Lecture 5

Heterogeneous Effects

Instrumental Variables

Causal Inference Using Graphs
August 13, 2019

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Acknowledgements

Daniel Arnon contributed to many of the slides from lecture 5 today.

Goals and Objectives for This Morning:

- Review IV with constant effects
- Introduce IV with heterogeneous effects
- Learning about compliers

Overview

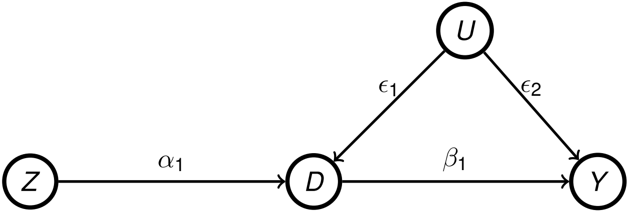
1. **Review of Constant Effects**
2. **Heterogeneous Effects**

1. **Review of Constant Effects**

2. Heterogeneous Effects

Consider the following path model:

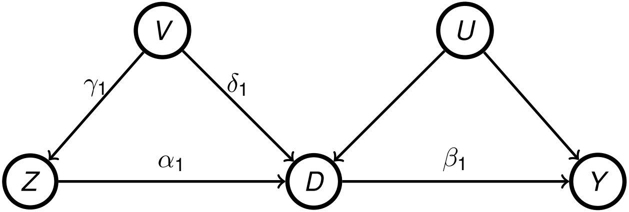
Figure: Confounding on *D* and *Y*



Discuss the Wald estimator and why it works.

Lets consider a few more path models:

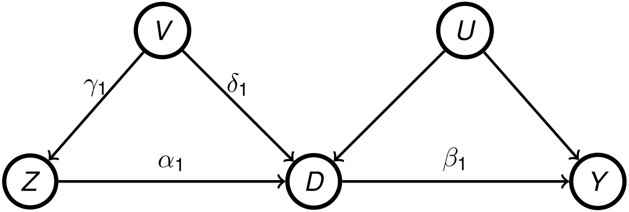
Figure: Confounding on Z, D, Y



Can we still calculate the effect of $D \rightarrow Y$?

Lets consider a few more path models:

Figure: Confounding on Z, D, Y

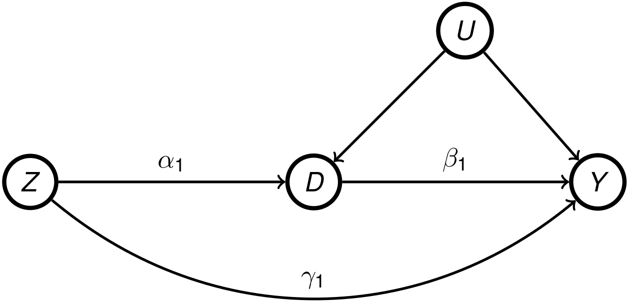


Can we still calculate the effect of $D \to Y$?

$$\frac{Y \sim Z}{D \sim Z} \underset{p}{\to} \frac{\alpha_1\beta_1 + \gamma_1\delta_1\beta_1}{\alpha_1 + \gamma_1\delta_1} = \frac{\beta_1(\gamma_1 + \delta_1)}{\gamma_1 + \delta_1} = \beta_1$$

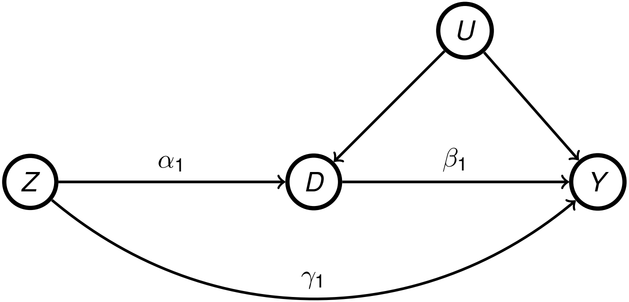
Let's consider one more DAG:

Figure: Direct Effect of Z on Y



Let's consider one more DAG:

Figure: Direct Effect of Z on Y

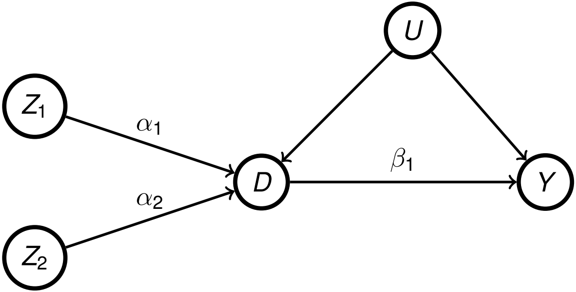


$$\frac{Y \sim Z}{D \sim Z} \xrightarrow[p]{} \frac{\alpha_1 \beta_1 + \gamma_1}{\alpha_1} = \beta_1 + \frac{\gamma_1}{\alpha_1}$$

Multiple Instruments

Consider the following DAG, with multiple instruments:

Figure: Multiple Instruments, No Exclusion Restriction Violation



1. **Estimating with Wald**: The other option is to use a Wald Estimator *for each* instrument, and to weight them by the strength of the instrument. Formally:

   Wald1: $\dfrac{Y \sim Z_1}{D \sim Z_1} = \dfrac{\alpha_1 \beta_1}{\alpha_1}$

   Wald2: $\dfrac{Y \sim Z_2}{D \sim Z_2} = \dfrac{\alpha_2 \beta_1}{\alpha_2}$

2. **Estimating with 2SLS**: 2SLS: ψWald1 $+ (1 - \psi)$Wald2.

