Lecture 3

Sketching with Path Analysis

Singleton Treatments

Causal Inference Using Graphs
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Acknowledgements

Daniel Arnon contributed to many of the slides from lectures 3 and 4 today.

Goals and Objectives for This Morning:

- Introduce path analysis with linear SEMs
- Using BDC with linear SEMs
- Instrumental variables with constant effects
- Other examples with constant effects

Overview

1. **Path Analysis**
   Potential outcomes and structural linear models
   Path Rules

2. **Instrumental Variables**

3. **Other examples**

The potential outcomes framework can be translated into the structural linear models framework in the following manner. Formally, potential outcomes are defined as:

$$\text{Potential outcome} = \begin{cases} Y_i(1), & ifD_i = 1 \\ Y_i(0), & ifD_i = 0 \end{cases}$$

Using the potential outcomes framework we can talk about both quantities: $Y_i(1)$ which is the potential outcome under treatment and $Y_i(0)$ which is the potential outcome under control. But since we only observe one of these two outcomes, the observed outcome Y_i can be rewritten as:

$$\begin{aligned} Y_i &= Y_i(1)D_i + Y_i(0)(1 - D_i) \\ &= Y_i(0) + (Y_i(1) - Y_i(0))D_i \\ &= \beta_{0i} + \beta_{1i}D_i \end{aligned}$$

$$
\begin{aligned}
Y_i &= E[\beta_{0i}] + \beta_{1i} D_i + \beta_{0i} - E[\beta_{0i}] \\
&= E[\beta_{0i}] + \beta_{1i} D_i + \epsilon_i \\
&= \beta_0 + \beta_1 D_i + \epsilon_i
\end{aligned}
$$

- to go from line 1 to line 2, we simply define $\epsilon_i = \beta_{0i} - E[\beta_{0i}]$.
- to go from line 2 to line 3 however relies on $\beta_{1i} = \beta_i \forall i$ (i.e., constant effects holds).

1 **Path Analysis**

Potential outcomes and structural linear models

Path Rules

2 **Instrumental Variables**

3 **Other examples**

Consider the following DAG (Directed Acyclic Graph), where by convention, we do not include error terms on the graph unless they are correlated, point into more than one variable, or are pointed into themselves.

$$Z \xrightarrow{\alpha_1} D \xrightarrow{\beta_1} Y$$

$$(1) D_i(z) = \alpha_0 + \alpha_1 z + \nu_i$$

$$(2) Y_i(d) = \beta_0 + \beta_1 d + \epsilon_i$$

$$Y_i(D_i(z)) = Y_i(z) = \beta_0 + \beta_1(\alpha_0 + \alpha_1 z + \nu_i) + \epsilon_i$$

$$Y_i(z) = \underbrace{\beta_0 + \beta_1\alpha_0}_{\text{The intercept}} + \underbrace{(\beta_1\alpha_1)}_{\text{The Effect}} z + \underbrace{\beta_1\nu_i + \epsilon_i}_{\text{Error}}$$

The total effect of Z on Y, is the *product* of the the path coefficients. It is important to note the lack of *i* subscripts on these coefficients.

$$
\begin{tikzcd}
Z \arrow[r, "\alpha_1"] \arrow[rr, bend right, "\beta_2"'] & D \arrow[r, "\beta_1"] & Y
\end{tikzcd}
$$

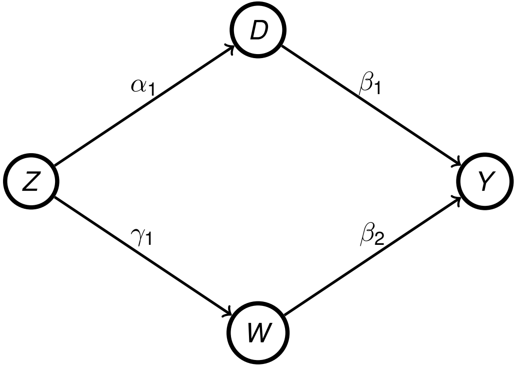
$$(1)\, D_i(z) = \alpha_0 + \alpha_1 z + \nu_i$$

$$(2)\, Y_i(d, z) = \beta_0 + \beta_1 d + \beta_2 z + \epsilon_i$$

$$Y_i(d_i(z), z) = Y_i(z) = \beta_0 + \beta_1(\alpha_0 + \alpha_1 Z + \nu_i) + \beta_2 Z + \epsilon_i$$

$$Y_i(z) = \underbrace{\beta_0 + \beta_1\alpha_0}_{\textit{Intercept}} + \underbrace{(\beta_1\alpha_1 + \beta_2)}_{\text{Effect}} z + \underbrace{\epsilon_i + \beta_1\nu_i}_{\text{Error}}$$

Just to make sure this is clear, lets look at this final DAG:



What is the total effect of Z on Y in terms of the path coefficients?

