Lecture 2

Structural and Counterfactual Causal Models

An Introduction

Causal Inference Using Graphs
August 6, 2019

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Goals and Objectives for This Afternoon:

- Discuss relationship between nonparametric structural equation models (NPSEM) and DAGs
- Present back-door criterion for control variable selection
- Present SWIGs as an alternative to DAGs and BDC.
- Present disjunctive criterion for control variable selection

Some Typical Advice on Control Variable Selection

In sum, if the variable we leave out is positively correlated with the variable we include and also has itself a positive effect on the dependent variable, then our mispecified model will produce an upward-biased estimator of the effect of X_1 on Y. [Ashenfelter, Levine, and Zimmerman, p. 189]

Some Typical Advice on Control Variable Selection

The short answer to the questions posed above is that the OLS estimates based on the equation containing too many explanatory variables will indeed be unbiased estimates of the true parameters. [Ashenfelter, Levine, and Zimmerman, p. 187]

Some Typical Advice on Control Variable Selection

...we cannot be sure that treatment assignment is strongly ignorable given the observed covariates because there may remain unmeasured covariates that affect both outcomes and treatment assignment. [Rosenbaum and Rubin 1984, p. 522]

Some Typical Advice on Control Variable Selection

control for all pre- treatment covariates [Rubin 2009]

The Typical Advice can be Misleading

In the rest of this lecture, we will explore why this sort of typical advice can be misleading, and present some alternative criteria for covariate/control variable selection.

Overview

1. **Nonparametric Structural Equation Models and DAGs**
2. **Back-door Criterion**
3. **SWIGs**
4. **Disjunctive Criterion**

Model Definition

Definition (Deterministic Nonparametric Structural Equation Model (DNPSEM))

A DNPSEM is a triple

$$M = \langle \mathbf{U}, \mathbf{V}, \mathbf{g} \rangle$$

where:

1. $\mathbf{U}$ is a set of exogenous variables
2. $\mathbf{V}$ is a set of endogenous variables, that are uniquely determined by the variables in the model—that is, variables in $\mathbf{U} \cup \mathbf{V}$; and
3. $\mathbf{g}$ is a set of functions (one for each endogenous variable) that maps $\mathbf{U} \cup \mathbf{V}$ onto $\mathbf{V}$.

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3. $\mathbf{g}$ is a set of functions (one for each endogenous variable) that maps $\mathbf{U} \cup \mathbf{V}$ onto $\mathbf{V}$.

Note: we will often find it convenient to represent different variables in $\mathbf{V}$ with the the subsets of variables $\mathbf{X}$,$\mathbf{Y}$, and $\mathbf{Z}$.

Example (Stroke Victims)

Suppose a group of stroke victims are admitted to a randomized placebo controlled trial where the active treatment is a blood thinning drug. After a period of time, patients are observed to be alive or dead.

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For this example, a DNPSEM for each individual would include two exogenous variables: one for treatment randomization and one that specifies response to treatment.

$$U_1 = \begin{cases} \text{heads} \\ \text{tails} \end{cases}$$

$$U_2 = \begin{cases} \text{always recover} \\ \text{never recover} \\ \text{saved by drug} \\ \text{killed by drug} \end{cases}$$

In this case, the possible values of U_2 might correspond to whether a stroke victim suffers from leaking or clotting type strokes and the severity of the condition.

Example (Stroke Victims)

... two endogenous variables: observed treatment (active/placebo) and observed outcome (alive/dead) and their respective functions.

$$X \leftarrow g_x(u_1)$$
$$Y \leftarrow g_y(x, u_2)$$

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We could express this model graphically as:

Example (Stroke Victims)

Observed X and Y for the possible values of U_1 and U_2.

$g_x(u_1)$	
U_1	X
heads	active
tails	placebo

$g_y(g_x(u_1), u_2)$	Y	
U_2	$U_1 =$ heads	$U_1 =$ tails
always recover		
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always recover	alive	alive
never recover	dead	dead
saved by drug	alive	dead
killed by drug	dead	alive

Submodel Definition

Definition (Submodel)

Let M be a DNPSEM, and let $\mathbf{X} \subset \mathbf{V}$ be a set of *endogenous* intervention variables. A submodel $M_{\mathbf{x}}$ of M is the DNPSEM

$$M_{\mathbf{x}} = \langle \mathbf{U}, \mathbf{V}, \mathbf{g}_{-\mathbf{x}} \rangle$$

where

$$\mathbf{g}_{-\mathbf{x}} = \{g_w : w \notin \mathbf{x}\}$$

Effectively, a submodel makes the previously endogenous intervention variables exogenous.

