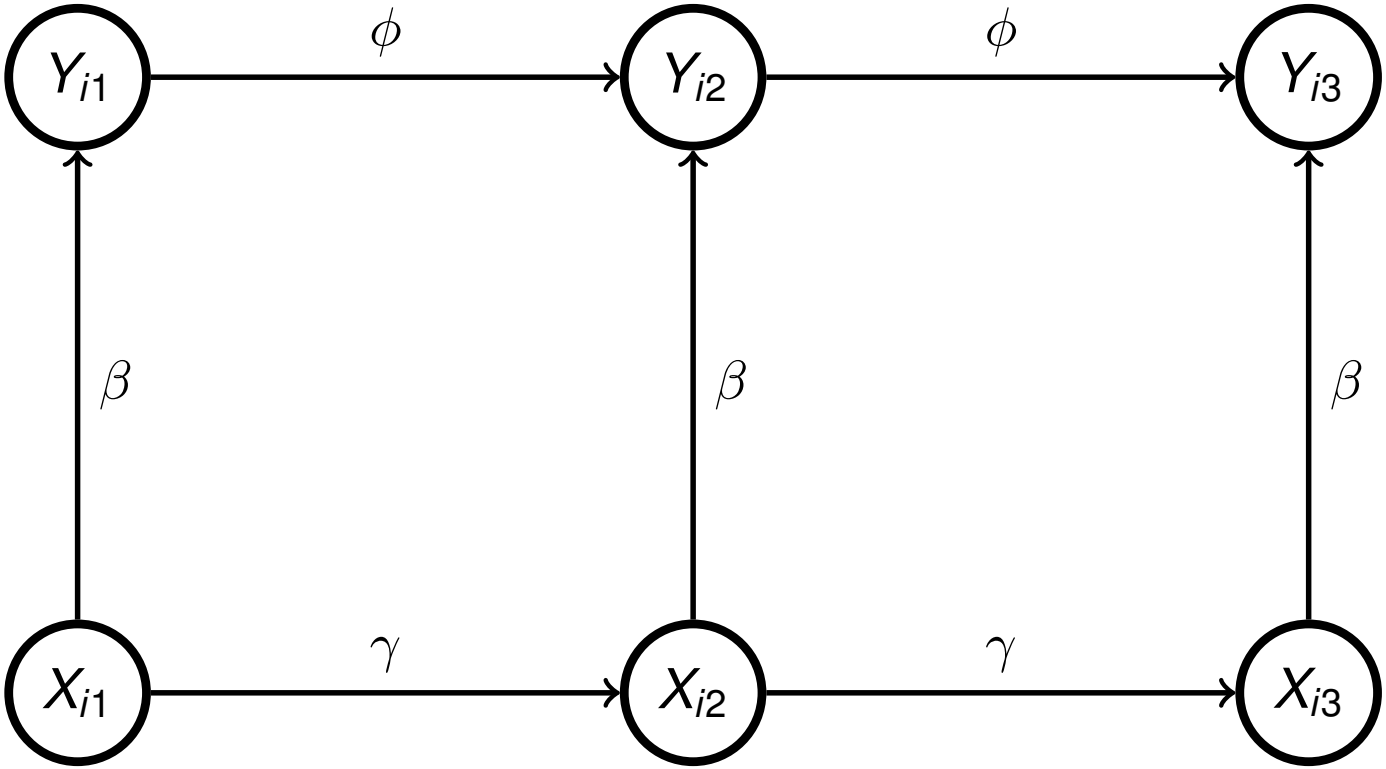
- Define some joint effects of X_2 and X_3 on Y_3 in potential outcomes and coefficients.
- Define some joint effects of X_1 and X_2 on Y_3 in potential outcomes and coefficients.



