

# Generalized Nonlinear Difference-in-Difference-in-Differences

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# DiD and DiDiD primer

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- ▶ Difference-in-Difference-in-Differences (DiDiD) adds a 3rd D to check/correct for violations of the DiD assumptions.



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- ▶ Difference-in-Difference-in-Differences (DiDiD) adds a 3rd D to check/correct for violations of the DiD assumptions. (e.g., 3rd D might be min wage jobs vs non-min wage jobs)

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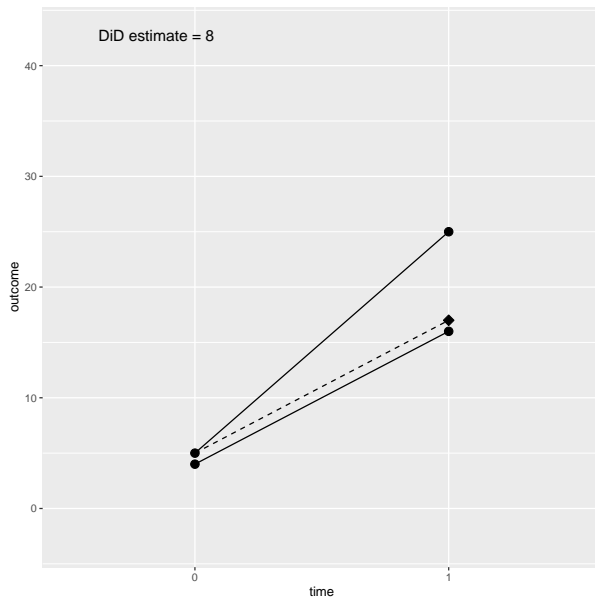
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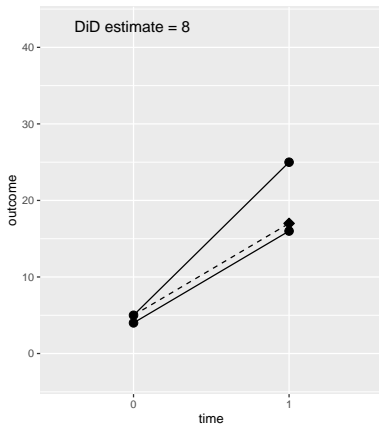
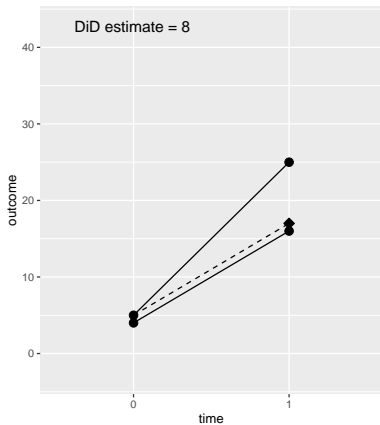
Let's work through the intuition behind CiC and NOC.



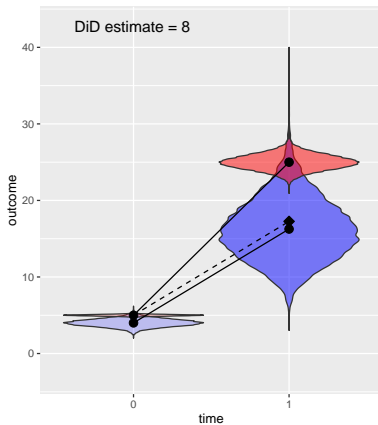
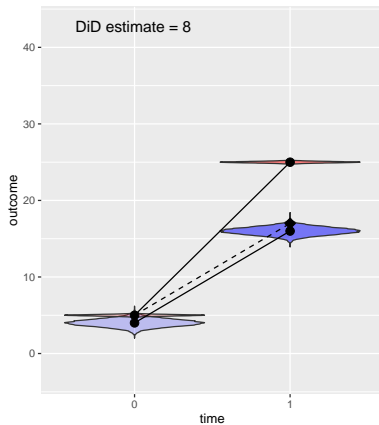
# Traditional DiD with one pre-treatment period



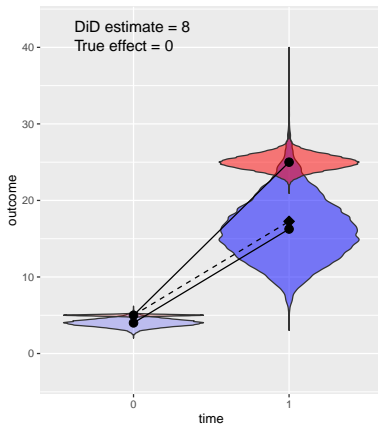
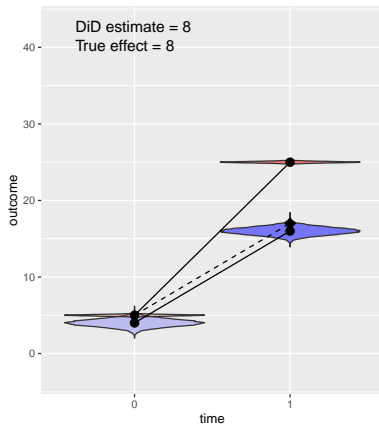
# Means don't tell us everything