1. Path Analysis
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2. **Instrumental Variables**

3. Other examples

One way to overcome the problem of unmeasured confounding variables is by using instrumental variables. As a precursor, consider the following path model:

Figure: No Confounding

$$Z \xrightarrow{\alpha_1} D \xrightarrow{\beta_1} Y$$

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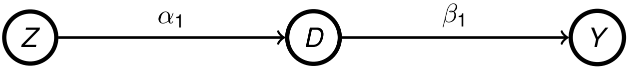
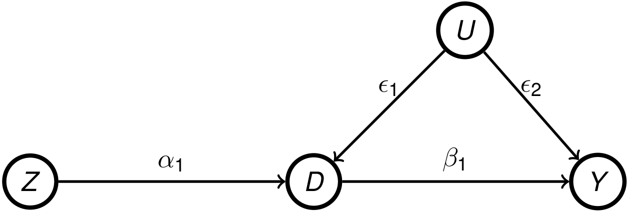


How can we estimate the effect of $D \rightarrow Y$? We can do this two ways:

1. $Y \sim D \underset{p}{\rightarrow} \beta_1$
2. $\dfrac{Y \sim Z}{D \sim Z} \underset{p}{\rightarrow} \dfrac{\alpha_1 \beta_1}{\alpha_1} = \beta_1$

But now consider the following path model:

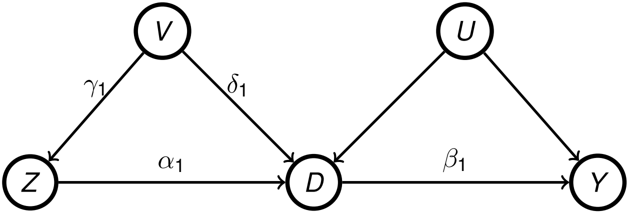
Figure: Confounding on *D* and *Y*



Now that *D* and *Y* have common cause confounding, method 1 from before no longer works (why?), but method 2 still works.

Lets consider a few more path models:

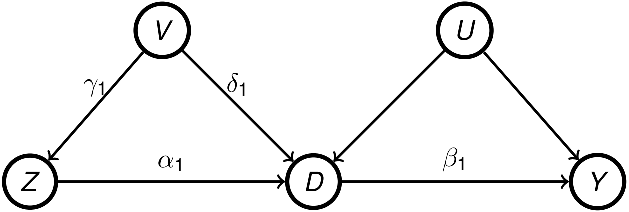
Figure: Confounding on Z, D, Y



Can we still calculate the effect of $D \rightarrow Y$?

Lets consider a few more path models:

Figure: Confounding on Z, D, Y

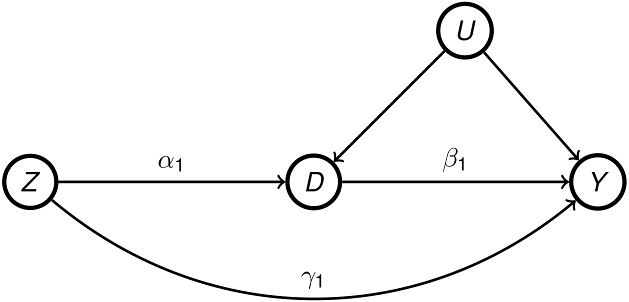


Can we still calculate the effect of $D \rightarrow Y$?

$$\frac{Y \sim Z}{D \sim Z} \underset{p}{\rightarrow} \frac{\alpha_1\beta_1 + \gamma_1\delta_1\beta_1}{\alpha_1 + \gamma_1\delta_1} = \frac{\beta_1(\gamma_1 + \delta_1)}{\gamma_1 + \delta_1} = \beta_1$$

