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NJIT Tigers

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Contest (1)

template.cpp

```cpp
#include <bits/stdc++.h>
using namespace std;

typedef long long ll;
typedef vector<int> vi;
typedef pair<int,int> pi;
typedef tuple<int, int, int> iii;

#define f first
#define s second
#define PB push_back
#define MP make_pair
#define MAX 100
#define LSOne(S) ((S) & -(S))
#define sz(x) int((x).size())
#define all(x) begin(x), end(x)

#define FOR(i,a,b) for(int i=(a),_b=(b); i<=_b; i++)
#define FORD(i,a,b) for(int i=(a),_b=(b); i>=_b; i--)
#define REP(i,a) for(int i=0,_a=(a); i<_a; i++)
#define DEBUG(x) { cout << #x << " = "; cout << (x) << endl; }
#define PR(a,n) { cout << #a << " = "; FOR(_,1,n) cout << a[_]
    << ' '; cout << endl; }
#define PR0(a,n) { cout << #a << " = "; REP(_,n) cout << a[_]
    << ' '; cout << endl; }

const int INF = 1e9 + 5;
const int MOD = 1000007;

int main()
{
    ios_base::sync_with_stdio(false);
    cin.tie(nullptr);

    return 0;
}
```

template2.cpp

```cpp
#include <bits/stdc++.h>
using namespace std;

#define rep(i, a, b) for(int i = a; i < (b); ++i)
#define all(x) begin(x), end(x)
#define sz(x) (int)(x).size()
typedef long long ll;
typedef pair<int, int> pii;
typedef vector<int> vi;

int main() {
  cin.tie(0)->sync_with_stdio(0);
  cin.exceptions(cin.failbit);
}
```

troubleshoot.txt

```
Pre-submit:
Write a few simple test cases if sample is not enough.
Are time limits close? If so, generate max cases.
Is the memory usage fine?
Could anything overflow?
Make sure to submit the right file.

Wrong answer:
Print your solution! Print debug output, as well.
Are you clearing all data structures between test cases?
Can your algorithm handle the whole range of input?
Read the full problem statement again.
Do you handle all corner cases correctly?
Have you understood the problem correctly?
Any uninitialized variables?
Any overflows?
Confusing N and M, i and j, etc.?
Are you sure your algorithm works?
What special cases have you not thought of?
Are you sure the STL functions you use work as you think?
Add some assertions, maybe resubmit.
Create some testcases to run your algorithm on.
Go through the algorithm for a simple case.
Go through this list again.
Explain your algorithm to a teammate.
Ask the teammate to look at your code.
Go for a small walk, e.g. to the toilet.
Is your output format correct? (including whitespace)
Rewrite your solution from the start or let a teammate do it.

Runtime error:
Have you tested all corner cases locally?
Any uninitialized variables?
Are you reading or writing outside the range of any vector?
Any assertions that might fail?
Any possible division by 0? (mod 0 for example)
Any possible infinite recursion?
Invalidated pointers or iterators?
Are you using too much memory?
Debug with resubmits (e.g. remapped signals, see Various).

Time limit exceeded:
Do you have any possible infinite loops?
What is the complexity of your algorithm?
Are you copying a lot of unnecessary data? (References)
How big is the input and output? (consider scanf)
Avoid vector, map. (use arrays/unordered_map)
What do your teammates think about your algorithm?

Memory limit exceeded:
What is the max amount of memory your algorithm should need?
Are you clearing all data structures between test cases?
```

Mathematics (2)

2.1 Equations

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The extremum is given by $x = -b/2a$.

$$\begin{aligned} ax + by &= e \\ cx + dy &= f \end{aligned} \Rightarrow \begin{aligned} x &= \frac{ed - bf}{ad - bc} \\ y &= \frac{af - ec}{ad - bc} \end{aligned}$$

In general, given an equation $Ax = b$, the solution to a variable x_i is given by

$$x_i = \frac{\det A_i'}{\det A}$$

where A_i' is A with the i'th column replaced by b.

2.2 Recurrences

If $a_n = c_1 a_{n-1} + \cdots + c_k a_{n-k}$, and $r_1, \ldots, r_k$ are distinct roots of $x^k - c_1 x^{k-1} - \cdots - c_k$, there are $d_1, \ldots, d_k$ s.t.

$$a_n = d_1 r_1^n + \cdots + d_k r_k^n.$$

Non-distinct roots r become polynomial factors, e.g. $a_n = (d_1 n + d_2) r^n$.

2.3 Trigonometry

$$\sin(v + w) = \sin v \cos w + \cos v \sin w$$
$$\cos(v + w) = \cos v \cos w - \sin v \sin w$$

$$\tan(v + w) = \frac{\tan v + \tan w}{1 - \tan v \tan w}$$
$$\sin v + \sin w = 2 \sin \frac{v + w}{2} \cos \frac{v - w}{2}$$
$$\cos v + \cos w = 2 \cos \frac{v + w}{2} \cos \frac{v - w}{2}$$

$$(V + W) \tan(v - w)/2 = (V - W) \tan(v + w)/2$$

where V, W are lengths of sides opposite angles v, w.

$$a \cos x + b \sin x = r \cos(x - \phi)$$
$$a \sin x + b \cos x = r \sin(x + \phi)$$

where $r = \sqrt{a^2 + b^2}, \phi = \text{atan2}(b, a)$.

2.4 Geometry

2.4.1 Triangles

Side lengths: a, b, c

Semiperimeter: $p = \frac{a + b + c}{2}$

Area: $A = \sqrt{p(p - a)(p - b)(p - c)}$

Circumradius: $R = \frac{abc}{4A}$

Inradius: $r = \frac{A}{p}$

Length of median (divides triangle into two equal-area triangles): $m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

Length of bisector (divides angles in two):

$$s_a = \sqrt{bc\left[1 - \left(\frac{a}{b + c}\right)^2\right]}$$

Law of sines: $\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c} = \frac{1}{2R}$

Law of cosines: $a^2 = b^2 + c^2 - 2bc \cos \alpha$

Law of tangents: $\frac{a + b}{a - b} = \frac{\tan \frac{\alpha + \beta}{2}}{\tan \frac{\alpha - \beta}{2}}$

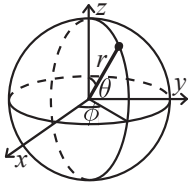
2.4.2 Quadrilaterals

With side lengths a, b, c, d, diagonals e, f, diagonals angle θ, area A and magic flux $F = b^2 + d^2 - a^2 - c^2$:

$$4A = 2ef \cdot \sin\theta = F \tan\theta = \sqrt{4e^2f^2 - F^2}$$

For cyclic quadrilaterals the sum of opposite angles is $180°$, $ef = ac + bd$, and $A = \sqrt{(p-a)(p-b)(p-c)(p-d)}$.

2.4.3 Spherical coordinates



$$\begin{array}{cc} x = r\sin\theta\cos\phi & r = \sqrt{x^2+y^2+z^2} \\ y = r\sin\theta\sin\phi & \theta = \operatorname{acos}(z/\sqrt{x^2+y^2+z^2}) \\ z = r\cos\theta & \phi = \operatorname{atan2}(y,x) \end{array}$$

2.5 Derivatives/Integrals

$$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan x = 1 + \tan^2 x \qquad \frac{d}{dx}\arctan x = \frac{1}{1+x^2}$$

$$\int \tan ax = -\frac{\ln|\cos ax|}{a} \qquad \int x\sin ax = \frac{\sin ax - ax\cos ax}{a^2}$$

$$\int e^{-x^2} = \frac{\sqrt{\pi}}{2}\operatorname{erf}(x) \qquad \int xe^{ax}dx = \frac{e^{ax}}{a^2}(ax-1)$$

Integration by parts:

$$\int_a^b f(x)g(x)dx = [F(x)g(x)]_a^b - \int_a^b F(x)g'(x)dx$$

2.6 Sums

$$c^a + c^{a+1} + \cdots + c^b = \frac{c^{b+1} - c^a}{c-1}, c \neq 1$$

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(2n+1)(n+1)}{6}$$

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$$

$$1^4 + 2^4 + 3^4 + \cdots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

2.7 Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots, (-\infty < x < \infty)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots, (-1 < x \leq 1)$$

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \ldots, (-1 \leq x \leq 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots, (-\infty < x < \infty)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots, (-\infty < x < \infty)$$

2.8 Probability theory

Let X be a discrete random variable with probability $p_X(x)$ of assuming the value x. It will then have an expected value (mean) $\mu = \mathbb{E}(X) = \sum_x xp_X(x)$ and variance $\sigma^2 = V(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = \sum_x (x - \mathbb{E}(X))^2 p_X(x)$ where σ is the standard deviation. If X is instead continuous it will have a probability density function $f_X(x)$ and the sums above will instead be integrals with $p_X(x)$ replaced by $f_X(x)$.

Expectation is linear:

$$\mathbb{E}(aX + bY) = a\mathbb{E}(X) + b\mathbb{E}(Y)$$

For independent X and Y,

$$V(aX + bY) = a^2V(X) + b^2V(Y).$$

2.8.1 Discrete distributions

Binomial distribution

The number of successes in n independent yes/no experiments, each which yields success with probability p is $\operatorname{Bin}(n,p)$, $n = 1, 2, \ldots, 0 \leq p \leq 1$.

$$p(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$\mu = np,\ \sigma^2 = np(1-p)$$

$\operatorname{Bin}(n,p)$ is approximately $\operatorname{Po}(np)$ for small p.

First success distribution

The number of trials needed to get the first success in independent yes/no experiments, each wich yields success with probability p is $\operatorname{Fs}(p)$, $0 \leq p \leq 1$.

$$p(k) = p(1-p)^{k-1},\ k = 1, 2, \ldots$$

$$\mu = \frac{1}{p},\ \sigma^2 = \frac{1-p}{p^2}$$

Poisson distribution

The number of events occurring in a fixed period of time t if these events occur with a known average rate κ and independently of the time since the last event is $\operatorname{Po}(\lambda)$, $\lambda = t\kappa$.

$$p(k) = e^{-\lambda}\frac{\lambda^k}{k!}, k = 0, 1, 2, \ldots$$

$$\mu = \lambda,\ \sigma^2 = \lambda$$

2.8.2 Continuous distributions

Uniform distribution

If the probability density function is constant between a and b and 0 elsewhere it is $\operatorname{U}(a,b)$, $a < b$.

$$f(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases}$$

$$\mu = \frac{a+b}{2},\ \sigma^2 = \frac{(b-a)^2}{12}$$

Exponential distribution

The time between events in a Poisson process is $\operatorname{Exp}(\lambda)$, $\lambda > 0$.

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$\mu = \frac{1}{\lambda},\ \sigma^2 = \frac{1}{\lambda^2}$$

Normal distribution

Most real random values with mean μ and variance σ^2 are well described by $\mathcal{N}(\mu, \sigma^2)$, $\sigma > 0$.

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

If $X_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$ and $X_2 \sim \mathcal{N}(\mu_2, \sigma_2^2)$ then

$$aX_1 + bX_2 + c \sim \mathcal{N}(\mu_1 + \mu_2 + c, a^2\sigma_1^2 + b^2\sigma_2^2)$$

Data structures (3)

OrderStatisticTree.h

Description: A set (not multiset!) with support for finding the n'th element, and finding the index of an element. To get a map, change `null_type`.
Time: $\mathcal{O}(\log N)$ d41d8c, 16 lines

```
#include <bits/extc++.h>
using namespace __gnu_pbds;

template<class T>
using Tree = tree<T, null_type, less<T>, rb_tree_tag,
    tree_order_statistics_node_update>;

void example() {
```

```
  Tree<int> t, t2; t.insert(8);
  auto it = t.insert(10).first;
  assert(it == t.lower_bound(9));
  assert(t.order_of_key(10) == 1);
  assert(t.order_of_key(11) == 2);
  assert(*t.find_by_order(0) == 8);
  t.join(t2); // assuming T < T2 or T > T2, merge t2 into t
}
```

HashMap.h

Description: Hash map with mostly the same API as unordered_map, but ~3x faster. Uses 1.5x memory. Initial capacity must be a power of 2 (if provided).

d41d8c, 7 lines

```
#include <bits/extc++.h>
// To use most bits rather than just the lowest ones:
struct chash { // large odd number for C
  const uint64_t C = ll(4e18 * acos(0)) | 71;
  ll operator()(ll x) const { return __builtin_bswap64(x*C); }
};
__gnu_pbds::gp_hash_table<ll,int,chash> h({},{},{},{},{1<<16});
```

SegmentTree.h

Description: Zero-indexed max-tree. Bounds are inclusive to the left and exclusive to the right. Can be changed by modifying T, f and unit.

Time: $\mathcal{O}(\log N)$

d41d8c, 19 lines

```
struct SegmentTree {
  typedef int T;
  static constexpr T unit = INT_MIN;
  T f(T a, T b) { return max(a, b); } // (any associative fn)
  vector<T> s; int n;
  Tree(int n = 0, T def = unit) : s(2*n, def), n(n) {}
  void update(int pos, T val) {
    for (s[pos += n] = val; pos /= 2;)
      s[pos] = f(s[pos * 2], s[pos * 2 + 1]);
  }
  T query(int b, int e) { // query [b, e)
    T ra = unit, rb = unit;
    for (b += n, e += n; b < e; b /= 2, e /= 2) {
      if (b % 2) ra = f(ra, s[b++]);
      if (e % 2) rb = f(s[--e], rb);
    }
    return f(ra, rb);
  }
};
```

LazySegmentTree.h

Description: Segment tree with ability to add or set values of large intervals, and compute max of intervals. Can be changed to other things. Use with a bump allocator for better performance, and SmallPtr or implicit indices to save memory.

Usage: `Node* tr = new Node(v, 0, sz(v));`

Time: $\mathcal{O}(\log N)$.

"../various/BumpAllocator.h" d41d8c, 50 lines

```
const int inf = 1e9;
struct Node {
  Node *l = 0, *r = 0;
  int lo, hi, mset = inf, madd = 0, val = -inf;
  Node(int lo,int hi):lo(lo),hi(hi){} // Large interval of -inf
  Node(vi& v, int lo, int hi) : lo(lo), hi(hi) {
    if (lo + 1 < hi) {
      int mid = lo + (hi - lo)/2;
      l = new Node(v, lo, mid); r = new Node(v, mid, hi);
      val = max(l->val, r->val);
    }
    else val = v[lo];
  }
  int query(int L, int R) {
    if (R <= lo || hi <= L) return -inf;
    if (L <= lo && hi <= R) return val;
    push();
    return max(l->query(L, R), r->query(L, R));
  }
  void set(int L, int R, int x) {
    if (R <= lo || hi <= L) return;
    if (L <= lo && hi <= R) mset = val = x, madd = 0;
    else {
      push(), l->set(L, R, x), r->set(L, R, x);
      val = max(l->val, r->val);
    }
  }
  void add(int L, int R, int x) {
    if (R <= lo || hi <= L) return;
    if (L <= lo && hi <= R) {
      if (mset != inf) mset += x;
      else madd += x;
      val += x;
    }
    else {
      push(), l->add(L, R, x), r->add(L, R, x);
      val = max(l->val, r->val);
    }
  }
  void push() {
    if (!l) {
      int mid = lo + (hi - lo)/2;
      l = new Node(lo, mid); r = new Node(mid, hi);
    }
    if (mset != inf)
      l->set(lo,hi,mset), r->set(lo,hi,mset), mset = inf;
    else if (madd)
      l->add(lo,hi,madd), r->add(lo,hi,madd), madd = 0;
  }
};
```

UnionFindRollback.h

Description: Disjoint-set data structure with undo. If undo is not needed, skip st, time() and rollback().

Usage: `int t = uf.time(); ...; uf.rollback(t);`

Time: $\mathcal{O}(\log(N))$

d41d8c, 21 lines

```
struct RollbackUF {
  vi e; vector<pii> st;
  RollbackUF(int n) : e(n, -1) {}
  int size(int x) { return -e[find(x)]; }
  int find(int x) { return e[x] < 0 ? x : find(e[x]); }
  int time() { return sz(st); }
  void rollback(int t) {
    for (int i = time(); i --> t;)
      e[st[i].first] = st[i].second;
    st.resize(t);
  }
  bool join(int a, int b) {
    a = find(a), b = find(b);
    if (a == b) return false;
    if (e[a] > e[b]) swap(a, b);
    st.push_back({a, e[a]});
    st.push_back({b, e[b]});
    e[a] += e[b]; e[b] = a;
    return true;
  }
};
```

SubMatrix.h

Description: Calculate submatrix sums quickly, given upper-left and lower-right corners (half-open).

Usage: `SubMatrix<int> m(matrix);`
`m.sum(0, 0, 2, 2); // top left 4 elements`

Time: $\mathcal{O}\left(N^2 + Q\right)$

d41d8c, 13 lines

```
template<class T>
struct SubMatrix {
  vector<vector<T>> p;
  SubMatrix(vector<vector<T>>& v) {
    int R = sz(v), C = sz(v[0]);
    p.assign(R+1, vector<T>(C+1));
    rep(r,0,R) rep(c,0,C)
      p[r+1][c+1] = v[r][c] + p[r][c+1] + p[r+1][c] - p[r][c];
  }
  T sum(int u, int l, int d, int r) {
    return p[d][r] - p[d][l] - p[u][r] + p[u][l];
  }
};
```

Matrix.h

Description: Basic operations on square matrices.

Usage: `Matrix<int, 3> A;`
`A.d = {{{{1,2,3}}, {{4,5,6}}, {{7,8,9}}}};`
`vector<int> vec = {1,2,3};`
`vec = (A^N) * vec;`

d41d8c, 26 lines

```
template<class T, int N> struct Matrix {
  typedef Matrix M;
  array<array<T, N>, N> d{};
  M operator*(const M& m) const {
    M a;
    rep(i,0,N) rep(j,0,N)
      rep(k,0,N) a.d[i][j] += d[i][k]*m.d[k][j];
    return a;
  }
  vector<T> operator*(const vector<T>& vec) const {
    vector<T> ret(N);
    rep(i,0,N) rep(j,0,N) ret[i] += d[i][j] * vec[j];
    return ret;
  }
  M operator^(ll p) const {
    assert(p >= 0);
    M a, b(*this);
    rep(i,0,N) a.d[i][i] = 1;
    while (p) {
      if (p&1) a = a*b;
      b = b*b;
      p >>= 1;
    }
    return a;
  }
};
```

LineContainer.h

Description: Container where you can add lines of the form kx+m, and query maximum values at points x. Useful for dynamic programming ("convex hull trick").