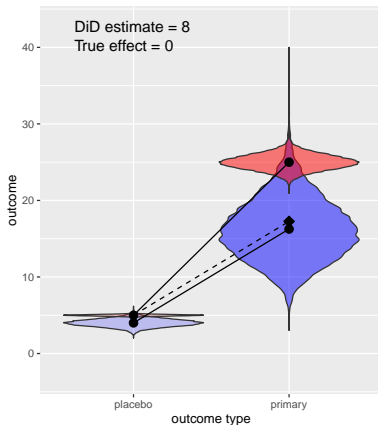
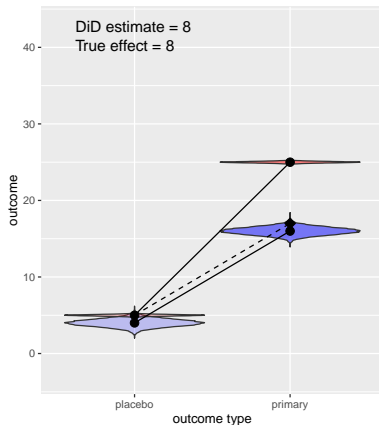
# Useful information in the distributions



# Useful information in the distributions



# X-axis doesn't have to be time



# Our approach to solving issues 1 and 2

We generalize DiDiD by extending the CiC procedure and the NOC procedure to a NOCNOC procedure.

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We generalize DiDiD by extending the CiC procedure and the NOC procedure to a NOCNOC procedure. This approach allows the following:

- ▶ nonlinear DiDiD that uses all the information
- ▶ generalized nonlinear DiDiD to accommodate placebo outcomes

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# Running example: effects of debate attendance on political efficacy in a 2017 transitional election in Nepal



## Treatment (A): attendance at debate screening ( $\approx 50$ min)

- ▶ Samriddhi Foundation organized candidate debates for each of 3 SMD seats in House of Representatives (parliament).
- ▶ Combinations of debate/discussions were randomly assigned among attendees of the debates.
- ▶ Here, we focus on the effects of attendance, **which was not randomized**.



# Sample

- ▶ Constituencies within Kanchanpur, Jhapa, Sunsari districts
- ▶ Random sample of adults in each constituency were invited to sign up for an event date of their choice.
- ▶ Reinterview rate of 90.7% of 777 baseline respondents.
- ▶ We analyze  $n = 705$  complete cases among those who signed up for Nov 21 - Nov 27.
- ▶  $A = 1$  for 30.4% of these cases.

## Primary outcome $Y$ : political efficacy

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Parameter of Interest (ATT):  $E[Y_{11}] - E[Y_{11}(0)]$  is the average effect of attendance on those that attended.

# Classic DiD approach

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Difference-in-differences:

$$\begin{aligned} &(\hat{E}[Y_{11}] - \hat{E}[Y_{10}]) - (\hat{E}[Y_{01}] - \hat{E}[Y_{00}]) = \\ &(\hat{E}[Y_{11}] - \hat{E}[Y_{01}]) - (\hat{E}[Y_{10}] - \hat{E}[Y_{00}]) = .041 \text{ (.023)} \end{aligned}$$

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Appears to show effect (90% CI), but relies on parallel trends assumption, or time-invariant bias assumption, which are not scale invariant. So try CiC.

## CiC procedure

Athey and Imbens (2006) presents a generalization of DiD using cdfs and inverse cdfs.

- ▶ DiD average estimate (re-written):

$$\hat{E}[Y_{11}] - (\hat{E}[Y_{01}] + (\hat{E}[Y_{10}] - \hat{E}[Y_{00}]))$$

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- ▶ CiC average estimate:

$$\frac{1}{n_{11}} \sum_{i=1}^{n_{11}} Y_{11,i} - \frac{1}{n_{10}} \sum_{i=1}^{n_{10}} \hat{F}_{Y_{01}}^{-1}(\hat{F}_{Y_{00}}(Y_{10,i}))$$