Example (Stroke Victims)

In our stroke victim example, we could create a submodel by breaking the random treatment assignment mechanism and forcing the subject to take one of the two values.

$$X \leftarrow x$$
$$Y \leftarrow g_y(x, u_2)$$

We could express this submodel graphically as:

Potential Outcome Definition

Definition (Potential Outcome)

Let M be a DNPSEM, and let $\mathbf{X} \subset \mathbf{V}$ be a set of *endogenous* intervention variables and let $\mathbf{Y} \subset \mathbf{V}$ be a disjoint set of outcome variables. A potential outcome $\mathbf{Y}(\mathbf{x}, \mathbf{u})$ is the unique output of the functions

$$\mathbf{g}_{\mathbf{y}} = \{g_w : w \in \mathbf{y}\}$$

in the submodel M_x.

This quantity can be seen as analogous to the potential outcomes that are taken to be primitive in the Neyman-Rubin causal model.

Example (Stroke Victims)

In the stroke victim example, each individual will have two potential outcomes corresponding to the two levels of treatment.

Potential Outcomes	$Y(x, u_2)$	
U_2	$X = 1$ (active)	$X = 0$ (placebo)
always recover	1 (alive)	1 (alive)
never recover	0 (dead)	0 (dead)
saved by drug		
killed by drug		

Individual (Total) Causal Effects

Definition (Individual Total Causal Effects (ICEs))

Let M be a DNPSEM, and index individual units by $i = 1, ..., n$. Let $Y_i(X_i = x, \mathbf{u})$ be the potential outcome for unit i when X_i is set to the value x. We define the indiviudal (total) causal effect for unit i to be:

$$Y_i(X_i = x, \mathbf{u}) - Y_i(X_i = x', \mathbf{u})$$

Notice the following about this definition:

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1. $\mathbf{u}$ is not indexed by i, hence it represents the background variables for all units. This leaves open the possibility for a type of dependence across units.
2. The intervention variable X_i is indexed by i, but the functional definition of the potential outcomes does not preclude the possibility that intervening on unit i will affect Y_i though the other units.

Example (Stroke Victims)

In the stroke victim example, each individual will have two potential outcomes corresponding to the two levels of treatment (assume all variables are indexed by i).

Potential Outcomes	$Y(x, u_2)$		
U_2	$X = 1$	$X = 0$	ICE
always recover	1 (alive)	1 (alive)	
never recover	0 (dead)	0 (dead)	
saved by drug	1 (alive)	0 (dead)	1
killed by drug	0 (dead)	1 (alive)	

Individual Joint Causal Effects

Definition (Individual Partial Causal Effects (IJCE))

Let M be a DNPSEM and index individual units by $i = 1, ..., n$. Let $Y_i(X_i = x, Z_i = z, \mathbf{u})$ be the potential outcome for unit i when X_i is set to the value x and Z_i is set to the value z and let $Y_i(X_i = x', Z_i = z', \mathbf{u})$ be the potential outcome for unit i when X_i is set to the value x' and Z_i is set to the value z' . We define the individual joint causal effect for unit i:

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Notice the following about this definition:

1. A controlled direct causal effect could be defined by fixing the second intervention variable at a particular value ($z = z'$) while changing the first intervention variable.
2. We could expand this definition to include many intervention variables and to allow changes over multiple intervention variables.