Time: $\mathcal{O}(\log N)$

d41d8c, 30 lines

```
struct Line {
  mutable ll k, m, p;
  bool operator<(const Line& o) const { return k < o.k; }
  bool operator<(ll x) const { return p < x; }
};

struct LineContainer : multiset<Line, less<>> {
  // (for doubles, use inf = 1/.0, div(a,b) = a/b)
  static const ll inf = LLONG_MAX;
  ll div(ll a, ll b) { // floored division
    return a / b - ((a ^ b) < 0 && a % b); }
  bool isect(iterator x, iterator y) {
```

```
    if (y == end()) return x->p = inf, 0;
    if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
    else x->p = div(y->m - x->m, x->k - y->k);
    return x->p >= y->p;
  }
  void add(ll k, ll m) {
    auto z = insert({k, m, 0}), y = z++, x = y;
    while (isect(y, z)) z = erase(z);
    if (x != begin() && isect(--x, y)) isect(x, y = erase(y));
    while ((y = x) != begin() && (--x)->p >= y->p)
      isect(x, erase(y));
  }
  ll query(ll x) {
    assert(!empty());
    auto l = *lower_bound(x);
    return l.k * x + l.m;
  }
};
```

Treap.h

Description: A short self-balancing tree. It acts as a sequential container with log-time splits/joins, and is easy to augment with additional data.
Time: $\mathcal{O}(\log N)$

d41d8c, 55 lines

```
struct Node {
  Node *l = 0, *r = 0;
  int val, y, c = 1;
  Node(int val) : val(val), y(rand()) {}
  void recalc();
};

int cnt(Node* n) { return n ? n->c : 0; }
void Node::recalc() { c = cnt(l) + cnt(r) + 1; }

template<class F> void each(Node* n, F f) {
  if (n) { each(n->l, f); f(n->val); each(n->r, f); }
}

pair<Node*, Node*> split(Node* n, int k) {
  if (!n) return {};
  if (cnt(n->l) >= k) { // "n->val >= k" for lower_bound(k)
    auto pa = split(n->l, k);
    n->l = pa.second;
    n->recalc();
    return {pa.first, n};
  } else {
    auto pa = split(n->r, k - cnt(n->l) - 1); // and just "k"
    n->r = pa.first;
    n->recalc();
    return {n, pa.second};
  }
}

Node* merge(Node* l, Node* r) {
  if (!l) return r;
  if (!r) return l;
  if (l->y > r->y) {
    l->r = merge(l->r, r);
    l->recalc();
    return l;
  } else {
    r->l = merge(l, r->l);
    r->recalc();
    return r;
  }
}

Node* ins(Node* t, Node* n, int pos) {
  auto pa = split(t, pos);
  return merge(merge(pa.first, n), pa.second);
}

// Example application: move the range [l, r) to index k
void move(Node*& t, int l, int r, int k) {
  Node *a, *b, *c;
  tie(a,b) = split(t, l); tie(b,c) = split(b, r - l);
  if (k <= l) t = merge(ins(a, b, k), c);
  else t = merge(a, ins(c, b, k - r));
}
```

FenwickTree.h

Description: Computes partial sums a[0] + a[1] + ... + a[pos - 1], and updates single elements a[i], taking the difference between the old and new value.
Time: Both operations are $\mathcal{O}(\log N)$.

d41d8c, 22 lines

```
struct FT {
  vector<ll> s;
  FT(int n) : s(n) {}
  void update(int pos, ll dif) { // a[pos] += dif
    for (; pos < sz(s); pos |= pos + 1) s[pos] += dif;
  }
  ll query(int pos) { // sum of values in [0, pos)
    ll res = 0;
    for (; pos > 0; pos &= pos - 1) res += s[pos-1];
    return res;
  }
  int lower_bound(ll sum) {// min pos st sum of [0, pos] >= sum
    // Returns n if no sum is >= sum, or -1 if empty sum is.
    if (sum <= 0) return -1;
    int pos = 0;
    for (int pw = 1 << 25; pw; pw >>= 1) {
      if (pos + pw <= sz(s) && s[pos + pw-1] < sum)
        pos += pw, sum -= s[pos-1];
    }
    return pos;
  }
};
```

FenwickTree2d.h

Description: Computes sums a[i,j] for all i<I, j<J, and increases single elements a[i,j]. Requires that the elements to be updated are known in advance (call fakeUpdate() before init()).
Time: $\mathcal{O}\left(\log^2 N\right)$. (Use persistent segment trees for $\mathcal{O}(\log N)$.)

"FenwickTree.h" d41d8c, 22 lines

```
struct FT2 {
  vector<vi> ys; vector<FT> ft;
  FT2(int limx) : ys(limx) {}
  void fakeUpdate(int x, int y) {
    for (; x < sz(ys); x |= x + 1) ys[x].push_back(y);
  }
  void init() {
    for (vi& v : ys) sort(all(v)), ft.emplace_back(sz(v));
  }
  int ind(int x, int y) {
    return (int)(lower_bound(all(ys[x]), y) - ys[x].begin()); }
  void update(int x, int y, ll dif) {
    for (; x < sz(ys); x |= x + 1)
      ft[x].update(ind(x, y), dif);
  }
  ll query(int x, int y) {
    ll sum = 0;
    for (; x; x &= x - 1)
      sum += ft[x-1].query(ind(x-1, y));
    return sum;
  }
};
```

RMQ.h

Description: Range Minimum Queries on an array. Returns min(V[a], V[a + 1], ... V[b - 1]) in constant time.
Usage: `RMQ rmq(values);`
`rmq.query(inclusive, exclusive);`
Time: $\mathcal{O}(|V| \log |V| + Q)$

d41d8c, 16 lines

```
template<class T>
struct RMQ {
  vector<vector<T>> jmp;
  RMQ(const vector<T>& V) : jmp(1, V) {
    for (int pw = 1, k = 1; pw * 2 <= sz(V); pw *= 2, ++k) {
      jmp.emplace_back(sz(V) - pw * 2 + 1);
      rep(j,0,sz(jmp[k]))
        jmp[k][j] = min(jmp[k - 1][j], jmp[k - 1][j + pw]);
    }
  }
  T query(int a, int b) {
    assert(a < b); // or return inf if a == b
    int dep = 31 - __builtin_clz(b - a);
    return min(jmp[dep][a], jmp[dep][b - (1 << dep)]);
  }
};
```

Numerical (4)

4.1 Polynomials and recurrences

Polynomial.h

d41d8c, 17 lines

```
struct Poly {
  vector<double> a;
  double operator()(double x) const {
    double val = 0;
    for (int i = sz(a); i--;) (val *= x) += a[i];
    return val;
  }
  void diff() {
    rep(i,1,sz(a)) a[i-1] = i*a[i];
    a.pop_back();
  }
  void divroot(double x0) {
    double b = a.back(), c; a.back() = 0;
    for(int i=sz(a)-1; i--;) c = a[i], a[i] = a[i+1]*x0+b, b=c;
    a.pop_back();
  }
};
```

PolyRoots.h

Description: Finds the real roots to a polynomial.
Usage: `polyRoots({{2,-3,1}},-1e9,1e9) // solve x^2-3x+2 = 0`
Time: $\mathcal{O}\left(n^2 \log(1/\epsilon)\right)$

"Polynomial.h" d41d8c, 23 lines

```
vector<double> polyRoots(Poly p, double xmin, double xmax) {
  if (sz(p.a) == 2) { return {-p.a[0]/p.a[1]}; }
  vector<double> ret;
  Poly der = p;
  der.diff();
  auto dr = polyRoots(der, xmin, xmax);
  dr.push_back(xmin-1);
  dr.push_back(xmax+1);
  sort(all(dr));
  rep(i,0,sz(dr)-1) {
    double l = dr[i], h = dr[i+1];
    bool sign = p(l) > 0;
    if (sign ^ (p(h) > 0)) {
      rep(it,0,60) { // while (h - l > 1e-8)
        double m = (l + h) / 2, f = p(m);
```

```
      if ((f <= 0) ^ sign) l = m;
      else h = m;
    }
    ret.push_back((l + h) / 2);
  }
 }
 return ret;
}
```

PolyInterpolate.h

Description: Given n points (x[i], y[i]), computes an n-1-degree polynomial p that passes through them: $p(x) = a[0] * x^0 + ... + a[n-1] * x^{n-1}$. For numerical precision, pick $x[k] = c * \cos(k/(n-1) * \pi), k = 0 \dots n-1$.
Time: $\mathcal{O}\left(n^2\right)$

d41d8c, 13 lines

```
typedef vector<double> vd;
vd interpolate(vd x, vd y, int n) {
 vd res(n), temp(n);
 rep(k,0,n-1) rep(i,k+1,n)
  y[i] = (y[i] - y[k]) / (x[i] - x[k]);
 double last = 0; temp[0] = 1;
 rep(k,0,n) rep(i,0,n) {
  res[i] += y[k] * temp[i];
  swap(last, temp[i]);
  temp[i] -= last * x[k];
 }
 return res;
}
```

4.2 Optimization

GoldenSectionSearch.h

Description: Finds the argument minimizing the function f in the interval $[a, b]$ assuming f is unimodal on the interval, i.e. has only one local minimum. The maximum error in the result is eps. Works equally well for maximization with a small change in the code. See TernarySearch.h in the Various chapter for a discrete version.
Usage: `double func(double x) { return 4+x+.3*x*x; }`
`double xmin = gss(-1000,1000,func);`
Time: $\mathcal{O}\left(\log((b-a)/\epsilon)\right)$

d41d8c, 14 lines

```
double gss(double a, double b, double (*f)(double)) {
 double r = (sqrt(5)-1)/2, eps = 1e-7;
 double x1 = b - r*(b-a), x2 = a + r*(b-a);
 double f1 = f(x1), f2 = f(x2);
 while (b-a > eps)
  if (f1 < f2) { //change to > to find maximum
   b = x2; x2 = x1; f2 = f1;
   x1 = b - r*(b-a); f1 = f(x1);
  } else {
   a = x1; x1 = x2; f1 = f2;
   x2 = a + r*(b-a); f2 = f(x2);
  }
 return a;
}
```

Integrate.h

Description: Simple integration of a function over an interval using Simpson's rule. The error should be proportional to h^4, although in practice you will want to verify that the result is stable to desired precision when epsilon changes.

d41d8c, 7 lines

```
template<class F>
double quad(double a, double b, F f, const int n = 1000) {
 double h = (b - a) / 2 / n, v = f(a) + f(b);
 rep(i,1,n*2)
  v += f(a + i*h) * (i&1 ? 4 : 2);
 return v * h / 3;
}
```

IntegrateAdaptive.h

Description: Fast integration using an adaptive Simpson's rule.
Usage: `double sphereVolume = quad(-1, 1, [](double x) {`
`return quad(-1, 1, [&](double y) {`
`return quad(-1, 1, [&](double z) {`
`return x*x + y*y + z*z < 1; });});});`

d41d8c, 15 lines

```
typedef double d;
#define S(a,b) (f(a) + 4*f((a+b) / 2) + f(b)) * (b-a) / 6

template <class F>
d rec(F& f, d a, d b, d eps, d S) {
 d c = (a + b) / 2;
 d S1 = S(a, c), S2 = S(c, b), T = S1 + S2;
 if (abs(T - S) <= 15 * eps || b - a < 1e-10)
  return T + (T - S) / 15;
 return rec(f, a, c, eps / 2, S1) + rec(f, c, b, eps / 2, S2);
}
template<class F>
d quad(d a, d b, F f, d eps = 1e-8) {
 return rec(f, a, b, eps, S(a, b));
}
```

4.3 Matrices

Determinant.h

Description: Calculates determinant of a matrix. Destroys the matrix.
Time: $\mathcal{O}\left(N^3\right)$

d41d8c, 15 lines

```
double det(vector<vector<double>>& a) {
 int n = sz(a); double res = 1;
 rep(i,0,n) {
  int b = i;
  rep(j,i+1,n) if (fabs(a[j][i]) > fabs(a[b][i])) b = j;
  if (i != b) swap(a[i], a[b]), res *= -1;
  res *= a[i][i];
  if (res == 0) return 0;
  rep(j,i+1,n) {
   double v = a[j][i] / a[i][i];
   if (v != 0) rep(k,i+1,n) a[j][k] -= v * a[i][k];
  }
 }
 return res;
}
```

IntDeterminant.h

Description: Calculates determinant using modular arithmetics. Modulos can also be removed to get a pure-integer version.
Time: $\mathcal{O}\left(N^3\right)$

d41d8c, 18 lines

```
const ll mod = 12345;
ll det(vector<vector<ll>>& a) {
 int n = sz(a); ll ans = 1;
 rep(i,0,n) {
  rep(j,i+1,n) {
   while (a[j][i] != 0) { // gcd step
    ll t = a[i][i] / a[j][i];
    if (t) rep(k,i,n)
     a[i][k] = (a[i][k] - a[j][k] * t) % mod;
    swap(a[i], a[j]);
    ans *= -1;
   }
  }
  ans = ans * a[i][i] % mod;
  if (!ans) return 0;
 }
 return (ans + mod) % mod;
}
```

SolveLinear.h

Description: Solves $A * x = b$. If there are multiple solutions, an arbitrary one is returned. Returns rank, or -1 if no solutions. Data in A and b is lost.
Time: $\mathcal{O}\left(n^2 m\right)$

d41d8c, 38 lines

```
typedef vector<double> vd;
const double eps = 1e-12;

int solveLinear(vector<vd>& A, vd& b, vd& x) {
 int n = sz(A), m = sz(x), rank = 0, br, bc;
 if (n) assert(sz(A[0]) == m);
 vi col(m); iota(all(col), 0);

 rep(i,0,n) {
  double v, bv = 0;
  rep(r,i,n) rep(c,i,m)
   if ((v = fabs(A[r][c])) > bv)
    br = r, bc = c, bv = v;
  if (bv <= eps) {
   rep(j,i,n) if (fabs(b[j]) > eps) return -1;
   break;
  }
  swap(A[i], A[br]);
  swap(b[i], b[br]);
  swap(col[i], col[bc]);
  rep(j,0,n) swap(A[j][i], A[j][bc]);
  bv = 1/A[i][i];
  rep(j,i+1,n) {
   double fac = A[j][i] * bv;
   b[j] -= fac * b[i];
   rep(k,i+1,m) A[j][k] -= fac*A[i][k];
  }
  rank++;
 }

 x.assign(m, 0);
 for (int i = rank; i--;) {
  b[i] /= A[i][i];
  x[col[i]] = b[i];
  rep(j,0,i) b[j] -= A[j][i] * b[i];
 }
 return rank; // (multiple solutions if rank < m)
}
```

Number theory (5)

5.1 Modular arithmetic

ModularArithmetic.h

Description: Operators for modular arithmetic. You need to set `mod` to some number first and then you can use the structure.

"euclid.h" d41d8c, 18 lines

```
const ll mod = 17; // change to something else
struct Mod {
 ll x;
 Mod(ll xx) : x(xx) {}
 Mod operator+(Mod b) { return Mod((x + b.x) % mod); }
 Mod operator-(Mod b) { return Mod((x - b.x + mod) % mod); }
 Mod operator*(Mod b) { return Mod((x * b.x) % mod); }
 Mod operator/(Mod b) { return *this * invert(b); }
 Mod invert(Mod a) {
  ll x, y, g = euclid(a.x, mod, x, y);
  assert(g == 1); return Mod((x + mod) % mod);
 }
 Mod operator^(ll e) {
  if (!e) return Mod(1);
  Mod r = *this ^ (e / 2); r = r * r;
  return e&1 ? *this * r : r;
```

```
  }
};
```

ModInverse.h

Description: Pre-computation of modular inverses. Assumes LIM $\leq$ mod and that mod is a prime.

d41d8c, 3 lines

```
const ll mod = 1000000007, LIM = 200000;
ll* inv = new ll[LIM] - 1; inv[1] = 1;
rep(i,2,LIM) inv[i] = mod - (mod / i) * inv[mod % i] % mod;
```

ModPow.h

d41d8c, 8 lines

```
const ll mod = 1000000007; // faster if const

ll modpow(ll b, ll e) {
  ll ans = 1;
  for (; e; b = b * b % mod, e /= 2)
    if (e & 1) ans = ans * b % mod;
  return ans;
}
```

ModLog.h

Description: Returns the smallest $x > 0$ s.t. $a^x = b \pmod m$, or -1 if no such x exists. modLog(a,1,m) can be used to calculate the order of a.
Time: $\mathcal{O}\left(\sqrt{m}\right)$

d41d8c, 11 lines

```
ll modLog(ll a, ll b, ll m) {
  ll n = (ll) sqrt(m) + 1, e = 1, f = 1, j = 1;
  unordered_map<ll, ll> A;
  while (j <= n && (e = f = e * a % m) != b % m)
    A[e * b % m] = j++;
  if (e == b % m) return j;
  if (__gcd(m, e) == __gcd(m, b))
    rep(i,2,n+2) if (A.count(e = e * f % m))
      return n * i - A[e];
  return -1;
}
```

ModSum.h

Description: Sums of mod'ed arithmetic progressions.
`modsum(to, c, k, m)` $= \sum_{i=0}^{\text{to}-1} (ki + c)\%m$. `divsum` is similar but for floored division.
Time: $\log(m)$, with a large constant.

d41d8c, 16 lines

```
typedef unsigned long long ull;
ull sumsq(ull to) { return to / 2 * ((to-1) | 1); }

ull divsum(ull to, ull c, ull k, ull m) {
  ull res = k / m * sumsq(to) + c / m * to;
  k %= m; c %= m;
  if (!k) return res;
  ull to2 = (to * k + c) / m;
  return res + (to - 1) * to2 - divsum(to2, m-1 - c, m, k);
}

ll modsum(ull to, ll c, ll k, ll m) {
  c = ((c % m) + m) % m;
  k = ((k % m) + m) % m;
  return to * c + k * sumsq(to) - m * divsum(to, c, k, m);
}
```

ModMulLL.h

Description: Calculate $a \cdot b \bmod c$ (or $a^b \bmod c$) for $0 \leq a, b \leq c \leq 7.2 \cdot 10^{18}$.
Time: $\mathcal{O}(1)$ for `modmul`, $\mathcal{O}(\log b)$ for `modpow`

d41d8c, 11 lines

```
typedef unsigned long long ull;
ull modmul(ull a, ull b, ull M) {
  ll ret = a * b - M * ull(1.L / M * a * b);
  return ret + M * (ret < 0) - M * (ret >= (ll)M);
}
ull modpow(ull b, ull e, ull mod) {
  ull ans = 1;
  for (; e; b = modmul(b, b, mod), e /= 2)
    if (e & 1) ans = modmul(ans, b, mod);
  return ans;
}
```

ModSqrt.h

Description: Tonelli-Shanks algorithm for modular square roots. Finds x s.t. $x^2 = a \pmod p$ ($-x$ gives the other solution).
Time: $\mathcal{O}\left(\log^2 p\right)$ worst case, $\mathcal{O}(\log p)$ for most p

"ModPow.h" d41d8c, 24 lines

```
ll sqrt(ll a, ll p) {
  a %= p; if (a < 0) a += p;
  if (a == 0) return 0;
  assert(modpow(a, (p-1)/2, p) == 1); // else no solution
  if (p % 4 == 3) return modpow(a, (p+1)/4, p);
  // a^(n+3)/8 or 2^(n+3)/8 * 2^(n-1)/4 works if p % 8 == 5
  ll s = p - 1, n = 2;
  int r = 0, m;
  while (s % 2 == 0)
    ++r, s /= 2;
  while (modpow(n, (p - 1) / 2, p) != p - 1) ++n;
  ll x = modpow(a, (s + 1) / 2, p);
  ll b = modpow(a, s, p), g = modpow(n, s, p);
  for (;; r = m) {
    ll t = b;
    for (m = 0; m < r && t != 1; ++m)
      t = t * t % p;
    if (m == 0) return x;
    ll gs = modpow(g, 1LL << (r - m - 1), p);
    g = gs * gs % p;
    x = x * gs % p;
    b = b * g % p;
  }
}
```

5.2 Primality

FastEratosthenes.h

Description: Prime sieve for generating all primes smaller than LIM.
Time: LIM=1e9 $\approx$ 1.5s

d41d8c, 20 lines

```
const int LIM = 1e6;
bitset<LIM> isPrime;
vi eratosthenes() {
  const int S = (int)round(sqrt(LIM)), R = LIM / 2;
  vi pr = {2}, sieve(S+1); pr.reserve(int(LIM/log(LIM)*1.1));
  vector<pii> cp;
  for (int i = 3; i <= S; i += 2) if (!sieve[i]) {
    cp.push_back({i, i * i / 2});
    for (int j = i * i; j <= S; j += 2 * i) sieve[j] = 1;
  }
  for (int L = 1; L <= R; L += S) {
    array<bool, S> block{};
    for (auto &[p, idx] : cp)
      for (int i=idx; i < S+L; idx = (i+=p)) block[i-L] = 1;
    rep(i,0,min(S, R - L))
      if (!block[i]) pr.push_back((L + i) * 2 + 1);
  }
  for (int i : pr) isPrime[i] = 1;
  return pr;
}
```

MillerRabin.h

Description: Deterministic Miller-Rabin primality test. Guaranteed to work for numbers up to $7 \cdot 10^{18}$; for larger numbers, use Python and extend A randomly.
Time: 7 times the complexity of $a^b \mod c$.