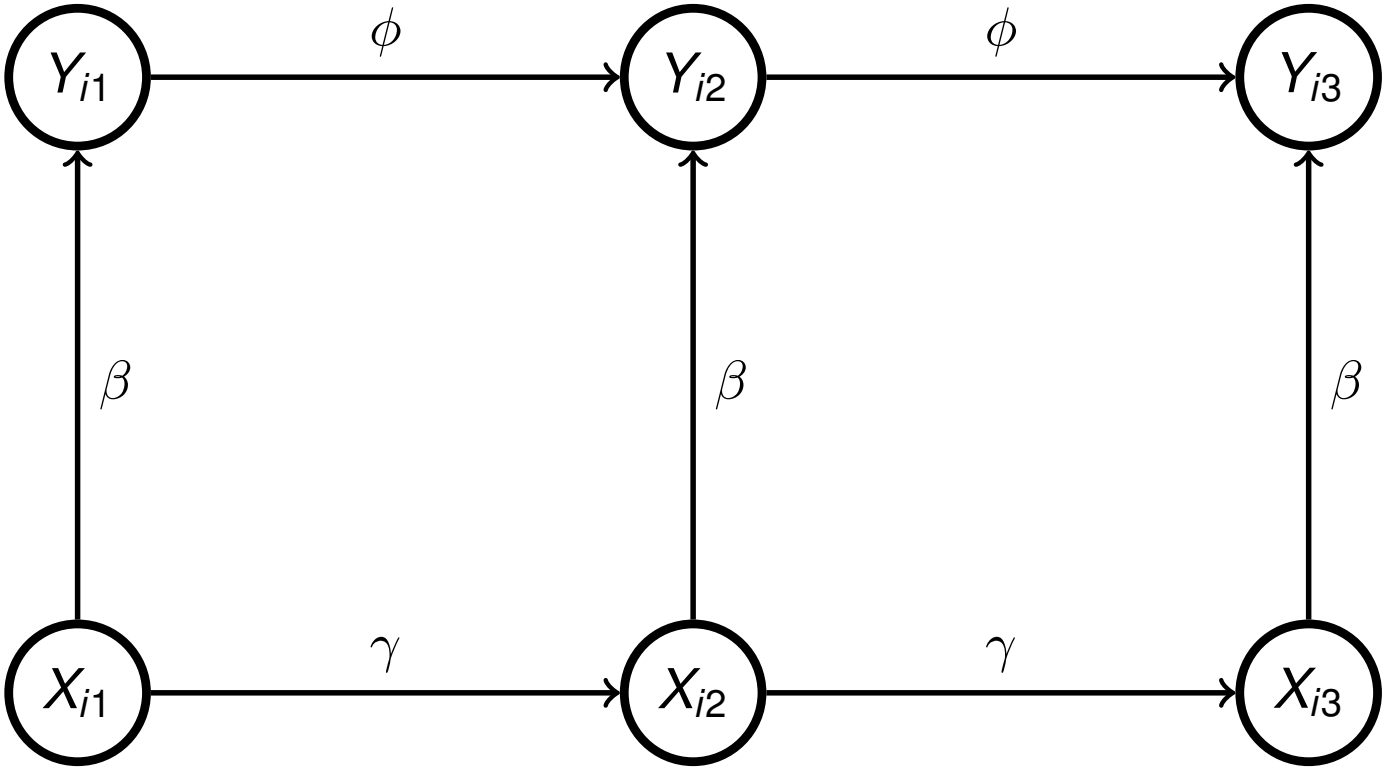
- Define some joint effects of X_2 and X_3 on Y_3 in potential outcomes and coefficients.
- Define some joint effects of X_1 and X_2 on Y_3 in potential outcomes and coefficients.
- Define some joint effects of X_1, X_2, and X_3 on Y_3 in potential outcomes and coefficients.