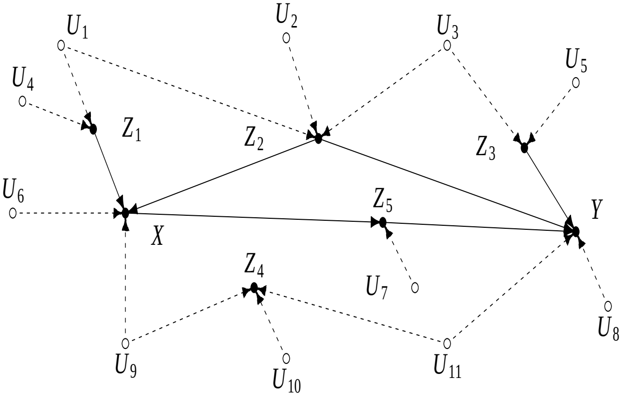
1 Nonparametric Structural Equation Models and DAGs

2 Back-door Criterion

3 SWIGs

4 Disjunctive Criterion

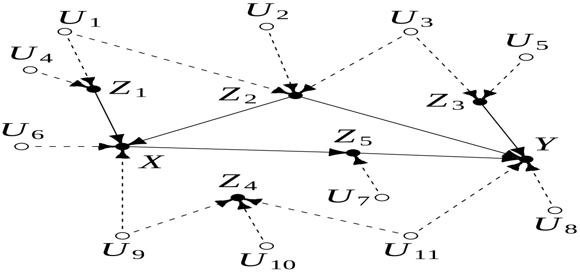
Directed Acyclic Graphs



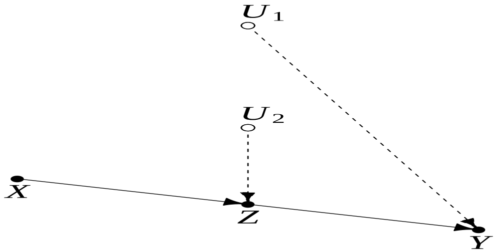
Choosing Conditioning Variables

Definition (Back-Door Criterion (Pearl (2000, p. 79)))

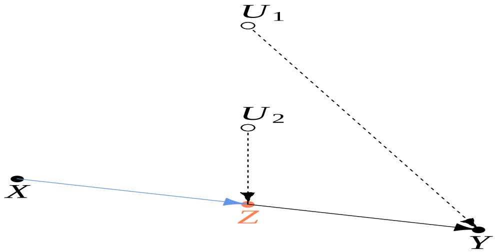
1. none of the conditioning variables is post-X; and
2. X is "d-separated" from Y by the conditioning variables in the graph formed by deleting all arrows out of X.



BD1: Don't condition on post-treatment variables



BD1: Don't condition on descendants of X



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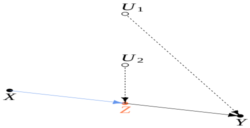


In sum, if the variable we leave out is positively correlated with the variable we include and also has itself a positive effect on the dependent variable, then our mispecified model will produce an upward-biased estimator of the effect of X_1 on Y. [Ashenfelter, Levine, and Zimmerman, p. 2003]

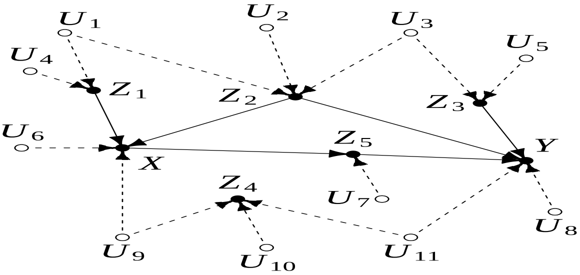
BD2: X is d-separated from Y by the conditioning variables in the graph formed by deleting all arrows out of X.

Definition (d-Separation (Pearl (2000, p. 16)))

1. X is d-Separated from Y if all paths are blocked
2. paths are blocked by "unconditioned colliders" or conditioned non-colliders

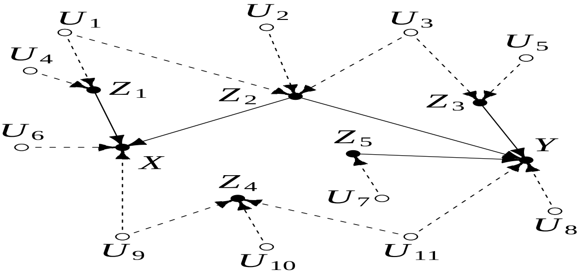
d-Separation allows conditional independence relations to be read from a DAG consistent with a probabilistic model.

Example: Sufficient Conditioning Sets



Remove arrows out of X.