"ModMulLL.h" d41d8c, 12 lines

```
bool isPrime(ull n) {
  if (n < 2 || n % 6 % 4 != 1) return (n | 1) == 3;
  ull A[] = {2, 325, 9375, 28178, 450775, 9780504, 1795265022},
      s = __builtin_ctzll(n-1), d = n >> s;
  for (ull a : A) {   // ^ count trailing zeroes
    ull p = modpow(a%n, d, n), i = s;
    while (p != 1 && p != n - 1 && a % n && i--)
      p = modmul(p, p, n);
    if (p != n-1 && i != s) return 0;
  }
  return 1;
}
```

Factor.h

Description: Pollard-rho randomized factorization algorithm. Returns prime factors of a number, in arbitrary order (e.g. 2299 -> {11, 19, 11}).
Time: $\mathcal{O}\left(n^{1/4}\right)$, less for numbers with small factors.

"ModMulLL.h", "MillerRabin.h" d41d8c, 18 lines

```
ull pollard(ull n) {
  auto f = [n](ull x) { return modmul(x, x, n) + 1; };
  ull x = 0, y = 0, t = 30, prd = 2, i = 1, q;
  while (t++ % 40 || __gcd(prd, n) == 1) {
    if (x == y) x = ++i, y = f(x);
    if ((q = modmul(prd, max(x,y) - min(x,y), n))) prd = q;
    x = f(x), y = f(f(y));
  }
  return __gcd(prd, n);
}
vector<ull> factor(ull n) {
  if (n == 1) return {};
  if (isPrime(n)) return {n};
  ull x = pollard(n);
  auto l = factor(x), r = factor(n / x);
  l.insert(l.end(), all(r));
  return l;
}
```

5.3 Divisibility

euclid.h

Description: Finds two integers x and y, such that $ax + by = \gcd(a, b)$. If you just need gcd, use the built in __gcd instead. If a and b are coprime, then x is the inverse of $a \pmod b$.

d41d8c, 5 lines

```
ll euclid(ll a, ll b, ll &x, ll &y) {
  if (!b) return x = 1, y = 0, a;
  ll d = euclid(b, a % b, y, x);
  return y -= a/b * x, d;
}
```

CRT.h

Description: Chinese Remainder Theorem.
`crt(a, m, b, n)` computes x such that $x \equiv a \pmod m$, $x \equiv b \pmod n$. If $|a| < m$ and $|b| < n$, x will obey $0 \leq x < \text{lcm}(m, n)$. Assumes $mn < 2^{62}$.
Time: $\log(n)$

"euclid.h" d41d8c, 7 lines

```
ll crt(ll a, ll m, ll b, ll n) {
  if (n > m) swap(a, b), swap(m, n);
  ll x, y, g = euclid(m, n, x, y);
  assert((a - b) % g == 0); // else no solution
  x = (b - a) % n * x % n / g * m + a;
  return x < 0 ? x + m*n/g : x;
}
```

5.3.1 Bézout's identity

For $a \neq, b \neq 0$, then $d = gcd(a, b)$ is the smallest positive integer for which there are integer solutions to

$$ax + by = d$$

If (x, y) is one solution, then all solutions are given by

$$\left(x + \frac{kb}{\gcd(a,b)}, y - \frac{ka}{\gcd(a,b)}\right), \quad k \in \mathbb{Z}$$

phiFunction.h

Description: *Euler's* ϕ function is defined as $\phi(n) :=$ # of positive integers $\leq n$ that are coprime with n. $\phi(1) = 1$, p prime $\Rightarrow \phi(p^k) = (p-1)p^{k-1}$, m, n coprime $\Rightarrow \phi(mn) = \phi(m)\phi(n)$. If $n = p_1^{k_1} p_2^{k_2} ... p_r^{k_r}$ then $\phi(n) = (p_1 - 1)p_1^{k_1-1} ... (p_r - 1)p_r^{k_r-1}$. $\phi(n) = n \cdot \prod_{p|n}(1 - 1/p)$.
$\sum_{d|n} \phi(d) = n$, $\sum_{1 \leq k \leq n, \gcd(k,n)=1} k = n\phi(n)/2, n > 1$
Euler's thm: a, n coprime $\Rightarrow a^{\phi(n)} \equiv 1 \pmod{n}$.
Fermat's little thm: p prime $\Rightarrow a^{p-1} \equiv 1 \pmod{p}$ $\forall a$.

d41d8c, 8 lines

```
const int LIM = 5000000;
int phi[LIM];

void calculatePhi() {
  rep(i,0,LIM) phi[i] = i&1 ? i : i/2;
  for (int i = 3; i < LIM; i += 2) if(phi[i] == i)
    for (int j = i; j < LIM; j += i) phi[j] -= phi[j] / i;
}
```

5.4 Fractions

ContinuedFractions.h

Description: Given N and a real number $x \geq 0$, finds the closest rational approximation p/q with $p, q \leq N$. It will obey $|p/q - x| \leq 1/qN$.
For consecutive convergents, $p_{k+1}q_k - q_{k+1}p_k = (-1)^k$. ($p_k/q_k$ alternates between $> x$ and $< x$.) If x is rational, y eventually becomes ∞; if x is the root of a degree 2 polynomial the a's eventually become cyclic.
Time: $\mathcal{O}(\log N)$

d41d8c, 21 lines

```
typedef double d; // for N ~ 1e7; long double for N ~ 1e9
pair<ll, ll> approximate(d x, ll N) {
  ll LP = 0, LQ = 1, P = 1, Q = 0, inf = LLONG_MAX; d y = x;
  for (;;) {
    ll lim = min(P ? (N-LP) / P : inf, Q ? (N-LQ) / Q : inf),
       a = (ll)floor(y), b = min(a, lim),
       NP = b*P + LP, NQ = b*Q + LQ;
    if (a > b) {
      // If b > a/2, we have a semi-convergent that gives us a
      // better approximation; if b = a/2, we *may* have one.
      // Return {P, Q} here for a more canonical approximation.
      return (abs(x - (d)NP / (d)NQ) < abs(x - (d)P / (d)Q)) ?
        make_pair(NP, NQ) : make_pair(P, Q);
    }
    if (abs(y = 1/(y - (d)a)) > 3*N) {
      return {NP, NQ};
    }
    LP = P; P = NP;
    LQ = Q; Q = NQ;
  }
}
```

FracBinarySearch.h

Description: Given f and N, finds the smallest fraction $p/q \in [0, 1]$ such that $f(p/q)$ is true, and $p, q \leq N$. You may want to throw an exception from f if it finds an exact solution, in which case N can be removed.

Usage: `fracBS([](Frac f) { return f.p>=3*f.q; }, 10); // {1,3}`
Time: $\mathcal{O}(\log(N))$

d41d8c, 25 lines

```
struct Frac { ll p, q; };

template<class F>
Frac fracBS(F f, ll N) {
  bool dir = 1, A = 1, B = 1;
  Frac lo{0, 1}, hi{1, 1}; // Set hi to 1/0 to search (0, N]
  if (f(lo)) return lo;
  assert(f(hi));
  while (A || B) {
    ll adv = 0, step = 1; // move hi if dir, else lo
    for (int si = 0; step; (step *= 2) >>= si) {
      adv += step;
      Frac mid{lo.p * adv + hi.p, lo.q * adv + hi.q};
      if (abs(mid.p) > N || mid.q > N || dir == !f(mid)) {
        adv -= step; si = 2;
      }
    }
    hi.p += lo.p * adv;
    hi.q += lo.q * adv;
    dir = !dir;
    swap(lo, hi);
    A = B; B = !!adv;
  }
  return dir ? hi : lo;
}
```

5.5 Pythagorean Triples

The Pythagorean triples are uniquely generated by

$$a = k \cdot (m^2 - n^2), \ b = k \cdot (2mn), \ c = k \cdot (m^2 + n^2),$$

with $m > n > 0$, $k > 0$, $m \perp n$, and either m or n even.

Combinatorial (6)

6.1 Permutations

6.1.1 Factorial

n	1	2	3	4	5	6	7	8	9	10
$n!$	1	2	6	24	120	720	5040	40320	362880	3628800
n	11	12	13	14	15	16	17			
$n!$	4.0e7	4.8e8	6.2e9	8.7e10	1.3e12	2.1e13	3.6e14			
n	20	25	30	40	50	100	150	171		
$n!$	2e18	2e25	3e32	8e47	3e64	9e157	6e262	>DBL_MAX		

IntPerm.h

Description: Permutation -> integer conversion. (Not order preserving.) Integer -> permutation can use a lookup table.
Time: $\mathcal{O}(n)$

d41d8c, 6 lines

```
int permToInt(vi& v) {
  int use = 0, i = 0, r = 0;
  for(int x:v) r = r * ++i + __builtin_popcount(use & -(1<<x)),
    use |= 1 << x;                    // (note: minus, not ~!)
  return r;
}
```

6.1.2 Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1) + D(n-2)) = nD(n-1) + (-1)^n = \left\lfloor \frac{n!}{e} \right\rceil$$

6.2 Partitions and subsets

6.2.1 Partition function

Number of ways of writing n as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1, \ p(n) = \sum_{k \in \mathbb{Z} \setminus \{0\}} (-1)^{k+1} p(n - k(3k-1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

n	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	~2e5	~2e8

6.2.2 Binomials

multinomial.h

Description: Computes $\binom{k_1 + \cdots + k_n}{k_1, k_2, \ldots, k_n} = \frac{(\sum k_i)!}{k_1! k_2! ... k_n!}$.

d41d8c, 6 lines

```
ll multinomial(vi& v) {
  ll c = 1, m = v.empty() ? 1 : v[0];
  rep(i,1,sz(v)) rep(j,0,v[i])
    c = c * ++m / (j+1);
  return c;
}
```

6.3 General purpose numbers

6.3.1 Labeled unrooted trees

on n vertices: n^{n-2}
on k existing trees of size n_i: $n_1 n_2 \cdots n_k n^{k-2}$
with degrees d_i: $(n-2)!/((d_1 - 1)! \cdots (d_n - 1)!)$

6.3.2 Catalan numbers

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1, \ C_{n+1} = \frac{2(2n+1)}{n+2} C_n, \ C_{n+1} = \sum C_i C_{n-i}$$

$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \ldots$

- sub-diagonal monotone paths in an $n \times n$ grid.
- strings with n pairs of parenthesis, correctly nested.
- binary trees with with $n + 1$ leaves (0 or 2 children).
- ordered trees with $n + 1$ vertices.
- ways a convex polygon with $n + 2$ sides can be cut into triangles by connecting vertices with straight lines.
- permutations of $[n]$ with no 3-term increasing subseq.

Graph (7)

7.1 Fundamentals

BellmanFord.h

Description: Calculates shortest paths from s in a graph that might have negative edge weights. Unreachable nodes get dist = inf; nodes reachable through negative-weight cycles get dist = -inf. Assumes $V^2 \max|w_i| < \sim 2^{63}$.
Time: $\mathcal{O}(VE)$

d41d8c, 23 lines

```
const ll inf = LLONG_MAX;
struct Ed { int a, b, w, s() { return a < b ? a : -a; }};
struct Node { ll dist = inf; int prev = -1; };

void bellmanFord(vector<Node>& nodes, vector<Ed>& eds, int s) {
  nodes[s].dist = 0;
  sort(all(eds), [](Ed a, Ed b) { return a.s() < b.s(); });

  int lim = sz(nodes) / 2 + 2; // /3+100 with shuffled vertices
  rep(i,0,lim) for (Ed ed : eds) {
    Node cur = nodes[ed.a], &dest = nodes[ed.b];
    if (abs(cur.dist) == inf) continue;
    ll d = cur.dist + ed.w;
    if (d < dest.dist) {
      dest.prev = ed.a;
      dest.dist = (i < lim-1 ? d : -inf);
    }
  }
  rep(i,0,lim) for (Ed e : eds) {
    if (nodes[e.a].dist == -inf)
      nodes[e.b].dist = -inf;
  }
}
```

FloydWarshall.h

Description: Calculates all-pairs shortest path in a directed graph that might have negative edge weights. Input is an distance matrix m, where $m[i][j] =$ `inf` if i and j are not adjacent. As output, $m[i][j]$ is set to the shortest distance between i and j, `inf` if no path, or `-inf` if the path goes through a negative-weight cycle.
Time: $\mathcal{O}(N^3)$

d41d8c, 12 lines

```
const ll inf = 1LL << 62;
void floydWarshall(vector<vector<ll>>& m) {
  int n = sz(m);
  rep(i,0,n) m[i][i] = min(m[i][i], 0LL);
  rep(k,0,n) rep(i,0,n) rep(j,0,n)
    if (m[i][k] != inf && m[k][j] != inf) {
      auto newDist = max(m[i][k] + m[k][j], -inf);
      m[i][j] = min(m[i][j], newDist);
    }
  rep(k,0,n) if (m[k][k] < 0) rep(i,0,n) rep(j,0,n)
    if (m[i][k] != inf && m[k][j] != inf) m[i][j] = -inf;
}
```

TopoSort.h

Description: Topological sorting. Given is an oriented graph. Output is an ordering of vertices, such that there are edges only from left to right. If there are cycles, the returned list will have size smaller than n – nodes reachable from cycles will not be returned.
Time: $\mathcal{O}(|V| + |E|)$

d41d8c, 14 lines

```
vi topoSort(const vector<vi>& gr) {
  vi indeg(sz(gr)), ret;
  for (auto& li : gr) for (int x : li) indeg[x]++;
  queue<int> q; // use priority_queue for lexic. largest ans.
  rep(i,0,sz(gr)) if (indeg[i] == 0) q.push(i);
  while (!q.empty()) {
    int i = q.front(); // top() for priority queue
    ret.push_back(i);
    q.pop();
    for (int x : gr[i])
      if (--indeg[x] == 0) q.push(x);
  }
  return ret;
}
```

7.2 Matching

hopcroftKarp.h

Description: Fast bipartite matching algorithm. Graph g should be a list of neighbors of the left partition, and *btoa* should be a vector full of -1's of the same size as the right partition. Returns the size of the matching. $btoa[i]$ will be the match for vertex i on the right side, or -1 if it's not matched.
Usage: `vi btoa(m, -1); hopcroftKarp(g, btoa);`
Time: $\mathcal{O}\left(\sqrt{V}E\right)$

d41d8c, 42 lines

```
bool dfs(int a, int L, vector<vi>& g, vi& btoa, vi& A, vi& B) {
  if (A[a] != L) return 0;
  A[a] = -1;
  for (int b : g[a]) if (B[b] == L + 1) {
    B[b] = 0;
    if (btoa[b] == -1 || dfs(btoa[b], L + 1, g, btoa, A, B))
      return btoa[b] = a, 1;
  }
  return 0;
}

int hopcroftKarp(vector<vi>& g, vi& btoa) {
  int res = 0;
  vi A(g.size()), B(btoa.size()), cur, next;
  for (;;) {
    fill(all(A), 0);
    fill(all(B), 0);
    cur.clear();
    for (int a : btoa) if(a != -1) A[a] = -1;
    rep(a,0,sz(g)) if(A[a] == 0) cur.push_back(a);
    for (int lay = 1;; lay++) {
      bool islast = 0;
      next.clear();
      for (int a : cur) for (int b : g[a]) {
        if (btoa[b] == -1) {
          B[b] = lay;
          islast = 1;
        }
        else if (btoa[b] != a && !B[b]) {
          B[b] = lay;
          next.push_back(btoa[b]);
        }
      }
      if (islast) break;
      if (next.empty()) return res;
      for (int a : next) A[a] = lay;
      cur.swap(next);
    }
    rep(a,0,sz(g))
      res += dfs(a, 0, g, btoa, A, B);
  }
}
```

DFSMatching.h

Description: Simple bipartite matching algorithm. Graph g should be a list of neighbors of the left partition, and *btoa* should be a vector full of -1's of the same size as the right partition. Returns the size of the matching. $btoa[i]$ will be the match for vertex i on the right side, or -1 if it's not matched.
Usage: `vi btoa(m, -1); dfsMatching(g, btoa);`
Time: $\mathcal{O}(VE)$

d41d8c, 22 lines

```
bool find(int j, vector<vi>& g, vi& btoa, vi& vis) {
  if (btoa[j] == -1) return 1;
  vis[j] = 1; int di = btoa[j];
  for (int e : g[di])
    if (!vis[e] && find(e, g, btoa, vis)) {
      btoa[e] = di;
      return 1;
    }
  return 0;
}
int dfsMatching(vector<vi>& g, vi& btoa) {
  vi vis;
  rep(i,0,sz(g)) {
    vis.assign(sz(btoa), 0);
    for (int j : g[i])
      if (find(j, g, btoa, vis)) {
        btoa[j] = i;
        break;
      }
  }
  return sz(btoa) - (int)count(all(btoa), -1);
}
```

MinimumVertexCover.h

Description: Finds a minimum vertex cover in a bipartite graph. The size is the same as the size of a maximum matching, and the complement is a maximum independent set.