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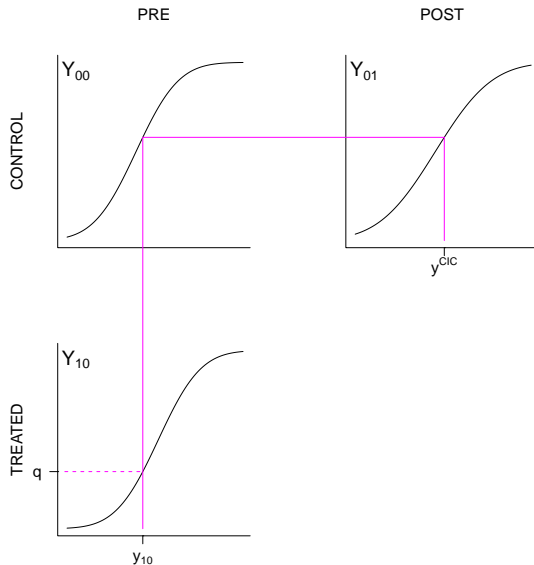
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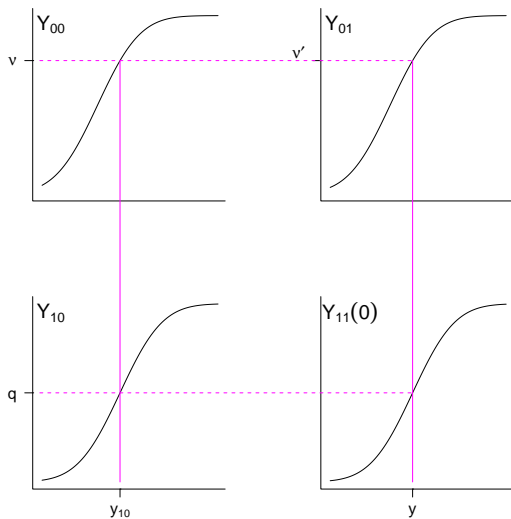
- ▶ Imputation can be done for each quantile ( $\hat{F}_{Y_{11}(0)}^{-1}(q)$ ).

# CiC procedure for imputing $Y(0)$ for quantile $q$



Relies on time-invariant confounding on the quantile scale.

## CiC equi-confounding assumption for quantile $q$



# Nepal CiC estimate

CiC estimate of ATT:

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Effect is robust, but maybe assumptions don't hold. Can't find 3rd D group, but can find placebo outcome.

# Placebo (negative) outcome: knowledge

*N*: Knowledge of government not presented in the debate (max possible score of 18)

- ▶ How many levels of government under the new constitution
- ▶ How many assemblies at the federal level
- ▶ Name responsibilities/powers of local government under the new constitution



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Should not be affected (much) by debate attendance, but may be affected by similar confounding factors.

# NOC procedure

Sofer et al. (2016) presents a difference-in-differences like procedure using placebo (negative) outcomes ( $N$ ) that are known to not be affected by the treatment but may be affected by the same confounding factors.

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- ▶ NOC average estimate:

$$\frac{1}{n_{11}} \sum_{i=1}^{n_{11}} Y_{11,i} - \frac{1}{n_{11}} \sum_{i=1}^{n_{11}} \hat{F}_{Y_{01}}^{-1}(\hat{F}_{N_{01}}(N_{11,i}))$$

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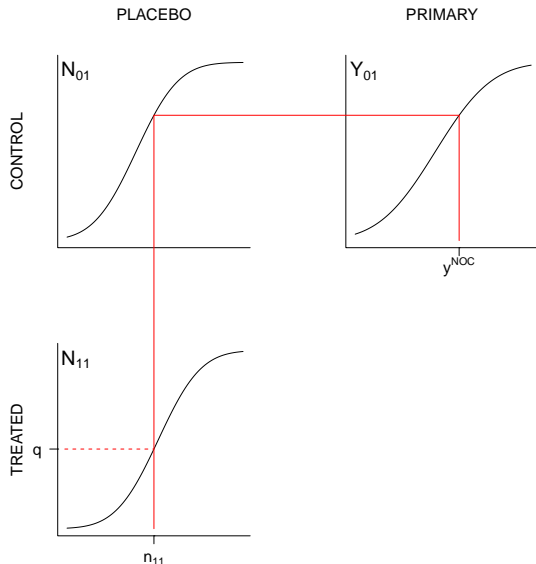
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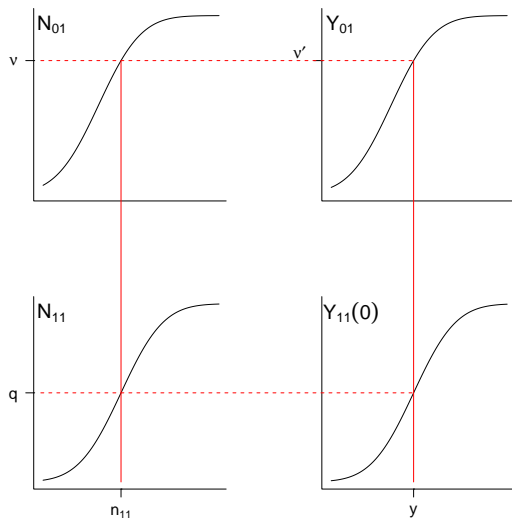
- ▶ Imputation can again be done for each quantile ( $\hat{F}_{Y_{11}(0)}^{-1}(q)$ ).