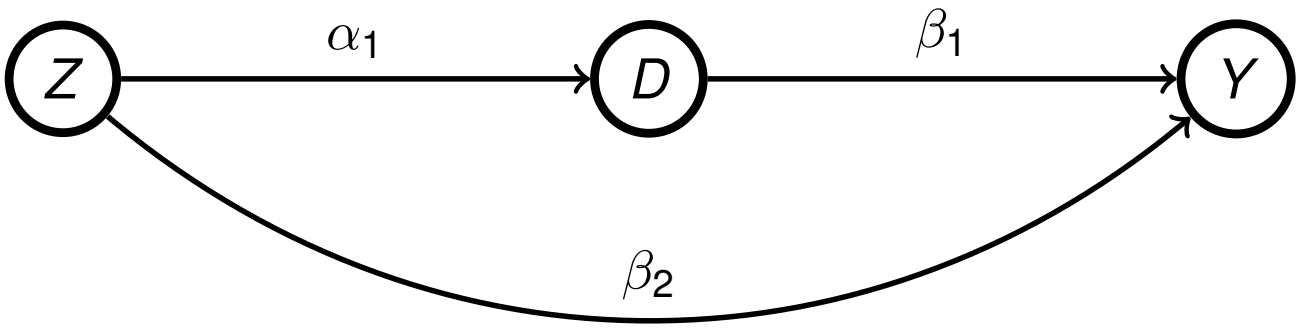
- Define some joint effects of X_2 and X_3 on Y_3 in potential outcomes and coefficients.



- Define some joint effects of X_2 and X_3 on Y_3 in potential outcomes and coefficients.
- Define some joint effects of X_1 and X_2 on Y_3 in potential outcomes and coefficients.

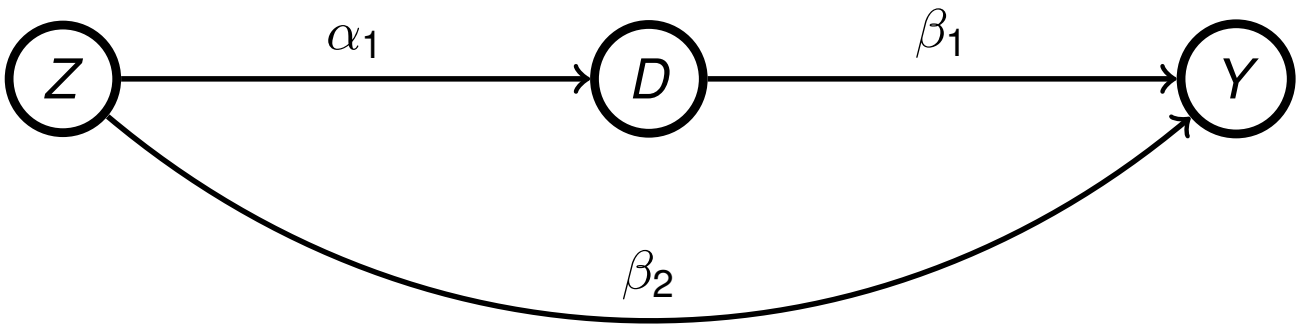


- Define some joint effects of X_2 and X_3 on Y_3 in potential outcomes and coefficients.
- Define some joint effects of X_1 and X_2 on Y_3 in potential outcomes and coefficients.
- Define some joint effects of X_1, X_2, and X_3 on Y_3 in potential outcomes and coefficients.



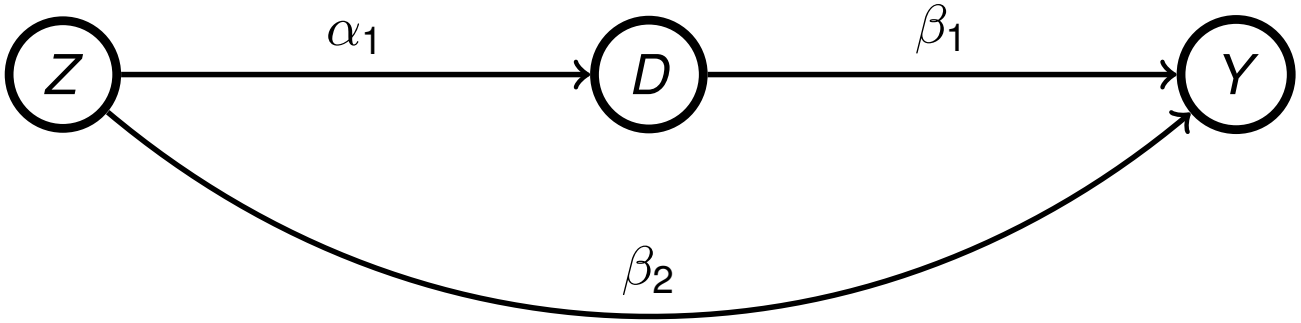
$$(1) D_i(z) = \alpha_0 + \alpha_1 z + \nu_i$$
$$(2) Y_i(d, z) = \beta_0 + \beta_1 d + \beta_2 z + \epsilon_i$$