"DFSMatching.h" d41d8c, 20 lines

```
vi cover(vector<vi>& g, int n, int m) {
  vi match(m, -1);
  int res = dfsMatching(g, match);
  vector<bool> lfound(n, true), seen(m);
  for (int it : match) if (it != -1) lfound[it] = false;
  vi q, cover;
  rep(i,0,n) if (lfound[i]) q.push_back(i);
  while (!q.empty()) {
    int i = q.back(); q.pop_back();
    lfound[i] = 1;
    for (int e : g[i]) if (!seen[e] && match[e] != -1) {
      seen[e] = true;
      q.push_back(match[e]);
    }
  }
  rep(i,0,n) if (!lfound[i]) cover.push_back(i);
  rep(i,0,m) if (seen[i]) cover.push_back(n+i);
  assert(sz(cover) == res);
  return cover;
}
```

WeightedMatching.h

Description: Given a weighted bipartite graph, matches every node on the left with a node on the right such that no nodes are in two matchings and the sum of the edge weights is minimal. Takes cost[N][M], where cost[i][j] = cost for L[i] to be matched with R[j] and returns (min cost, match), where L[i] is matched with R[match[i]]. Negate costs for max cost. Requires $N \leq M$.
Time: $\mathcal{O}\left(N^2M\right)$

d41d8c, 31 lines

```
pair<int, vi> hungarian(const vector<vi> &a) {
  if (a.empty()) return {0, {}};
  int n = sz(a) + 1, m = sz(a[0]) + 1;
  vi u(n), v(m), p(m), ans(n - 1);
  rep(i,1,n) {
    p[0] = i;
    int j0 = 0; // add "dummy" worker 0
    vi dist(m, INT_MAX), pre(m, -1);
    vector<bool> done(m + 1);
    do { // dijkstra
      done[j0] = true;
      int i0 = p[j0], j1, delta = INT_MAX;
```

```
      rep(j,1,m) if (!done[j]) {
        auto cur = a[i0 - 1][j - 1] - u[i0] - v[j];
        if (cur < dist[j]) dist[j] = cur, pre[j] = j0;
        if (dist[j] < delta) delta = dist[j], j1 = j;
      }
      rep(j,0,m) {
        if (done[j]) u[p[j]] += delta, v[j] -= delta;
        else dist[j] -= delta;
      }
      j0 = j1;
    } while (p[j0]);
    while (j0) { // update alternating path
      int j1 = pre[j0];
      p[j0] = p[j1], j0 = j1;
    }
  }
  rep(j,1,m) if (p[j]) ans[p[j] - 1] = j - 1;
  return {-v[0], ans}; // min cost
}
```

GeneralMatching.h

Description: Matching for general graphs. Fails with probability N/mod.
Time: $\mathcal{O}(N^3)$

"../numerical/MatrixInverse-mod.h" d41d8c, 40 lines

```
vector<pii> generalMatching(int N, vector<pii>& ed) {
  vector<vector<ll>> mat(N, vector<ll>(N)), A;
  for (pii pa : ed) {
    int a = pa.first, b = pa.second, r = rand() % mod;
    mat[a][b] = r, mat[b][a] = (mod - r) % mod;
  }

  int r = matInv(A = mat), M = 2*N - r, fi, fj;
  assert(r % 2 == 0);

  if (M != N) do {
    mat.resize(M, vector<ll>(M));
    rep(i,0,N) {
      mat[i].resize(M);
      rep(j,N,M) {
        int r = rand() % mod;
        mat[i][j] = r, mat[j][i] = (mod - r) % mod;
      }
    }
  } while (matInv(A = mat) != M);

  vi has(M, 1); vector<pii> ret;
  rep(it,0,M/2) {
    rep(i,0,M) if (has[i])
      rep(j,i+1,M) if (A[i][j] && mat[i][j]) {
        fi = i; fj = j; goto done;
    } assert(0); done:
    if (fj < N) ret.emplace_back(fi, fj);
    has[fi] = has[fj] = 0;
    rep(sw,0,2) {
      ll a = modpow(A[fi][fj], mod-2);
      rep(i,0,M) if (has[i] && A[i][fj]) {
        ll b = A[i][fj] * a % mod;
        rep(j,0,M) A[i][j] = (A[i][j] - A[fi][j] * b) % mod;
      }
      swap(fi,fj);
    }
  }
  return ret;
}
```

7.3 DFS algorithms

SCC.h

Description: Finds strongly connected components in a directed graph. If vertices u, v belong to the same component, we can reach u from v and vice versa.
Usage: `scc(graph, [&](vi& v) { ... })` visits all components in reverse topological order. `comp[i]` holds the component index of a node (a component only has edges to components with lower index). `ncomps` will contain the number of components.
Time: $\mathcal{O}(E+V)$

d41d8c, 24 lines

```
vi val, comp, z, cont;
int Time, ncomps;
template<class G, class F> int dfs(int j, G& g, F& f) {
  int low = val[j] = ++Time, x; z.push_back(j);
  for (auto e : g[j]) if (comp[e] < 0)
    low = min(low, val[e] ?: dfs(e,g,f));

  if (low == val[j]) {
    do {
      x = z.back(); z.pop_back();
      comp[x] = ncomps;
      cont.push_back(x);
    } while (x != j);
    f(cont); cont.clear();
    ncomps++;
  }
  return val[j] = low;
}
template<class G, class F> void scc(G& g, F f) {
  int n = sz(g);
  val.assign(n, 0); comp.assign(n, -1);
  Time = ncomps = 0;
  rep(i,0,n) if (comp[i] < 0) dfs(i, g, f);
}
```

BiconnectedComponents.h

Description: Finds all biconnected components in an undirected graph, and runs a callback for the edges in each. In a biconnected component there are at least two distinct paths between any two nodes. Note that a node can be in several components. An edge which is not in a component is a bridge, i.e., not part of any cycle.
Usage: `int eid = 0; ed.resize(N);`
`for each edge (a,b) {`
`ed[a].emplace_back(b, eid);`
`ed[b].emplace_back(a, eid++); }`
`bicomps([&](const vi& edgelist) {...});`
Time: $\mathcal{O}(E+V)$

d41d8c, 33 lines

```
vi num, st;
vector<vector<pii>> ed;
int Time;
template<class F>
int dfs(int at, int par, F& f) {
  int me = num[at] = ++Time, e, y, top = me;
  for (auto pa : ed[at]) if (pa.second != par) {
    tie(y, e) = pa;
    if (num[y]) {
      top = min(top, num[y]);
      if (num[y] < me)
        st.push_back(e);
    } else {
      int si = sz(st);
      int up = dfs(y, e, f);
      top = min(top, up);
      if (up == me) {
        st.push_back(e);
        f(vi(st.begin() + si, st.end()));
        st.resize(si);
      }
      else if (up < me) st.push_back(e);
      else { /* e is a bridge */ }
    }
  }
  return top;
}

template<class F>
void bicomps(F f) {
  num.assign(sz(ed), 0);
  rep(i,0,sz(ed)) if (!num[i]) dfs(i, -1, f);
}
```

EulerWalk.h

Description: Eulerian undirected/directed path/cycle algorithm. Input should be a vector of (dest, global edge index), where for undirected graphs, forward/backward edges have the same index. Returns a list of nodes in the Eulerian path/cycle with src at both start and end, or empty list if no cycle/path exists. To get edge indices back, add .second to s and ret.
Time: $\mathcal{O}(V+E)$

d41d8c, 15 lines

```
vi eulerWalk(vector<vector<pii>>& gr, int nedges, int src=0) {
  int n = sz(gr);
  vi D(n), its(n), eu(nedges), ret, s = {src};
  D[src]++; // to allow Euler paths, not just cycles
  while (!s.empty()) {
    int x = s.back(), y, e, &it = its[x], end = sz(gr[x]);
    if (it == end){ ret.push_back(x); s.pop_back(); continue; }
    tie(y, e) = gr[x][it++];
    if (!eu[e]) {
      D[x]--, D[y]++;
      eu[e] = 1; s.push_back(y);
    }}
  for (int x : D) if (x < 0 || sz(ret) != nedges+1) return {};
  return {ret.rbegin(), ret.rend()};
}
```

7.4 Coloring

EdgeColoring.h

Description: Given a simple, undirected graph with max degree D, computes a $(D+1)$-coloring of the edges such that no neighboring edges share a color. (D-coloring is NP-hard, but can be done for bipartite graphs by repeated matchings of max-degree nodes.)
Time: $\mathcal{O}(NM)$

d41d8c, 31 lines

```
vi edgeColoring(int N, vector<pii> eds) {
  vi cc(N + 1), ret(sz(eds)), fan(N), free(N), loc;
  for (pii e : eds) ++cc[e.first], ++cc[e.second];
  int u, v, ncols = *max_element(all(cc)) + 1;
  vector<vi> adj(N, vi(ncols, -1));
  for (pii e : eds) {
    tie(u, v) = e;
    fan[0] = v;
    loc.assign(ncols, 0);
    int at = u, end = u, d, c = free[u], ind = 0, i = 0;
    while (d = free[v], !loc[d] && (v = adj[u][d]) != -1)
      loc[d] = ++ind, cc[ind] = d, fan[ind] = v;
    cc[loc[d]] = c;
    for (int cd = d; at != -1; cd ^= c ^ d, at = adj[at][cd])
      swap(adj[at][cd], adj[end = at][cd ^ c ^ d]);
    while (adj[fan[i]][d] != -1) {
      int left = fan[i], right = fan[++i], e = cc[i];
      adj[u][e] = left;
      adj[left][e] = u;
      adj[right][e] = -1;
      free[right] = e;
    }
    adj[u][d] = fan[i];
```

```
    adj[fan[i]][d] = u;
    for (int y : {fan[0], u, end})
      for (int& z = free[y] = 0; adj[y][z] != -1; z++);
  }
  rep(i,0,sz(eds))
    for (tie(u, v) = eds[i]; adj[u][ret[i]] != v;) ++ret[i];
  return ret;
}
```

7.5 Trees

BinaryLifting.h

Description: Calculate power of two jumps in a tree, to support fast upward jumps and LCAs. Assumes the root node points to itself.
Time: construction $\mathcal{O}(N \log N)$, queries $\mathcal{O}(\log N)$

d41d8c, 25 lines

```
vector<vi> treeJump(vi& P){
  int on = 1, d = 1;
  while(on < sz(P)) on *= 2, d++;
  vector<vi> jmp(d, P);
  rep(i,1,d) rep(j,0,sz(P))
    jmp[i][j] = jmp[i-1][jmp[i-1][j]];
  return jmp;
}

int jmp(vector<vi>& tbl, int nod, int steps){
  rep(i,0,sz(tbl))
    if(steps&(1<<i)) nod = tbl[i][nod];
  return nod;
}

int lca(vector<vi>& tbl, vi& depth, int a, int b) {
  if (depth[a] < depth[b]) swap(a, b);
  a = jmp(tbl, a, depth[a] - depth[b]);
  if (a == b) return a;
  for (int i = sz(tbl); i--;) {
    int c = tbl[i][a], d = tbl[i][b];
    if (c != d) a = c, b = d;
  }
  return tbl[0][a];
}
```

LCA.h

Description: Data structure for computing lowest common ancestors in a tree (with 0 as root). C should be an adjacency list of the tree, either directed or undirected.
Time: $\mathcal{O}(N \log N + Q)$

"../data-structures/RMQ.h" d41d8c, 21 lines

```
struct LCA {
  int T = 0;
  vi time, path, ret;
  RMQ<int> rmq;

  LCA(vector<vi>& C) : time(sz(C)), rmq((dfs(C,0,-1), ret)) {}
  void dfs(vector<vi>& C, int v, int par) {
    time[v] = T++;
    for (int y : C[v]) if (y != par) {
      path.push_back(v), ret.push_back(time[v]);
      dfs(C, y, v);
    }
  }

  int lca(int a, int b) {
    if (a == b) return a;
    tie(a, b) = minmax(time[a], time[b]);
    return path[rmq.query(a, b)];
  }
  //dist(a,b){return depth[a] + depth[b] - 2*depth[lca(a,b)];}
};
```

CompressTree.h

Description: Given a rooted tree and a subset S of nodes, compute the minimal subtree that contains all the nodes by adding all (at most $|S|-1$) pairwise LCA's and compressing edges. Returns a list of (par, orig_index) representing a tree rooted at 0. The root points to itself.
Time: $\mathcal{O}(|S| \log |S|)$

"LCA.h" d41d8c, 21 lines

```
typedef vector<pair<int, int>> vpi;
vpi compressTree(LCA& lca, const vi& subset) {
  static vi rev; rev.resize(sz(lca.time));
  vi li = subset, &T = lca.time;
  auto cmp = [&](int a, int b) { return T[a] < T[b]; };
  sort(all(li), cmp);
  int m = sz(li)-1;
  rep(i,0,m) {
    int a = li[i], b = li[i+1];
    li.push_back(lca.lca(a, b));
  }
  sort(all(li), cmp);
  li.erase(unique(all(li)), li.end());
  rep(i,0,sz(li)) rev[li[i]] = i;
  vpi ret = {pii(0, li[0])};
  rep(i,0,sz(li)-1) {
    int a = li[i], b = li[i+1];
    ret.emplace_back(rev[lca.lca(a, b)], b);
  }
  return ret;
}
```

LinkCutTree.h

Description: Represents a forest of unrooted trees. You can add and remove edges (as long as the result is still a forest), and check whether two nodes are in the same tree.
Time: All operations take amortized $\mathcal{O}(\log N)$.

d41d8c, 90 lines

```
struct Node { // Splay tree. Root's pp contains tree's parent.
  Node *p = 0, *pp = 0, *c[2];
  bool flip = 0;
  Node() { c[0] = c[1] = 0; fix(); }
  void fix() {
    if (c[0]) c[0]->p = this;
    if (c[1]) c[1]->p = this;
    // (+ update sum of subtree elements etc. if wanted)
  }
  void pushFlip() {
    if (!flip) return;
    flip = 0; swap(c[0], c[1]);
    if (c[0]) c[0]->flip ^= 1;
    if (c[1]) c[1]->flip ^= 1;
  }
  int up() { return p ? p->c[1] == this : -1; }
  void rot(int i, int b) {
    int h = i ^ b;
    Node *x = c[i], *y = b == 2 ? x : x->c[h], *z = b ? y : x;
    if ((y->p = p)) p->c[up()] = y;
    c[i] = z->c[i ^ 1];
    if (b < 2) {
      x->c[h] = y->c[h ^ 1];
      z->c[h ^ 1] = b ? x : this;
    }
    y->c[i ^ 1] = b ? this : x;
    fix(); x->fix(); y->fix();
    if (p) p->fix();
    swap(pp, y->pp);
  }
  void splay() {
    for (pushFlip(); p; ) {
      if (p->p) p->p->pushFlip();
      p->pushFlip(); pushFlip();
      int c1 = up(), c2 = p->up();
      if (c2 == -1) p->rot(c1, 2);
      else p->p->rot(c2, c1 != c2);
    }
  }
  Node* first() {
    pushFlip();
    return c[0] ? c[0]->first() : (splay(), this);
  }
};

struct LinkCut {
  vector<Node> node;
  LinkCut(int N) : node(N) {}

  void link(int u, int v) { // add an edge (u, v)
    assert(!connected(u, v));
    makeRoot(&node[u]);
    node[u].pp = &node[v];
  }
  void cut(int u, int v) { // remove an edge (u, v)
    Node *x = &node[u], *top = &node[v];
    makeRoot(top); x->splay();
    assert(top == (x->pp ?: x->c[0]));
    if (x->pp) x->pp = 0;
    else {
      x->c[0] = top->p = 0;
      x->fix();
    }
  }
  bool connected(int u, int v) { // are u, v in the same tree?
    Node* nu = access(&node[u])->first();
    return nu == access(&node[v])->first();
  }
  void makeRoot(Node* u) {
    access(u);
    u->splay();
    if(u->c[0]) {
      u->c[0]->p = 0;
      u->c[0]->flip ^= 1;
      u->c[0]->pp = u;
      u->c[0] = 0;
      u->fix();
    }
  }
  Node* access(Node* u) {
    u->splay();
    while (Node* pp = u->pp) {
      pp->splay(); u->pp = 0;
      if (pp->c[1]) {
        pp->c[1]->p = 0; pp->c[1]->pp = pp; }
      pp->c[1] = u; pp->fix(); u = pp;
    }
    return u;
  }
};
```

DirectedMST.h

Description: Finds a minimum spanning tree/arborescence of a directed graph, given a root node. If no MST exists, returns -1.
Time: $\mathcal{O}(E \log V)$

"../data-structures/UnionFindRollback.h" d41d8c, 60 lines

```
struct Edge { int a, b; ll w; };
struct Node {
  Edge key;
  Node *l, *r;
  ll delta;
  void prop() {
    key.w += delta;
```

```
    if (l) l->delta += delta;
    if (r) r->delta += delta;
    delta = 0;
  }
  Edge top() { prop(); return key; }
};
Node *merge(Node *a, Node *b) {
  if (!a || !b) return a ?: b;
  a->prop(), b->prop();
  if (a->key.w > b->key.w) swap(a, b);
  swap(a->l, (a->r = merge(b, a->r)));
  return a;
}
void pop(Node*& a) { a->prop(); a = merge(a->l, a->r); }

pair<ll, vi> dmst(int n, int r, vector<Edge>& g) {
  RollbackUF uf(n);
  vector<Node*> heap(n);
  for (Edge e : g) heap[e.b] = merge(heap[e.b], new Node{e});
  ll res = 0;
  vi seen(n, -1), path(n), par(n);
  seen[r] = r;
  vector<Edge> Q(n), in(n, {-1,-1}), comp;
  deque<tuple<int, int, vector<Edge>>> cycs;
  rep(s,0,n) {
    int u = s, qi = 0, w;
    while (seen[u] < 0) {
      if (!heap[u]) return {-1,{}};
      Edge e = heap[u]->top();
      heap[u]->delta -= e.w, pop(heap[u]);
      Q[qi] = e, path[qi++] = u, seen[u] = s;
      res += e.w, u = uf.find(e.a);
      if (seen[u] == s) {
        Node* cyc = 0;
        int end = qi, time = uf.time();
        do cyc = merge(cyc, heap[w = path[--qi]]);
        while (uf.join(u, w));
        u = uf.find(u), heap[u] = cyc, seen[u] = -1;
        cycs.push_front({u, time, {&Q[qi], &Q[end]}});
      }
    }
    rep(i,0,qi) in[uf.find(Q[i].b)] = Q[i];
  }

  for (auto& [u,t,comp] : cycs) { // restore sol (optional)
    uf.rollback(t);
    Edge inEdge = in[u];
    for (auto& e : comp) in[uf.find(e.b)] = e;
    in[uf.find(inEdge.b)] = inEdge;
  }
  rep(i,0,n) par[i] = in[i].a;
  return {res, par};
}
```

7.6 Math

7.6.1 Number of Spanning Trees

Create an $N \times N$ matrix `mat`, and for each edge $a \to b \in G$, do `mat[a][b]--, mat[b][b]++` (and `mat[b][a]--, mat[a][a]++` if G is undirected). Remove the ith row and column and take the determinant; this yields the number of directed spanning trees rooted at i (if G is undirected, remove any row/column).