# NOC procedure for imputing $Y(0)$ for quantile $q$



Relies on variable-invariant confounding on the quantile scale.

# Equi-confounding assumption



# NOC procedure

NOC estimate of ATT:

$$\widehat{NOC} = .004 (.029)$$

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Lack of effect contradicts previous findings, but is the variable-invariant confounding assumption reasonable?



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# NOCNOC procedure

- ▶ no treatment effect in pre-treatment period, so figure out  $q'$  of  $N_{10}$  that would have produced  $q$  of  $Y_{10}$  for NOC in the pre-treatment period

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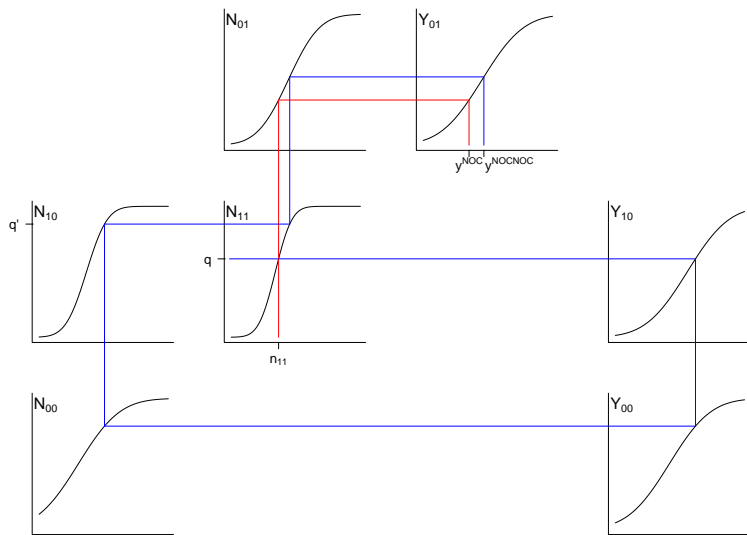
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- ▶ use  $q'$  instead of  $q$  in the posttreatment NOC procedure
- ▶ relies on time-invariant, variable-variant confounding assumption
- ▶ NOCNOC average estimate:

$$\frac{1}{k_{11}} \sum_{i=1}^{k_{11}} Y_{11,i} - \frac{1}{k_{10}} \sum_{i=1}^{k_{10}} \hat{F}_{Y_{01}}^{-1}(\hat{F}_{N_{01}}(\hat{F}_{N_{11}}^{-1}(\hat{F}_{N_{10}}(\hat{F}_{N_{00}}^{-1}(\hat{F}_{Y_{00}}^{-1}(\hat{F}_{Y_{00}}(Y_{10,i})))))))$$

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Reduced effect so less confidence in finding.

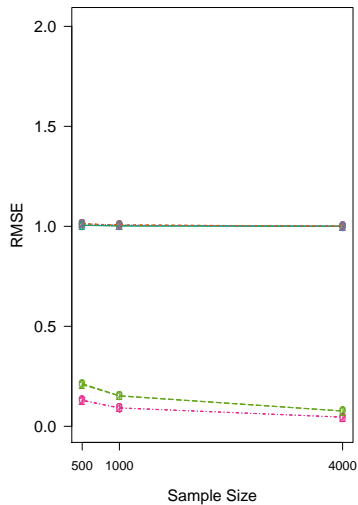
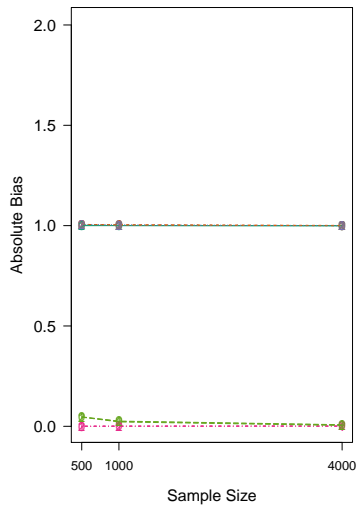
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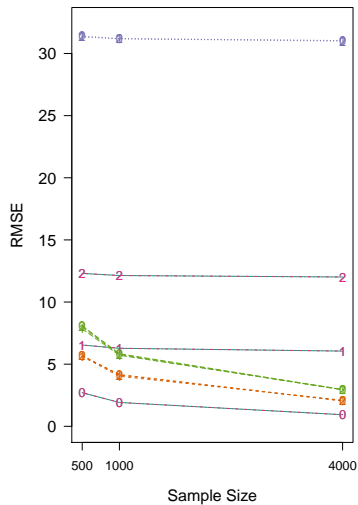
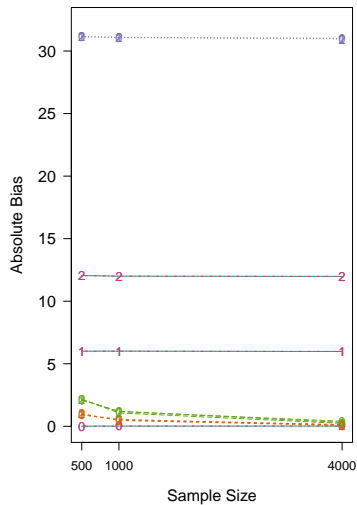
SIMULATION RESULTS

