Causal Graphs: Problem Set 3 Solution

Instrumental Variables

1. Assume that Z is a binary instrument, D is a binary treatment, and Y is a generic outcome. Assume that SUTVA and randomization of Z hold. (Note that we have not assumed monotonicity or the exclusion restriction yet, although this will change later in the problem.)

   a. If we re-parameterize $D_i(0)$ and $D_i(1)$ as:

   $$D_i(z) = \alpha_{i0} + \alpha_{i1} \cdot z$$

   then what values do α_{i0} and α_{i1} take for always takers, never takers, defiers and compliers.

   always takers ($\alpha_{i0} = 1$ and $\alpha_{i1} = 0$)
   never takers ($\alpha_{i0} = 0$ and $\alpha_{i1} = 0$)
   defiers ($\alpha_{i0} = 1$ and $\alpha_{i1} = -1$)
   compliers ($\alpha_{i0} = 0$ and $\alpha_{i1} = 1$)

   b. Characterize the monotonicity assumption within this new parameterization. $\alpha_{i1} \geq 0$

   c. Characterize the non-zero average causal effect of Z on D assumption within this new parameterization. $E[\alpha_{i1}] \neq 0$

   d. Suppose we parameterize $Y_i(0,0)$, $Y_i(0,1)$, $Y_i(1,0)$, and $Y_i(1,1)$ as:

   $$Y_i(z, d) = \beta_{i0} + \beta_{i1} \cdot z + \beta_{i2} \cdot d + \beta_{i3} \cdot z \cdot d$$

   Does this parameterization represent any additional assumptions? Explain.

   This parameterization does not represent any additional assumptions. We have enough betas to represent all joint potential outcomes.
   $Y_i(0,0) = \beta_{i0}$
   $Y_i(0,1) = \beta_{i0} + \beta_{i2}$
   $Y_i(1,0) = \beta_{i0} + \beta_{i1}$
   $Y_i(1,1) = \beta_{i0} + \beta_{i1} + \beta_{i2} + \beta_{i3}$

   e. Characterize the exclusion restriction within this parameterization. $\beta_{i1} = 0$ and $\beta_{i3} = 0$

   f. When the exclusion restriction holds as well as non-zero average causal effect of Z on D in addition to SUTVA and the randomization of Z, write the IV estimand using this parameterization.

   $$\frac{E[\alpha_{i1} \cdot \beta_{i2}]}{E[\alpha_{i1}]}$$

   This is the average effect of D on Y among those with $\alpha_{i1} = 1$. In other words, this is the complier average causal effect.

g. Write the IV estimand using this parameterization when the exclusion restriction doesn't hold.

$$\frac{E[\beta_{i1} + \alpha_{i1} \cdot \beta_{i2} + \beta_{i3}(\alpha_{i0} + \alpha_{i1})]}{E[\alpha_{i1}]}$$

h. Suppose now that D takes three values (0,1, and 2). If we maintain the use of our new parameterization, what new assumptions have we made about the joint potential outcomes $Y_i(z, d)$ for $z = 0, 1$ and $d = 0, 1, 2$? Linearity, that $Y_i(z, 2) - Y_i(z, 1) = Y_i(z, 1) - Y_i(z, 0)$ for $z = 0, 1$

i. If these new assumptions hold, as well as the monotonicity, non-zero effect, and exclusion restrictions (as specified in our parameterization), what is the IV estimand and how can we interpret it?

$$\frac{E[\alpha_{i1} \cdot \beta_{i2}]}{E[\alpha_{i1}]}$$

This is now the weighted average effect of D on Y where the weights are determined by the size of the first stage effect α_{i1}. However, now α_{i1} can take the values $0, 1, 2$, so some observations get weight zero, some get a weight of one, and some get a weight of two.

2. Mediation Question

a. Download the replication files for "Unpacking the Black Box of Causality," Imai et al. (2011) and replicate the sensitivity analysis of Brader, Valentino, and Suhay (2008) done in this paper.

b. Download the replication files for "Identification and Sensitivity Analysis for Multiple Causal Mechanisms," Imai and Yamamoto (2013) and replicate the sensitivity analysis of Brader, Valentino, Suhay (2008) done in this paper.

c. Compare and contrast these results.