7.6.2 Erdős–Gallai theorem

A simple graph with node degrees $d_1 \geq \cdots \geq d_n$ exists iff $d_1 + \cdots + d_n$ is even and for every $k = 1 \ldots n$,

$$\sum_{i=1}^{k} d_i \leq k(k-1) + \sum_{i=k+1}^{n} \min(d_i, k).$$

Geometry (8)

8.1 Geometric primitives

Point.h

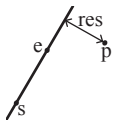
Description: Class to handle points in the plane. T can be e.g. double or long long. (Avoid int.)

d41d8c, 28 lines

```
template <class T> int sgn(T x) { return (x > 0) - (x < 0); }
template<class T>
struct Point {
  typedef Point P;
  T x, y;
  explicit Point(T x=0, T y=0) : x(x), y(y) {}
  bool operator<(P p) const { return tie(x,y) < tie(p.x,p.y); }
  bool operator==(P p) const { return tie(x,y)==tie(p.x,p.y); }
  P operator+(P p) const { return P(x+p.x, y+p.y); }
  P operator-(P p) const { return P(x-p.x, y-p.y); }
  P operator*(T d) const { return P(x*d, y*d); }
  P operator/(T d) const { return P(x/d, y/d); }
  T dot(P p) const { return x*p.x + y*p.y; }
  T cross(P p) const { return x*p.y - y*p.x; }
  T cross(P a, P b) const { return (a-*this).cross(b-*this); }
  T dist2() const { return x*x + y*y; }
  double dist() const { return sqrt((double)dist2()); }
  // angle to x-axis in interval [-pi, pi]
  double angle() const { return atan2(y, x); }
  P unit() const { return *this/dist(); } // makes dist()=1
  P perp() const { return P(-y, x); } // rotates +90 degrees
  P normal() const { return perp().unit(); }
  // returns point rotated 'a' radians ccw around the origin
  P rotate(double a) const {
    return P(x*cos(a)-y*sin(a),x*sin(a)+y*cos(a)); }
  friend ostream& operator<<(ostream& os, P p) {
    return os << "(" << p.x << "," << p.y << ")"; }
};
```

lineDistance.h

Description:
Returns the signed distance between point p and the line containing points a and b. Positive value on left side and negative on right as seen from a towards b. a==b gives nan. P is supposed to be Point<T> or Point3D<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long. Using Point3D will always give a non-negative distance. For Point3D, call .dist on the result of the cross product.

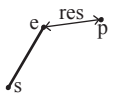


"Point.h" d41d8c, 4 lines

```
template<class P>
double lineDist(const P& a, const P& b, const P& p) {
  return (double)(b-a).cross(p-a)/(b-a).dist();
}
```

SegmentDistance.h

Description:
Returns the shortest distance between point p and the line segment from point s to e.



Usage:

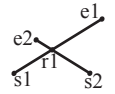
```
Point<double> a, b(2,2), p(1,1);
bool onSegment = segDist(a,b,p) < 1e-10;
```

"Point.h" d41d8c, 6 lines

```
typedef Point<double> P;
double segDist(P& s, P& e, P& p) {
  if (s==e) return (p-s).dist();
  auto d = (e-s).dist2(), t = min(d,max(.0,(p-s).dot(e-s)));
  return ((p-s)*d-(e-s)*t).dist()/d;
}
```

SegmentIntersection.h

Description:
If a unique intersection point between the line segments going from s1 to e1 and from s2 to e2 exists then it is returned. If no intersection point exists an empty vector is returned. If infinitely many exist a vector with 2 elements is returned, containing the endpoints of the common line segment. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long.



Usage:

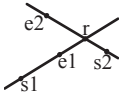
```
vector<P> inter = segInter(s1,e1,s2,e2);
if (sz(inter)==1)
cout << "segments intersect at " << inter[0] << endl;
```

"Point.h", "OnSegment.h" d41d8c, 13 lines

```
template<class P> vector<P> segInter(P a, P b, P c, P d) {
  auto oa = c.cross(d, a), ob = c.cross(d, b),
       oc = a.cross(b, c), od = a.cross(b, d);
  // Checks if intersection is single non-endpoint point.
  if (sgn(oa) * sgn(ob) < 0 && sgn(oc) * sgn(od) < 0)
    return {(a * ob - b * oa) / (ob - oa)};
  set<P> s;
  if (onSegment(c, d, a)) s.insert(a);
  if (onSegment(c, d, b)) s.insert(b);
  if (onSegment(a, b, c)) s.insert(c);
  if (onSegment(a, b, d)) s.insert(d);
  return {all(s)};
}
```

lineIntersection.h

Description:
If a unique intersection point of the lines going through s1,e1 and s2,e2 exists {1, point} is returned. If no intersection point exists {0, (0,0)} is returned and if infinitely many exists {-1, (0,0)} is returned. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or ll.



Usage:

```
auto res = lineInter(s1,e1,s2,e2);
if (res.first == 1)
cout << "intersection point at " << res.second << endl;
```

"Point.h" d41d8c, 8 lines

```
template<class P>
pair<int, P> lineInter(P s1, P e1, P s2, P e2) {
  auto d = (e1 - s1).cross(e2 - s2);
  if (d == 0) // if parallel
    return {-(s1.cross(e1, s2) == 0), P(0, 0)};
  auto p = s2.cross(e1, e2), q = s2.cross(e2, s1);
  return {1, (s1 * p + e1 * q) / d};
}
```

sideOf.h

Description: Returns where p is as seen from s towards e. 1/0/-1 $\Leftrightarrow$ left/on line/right. If the optional argument eps is given 0 is returned if p is within distance eps from the line. P is supposed to be Point<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long.

Usage: `bool left = sideOf(p1,p2,q)==1;`

"Point.h" d41d8c, 9 lines
```
template<class P>
int sideOf(P s, P e, P p) { return sgn(s.cross(e, p)); }

template<class P>
int sideOf(const P& s, const P& e, const P& p, double eps) {
  auto a = (e-s).cross(p-s);
  double l = (e-s).dist()*eps;
  return (a > l) - (a < -l);
}
```

OnSegment.h

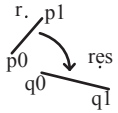
Description: Returns true iff p lies on the line segment from s to e. Use `(segDist(s,e,p)<=epsilon)` instead when using Point<double>.

"Point.h" d41d8c, 3 lines
```
template<class P> bool onSegment(P s, P e, P p) {
  return p.cross(s, e) == 0 && (s - p).dot(e - p) <= 0;
}
```

linearTransformation.h

Description:

Apply the linear transformation (translation, rotation and scaling) which takes line p0-p1 to line q0-q1 to point r.



"Point.h" d41d8c, 6 lines
```
typedef Point<double> P;
P linearTransformation(const P& p0, const P& p1,
    const P& q0, const P& q1, const P& r) {
  P dp = p1-p0, dq = q1-q0, num(dp.cross(dq), dp.dot(dq));
  return q0 + P((r-p0).cross(num), (r-p0).dot(num))/dp.dist2();
}
```

Angle.h

Description: A class for ordering angles (as represented by int points and a number of rotations around the origin). Useful for rotational sweeping. Sometimes also represents points or vectors.

Usage: `vector<Angle> v = {w[0], w[0].t360() ...}; // sorted`
`int j = 0; rep(i,0,n) { while (v[j] < v[i].t180()) ++j; }`
`// sweeps j such that (j-i) represents the number of positively oriented triangles with vertices at 0 and i`

d41d8c, 35 lines
```
struct Angle {
  int x, y;
  int t;
  Angle(int x, int y, int t=0) : x(x), y(y), t(t) {}
  Angle operator-(Angle b) const { return {x-b.x, y-b.y, t}; }
  int half() const {
    assert(x || y);
    return y < 0 || (y == 0 && x < 0);
  }
  Angle t90() const { return {-y, x, t + (half() && x >= 0)}; }
  Angle t180() const { return {-x, -y, t + half()}; }
  Angle t360() const { return {x, y, t + 1}; }
};
bool operator<(Angle a, Angle b) {
  // add a.dist2() and b.dist2() to also compare distances
  return make_tuple(a.t, a.half(), a.y * (ll)b.x) <
         make_tuple(b.t, b.half(), a.x * (ll)b.y);
}

// Given two points, this calculates the smallest angle between
// them, i.e., the angle that covers the defined line segment.
pair<Angle, Angle> segmentAngles(Angle a, Angle b) {
  if (b < a) swap(a, b);
  return (b < a.t180() ?
          make_pair(a, b) : make_pair(b, a.t360()));
}
Angle operator+(Angle a, Angle b) { // point a + vector b
  Angle r(a.x + b.x, a.y + b.y, a.t);
  if (a.t180() < r) r.t--;
  return r.t180() < a ? r.t360() : r;
}
Angle angleDiff(Angle a, Angle b) { // angle b - angle a
  int tu = b.t - a.t; a.t = b.t;
  return {a.x*b.x + a.y*b.y, a.x*b.y - a.y*b.x, tu - (b < a)};
}
```

8.2 Circles

CircleIntersection.h

Description: Computes the pair of points at which two circles intersect. Returns false in case of no intersection.

"Point.h" d41d8c, 11 lines
```
typedef Point<double> P;
bool circleInter(P a,P b,double r1,double r2,pair<P, P>* out) {
  if (a == b) { assert(r1 != r2); return false; }
  P vec = b - a;
  double d2 = vec.dist2(), sum = r1+r2, dif = r1-r2,
         p = (d2 + r1*r1 - r2*r2)/(d2*2), h2 = r1*r1 - p*p*d2;
  if (sum*sum < d2 || dif*dif > d2) return false;
  P mid = a + vec*p, per = vec.perp() * sqrt(fmax(0, h2) / d2);
  *out = {mid + per, mid - per};
  return true;
}
```

CircleTangents.h

Description: Finds the external tangents of two circles, or internal if r2 is negated. Can return 0, 1, or 2 tangents – 0 if one circle contains the other (or overlaps it, in the internal case, or if the circles are the same); 1 if the circles are tangent to each other (in which case .first = .second and the tangent line is perpendicular to the line between the centers). .first and .second give the tangency points at circle 1 and 2 respectively. To find the tangents of a circle with a point set r2 to 0.

"Point.h" d41d8c, 13 lines
```
template<class P>
vector<pair<P, P>> tangents(P c1, double r1, P c2, double r2) {
  P d = c2 - c1;
  double dr = r1 - r2, d2 = d.dist2(), h2 = d2 - dr * dr;
  if (d2 == 0 || h2 < 0)  return {};
  vector<pair<P, P>> out;
  for (double sign : {-1, 1}) {
    P v = (d * dr + d.perp() * sqrt(h2) * sign) / d2;
    out.push_back({c1 + v * r1, c2 + v * r2});
  }
  if (h2 == 0) out.pop_back();
  return out;
}
```

CirclePolygonIntersection.h

Description: Returns the area of the intersection of a circle with a ccw polygon.

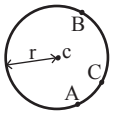
Time: $\mathcal{O}(n)$

"../../content/geometry/Point.h" d41d8c, 19 lines
```
typedef Point<double> P;
#define arg(p, q) atan2(p.cross(q), p.dot(q))
double circlePoly(P c, double r, vector<P> ps) {
  auto tri = [&](P p, P q) {
    auto r2 = r * r / 2;
    P d = q - p;
    auto a = d.dot(p)/d.dist2(), b = (p.dist2()-r*r)/d.dist2();
    auto det = a * a - b;
    if (det <= 0) return arg(p, q) * r2;
    auto s = max(0., -a-sqrt(det)), t = min(1., -a+sqrt(det));
    if (t < 0 || 1 <= s) return arg(p, q) * r2;
    P u = p + d * s, v = p + d * t;
    return arg(p,u) * r2 + u.cross(v)/2 + arg(v,q) * r2;
  };
  auto sum = 0.0;
  rep(i,0,sz(ps))
    sum += tri(ps[i] - c, ps[(i + 1) % sz(ps)] - c);
  return sum;
}
```

circumcircle.h

Description:

The circumcirle of a triangle is the circle intersecting all three vertices. ccRadius returns the radius of the circle going through points A, B and C and ccCenter returns the center of the same circle.



"Point.h" d41d8c, 9 lines
```
typedef Point<double> P;
double ccRadius(const P& A, const P& B, const P& C) {
  return (B-A).dist()*(C-B).dist()*(A-C).dist()/
      abs((B-A).cross(C-A))/2;
}
P ccCenter(const P& A, const P& B, const P& C) {
  P b = C-A, c = B-A;
  return A + (b*c.dist2()-c*b.dist2()).perp()/b.cross(c)/2;
}
```

MinimumEnclosingCircle.h

Description: Computes the minimum circle that encloses a set of points.

Time: expected $\mathcal{O}(n)$

"circumcircle.h" d41d8c, 17 lines
```
pair<P, double> mec(vector<P> ps) {
  shuffle(all(ps), mt19937(time(0)));
  P o = ps[0];
  double r = 0, EPS = 1 + 1e-8;
  rep(i,0,sz(ps)) if ((o - ps[i]).dist() > r * EPS) {
    o = ps[i], r = 0;
    rep(j,0,i) if ((o - ps[j]).dist() > r * EPS) {
      o = (ps[i] + ps[j]) / 2;
      r = (o - ps[i]).dist();
      rep(k,0,j) if ((o - ps[k]).dist() > r * EPS) {
        o = ccCenter(ps[i], ps[j], ps[k]);
        r = (o - ps[i]).dist();
      }
    }
  }
  return {o, r};
}
```

8.3 Polygons

InsidePolygon.h

Description: Returns true if p lies within the polygon. If strict is true, it returns false for points on the boundary. The algorithm uses products in intermediate steps so watch out for overflow.

Usage: `vector<P> v = {P{4,4}, P{1,2}, P{2,1}};`
`bool in = inPolygon(v, P{3, 3}, false);`

Time: $\mathcal{O}(n)$

"Point.h", "OnSegment.h", "SegmentDistance.h" d41d8c, 11 lines
```
template<class P>
bool inPolygon(vector<P> &p, P a, bool strict = true) {
  int cnt = 0, n = sz(p);
  rep(i,0,n) {
    P q = p[(i + 1) % n];
    if (onSegment(p[i], q, a)) return !strict;
    //or: if (segDist(p[i], q, a) <= eps) return !strict;
    cnt ^= ((a.y<p[i].y) - (a.y<q.y)) * a.cross(p[i], q) > 0;
  }
  return cnt;
}
```

PolygonArea.h

Description: Returns twice the signed area of a polygon. Clockwise enumeration gives negative area. Watch out for overflow if using int as T!

"Point.h" d41d8c, 6 lines

```
template<class T>
T polygonArea2(vector<Point<T>>& v) {
  T a = v.back().cross(v[0]);
  rep(i,0,sz(v)-1) a += v[i].cross(v[i+1]);
  return a;
}
```

PolygonCenter.h

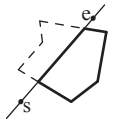
Description: Returns the center of mass for a polygon.
Time: $\mathcal{O}(n)$

"Point.h" d41d8c, 9 lines

```
typedef Point<double> P;
P polygonCenter(const vector<P>& v) {
  P res(0, 0); double A = 0;
  for (int i = 0, j = sz(v) - 1; i < sz(v); j = i++) {
    res = res + (v[i] + v[j]) * v[j].cross(v[i]);
    A += v[j].cross(v[i]);
  }
  return res / A / 3;
}
```

PolygonCut.h

Description:
Returns a vector with the vertices of a polygon with everything to the left of the line going from s to e cut away.
Usage: vector<P> p = ...;
p = polygonCut(p, P(0,0), P(1,0));



"Point.h", "lineIntersection.h" d41d8c, 13 lines

```
typedef Point<double> P;
vector<P> polygonCut(const vector<P>& poly, P s, P e) {
  vector<P> res;
  rep(i,0,sz(poly)) {
    P cur = poly[i], prev = i ? poly[i-1] : poly.back();
    bool side = s.cross(e, cur) < 0;
    if (side != (s.cross(e, prev) < 0))
      res.push_back(lineInter(s, e, cur, prev).second);
    if (side)
      res.push_back(cur);
  }
  return res;
}
```

ConvexHull.h

Description:
Returns a vector of the points of the convex hull in counterclockwise order. Points on the edge of the hull between two other points are not considered part of the hull.
Time: $\mathcal{O}(n \log n)$

"Point.h" d41d8c, 13 lines

```
typedef Point<ll> P;
vector<P> convexHull(vector<P> pts) {
  if (sz(pts) <= 1) return pts;
  sort(all(pts));
  vector<P> h(sz(pts)+1);
  int s = 0, t = 0;
  for (int it = 2; it--; s = --t, reverse(all(pts)))
    for (P p : pts) {
      while (t >= s + 2 && h[t-2].cross(h[t-1], p) <= 0) t--;
      h[t++] = p;
    }
  return {h.begin(), h.begin() + t - (t == 2 && h[0] == h[1])};
}
```

HullDiameter.h

Description: Returns the two points with max distance on a convex hull (ccw, no duplicate/collinear points).
Time: $\mathcal{O}(n)$

"Point.h" d41d8c, 12 lines

```
typedef Point<ll> P;
array<P, 2> hullDiameter(vector<P> S) {
  int n = sz(S), j = n < 2 ? 0 : 1;
  pair<ll, array<P, 2>> res({0, {S[0], S[0]}});
  rep(i,0,j)
    for (;; j = (j + 1) % n) {
      res = max(res, {(S[i] - S[j]).dist2(), {S[i], S[j]}});
      if ((S[(j + 1) % n] - S[j]).cross(S[i + 1] - S[i]) >= 0)
        break;
    }
  return res.second;
}
```

PointInsideHull.h

Description: Determine whether a point t lies inside a convex hull (CCW order, with no collinear points). Returns true if point lies within the hull. If strict is true, points on the boundary aren't included.
Time: $\mathcal{O}(\log N)$