   Where $\psi = \dfrac{\alpha_1 \text{Cov}(D, Z_1)}{\alpha_1 \text{Cov}(D, Z_1) + \alpha_2 \text{Cov}(D, Z_2)}$

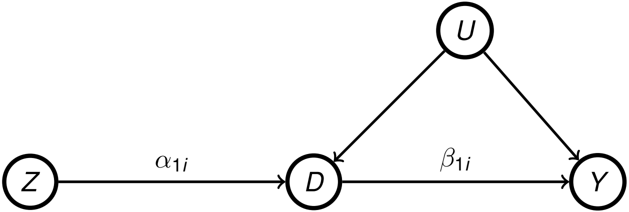
1 Review of Constant Effects

2 **Heterogeneous Effects**

		$d_i(0)$	
		0	1
$d_i(1)$	0	Never	Defier
	1	Complier	Always

Table: Principal strata for compliance behavior

Figure: Heterogenous Effects with Confounding on D and Y



		$d_i(0)$	
		0	1
$d_i(1)$	0	Never $\alpha_{1i} = 0$	Defier $\alpha_{1i} = -1$
	1	Complier $\alpha_{1i} = 1$	Always $\alpha_{1i} = 0$

Table: Principal strata and monotonicity

$$\frac{E[\alpha_{1i}\beta_{1i}]}{E[\alpha_{1i}]}$$

$$\frac{E[\alpha_{1i}\beta_{1i}]}{E[\alpha_{1i}]}$$

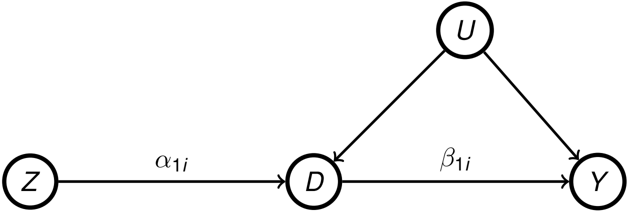
$$= \frac{E[\beta_{1i}|\alpha_{1i} = 1]Pr(\alpha_{1i} = 1) - E[\beta_{1i}|\alpha_{1i} = -1]Pr(\alpha_{1i} = -1)}{Pr(\alpha_{1i} = 1) - Pr(\alpha_{1i} = -1)}$$

$$\frac{E[\alpha_{1i}\beta_{1i}]}{E[\alpha_{1i}]}$$

$$= \frac{E[\beta_{1i}|\alpha_{1i}=1]Pr(\alpha_{1i}=1) - E[\beta_{1i}|\alpha_{1i}=-1]Pr(\alpha_{1i}=-1)}{Pr(\alpha_{1i}=1) - Pr(\alpha_{1i}=-1)}$$

$$= E[\beta_{1i}|\alpha_{1i}=1]\left(\frac{Pr(\alpha_{1i}=1) - \frac{E[\beta_{1i}|\alpha_{1i}=-1]}{E[\beta_{1i}|\alpha_{1i}=1]}Pr(\alpha_{1i}=-1)}{Pr(\alpha_{1i}=1) - Pr(\alpha_{1i}=-1)}\right)$$

Figure: Heterogenous Effects with Confounding on *D* and *Y*, Continuous Treatment



$$D_i = \alpha_0 + \alpha_{1i} Z_i + \epsilon_i$$

,

$$Y_i = \gamma_0 + \beta_{1i} D_i + \nu_i$$

What are compliers now?

$$\frac{E[\alpha_{1i}\beta_{1i}]}{E[\alpha_{1i}]} = E[\frac{\alpha_{1i}}{E[\alpha_{1i}]}\beta_{1i}]$$

$$\frac{\frac{1}{n}\sum\alpha_{1i}\beta_i}{\frac{1}{n}\sum\alpha_{1i}} = \frac{1}{n}\sum\frac{\alpha_{1i}\beta_i}{\bar{\alpha_1}}$$

Learning about compliers for one-sided noncompliance (binary treatment)

One sided non-compliance refers to a case a patient cannot get a drug without being assigned to treatment, i.e. there are no always-takers. There are only compliers and never-takers.

$$\frac{E[\alpha_{1i}\beta_{1i}]}{E[\alpha_{1i}]} = \frac{Pr(\alpha_{1i} = 1)E[\beta_i]|\alpha_{1i} = 1}{Pr(\alpha_{1i} = 1)} = E[\beta_i|\alpha_{1i} = 1]$$

Because under one-sided non-compliance, we know for every treated individual whether they are compliers or never takers.

Learning about compliers for monotonicity (binary treatment)

$$E[g(x_i)|\alpha_{1i} = 1, D_{1i} > D_{0i}] = \frac{E[\kappa_i g(x_i)]}{E[\kappa_i]}$$

Where:

$$\kappa_i = 1 - \underbrace{\frac{D_i(1 - Z_i)}{1 - Pr(Z_i = 1|X_i)}}_{\text{D=1, Z=0}\rightarrow\text{always-taker}} - \underbrace{\frac{(1 - D_i)Z_i}{Pr(Z_i = 1|X_i)}}_{\text{Z=1, D=0}\rightarrow\text{never-taker}}$$

In this equation, $\kappa_i = 1$ for compliers. For identifiable always-takers and never-takers, the κ equation gives negative values. The equation identifies who looks like they would have been always-takers and never takers based on their covariate characteristics. $E[\alpha_i] = E[\kappa_i]$ = proportion of compliers ($Pr(D_1 > D_0)$).

Learning about weights with continuous treatment

Ideas?

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This afternoon, mediation analysis and more with heterogeneous effects.