Let's consider one more DAG:

Figure: Direct Effect of Z on Y



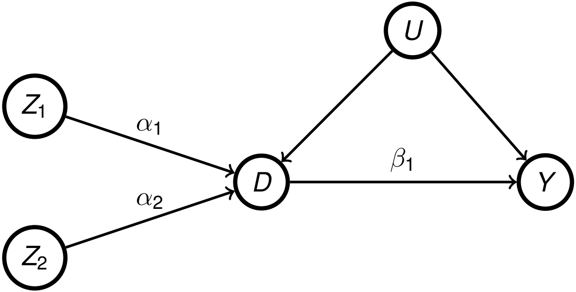
$$\frac{Y \sim Z}{D \sim Z} \underset{p}{\rightarrow} \frac{\alpha_1 \beta_1 + \gamma_1}{\alpha_1} = \beta_1 + \frac{\gamma_1}{\alpha_1}$$

The additional term $\frac{\gamma_1}{\alpha_1}$ is the additional bias from the direct effect of Z on Y.

Multiple Instruments

Consider the following DAG, with multiple instruments:

Figure: Multiple Instruments, No Exclusion Restriction Violation



1. **Estimating with Wald**: The other option is to use a Wald Estimator *for each* instrument, and to weight them by the strength of the instrument. Formally:

   Wald1: $\dfrac{Y \sim Z_1}{D \sim Z_1} = \dfrac{\alpha_1 \beta_1}{\alpha_1}$

   Wald2: $\dfrac{Y \sim Z_2}{D \sim Z_2} = \dfrac{\alpha_2 \beta_1}{\alpha_2}$

2. **Estimating with 2SLS**: 2SLS: ψWald1 $+ (1 - \psi)$Wald2.

   Where $\psi = \dfrac{\alpha_1 \text{Cov}(D, Z_1)}{\alpha_1 \text{Cov}(D, Z_1) + \alpha_2 \text{Cov}(D, Z_2)}$

Adding Covariates

Why add pre-instrument covariates? Assumptions may hold conditionally on covariates.

How to do it for exclusion restriction? Don't, because of likely confounding of the post-instrument variable.

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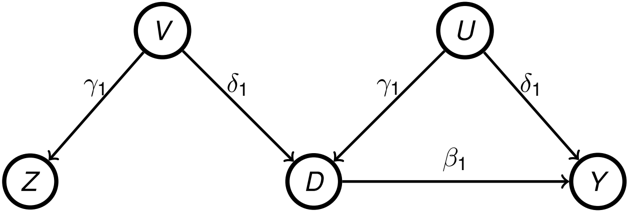
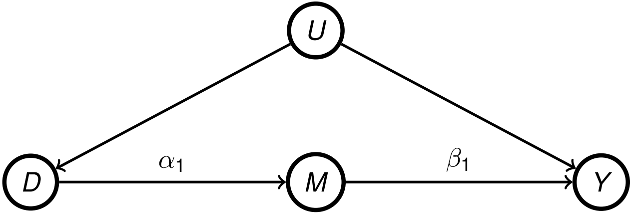
How to estimate the effect of X_1 on Y_3?

$$
\begin{array}{ccccc}
Y_{i1} & \xrightarrow{\phi} & Y_{i2} & \xrightarrow{\phi} & Y_{i3} \\
\uparrow \beta & & \uparrow \beta & & \uparrow \beta \\
X_{i1} & & X_{i2} & & X_{i3}
\end{array}
$$

Lets consider a few more path models:

Figure: Confounding on Z, D, Y

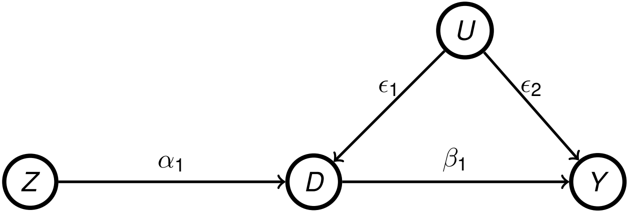




Can we identify the effect of D on Y?

Suppose we didn't know that Z was an instrument?

Figure: Confounding on D and Y



What if we regress Y on D? What if we regress Y on D and Z?

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This afternoon we will sketch with joint treatments.