# Simulation with linear DiDiD Model



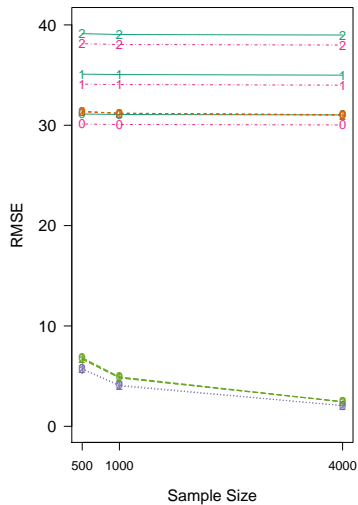
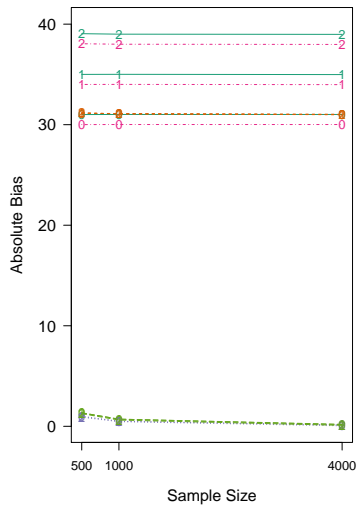
Linear DiD   CiC   NOC   Linear DiDiD   NOCNO

# Simulation with CiC Model



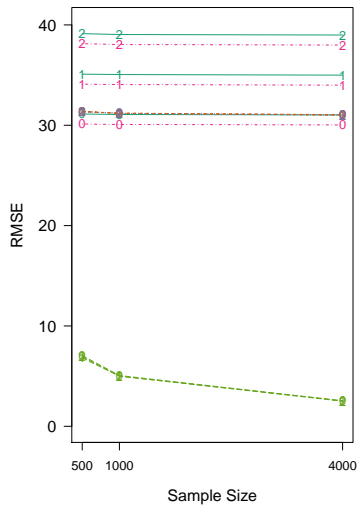
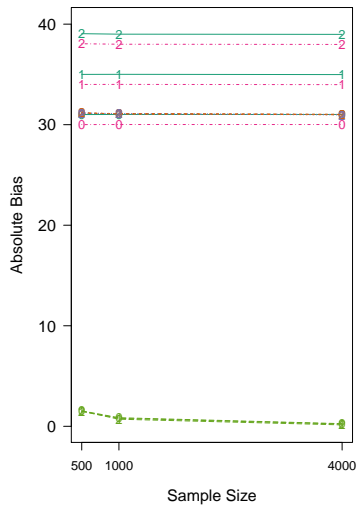
Linear DiD   CiC   NOC   Linear DiDiD   NOCNOc

# Simulation with NOC Model



Linear DiD CiC NOC Linear DiDiD NOCNO

# Simulation with NOCNOC Model



Linear DiD CiC NOC Linear DiDiD NOCNOC

# Conclusion and Future Work

- ▶ Generalized nonlinear DiDiD allows:
  - ▶ robustness check of linear DiDiD
  - ▶ use of placebo outcomes in lieu of third differencing group



# Conclusion and Future Work

- ▶ Generalized nonlinear DiDiD allows:
  - ▶ robustness check of linear DiDiD
  - ▶ use of placebo outcomes in lieu of third differencing group
- ▶ Inclusion of covariates almost done
- ▶ Nonparametric analysis almost done (outcomes need only be ranked, not fully measured)