"Point.h", "sideOf.h", "OnSegment.h" d41d8c, 14 lines

```
typedef Point<ll> P;

bool inHull(const vector<P>& l, P p, bool strict = true) {
  int a = 1, b = sz(l) - 1, r = !strict;
  if (sz(l) < 3) return r && onSegment(l[0], l.back(), p);
  if (sideOf(l[0], l[a], l[b]) > 0) swap(a, b);
  if (sideOf(l[0], l[a], p) >= r || sideOf(l[0], l[b], p)<= -r)
    return false;
  while (abs(a - b) > 1) {
    int c = (a + b) / 2;
    (sideOf(l[0], l[c], p) > 0 ? b : a) = c;
  }
  return sgn(l[a].cross(l[b], p)) < r;
}
```

LineHullIntersection.h

Description: Line-convex polygon intersection. The polygon must be ccw and have no collinear points. lineHull(line, poly) returns a pair describing the intersection of a line with the polygon: $\bullet$ $(-1,-1)$ if no collision, $\bullet$ $(i,-1)$ if touching the corner i, $\bullet$ (i,i) if along side $(i,i+1)$, $\bullet$ (i,j) if crossing sides $(i,i+1)$ and $(j,j+1)$. In the last case, if a corner i is crossed, this is treated as happening on side $(i,i+1)$. The points are returned in the same order as the line hits the polygon. `extrVertex` returns the point of a hull with the max projection onto a line.
Time: $\mathcal{O}(\log n)$

"Point.h" d41d8c, 39 lines

```
#define cmp(i,j) sgn(dir.perp().cross(poly[(i)%n]-poly[(j)%n]))
#define extr(i) cmp(i + 1, i) >= 0 && cmp(i, i - 1 + n) < 0
template <class P> int extrVertex(vector<P>& poly, P dir) {
  int n = sz(poly), lo = 0, hi = n;
  if (extr(0)) return 0;
  while (lo + 1 < hi) {
    int m = (lo + hi) / 2;
    if (extr(m)) return m;
    int ls = cmp(lo + 1, lo), ms = cmp(m + 1, m);
    (ls < ms || (ls == ms && ls == cmp(lo, m)) ? hi : lo) = m;
  }
  return lo;
}

#define cmpL(i) sgn(a.cross(poly[i], b))
template <class P>
array<int, 2> lineHull(P a, P b, vector<P>& poly) {
  int endA = extrVertex(poly, (a - b).perp());
  int endB = extrVertex(poly, (b - a).perp());
  if (cmpL(endA) < 0 || cmpL(endB) > 0)
    return {-1, -1};
  array<int, 2> res;
  rep(i,0,2) {
    int lo = endB, hi = endA, n = sz(poly);
    while ((lo + 1) % n != hi) {
      int m = ((lo + hi + (lo < hi ? 0 : n)) / 2) % n;
      (cmpL(m) == cmpL(endB) ? lo : hi) = m;
    }
    res[i] = (lo + !cmpL(hi)) % n;
    swap(endA, endB);
  }
  if (res[0] == res[1]) return {res[0], -1};
  if (!cmpL(res[0]) && !cmpL(res[1]))
    switch ((res[0] - res[1] + sz(poly) + 1) % sz(poly)) {
      case 0: return {res[0], res[0]};
      case 2: return {res[1], res[1]};
    }
  return res;
}
```

8.4 Misc. Point Set Problems

ClosestPair.h

Description: Finds the closest pair of points.
Time: $\mathcal{O}(n \log n)$

"Point.h" d41d8c, 17 lines

```
typedef Point<ll> P;
pair<P, P> closest(vector<P> v) {
  assert(sz(v) > 1);
  set<P> S;
  sort(all(v), [](P a, P b) { return a.y < b.y; });
  pair<ll, pair<P, P>> ret{LLONG_MAX, {P(), P()}};
  int j = 0;
  for (P p : v) {
    P d{1 + (ll)sqrt(ret.first), 0};
    while (v[j].y <= p.y - d.x) S.erase(v[j++]);
    auto lo = S.lower_bound(p - d), hi = S.upper_bound(p + d);
    for (; lo != hi; ++lo)
      ret = min(ret, {(*lo - p).dist2(), {*lo, p}});
    S.insert(p);
  }
  return ret.second;
}
```

8.5 3D

PolyhedronVolume.h

Description: Magic formula for the volume of a polyhedron. Faces should point outwards.

d41d8c, 6 lines

```
template<class V, class L>
double signedPolyVolume(const V& p, const L& trilist) {
  double v = 0;
  for (auto i : trilist) v += p[i.a].cross(p[i.b]).dot(p[i.c]);
  return v / 6;
}
```

Point3D.h

Description: Class to handle points in 3D space. T can be e.g. double or long long.

d41d8c, 32 lines

```
template<class T> struct Point3D {
  typedef Point3D P;
  typedef const P& R;
  T x, y, z;
  explicit Point3D(T x=0, T y=0, T z=0) : x(x), y(y), z(z) {}
  bool operator<(R p) const {
    return tie(x, y, z) < tie(p.x, p.y, p.z); }
```

```
  bool operator==(R p) const {
    return tie(x, y, z) == tie(p.x, p.y, p.z); }
  P operator+(R p) const { return P(x+p.x, y+p.y, z+p.z); }
  P operator-(R p) const { return P(x-p.x, y-p.y, z-p.z); }
  P operator*(T d) const { return P(x*d, y*d, z*d); }
  P operator/(T d) const { return P(x/d, y/d, z/d); }
  T dot(R p) const { return x*p.x + y*p.y + z*p.z; }
  P cross(R p) const {
    return P(y*p.z - z*p.y, z*p.x - x*p.z, x*p.y - y*p.x);
  }
  T dist2() const { return x*x + y*y + z*z; }
  double dist() const { return sqrt((double)dist2()); }
  //Azimuthal angle (longitude) to x-axis in interval [-pi, pi]
  double phi() const { return atan2(y, x); }
  //Zenith angle (latitude) to the z-axis in interval [0, pi]
  double theta() const { return atan2(sqrt(x*x+y*y),z); }
  P unit() const { return *this/(T)dist(); } //makes dist()=1
  //returns unit vector normal to *this and p
  P normal(P p) const { return cross(p).unit(); }
  //returns point rotated 'angle' radians ccw around axis
  P rotate(double angle, P axis) const {
    double s = sin(angle), c = cos(angle); P u = axis.unit();
    return u*dot(u)*(1-c) + (*this)*c - cross(u)*s;
  }
};
```

3dHull.h

Description: Computes all faces of the 3-dimension hull of a point set. *No four points must be coplanar*, or else random results will be returned. All faces will point outwards.
Time: $\mathcal{O}(n^2)$

"Point3D.h" d41d8c, 49 lines

```
typedef Point3D<double> P3;

struct PR {
  void ins(int x) { (a == -1 ? a : b) = x; }
  void rem(int x) { (a == x ? a : b) = -1; }
  int cnt() { return (a != -1) + (b != -1); }
  int a, b;
};

struct F { P3 q; int a, b, c; };

vector<F> hull3d(const vector<P3>& A) {
  assert(sz(A) >= 4);
  vector<vector<PR>> E(sz(A), vector<PR>(sz(A), {-1, -1}));
#define E(x,y) E[f.x][f.y]
  vector<F> FS;
  auto mf = [&](int i, int j, int k, int l) {
    P3 q = (A[j] - A[i]).cross((A[k] - A[i]));
    if (q.dot(A[l]) > q.dot(A[i]))
      q = q * -1;
    F f{q, i, j, k};
    E(a,b).ins(k); E(a,c).ins(j); E(b,c).ins(i);
    FS.push_back(f);
  };
  rep(i,0,4) rep(j,i+1,4) rep(k,j+1,4)
    mf(i, j, k, 6 - i - j - k);

  rep(i,4,sz(A)) {
    rep(j,0,sz(FS)) {
      F f = FS[j];
      if(f.q.dot(A[i]) > f.q.dot(A[f.a])) {
        E(a,b).rem(f.c);
        E(a,c).rem(f.b);
        E(b,c).rem(f.a);
        swap(FS[j--], FS.back());
        FS.pop_back();
      }
    }
    int nw = sz(FS);
    rep(j,0,nw) {
      F f = FS[j];
#define C(a, b, c) if (E(a,b).cnt() != 2) mf(f.a, f.b, i, f.c);
      C(a, b, c); C(a, c, b); C(b, c, a);
    }
  }
  for (F& it : FS) if ((A[it.b] - A[it.a]).cross(
    A[it.c] - A[it.a]).dot(it.q) <= 0) swap(it.c, it.b);
  return FS;
};
```

sphericalDistance.h

Description: Returns the shortest distance on the sphere with radius radius between the points with azimuthal angles (longitude) f1 (ϕ_1) and f2 (ϕ_2) from x axis and zenith angles (latitude) t1 (θ_1) and t2 (θ_2) from z axis (0 = north pole). All angles measured in radians. The algorithm starts by converting the spherical coordinates to cartesian coordinates so if that is what you have you can use only the two last rows. dx*radius is then the difference between the two points in the x direction and d*radius is the total distance between the points.

d41d8c, 8 lines

```
double sphericalDistance(double f1, double t1,
    double f2, double t2, double radius) {
  double dx = sin(t2)*cos(f2) - sin(t1)*cos(f1);
  double dy = sin(t2)*sin(f2) - sin(t1)*sin(f1);
  double dz = cos(t2) - cos(t1);
  double d = sqrt(dx*dx + dy*dy + dz*dz);
  return radius*2*asin(d/2);
}
```

Strings (9)

KMP.h

Description: pi[x] computes the length of the longest prefix of s that ends at x, other than s[0...x] itself (abacaba -> 0010123). Can be used to find all occurrences of a string.
Time: $\mathcal{O}(n)$

d41d8c, 16 lines

```
vi pi(const string& s) {
  vi p(sz(s));
  rep(i,1,sz(s)) {
    int g = p[i-1];
    while (g && s[i] != s[g]) g = p[g-1];
    p[i] = g + (s[i] == s[g]);
  }
  return p;
}

vi match(const string& s, const string& pat) {
  vi p = pi(pat + '\0' + s), res;
  rep(i,sz(p)-sz(s),sz(p))
    if (p[i] == sz(pat)) res.push_back(i - 2 * sz(pat));
  return res;
}
```

Zfunc.h

Description: z[x] computes the length of the longest common prefix of s[i:] and s, except z[0] = 0. (abacaba -> 0010301)
Time: $\mathcal{O}(n)$

d41d8c, 12 lines

```
vi Z(const string& S) {
  vi z(sz(S));
  int l = -1, r = -1;
  rep(i,1,sz(S)) {
    z[i] = i >= r ? 0 : min(r - i, z[i - l]);
    while (i + z[i] < sz(S) && S[i + z[i]] == S[z[i]])
      z[i]++;
    if (i + z[i] > r)
      l = i, r = i + z[i];
  }
  return z;
}
```

Manacher.h

Description: For each position in a string, computes p[0][i] = half length of longest even palindrome around pos i, p[1][i] = longest odd (half rounded down).
Time: $\mathcal{O}(N)$

d41d8c, 13 lines

```
array<vi, 2> manacher(const string& s) {
  int n = sz(s);
  array<vi,2> p = {vi(n+1), vi(n)};
  rep(z,0,2) for (int i=0,l=0,r=0; i < n; i++) {
    int t = r-i+!z;
    if (i<r) p[z][i] = min(t, p[z][l+t]);
    int L = i-p[z][i], R = i+p[z][i]-!z;
    while (L>=1 && R+1<n && s[L-1] == s[R+1])
      p[z][i]++, L--, R++;
    if (R>r) l=L, r=R;
  }
  return p;
}
```

MinRotation.h

Description: Finds the lexicographically smallest rotation of a string.
Usage: `rotate(v.begin(), v.begin()+minRotation(v), v.end());`
Time: $\mathcal{O}(N)$

d41d8c, 8 lines

```
int minRotation(string s) {
  int a=0, N=sz(s); s += s;
  rep(b,0,N) rep(k,0,N) {
    if (a+k == b || s[a+k] < s[b+k]) {b += max(0, k-1); break;}
    if (s[a+k] > s[b+k]) { a = b; break; }
  }
  return a;
}
```

SuffixArray.h

Description: Builds suffix array for a string. `sa[i]` is the starting index of the suffix which is i'th in the sorted suffix array. The returned vector is of size $n+1$, and `sa[0] = n`. The `lcp` array contains longest common prefixes for neighbouring strings in the suffix array: `lcp[i] = lcp(sa[i], sa[i-1])`, `lcp[0] = 0`. The input string must not contain any zero bytes.
Time: $\mathcal{O}(n \log n)$

d41d8c, 23 lines

```
struct SuffixArray {
  vi sa, lcp;
  SuffixArray(string& s, int lim=256) { // or basic_string<int>
    int n = sz(s) + 1, k = 0, a, b;
    vi x(all(s)+1), y(n), ws(max(n, lim)), rank(n);
    sa = lcp = y, iota(all(sa), 0);
    for (int j = 0, p = 0; p < n; j = max(1, j * 2), lim = p) {
      p = j, iota(all(y), n - j);
      rep(i,0,n) if (sa[i] >= j) y[p++] = sa[i] - j;
      fill(all(ws), 0);
      rep(i,0,n) ws[x[i]]++;
      rep(i,1,lim) ws[i] += ws[i - 1];
      for (int i = n; i--;) sa[--ws[x[y[i]]]] = y[i];
      swap(x, y), p = 1, x[sa[0]] = 0;
      rep(i,1,n) a = sa[i - 1], b = sa[i], x[b] =
        (y[a] == y[b] && y[a + j] == y[b + j]) ? p - 1 : p++;
    }
    rep(i,1,n) rank[sa[i]] = i;
    for (int i = 0, j; i < n - 1; lcp[rank[i++]] = k)
      for (k && k--, j = sa[rank[i] - 1];
```

```
      s[i + k] == s[j + k]; k++);
  }
};
```

SuffixTree.h

Description: Ukkonen's algorithm for online suffix tree construction. Each node contains indices [l, r) into the string, and a list of child nodes. Suffixes are given by traversals of this tree, joining [l, r) substrings. The root is 0 (has l = -1, r = 0), non-existent children are -1. To get a complete tree, append a dummy symbol – otherwise it may contain an incomplete path (still useful for substring matching, though).
Time: $\mathcal{O}(26N)$

d41d8c, 50 lines

```
struct SuffixTree {
  enum { N = 200010, ALPHA = 26 }; // N ~ 2*maxlen+10
  int toi(char c) { return c - 'a'; }
  string a; // v = cur node, q = cur position
  int t[N][ALPHA],l[N],r[N],p[N],s[N],v=0,q=0,m=2;

  void ukkadd(int i, int c) { suff:
    if (r[v]<=q) {
      if (t[v][c]==-1) { t[v][c]=m;  l[m]=i;
        p[m++]=v; v=s[v]; q=r[v];  goto suff; }
      v=t[v][c]; q=l[v];
    }
    if (q==-1 || c==toi(a[q])) q++; else {
      l[m+1]=i;  p[m+1]=m;  l[m]=l[v];  r[m]=q;
      p[m]=p[v];  t[m][c]=m+1;  t[m][toi(a[q])]=v;
      l[v]=q;  p[v]=m;  t[p[m]][toi(a[l[m]])]=m;
      v=s[p[m]];  q=l[m];
      while (q<r[m]) { v=t[v][toi(a[q])];  q+=r[v]-l[v]; }
      if (q==r[m])  s[m]=v;  else s[m]=m+2;
      q=r[v]-(q-r[m]);  m+=2;  goto suff;
    }
  }

  SuffixTree(string a) : a(a) {
    fill(r,r+N,sz(a));
    memset(s, 0, sizeof s);
    memset(t, -1, sizeof t);
    fill(t[1],t[1]+ALPHA,0);
    s[0] = 1; l[0] = l[1] = -1; r[0] = r[1] = p[0] = p[1] = 0;
    rep(i,0,sz(a)) ukkadd(i, toi(a[i]));
  }

  // example: find longest common substring (uses ALPHA = 28)
  pii best;
  int lcs(int node, int i1, int i2, int olen) {
    if (l[node] <= i1 && i1 < r[node]) return 1;
    if (l[node] <= i2 && i2 < r[node]) return 2;
    int mask = 0, len = node ? olen + (r[node] - l[node]) : 0;
    rep(c,0,ALPHA) if (t[node][c] != -1)
      mask |= lcs(t[node][c], i1, i2, len);
    if (mask == 3)
      best = max(best, {len, r[node] - len});
    return mask;
  }
  static pii LCS(string s, string t) {
    SuffixTree st(s + (char)('z' + 1) + t + (char)('z' + 2));
    st.lcs(0, sz(s), sz(s) + 1 + sz(t), 0);
    return st.best;
  }
};
```

Hashing.h

Description: Self-explanatory methods for string hashing.

d41d8c, 44 lines

```
// Arithmetic mod 2^64-1. 2x slower than mod 2^64 and more
// code, but works on evil test data (e.g. Thue-Morse, where
// ABBA... and BAAB... of length 2^10 hash the same mod 2^64).
// "typedef ull H;" instead if you think test data is random,
// or work mod 10^9+7 if the Birthday paradox is not a problem.
typedef uint64_t ull;
struct H {
  ull x; H(ull x=0) : x(x) {}
  H operator+(H o) { return x + o.x + (x + o.x < x); }
  H operator-(H o) { return *this + ~o.x; }
  H operator*(H o) { auto m = (__uint128_t)x * o.x;
    return H((ull)m) + (ull)(m >> 64); }
  ull get() const { return x + !~x; }
  bool operator==(H o) const { return get() == o.get(); }
  bool operator<(H o) const { return get() < o.get(); }
};
static const H C = (ll)1e11+3; // (order ~ 3e9; random also ok)

struct HashInterval {
  vector<H> ha, pw;
  HashInterval(string& str) : ha(sz(str)+1), pw(ha) {
    pw[0] = 1;
    rep(i,0,sz(str))
      ha[i+1] = ha[i] * C + str[i],
      pw[i+1] = pw[i] * C;
  }
  H hashInterval(int a, int b) { // hash [a, b)
    return ha[b] - ha[a] * pw[b - a];
  }
};

vector<H> getHashes(string& str, int length) {
  if (sz(str) < length) return {};
  H h = 0, pw = 1;
  rep(i,0,length)
    h = h * C + str[i], pw = pw * C;
  vector<H> ret = {h};
  rep(i,length,sz(str)) {
    ret.push_back(h = h * C + str[i] - pw * str[i-length]);
  }
  return ret;
}

H hashString(string& s){H h{}; for(char c:s) h=h*C+c;return h;}
```