$$Y_i(D_i(z'), z) = \beta_0 + \beta_1(\alpha_0 + \alpha_1 z' + \nu_i) + \beta_2 z + \epsilon_i$$
$$Y_i(D_i(z), z) = \beta_0 + \beta_1(\alpha_0 + \alpha_1 z + \nu_i) + \beta_2 z + \epsilon_i$$
$$Y_i(D_i(z'), z') = \beta_0 + \beta_1(\alpha_0 + \alpha_1 z' + \nu_i) + \beta_2 z' + \epsilon_i$$
$$Y_i(D_i(z), z') = \beta_0 + \beta_1(\alpha_0 + \alpha_1 z + \nu_i) + \beta_2 z' + \epsilon_i$$



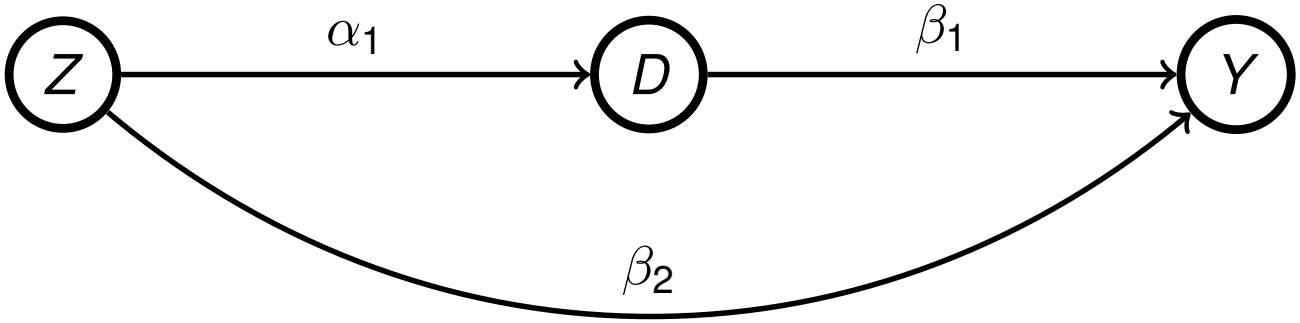
$$(1) D_i(z) = \alpha_0 + \alpha_1 z + \nu_i$$
$$(2) Y_i(d, z) = \beta_0 + \beta_1 d + \beta_2 z + \epsilon_i$$

$$Y_i(D_i(1), 0) = \beta_0 + \beta_1(\alpha_0 + \alpha_1 1 + \nu_i) + \beta_2 0 + \epsilon_i$$
$$Y_i(D_i(0), 0) = \beta_0 + \beta_1(\alpha_0 + \alpha_1 0 + \nu_i) + \beta_2 0 + \epsilon_i$$
$$Y_i(D_i(1), 1) = \beta_0 + \beta_1(\alpha_0 + \alpha_1 1 + \nu_i) + \beta_2 1 + \epsilon_i$$
$$Y_i(D_i(0), 1) = \beta_0 + \beta_1(\alpha_0 + \alpha_1 0 + \nu_i) + \beta_2 1 + \epsilon_i$$