Thank You

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## Extension: ATC with Placebo Outcomes $N$

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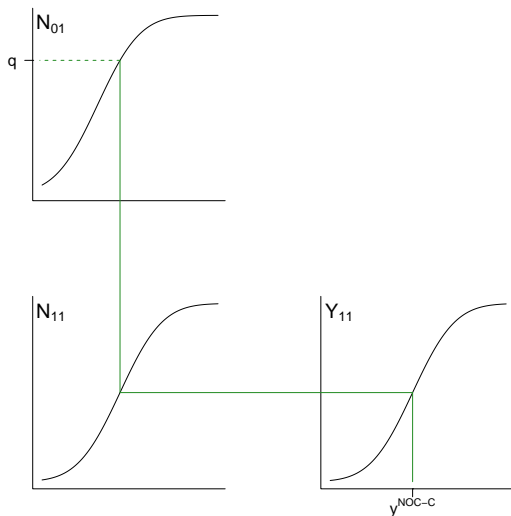
NOC estimator for ATC:

$$\frac{1}{k_{01}} \sum_{i=1}^{k_{01}} \hat{F}_{Y_{11}}^{-1}(\hat{F}_{N_{11}}(N_{01,i})) - \hat{E}[Y_{01}]$$

where  $k_{01}$  is the number of control units in the post-treatment period for the placebo outcome.

# NOC procedure for imputing $Y_{01}(1)$ for quantile $q$

Imputation can be done at each quantile  $\hat{F}_{Y_{01}(1)}^{-1}(q)$



## Extension: ATT with Surrogate Outcomes

Proxy/surrogate: no confounding, but effect on  $S$  is the same as the effect on  $Y$  on the quantile scale.

Surrogate estimator for ATT:

$$\hat{E}[Y_{11}] - \frac{1}{n_{01}} \sum_{i=1}^{n_{01}} \hat{F}_{Y_{11}}^{-1}(\hat{F}_{S_{11}}(S_{01,i}))$$

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1. Compare with NOC estimator for ATC:

$$\frac{1}{k_{01}} \sum_{i=1}^{k_{01}} \hat{F}_{Y_{11}}^{-1}(\hat{F}_{N_{11}}(N_{01,i})) - \hat{E}[Y_{01}]$$



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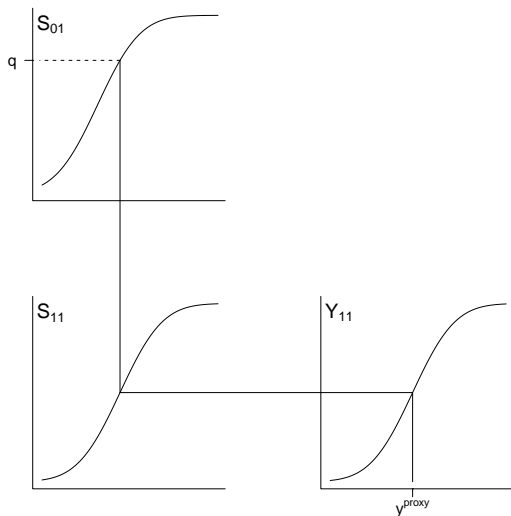
Surrogate estimator for ATT:

$$\hat{E}[Y_{11}] - \frac{1}{n_{01}} \sum_{i=1}^{n_{01}} \hat{F}_{Y_{11}}^{-1}(\hat{F}_{S_{11}}(S_{01,i}))$$

2. No primary outcome measures on the control used; neither  $Y_{01}$  nor  $Y_{00}$  are included in the formula.

# Surrogate ATT estimator for imputing $Y(0)$ for quantile $q$

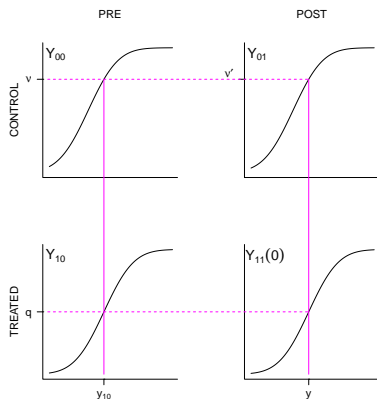
Imputation can be done at each quantile  $\hat{F}_{Y_{11}(0)}^{-1}(q)$



# Key assumptions of CiC

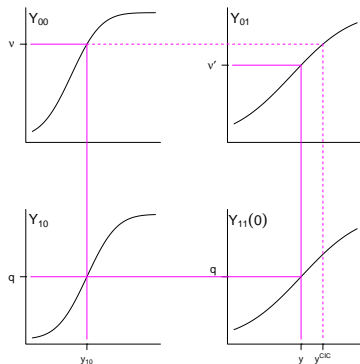
# Key assumptions of CiC

1. for full counterfactual distribution, assume equi-confounding for each quantile  $q$