Example: Sufficient Conditioning Sets



Recall that paths are blocked by “unconditioned colliders” or conditioned non-colliders

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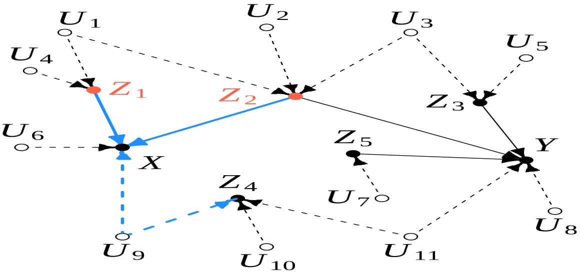

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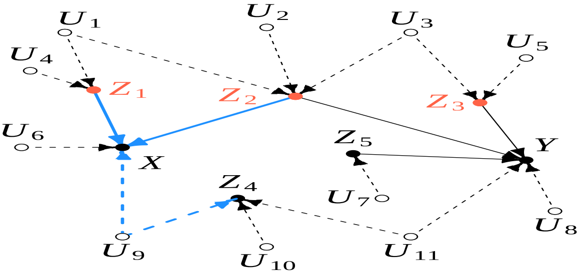

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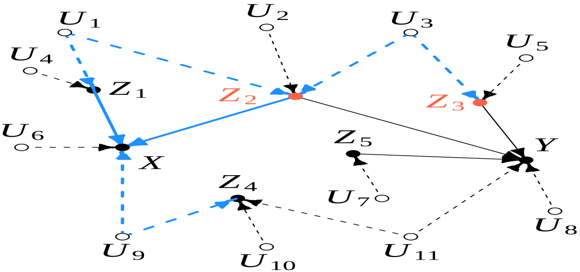

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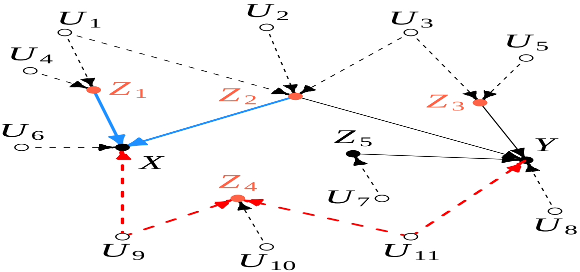
Example: Non-sufficient Conditioning Sets



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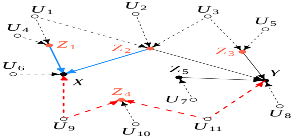
Example: Non-sufficient Conditioning Sets



The short answer to the questions posed above is that the OLS estimates based on the equation containing too many explanatory variables will indeed be unbiased estimates of the true parameters. [Ashenfelter, Levine, and Zimmerman, p. 187]

Section Conclusions

- Knowledge of the temporal order of observed variables is not sufficient to correctly specify $\mathbf{Z}$
  - Adjusting for “pre-treatment in time” variables can induce bias
  - Adjusting for “post-treatment in time” variables can eliminate bias
- There can be multiple *non-nested* (and even *disjoint*) sufficient conditioning sets
- Obviously, some models will have no sufficient observable conditioning set

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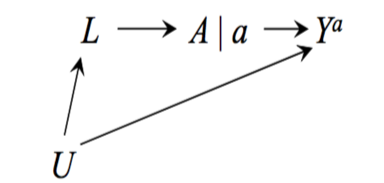
One issue with the BDC is that it doesn't directly relate to statements about potential outcomes (e.g, $Y^a \perp\!\!\!\perp A|L$). SWIGs provide a more direct bridge between the potential outcomes approach and graphs.

Definition (Single World Intervention Graph)

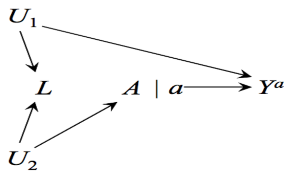
See Richardson and Robins (2013)

Example



SWIG Example 2

Example



An Easier Criterion

With a single graphical structure, the BDC and SWIGs allow us to find all possible sufficient sets, or if a set of plausible graphs is proposed, the BDC and SWIGs allow us to see if there exists a set that is sufficient for all graphs.

What if we don't want to propose graphs?