AhoCorasick.h

Description: Aho-Corasick automaton, used for multiple pattern matching. Initialize with AhoCorasick ac(patterns); the automaton start node will be at index 0. find(word) returns for each position the index of the longest word that ends there, or -1 if none. findAll(−, word) finds all words (up to $N\sqrt{N}$ many if no duplicate patterns) that start at each position (shortest first). Duplicate patterns are allowed; empty patterns are not. To find the longest words that start at each position, reverse all input. For large alphabets, split each symbol into chunks, with sentinel bits for symbol boundaries.
Time: construction takes $\mathcal{O}(26N)$, where N = sum of length of patterns. find(x) is $\mathcal{O}(N)$, where N = length of x. findAll is $\mathcal{O}(NM)$.

d41d8c, 66 lines

```
struct AhoCorasick {
  enum {alpha = 26, first = 'A'}; // change this!
  struct Node {
    // (nmatches is optional)
    int back, next[alpha], start = -1, end = -1, nmatches = 0;
    Node(int v) { memset(next, v, sizeof(next)); }
  };
  vector<Node> N;
  vi backp;
  void insert(string& s, int j) {
    assert(!s.empty());
    int n = 0;
    for (char c : s) {
      int& m = N[n].next[c - first];
      if (m == -1) { n = m = sz(N); N.emplace_back(-1); }
      else n = m;
    }
    if (N[n].end == -1) N[n].start = j;
    backp.push_back(N[n].end);
    N[n].end = j;
    N[n].nmatches++;
  }
  AhoCorasick(vector<string>& pat) : N(1, -1) {
    rep(i,0,sz(pat)) insert(pat[i], i);
    N[0].back = sz(N);
    N.emplace_back(0);

    queue<int> q;
    for (q.push(0); !q.empty(); q.pop()) {
      int n = q.front(), prev = N[n].back;
      rep(i,0,alpha) {
        int &ed = N[n].next[i], y = N[prev].next[i];
        if (ed == -1) ed = y;
        else {
          N[ed].back = y;
          (N[ed].end == -1 ? N[ed].end : backp[N[ed].start])
            = N[y].end;
          N[ed].nmatches += N[y].nmatches;
          q.push(ed);
        }
      }
    }
  }
  vi find(string word) {
    int n = 0;
    vi res; // ll count = 0;
    for (char c : word) {
      n = N[n].next[c - first];
      res.push_back(N[n].end);
      // count += N[n].nmatches;
    }
    return res;
  }
  vector<vi> findAll(vector<string>& pat, string word) {
    vi r = find(word);
    vector<vi> res(sz(word));
    rep(i,0,sz(word)) {
      int ind = r[i];
      while (ind != -1) {
        res[i - sz(pat[ind]) + 1].push_back(ind);
        ind = backp[ind];
      }
    }
    return res;
  }
};
```

Various (10)

10.1 Intervals

IntervalContainer.h

Description: Add and remove intervals from a set of disjoint intervals. Will merge the added interval with any overlapping intervals in the set when adding. Intervals are [inclusive, exclusive).
Time: $\mathcal{O}(\log N)$

d41d8c, 23 lines

```
set<pii>::iterator addInterval(set<pii>& is, int L, int R) {
  if (L == R) return is.end();
  auto it = is.lower_bound({L, R}), before = it;
  while (it != is.end() && it->first <= R) {
    R = max(R, it->second);
```

```
    before = it = is.erase(it);
  }
  if (it != is.begin() && (--it)->second >= L) {
    L = min(L, it->first);
    R = max(R, it->second);
    is.erase(it);
  }
  return is.insert(before, {L,R});
}

void removeInterval(set<pii>& is, int L, int R) {
  if (L == R) return;
  auto it = addInterval(is, L, R);
  auto r2 = it->second;
  if (it->first == L) is.erase(it);
  else (int&)it->second = L;
  if (R != r2) is.emplace(R, r2);
}
```

IntervalCover.h

Description: Compute indices of smallest set of intervals covering another interval. Intervals should be [inclusive, exclusive). To support [inclusive, inclusive], change (A) to add `|| R.empty()`. Returns empty set on failure (or if G is empty).
Time: $\mathcal{O}(N \log N)$

d41d8c, 19 lines

```
template<class T>
vi cover(pair<T, T> G, vector<pair<T, T>> I) {
  vi S(sz(I)), R;
  iota(all(S), 0);
  sort(all(S), [&](int a, int b) { return I[a] < I[b]; });
  T cur = G.first;
  int at = 0;
  while (cur < G.second) { // (A)
    pair<T, int> mx = make_pair(cur, -1);
    while (at < sz(I) && I[S[at]].first <= cur) {
      mx = max(mx, make_pair(I[S[at]].second, S[at]));
      at++;
    }
    if (mx.second == -1) return {};
    cur = mx.first;
    R.push_back(mx.second);
  }
  return R;
}
```

ConstantIntervals.h

Description: Split a monotone function on [from, to) into a minimal set of half-open intervals on which it has the same value. Runs a callback g for each such interval.
Usage: `constantIntervals(0, sz(v), [&](int x){return v[x];}, [&](int lo, int hi, T val){...});`
Time: $\mathcal{O}\left(k \log \frac{n}{k}\right)$

d41d8c, 19 lines

```
template<class F, class G, class T>
void rec(int from, int to, F& f, G& g, int& i, T& p, T q) {
  if (p == q) return;
  if (from == to) {
    g(i, to, p);
    i = to; p = q;
  } else {
    int mid = (from + to) >> 1;
    rec(from, mid, f, g, i, p, f(mid));
    rec(mid+1, to, f, g, i, p, q);
  }
}
template<class F, class G>
void constantIntervals(int from, int to, F f, G g) {
  if (to <= from) return;
  int i = from; auto p = f(i), q = f(to-1);
  rec(from, to-1, f, g, i, p, q);
  g(i, to, q);
}
```

10.2 Misc. algorithms

TernarySearch.h

Description: Find the smallest i in $[a, b]$ that maximizes $f(i)$, assuming that $f(a) < \ldots < f(i) \geq \cdots \geq f(b)$. To reverse which of the sides allows non-strict inequalities, change the $<$ marked with (A) to $<=$, and reverse the loop at (B). To minimize f, change it to $>$, also at (B).
Usage: `int ind = ternSearch(0,n-1,[&](int i){return a[i];});`
Time: $\mathcal{O}(\log(b-a))$

d41d8c, 11 lines

```
template<class F>
int ternSearch(int a, int b, F f) {
  assert(a <= b);
  while (b - a >= 5) {
    int mid = (a + b) / 2;
    if (f(mid) < f(mid+1)) a = mid; // (A)
    else b = mid+1;
  }
  rep(i,a+1,b+1) if (f(a) < f(i)) a = i; // (B)
  return a;
}
```

LIS.h

Description: Compute indices for the longest increasing subsequence.
Time: $\mathcal{O}(N \log N)$

d41d8c, 17 lines

```
template<class I> vi lis(const vector<I>& S) {
  if (S.empty()) return {};
  vi prev(sz(S));
  typedef pair<I, int> p;
  vector<p> res;
  rep(i,0,sz(S)) {
    // change 0 -> i for longest non-decreasing subsequence
    auto it = lower_bound(all(res), p{S[i], 0});
    if (it == res.end()) res.emplace_back(), it = res.end()-1;
    *it = {S[i], i};
    prev[i] = it == res.begin() ? 0 : (it-1)->second;
  }
  int L = sz(res), cur = res.back().second;
  vi ans(L);
  while (L--) ans[L] = cur, cur = prev[cur];
  return ans;
}
```

FastKnapsack.h

Description: Given N non-negative integer weights w and a non-negative target t, computes the maximum S <= t such that S is the sum of some subset of the weights.
Time: $\mathcal{O}(N \max(w_i))$

d41d8c, 16 lines

```
int knapsack(vi w, int t) {
  int a = 0, b = 0, x;
  while (b < sz(w) && a + w[b] <= t) a += w[b++];
  if (b == sz(w)) return a;
  int m = *max_element(all(w));
  vi u, v(2*m, -1);
  v[a+m-t] = b;
  rep(i,b,sz(w)) {
    u = v;
    rep(x,0,m) v[x+w[i]] = max(v[x+w[i]], u[x]);
    for (x = 2*m; --x > m;) rep(j, max(0,u[x]), v[x])
      v[x-w[j]] = max(v[x-w[j]], j);
  }
  for (a = t; v[a+m-t] < 0; a--) ;
  return a;
}
```

10.3 Dynamic programming

KnuthDP.h

Description: When doing DP on intervals: $a[i][j] = \min_{i<k<j}(a[i][k] + a[k][j]) + f(i,j)$, where the (minimal) optimal k increases with both i and j, one can solve intervals in increasing order of length, and search $k = p[i][j]$ for $a[i][j]$ only between $p[i][j-1]$ and $p[i+1][j]$. This is known as Knuth DP. Sufficient criteria for this are if $f(b,c) \leq f(a,d)$ and $f(a,c) + f(b,d) \leq f(a,d) + f(b,c)$ for all $a \leq b \leq c \leq d$. Consider also: LineContainer (ch. Data structures), monotone queues, ternary search.
Time: $\mathcal{O}\left(N^2\right)$

DivideAndConquerDP.h

Description: Given $a[i] = \min_{lo(i)\leq k<hi(i)}(f(i,k))$ where the (minimal) optimal k increases with i, computes $a[i]$ for $i = L..R-1$.
Time: $\mathcal{O}\left((N + (hi - lo))\log N\right)$

d41d8c, 18 lines

```
struct DP { // Modify at will:
  int lo(int ind) { return 0; }
  int hi(int ind) { return ind; }
  ll f(int ind, int k) { return dp[ind][k]; }
  void store(int ind, int k, ll v) { res[ind] = pii(k, v); }

  void rec(int L, int R, int LO, int HI) {
    if (L >= R) return;
    int mid = (L + R) >> 1;
    pair<ll, int> best(LLONG_MAX, LO);
    rep(k, max(LO,lo(mid)), min(HI,hi(mid)))
      best = min(best, make_pair(f(mid, k), k));
    store(mid, best.second, best.first);
    rec(L, mid, LO, best.second+1);
    rec(mid+1, R, best.second, HI);
  }
  void solve(int L, int R) { rec(L, R, INT_MIN, INT_MAX); }
};
```

10.4 Debugging tricks

- `signal(SIGSEGV, [](int) { _Exit(0); });` converts segfaults into Wrong Answers. Similarly one can catch SIGABRT (assertion failures) and SIGFPE (zero divisions). `_GLIBCXX_DEBUG` failures generate SIGABRT (or SIGSEGV on gcc 5.4.0 apparently).
- `feenableexcept(29);` kills the program on NaNs (1), 0-divs (4), infinities (8) and denormals (16).

10.5 Optimization tricks

`__builtin_ia32_ldmxcsr(40896);` disables denormals (which make floats 20x slower near their minimum value).

10.5.1 Bit hacks

- `x & -x` is the least bit in `x`.
- `for (int x = m; x; ) { --x &= m; ... }` loops over all subset masks of `m` (except `m` itself).
- `c = x&-x, r = x+c; (((r^x) >> 2)/c) | r` is the next number after `x` with the same number of bits set.
- `rep(b,0,K) rep(i,0,(1 << K))`

`if (i & 1 << b) D[i] += D[i^(1 << b)];` computes all sums of subsets.

NEW (11)

11.1 Trie

trie.h

Description: Trie

Time: $\mathcal{O}(lengthofwords)$

d41d8c, 99 lines

```
struct Node {
    Node *child[MAX];
    int countWord;
};

Node* newNode(){
    Node* node = new Node;
    node->countWord = 0;
    for (int i = 0; i < MAX; i++){
        node->child[i] = NULL;
    }
    return node;
}

// add a new word to the trie structure
void addWord(Node* root, string s){
    int ch;
    Node* temp = root;
    for (int i = 0; i < s.size(); i++){
        ch = s[i] - 'a';
        if (temp->child[ch] == NULL){
            temp->child[ch] = newNode(); // add the ch
                character to the trie
        }
        temp = temp->child[ch];
    }
    temp->countWord++;
}

// find a particular word in the trie structure
bool findWord(Node* root, string s){
    int ch;
    Node* temp = root;
    for (int i = 0; i < s.size(); i++){
        ch = s[i] - 'a';
        if (temp->child[ch] == NULL){
            return false;
        }
        temp = temp->child[ch];
    }
    return temp->countWord > 0;
}

//delete a word from a trie
bool isWord(Node* node){
    return (node->countWord != 0);
}
    // To check whether there are words at deeper levels
bool isEmpty(Node* node){
    for (int i = 0; i < MAX; i++){
        if (node->child[i] != NULL){
            return false;
        }
    }
    return true;
}
```

```
bool removeWord(Node* root, string s, int level = 0){
    if (!root){
        return false;
    }

    if (level == s.length()){
        if (root->countWord > 0){
            root->countWord--;
            return true;
        }
        return false;
    }

    int ch = s[level] - 'a';
    int flag = removeWord(root->child[ch], s, level + 1);
    if (flag && !isWord(root->child[ch]) && isEmpty(root->child
        [ch])){
        delete root->child[ch];
        root->child[ch] = NULL;
    }
    return flag;
}

int main()
{
    ios_base::sync_with_stdio(false);
    cin.tie(nullptr);

    int n, q, weight;
    string word, t;
    Node* root = newNode();

    cin >> n >> q;
    for (int i = 0; i < n; i++){
        cin >> word >> weight;
        addWord(root, word, weight);
    }

    for (int i = 0; i < q; i++){
        cin >> t;
        cout << find(root, t) << endl;
    }
    return 0;
}
```

11.2 Graph

11.2.1 DFS

dfs.h

Description: DFS

Time: $\mathcal{O}(V+E)$

d41d8c, 11 lines

```
vector<int> adj[N];
bool visited[N];

void dfs(int s){
    if (visited[s]) return;
    visited[s] = true;
    // process node s
    for (auto u : adj[s]){
        dfs(u);
    }
}
```

11.2.2 BFS

bfs.h

Description: BFS

Time: $\mathcal{O}(V+E)$

d41d8c, 90 lines

```
int V, E;
bool visited[MAX];
int path[MAX];
vi graph[MAX]; // adjacency List, an array of vectors

void BFS(int s){
    // initialize visited array and path array
    for (int i = 0; i < V; i++){
        visited[i] = false;
        path[i] = -1;
    }
    queue<int> q;
    visited[s] = true; // start BFS from s
    q.push(s);

    while (!q.empty()){
        int u = q.front();
        q.pop();
        for (int i = 0; i < graph[u].size(); i++){ // traverse
            through Vertex that are adjacent to u
            int v = graph[u][i];
            if (!visited[v]){
                visited[v] = true;
                q.push(v);
                path[v] = u;
            }
        }
    }
}

void printPath(int s, int f){
    int b[MAX]; // save the vertex that we have been to
    int m = 0;
    if (f == s){
        cout << s;
        return;
    }
    if (path[f] == -1){
        cout << "No path" << endl;
        return;
    }

    while(true){
        b[m++] = f;
        f = path[f]; // trace back to previous vertex
        if (f == s){ // found
            b[m++] = s;
            break;
        }
    }
    for (int i = m - 1; i >= 0; i--){ // print path
        cout << b[i] << " ";
    }
}

void printPathRecursion(int s, int f){
    if (s == f){ // base case 1
        cout << f << " ";
    }
    else{
        if (path[f] == -1){ // base case 2
            cout << "No path" << endl;
        }
        else{ // recursive case
            printPathRecursion(s, path[f]);
            cout << f << " ";
        }
    }
```

```
}

int main()
{
    ios_base::sync_with_stdio(false);
    cin.tie(nullptr);

    int u, v;

    // read graph input (Edge List)
    cin >> V >> E;
    for (int i = 0; i < E; i++){
        cin >> u >> v;
        graph[u].push_back(v);
        graph[v].push_back(u);
    }
    int s = 0; // start point
    int f = 6; // desired destination
    BFS(s);
    printPathRecursion(s, f);

    return 0;
}
```

11.2.3 Flood Fill

floodfill.h

Description: Flood Fill

d41d8c, 81 lines

```
int m, n; // row, col
bool visited[MAX][MAX];
string maze[MAX];

const int dr[] = {0, 0, 1, -1};
const int dc[] = {1, -1, 0, 0};

struct Cell {
    int r, c;
};

bool isValid(int r, int c){
    return r >= 0 && r < m && c >= 0 && c < n;
}

bool BFS(Cell s, Cell f){
    queue<Cell> q;
    visited[s.r][s.c] = true;
    q.push(s);

    while(!q.empty()){
        Cell u = q.front();
        q.pop();

        if (u.r == f.r && u.c == f.c){
            return true;
        }

        for (int i = 0; i < 4; i++){ // traverse through nodes
             that are adjacent to the current node
            int r = u.r + dr[i];
            int c = u.c + dc[i];

            if (isValid(r, c) && maze[r][c] == '.' && !visited[
                 r][c]){
                visited[r][c] = true;
                q.push((Cell) {r, c});
            }
        }
    }
    return false;
}

int main()
{
    ios_base::sync_with_stdio(false);
    cin.tie(nullptr);

    int t;
    cin >> t;

    while (t--){
        cin >> m >> n;

        for (int i = 0; i < m; i++){
            cin >> maze[i]; // read the maze
        }

        vector<Cell> entrance; // store Cells that are entrance

        // init visited array and check for entrance at the
             same time
        for (int i = 0; i < m; i++){
            for (int j = 0; j < n; j++){
                visited[i][j] = false;
                if (maze[i][j] == '.' && (i == 0 || j == 0 || i
                     == m - 1 || j == n - 1)){
                    entrance.push_back((Cell) {i, j});
                }
            }
        }

        if (entrance.size() != 2){
            cout << "invalid" << endl;
        }
        else{
            Cell s = entrance[0];
            Cell f = entrance[1];
            cout << (BFS(s, f) ? "valid" : "invalid") << endl;
        }
    }

    return 0;
}
```

11.2.4 Bipartite graph check

bipartitegraph.h

Description: Check bipartite, A graph is bipartite if it is possible to color its nodes using two colors in such a way that no adjacent nodes have the same color. It turns out that a graph is bipartite exactly when it does not have a cycle with an odd number of edges

d41d8c, 44 lines

```
// BFS implementation
int n, l;
int color[maxN];
vector <int> g[maxN];

bool checkBipartiteGraph() {
    fill_n(color, n + 1, -1);

    queue <int> q;
    q.push(0);
    color[0] = 0;
    while (!q.empty()) {
        int u = q.front();
        q.pop();
        for (auto v : g[u]) {
            if (color[v] == color[u]) return false;
            if (color[v] == -1) {
                color[v] = !color[u];
                q.push(v);
            }
        }
    }
    return true;
}

int main() {
    while (cin >> n){
        if (!n) return 0;

        cin >> l;
        while (l--) {
            int u, v;
            cin >> u >> v;
            g[u].push_back(v);
            g[v].push_back(u);
        }
        if (!checkBipartiteGraph()) cout << "NOT ";
        cout << "BICOLORABLE.\n";

        for (int i = 0; i < n; ++i) g[i].clear();
    }
}

// Same idea for DFS implementation
```

11.2.5 Cycle Check

cyclecheck.h

Description: Cycle Check

d41d8c, 27 lines

```
void cycleCheck(int u) { // check edge properties
    dfs_num[u] = EXPLORED; // color u as EXPLORED
    for (auto &[v, w] : AL[u]) { // C++17 style, w ignored
        printf("Edge (%d, %d) is a ", u, v);
        if (dfs_num[v] == UNVISITED) { // EXPLORED->UNVISITED
            printf("Tree Edge\n");
            dfs_parent[v] = u; // a tree edge u->v
            cycleCheck(v);
        }
        else if (dfs_num[v] == EXPLORED) { // EXPLORED->
             EXPLORED
            if (v == dfs_parent[u]) // differentiate them
                printf("Bidirectional Edge\n"); // a trivial
                     cycle
            else
                printf("Back Edge (Cycle)\n"); // a non trivial
                     cycle
        }
        else if (dfs_num[v] == VISITED) // EXPLORED->VISITED
            printf("Forward/Cross Edge\n"); // rare application
    }
    dfs_num[u] = VISITED; // color u as VISITED/DONE
}

// inside int main()
dfs_num.assign(V, UNVISITED);
dfs_parent.assign(V, -1);
for (int u = 0; u < V; ++u)
    if (dfs_num[u] == UNVISITED)
        cycleCheck(u);
```

11.2.6 Shortest Cycle

shortestcycle.h

Description: From a vertex u (source), Find the length of the shortest cycle includes that u vertex.