# Key assumptions of CiC

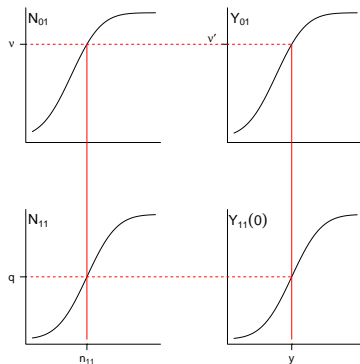
1. for full counterfactual distribution, assume equi-confounding for each quantile  $q$
2. for ATT, only need equi-confounding to hold on average ( $E_q[y - y^{CiC}] = 0$ )



# Key assumption of NOC

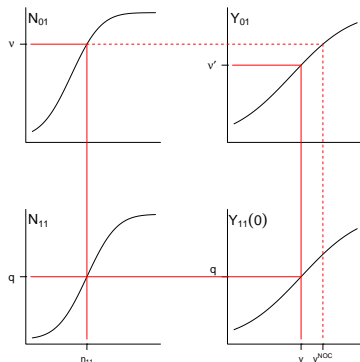
# Key assumption of NOC

1. for full counterfactual distribution, assume equi-confounding for each quantile  $q$



# Key assumption of NOC

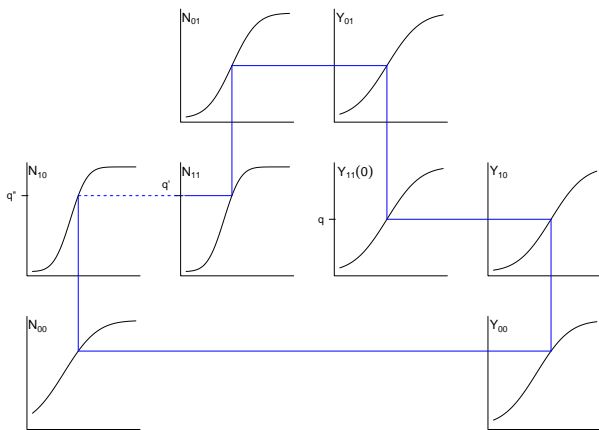
1. for full counterfactual distribution, assume equi-confounding for each quantile  $q$
2. for ATT, only need equi-confounding to hold on average ( $E_q[y - y^{NOC}] = 0$ )





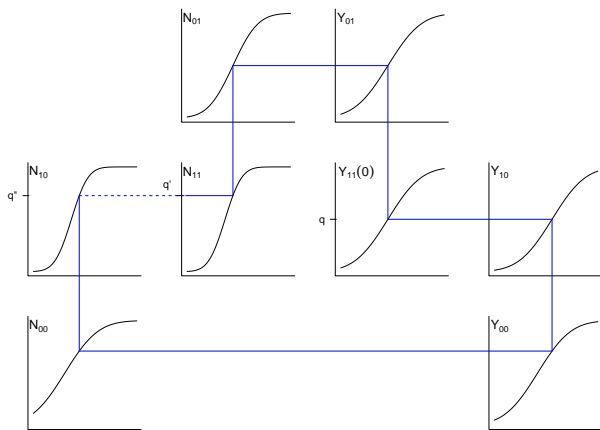
# Key assumption of NOCNOC

1. assume the  $q''$  that leads to  $F_{Y_{10}}(q)$  in pre-treatment period equals the  $q'$  that leads to  $F_{Y_{11}(0)}(q)$  in post-treatment period



# Key assumption of NOCNOC

1. assume the  $q''$  that leads to  $F_{Y_{10}}(q)$  in pre-treatment period equals the  $q'$  that leads to  $F_{Y_{11}(0)}(q)$  in post-treatment period
2. for ATT only need to hold on average  
 $E_q[y_{11}(0) - y^{NOCNOC}] = 0$



# Reduced form NOCNO assumptions

Time-invariant variable-variant confounding:

$$F_{N_{11}}(F_{N_{01}}^{-1}(F_{Y_{01}}(F_{Y_{11}(0)}^{-1}(q)))) = F_{N_{10}}(F_{N_{00}}^{-1}(F_{Y_{00}}(F_{Y_{10}}^{-1}(q)))) , q \in [0, 1]$$