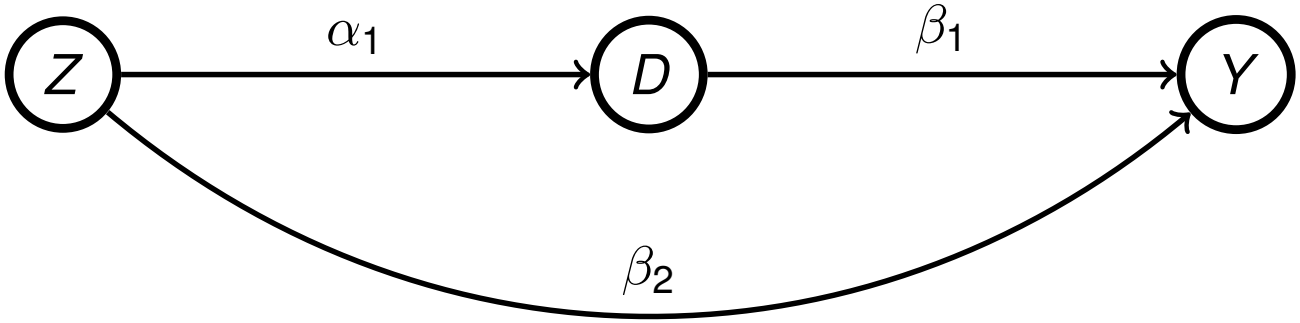
$$Y_i(D_i(1), 0) = \beta_0 + \beta_1(\alpha_0 + \alpha_1 1 + \nu_i) + \beta_2 0 + \epsilon_i$$
$$Y_i(D_i(0), 0) = \beta_0 + \beta_1(\alpha_0 + \alpha_1 0 + \nu_i) + \beta_2 0 + \epsilon_i$$
$$Y_i(D_i(1), 1) = \beta_0 + \beta_1(\alpha_0 + \alpha_1 1 + \nu_i) + \beta_2 1 + \epsilon_i$$
$$Y_i(D_i(0), 1) = \beta_0 + \beta_1(\alpha_0 + \alpha_1 0 + \nu_i) + \beta_2 1 + \epsilon_i$$

- Write some direct effects in terms of potential outcomes and coefficients.



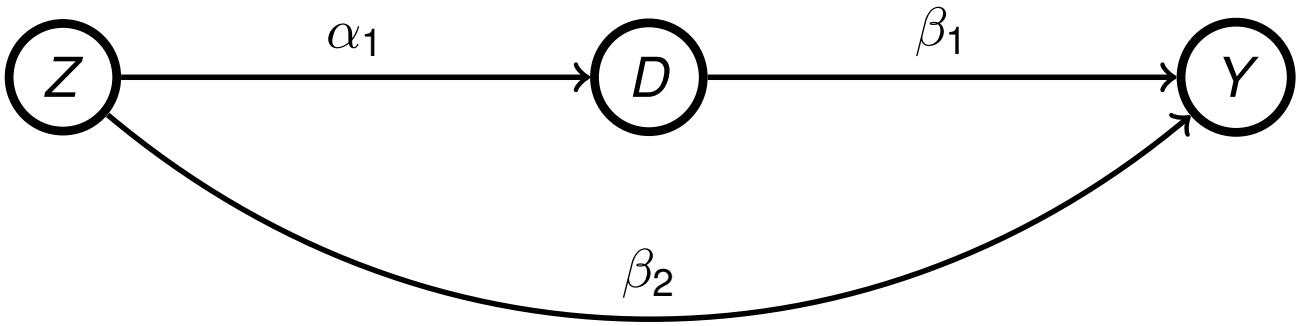
$$Y_i(D_i(1), 0) = \beta_0 + \beta_1(\alpha_0 + \alpha_1 1 + \nu_i) + \beta_2 0 + \epsilon_i$$
$$Y_i(D_i(0), 0) = \beta_0 + \beta_1(\alpha_0 + \alpha_1 0 + \nu_i) + \beta_2 0 + \epsilon_i$$
$$Y_i(D_i(1), 1) = \beta_0 + \beta_1(\alpha_0 + \alpha_1 1 + \nu_i) + \beta_2 1 + \epsilon_i$$
$$Y_i(D_i(0), 1) = \beta_0 + \beta_1(\alpha_0 + \alpha_1 0 + \nu_i) + \beta_2 1 + \epsilon_i$$

- Write some direct effects in terms of potential outcomes and coefficients.
- Write some indirect effects in terms of potential outcomes and coefficients.