Disjunctive Criterion

Definition (Disjunctive Criterion)

Let C be a set of measured pretreatment covariates. Let $S \subseteq C$ be the subset of C whose elements are either causes of A or of Y or of both. If there is any set $W \subseteq C$ such that W blocks all backdoor paths from A to Y then S does also and thus $Y_a \perp\!\!\!\perp A | S$.

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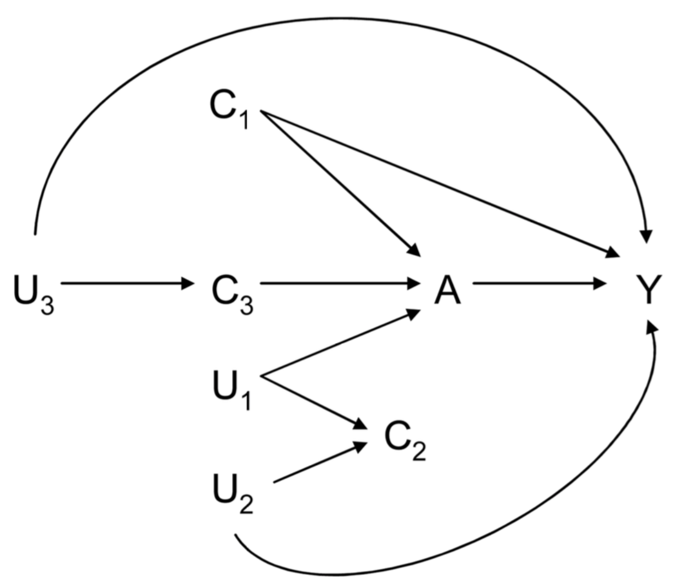
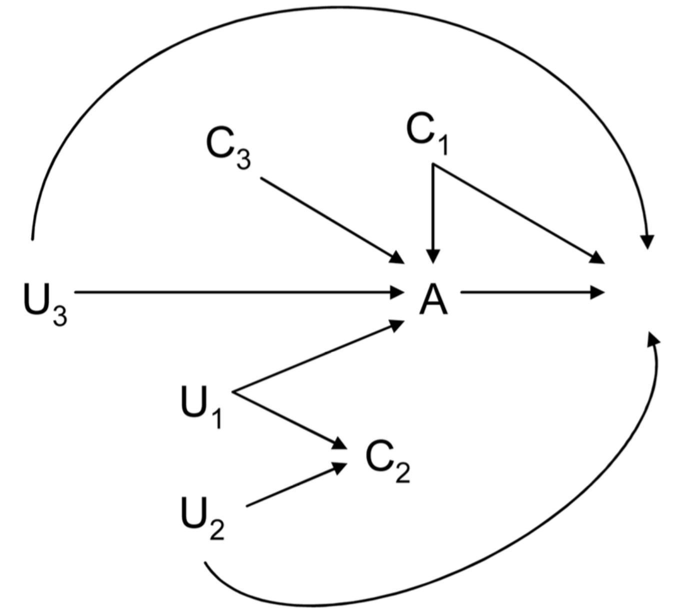
Note: while the disjunctive criterion does not require the specification of a graph or graphs, it does require assumptions about the true DAG.

Example (Figure 1 VS)



Example (Figure 2 VS)



Propositions 1 and 2

Definition (Propositions 1 and 2)

Suppose that for some set S, $Y_a \perp\!\!\!\perp A|S$ and that under some ordering of the elements of S, $(S_1, \ldots, S_n)$, and for some k, $Y \perp\!\!\!\perp S_{i+1}|(A, S_1, \ldots, S_i)$ for $i = k, \ldots, n-1$, then $Y_a \perp\!\!\!\perp A|(S_1, \ldots, S_k)$.

Suppose that for some set S, $Y_a \perp\!\!\!\perp A|S$, that the distribution of (Y, A, S) is faithful to the underlying causal diagram G, and that under some ordering of the elements of S, $(S_1, \ldots, S_n)$, and for some k, $Y \perp\!\!\!\perp S_i|(A, S_1, \ldots, S_k)$ for $i = k+1, \ldots, n$ then $Y_a \perp\!\!\!\perp A|(S_1, \ldots, S_k)$.

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End of slides for Day 1. Examples on board or R for remainder of day.