Support condition:

if  $0 < f_{Y_{10}}(y_{10})$ , then  $0 < F_{Y_{00}}(y_{10}) < 1$ ,

$0 < F_{N_{10}}(F_{N_{00}}^{-1}(F_{Y_{00}}(y_{10}))) < 1$ , and

$0 < F_{N_{01}}(F_{N_{11}}^{-1}(F_{N_{10}}(F_{N_{00}}^{-1}(F_{Y_{00}}(y_{10})))))) < 1$

# Generative model NOCNOC assumptions

No effect of treatment on placebo outcome:

$$N_{at}(a) = N_{at} \text{ for } a = 0, 1$$

$$Y_{at}(a) = Y_{at} \text{ if } A = a$$

Unconfounded conditional on unmeasured  $U_t$  and  $W_t$ :

$$A_t \perp\!\!\!\perp Y_t(0) | U_t \text{ for } t = 0, 1$$

$$A_t \perp\!\!\!\perp N_t | W_t \text{ for } t = 0, 1$$

Control potential outcomes monotone in  $U_t$  and  $W_t$ :

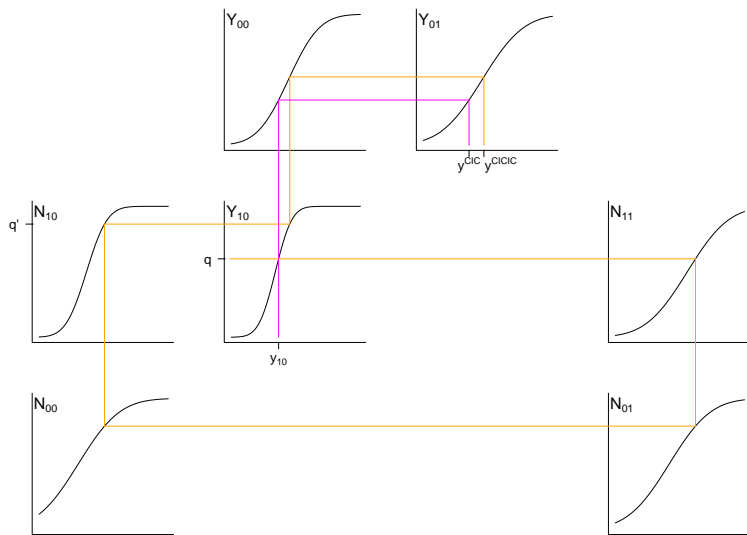
$$Y_{at}(0) = h_{yt}(U_{at}) \text{ where } h_{yt}(U_{at}) \text{ is monotone increasing}$$

$$N_{at} = h_{nt}(W_{at}) \text{ where } h_{nt}(W_{at}) \text{ is monotone increasing}$$

Time-invariant unobserved variable-variant confounding:

$$F_{W_{11}}(F_{W_{01}}^{-1}(F_{U_{01}}(F_{U_{11}(0)}^{-1}(q)))) = F_{W_{10}}(F_{W_{00}}^{-1}(F_{U_{00}}(F_{U_{10}}^{-1}(q)))), q \in [0, 1]$$

# CiCiC procedure

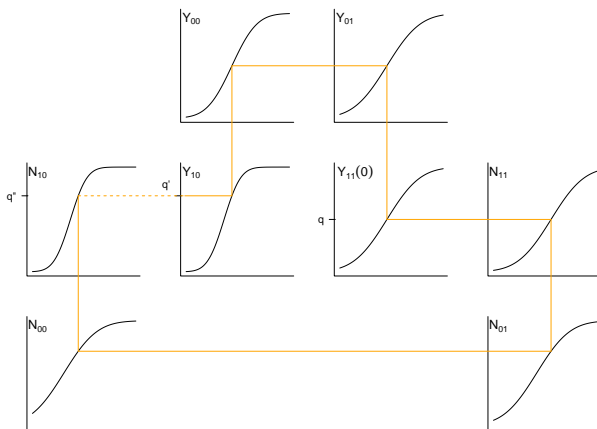


## Key assumption of CICIC

1. assume that if  $q'$  leads to  $F_{N_{11}(q)}$  for the negative outcome, then  $q'$  leads to  $F_{Y_{11}(q)}(0)$  for the outcome

# Key assumption of CICIC

1. assume that if  $q'$  leads to  $F_{N_{11}}(q)$  for the negative outcome, then  $q'$  leads to  $F_{Y_{11}(q)}(0)$  for the outcome
2. for ATT only need to hold on average  $E_q[y_{11}(0) - y^{CiCiC}] = 0$



# Efficacy Measure

1. I feel I can influence political decisions that affect my life.  
(Strongly agree to strongly disagree, 5 point scale)
2. I feel I am as well-informed about politics and government as most people. (Strongly agree to strongly disagree, 5 point scale)
3. Which of the following is closer to your view? (a) Politics is complicated and I usually do not understand what politicians are doing, (b) Most of the time I understand what politicians are doing