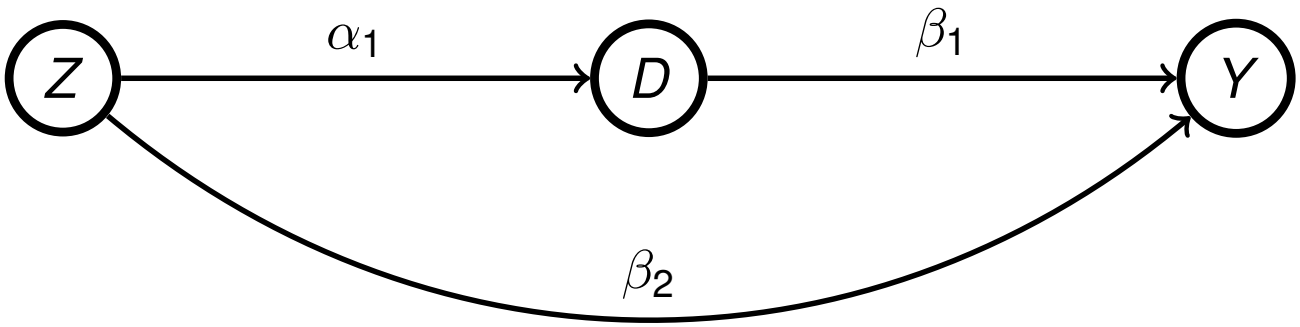
$$Y_i(D_i(1),0) = \beta_0 + \beta_1(\alpha_0 + \alpha_1 1 + \nu_i) + \beta_2 0 + \epsilon_i$$
$$Y_i(D_i(0),0) = \beta_0 + \beta_1(\alpha_0 + \alpha_1 0 + \nu_i) + \beta_2 0 + \epsilon_i$$
$$Y_i(D_i(1),1) = \beta_0 + \beta_1(\alpha_0 + \alpha_1 1 + \nu_i) + \beta_2 1 + \epsilon_i$$
$$Y_i(D_i(0),1) = \beta_0 + \beta_1(\alpha_0 + \alpha_1 0 + \nu_i) + \beta_2 1 + \epsilon_i$$

- Write some direct effects in terms of potential outcomes and coefficients.
- Write some indirect effects in terms of potential outcomes and coefficients.
- Which direct and indirect effects sum to the total effect of Z on Y?



$$Y_i(D_i(1), 0) = \beta_0 + \beta_1(\alpha_0 + \alpha_1 1 + \nu_i) + \beta_2 0 + \epsilon_i$$
$$Y_i(D_i(0), 0) = \beta_0 + \beta_1(\alpha_0 + \alpha_1 0 + \nu_i) + \beta_2 0 + \epsilon_i$$
$$Y_i(D_i(1), 1) = \beta_0 + \beta_1(\alpha_0 + \alpha_1 1 + \nu_i) + \beta_2 1 + \epsilon_i$$
$$Y_i(D_i(0), 1) = \beta_0 + \beta_1(\alpha_0 + \alpha_1 0 + \nu_i) + \beta_2 1 + \epsilon_i$$

- In this model, how can we identify these direct and indirect effects?



$$Y_i(D_i(1), 0) = \beta_0 + \beta_1(\alpha_0 + \alpha_1 1 + \nu_i) + \beta_2 0 + \epsilon_i$$
$$Y_i(D_i(0), 0) = \beta_0 + \beta_1(\alpha_0 + \alpha_1 0 + \nu_i) + \beta_2 0 + \epsilon_i$$
$$Y_i(D_i(1), 1) = \beta_0 + \beta_1(\alpha_0 + \alpha_1 1 + \nu_i) + \beta_2 1 + \epsilon_i$$
$$Y_i(D_i(0), 1) = \beta_0 + \beta_1(\alpha_0 + \alpha_1 0 + \nu_i) + \beta_2 1 + \epsilon_i$$

- In this model, how can we identify these direct and indirect effects?
- What happens as we complicate the model (interactions, heterogeneous effects, etc.).

Goals and Objectives for This Afternoon:

- Introduce the use of BDC with path analysis for joint treatments.
- Present TSCS examples.
- Discuss mediation with constant effects and preview mediation with non-constant effects.