Time: $\mathcal{O}(\log N)$ for querry and point update

d41d8c, 46 lines

```
const int maxN  = 210;

int n;
int visit[maxN], d[maxN];
vector <int> g[maxN];

int bfs(int s) {
    fill_n(d, n + 1, 0);
    fill_n(visit, n + 1, false);

    queue <int> q;
    q.push(s);
    visit[s] = true;
    while (!q.empty()) {
        int u = q.front();
        q.pop();
        for (auto v : g[u]) {

            // If get back to source, return cycle length and
                end BFS
            if (v == s) return d[u] + 1;

            if (!visit[v]) {
                d[v] = d[u] + 1;
                visit[v] = true;
                q.push(v);
            }
        }
    }
    return 0;
}

int main() {
    cin >> n;
    for (int i = 1; i <= n; ++i)
        for (int j = 1; j <= n; ++j) {
            int h;
            cin >> h;
            if (h) g[i].push_back(j);
        }

    for (int i = 1; i <= n; ++i) {
        int ans = bfs(i);
        if (ans) cout << ans << '\n';
        else cout << "NO WAY\n";
    }
}
```

11.3 SegmentTree

11.3.1 Lazysegtree

lazysegtree.h

Description: Lazy-SegTree, version range max queries

Time: $\mathcal{O}(\log N)$ for both range queries and updates a range

d41d8c, 54 lines

```
const int inf  = 1e9 + 7;
const int maxN = 1e5 + 7;

int n, q;
int a[maxN];
long long st[4 * maxN], lazy[4 * maxN];

void build(int id, int l, int r) {
    if (l == r) {
        st[id] = a[l];
        return;
    }
    int mid = l + r >> 1;
    build(2 * id, l, mid);
    build(2 * id + 1, mid + 1, r);
    st[id] = max(st[2 * id], st[2 * id + 1]);
}

void fix(int id, int l, int r) {
    if (!lazy[id]) return;
    st[id] += lazy[id];

    if (l != r){
        lazy[2 * id] += lazy[id];
        lazy[2 * id + 1] += lazy[id];
    }

    lazy[id] = 0;
}

void update(int id, int l, int r, int u, int v, int val) {
    fix(id, l, r);
    if (l >  v || r <  u) return;
    if (l >= u && r <= v) {
        lazy[id] += val;
        fix(id, l, r);
        return;
    }
    int mid = l + r >> 1;
    update(2 * id, l, mid, u, v, val);
    update(2 * id + 1, mid + 1, r, u, v, val);
    st[id] = max(st[2 * id], st[2 * id + 1]);
}

long long get(int id, int l, int r, int u, int v) {
    fix(id, l, r);
    if (l >  v || r <  u) return -inf;
    if (l >= u && r <= v) return st[id];

    int mid  = l + r >> 1;
    long long get1 = get(2 * id, l, mid, u, v);
    long long get2 = get(2 * id + 1, mid + 1, r, u, v);
    return max(get1, get2);
}
```

11.3.2 GSS problem

gss.h

Description: Find max sum of a subrange in the range[x, y]

d41d8c, 60 lines

```
const int inf  = 1e9 + 7;
const int maxN = 5e4 + 7;

// Information stored in each node
struct node {
    int pre, suf, sum, maxsum;

    static node base() { return { -inf, -inf, 0, -inf }; }

    // merge two node
    static node merge(const node& a, const node& b)  {
        node res;
        res.pre = max(a.pre, b.pre + a.sum);
        res.suf = max(b.suf, a.suf + b.sum);
        res.sum = a.sum + b.sum;
        res.maxsum = max(a.maxsum, b.maxsum);
        res.maxsum = max(res.maxsum, a.suf + b.pre);
        return res;
    }
};

int n, m;
int a[maxN];
node st[4 * maxN];

// Build segtree
void build(int id, int l ,int r) {
    if (l == r) {
        st[id] = { a[l], a[l], a[l], a[l] };
        return;
    }
    int mid = l + r >> 1;
    build(2 * id, l, mid);
    build(2 * id + 1, mid + 1, r);
    st[id] = node::merge(st[2 * id], st[2 * id + 1]);
}

// Querry result
node get(int id, int l, int r, int u, int v){
    if (l >  v || r <  u) return node::base();
    if (l >= u && r <= v) return st[id];

    int mid = l + r >> 1;
    node g1 = get(2 * id, l, mid, u, v);
    node g2 = get(2 * id + 1, mid + 1, r, u, v);
    return node::merge(g1, g2);
}

int main() {
    cin >> n;
    for (int i = 1; i <= n; ++i) cin >> a[i];
    build(1, 1, n);

    cin >> m;
    while (m--) {
        int x, y;
        cin >> x >> y;
        cout << get(1, 1, n, x, y).maxsum << '\n';
    }
}
```

11.3.3 Seg Tree

segtree.h

Description: SegTree, version range min queries

Time: $\mathcal{O}(\log N)$ for querry and point update

d41d8c, 66 lines

```
const int inf  = 1e9 + 7;
const int maxN = 1e5 + 7;

int n, q;
int a[maxN];
int st[4 * maxN];

void build(int id, int l, int r) {
    if (l == r) {
        st[id] = a[l];
        return;
    }

    int mid = l + r >> 1; // (l + r) / 2
    build(2 * id, l, mid);
    build(2 * id + 1, mid + 1, r);

    st[id] = min(st[2 * id], st[2 * id + 1]);
}

void update(int id, int l, int r, int i, int val) {
    // i is outside [l, r], ignore id
    if (l > i || r < i) return;

    // No children
    if (l == r) {
```

```
        st[id] = val;
        return;
    }

    // Call recursion to solve for children of id
    int mid = l + r >> 1; // (l + r) / 2
    update(2 * id, l, mid, i, val);
    update(2 * id + 1, mid + 1, r, i, val);

    // Update min of [l, r] according to 2 of its children
    st[id] = min(st[2 * id], st[2 * id + 1]);
}

int get(int id, int l, int r, int u, int v) {
    // [u, v] is not intersecting with [l, r]
    if (l >  v || r <  u) return inf;

    // [l, r] is completely inside [u, v]
    if (l >= u && r <= v) return st[id];

    int mid = l + r >> 1; // (l + r) / 2
    int get1 = get(2 * id, l, mid, u, v);
    int get2 = get(2 * id + 1, mid + 1, r, u, v);

    return min(get1, get2);
}

int main() {
    cin >> n;
    for (int i = 1; i <= n; ++i) cin >> a[i];
    build(1, 1, n);

    cin >> q;
    while (q--) {
        int type, x, y;
        cin >> type >> x >> y;
        if (type == 1) update(1, 1, n, x, y); // Assign y for
            element at index x
        else cout << get(1, 1, n, x, y) << '\n'; // RMQ(x, y)
    }
}
```

11.4 Disjoint Set Union

dsu.h

Description: DSU

d41d8c, 27 lines

```
struct DSU {
    vector<int> lab;

    DSU(int n) : lab(n+1, -1) {}

    int getRoot(int u) {
        if (lab[u] < 0) return u;
        return lab[u] = getRoot(lab[u]);
    }

    bool merge(int u, int v) {
        u = getRoot(u); v = getRoot(v);
        if (u == v) return false;
        if (lab[u] > lab[v]) swap(u, v);
        lab[u] += lab[v];
        lab[v] = u;
        return true;
    }

    bool same_component(int u, int v) {
        return getRoot(u) == getRoot(v);
    }
    int component_size(int u) {
        return -lab[getRoot(u)];
    }
};
```

11.5 FenwickTree

fenwicktree.h

Description: Fenwick Tree, solve Range Sum Querry problem, 1-based index

Time: $\mathcal{O}(\log N)$ for both querry and update

d41d8c, 19 lines

```
int tree[N];

// get sum [1->k]
int sum(int k){
    int s = 0;
    while (k >= 1){
        s += tree[k];
        k -= (k & -k);
    }
    return s;
}

//update point
void update(int k, int x) {
    while (k <= n){
        tree[k] += x;
        k += (k & -k);
    }
}
```

11.6 Bitwise

bitwise.h

Description: Bit manipulation

d41d8c, 80 lines

```
/*
1) To multiply/divide an integer by 2, we only need to shift
     all8 bits in the integer
left/right, respectively. Notice that the truncation in the
     shift right operation
automatically rounds the division-by-2 down, e.g., 17/2 = 8.

S = 34 (base 10) = 100010 (base 2)
S = S<<1 = S*2 = 68 (base 10) = 1000100 (base 2)
S = S>>2 = S/4 = 17 (base 10) = 10001 (base 2)
S = S>>1 = S/2 = 8 (base 10) = 1000 (base 2) <- LSB is gone
(LSB = Least Significant Bit)

*/

/*
2) To set/turn on the j-th item (0-based indexing) of the set,
use the bitwise OR operation S |= (1<<j).

S = 34 (base 10) = 100010 (base 2)
j = 3, 1<<j = 001000 <- bit 1 is shifted to the left 3 times
---------- OR (true if either of the bits is true)
S = 42 (base 10) = 101010 (base 2) // update S to this new
     value 42
*/

/*
3) To check if the j-th item of the set is on,
use the bitwise AND operation T = S & (1<<j).
If T=0, then the j-th item of the set is off.
If T != 0 (to be precise, T = (1<<j)), then the j-th item of
     the set is on.

S = 42 (base 10) = 101010 (base 2)
j = 3, 1<<j = 001000 <- bit 1 is shifted to the left 3 times
---------- AND (only true if both bits are true)
T = 8 (base 10) = 001000 (base 2) -> not zero, the 3rd item is
     on
*/

/*
4) To clear/turn off the j-th item of the set,
use the bitwise AND operation S &= ~(1<<j).

S = 42 (base 10) = 101010 (base 2)
j = 1, ~(1<<j) = 111101 <- ~ is the bitwise NOT operation
---------- AND
S = 40 (base 10) = 101000 (base 2) // update S to this new
     value 40
*/

/*
5) To toggle (flip the status of) the j-th item of the set,
use the bitwise XOR operation S ^= (1<<j).

S = 40 (base 10) = 101000 (base 2)
j = 2, (1<<j) = 000100 <- bit 1 is shifted to the left 2 times
---------- XOR <- true if both bits are different
S = 44 (base 10) = 101100 (base 2) // update S to this new
     value 44
*/

/*
6) To get the value of the least significant bit of S that is
     on (first from the right),
use T = ((S) & -(S)). This operation is abbreviated as LSOne(S)
     .

Notice that T = LSOne(S) is a power of 2, i.e., 2^j .
To get the actual index j (from the right), we can use
     __builtin_ctz(T) below.
*/

/*
7) To turn on all bits in a set of size n, use S = (1<<n) - 1
*/

/*
8) To enumerate all proper subsets of a given a bitmask, e.g.,
     if mask = (18)10 = (10010)2,
then its proper subsets are {(18)10 = (10010)2, (16)10 =
     (10000)2, (2)10 = (00010)2},
we can use:
int mask = 18;
for (int subset = mask; subset; subset = (mask & (subset-1)))
     cout << subset << "\n";
*/

/*
__builtin_popcount(S) to count how many bits that are on in S
     and
__builtin_ctz(S) to count how many trailing zeroes in S.
*/
```

11.7 Binary Lifting

binarylifting.h

Description: Binary Lifting, find kth ancestor of a node in a tree

d41d8c, 13 lines

```
int par[N], up[N][17];
void preprocess() {
    for (int u = 1; u <= n; ++u) up[u][0] = par[u];
    for (int j = 1; j < 17; ++j)
```

```
        for (int u = 1; u <= n; ++u)
            up[u][j] = up[up[u][j - 1]][j - 1];
}

int ancestor_k(int u, int k) {
    for (int j = 0; (1 << j) <= k; ++j)
        if (k >> j & 1) u = up[u][j];
    return u;
}
```

11.7.1 Find kth Ancestor, dist ¡= x

binarylifting2.h

Description: find the furthest ancestor of a node in which dist <= x
Time: $\mathcal{O}\left(N/logN + Q\log^2 N\right)$ d41d8c, 39 lines

```
// Algo 1
int dist[N][17];
int calc_dist(int u, int k) {
    int sum = 0;
    for (int j = 0; (1 << j) <= k; ++j)
        if (k >> j & 1) {
            sum += dist[u][j];
            u = up[u][j];
        }
    return sum;
}

    // binary search to find ans
int solve(int u, int x) {
    int lo = 0, hi = h[u], mid, ans = 0;
    while (lo <= hi) {
        mid = (lo + hi) / 2;
        if (calc_dist(u, mid) <= x) {
            ans = mid;
            lo = mid + 1;
        }
        else hi = mid - 1;
    }
    return ancestor_k(u, ans);
}

// Algo 2 (Efficient)
int dist[N][17];
int solve(int u, int x) {
    int now_dist = 0, k = 0;
    for (int j = __lg(h[u]); j >= 0; --j) {
        if (h[u] >= (1 << j) && now_dist + dist[u][j] <= x) {
            now_dist += dist[u][j];
            k |= 1 << j;
            u = up[u][j];
        }
    }
    return u;
}
```

11.7.2 LCA - Binary Lifting

lcabl.h

Description: LCA, Binary Lifting
Time: $\mathcal{O}\left(M\log N\right)$ for both range queries and updates a range d41d8c, 34 lines

```
// init up
void dfs(int u) {
    for (int v : g[u]) {
        if (v == up[u][0]) continue;
        h[v] = h[u] + 1;

        up[v][0] = u;
        for (int j = 1; j < 20; ++j)
            up[v][j] = up[up[v][j - 1]][j - 1];

        dfs(v);
    }
}

int h[N], up[N][20];
int lca(int u, int v) {
    if (h[u] != h[v]) {
        if (h[u] < h[v]) swap(u, v);

        // find ancestor u' of u in which h(u') = h(v)
        int k = h[u] - h[v];
        for (int j = 0; (1 << j) <= k; ++j)
            if (k >> j & 1) // if jth bit of k is 1
                u = up[u][j];
    }
    if (u == v) return u;

    // Find LCA(u,v)
    int k = __lg(h[u]);
    for (int j = k; j >= 0; --j)
        if (up[u][j] != up[v][j]) // if (1<<j)th ancestors of u
                                  // and v are different
            u = up[u][j], v = up[v][j];
    return up[u][0];
}
```

11.7.3 Dynamic LCA

dynamicLCA.h

Description: Dynamic LCA (find LCA(u,v) with different roots) d41d8c, 66 lines

```
const int N = 1e5 + 9;
int n;
vector<int> g[N];

int h[N], up[N][17];
void dfs(int u) {
    for (int v : g[u]) {
        if (v == up[u][0]) continue;
        h[v] = h[u] + 1;

        up[v][0] = u;
        for (int j = 1; j < 17; ++j)
            up[v][j] = up[up[v][j - 1]][j - 1];

        dfs(v);
    }
}

int lca(int u, int v) {
    if (h[u] != h[v]) {
        if (h[u] < h[v]) swap(u, v);

        int k = h[u] - h[v];
        for (int j = 0; (1 << j) <= k; ++j)
            if (k >> j & 1)
                u = up[u][j];
    }
    if (u == v) return u;

    int k = __lg(h[u]);
    for (int j = k; j >= 0; --j)
        if (up[u][j] != up[v][j])
            u = up[u][j], v = up[v][j];
    return up[u][0];
}

int main() {
    cin.tie(NULL)->sync_with_stdio(false);
    while (cin >> n, n) {
        for (int i = 1; i <= n; ++i) g[i].clear();
        for (int i = 1, u, v; i < n; ++i) {
            cin >> u >> v;
            g[u].push_back(v);
            g[v].push_back(u);
        }

        // use 1 as fixed root
        dfs(1);

        char c;
        int m, root(1), u, v; cin >> m; while (m--) {
            cin >> c;

            // fing LCA(u,v) with this root
            if (c == '!') cin >> root;
            else {
                cin >> u >> v;
                // ans is one of these
                int uv = lca(u, v);
                int ur = lca(u, root);
                int vr = lca(v, root);
                cout << (uv ^ ur ^ vr) << '\n';
            }
        }
    }
}
```

11.8 Shortest Path

11.8.1 Dijkstra

dijkstra.h

Description: Dijkstra
Time: $\mathcal{O}\left(M\log N\right)$ d41d8c, 64 lines

```
vector<vector<pi>> graph;
vi dist(MAX, INF);
int path[MAX];

struct option
{
    bool operator() (const pi &a, const pi &b) const
    {
        return a.S > b.S;
    }
};

void Dijkstra(int s){
    priority_queue<pi, vector<pi>, option> pq;
    pq.push(MP(s, 0)); // (vertex, current sp)
    dist[s] = 0;

    while(!pq.empty()){
        pi top = pq.top();
        pq.pop();
        int u = top.F;
        int w = top.S; // current sp
        if (dist[u] != w){
            continue;
        }

        for (int i = 0; i < graph[u].size(); i++){
            pi nb = graph[u][i];
            if (w + nb.S < dist[nb.F]){
                dist[nb.F] = w + nb.S;
                pq.push(pi(nb.F, dist[nb.F]));
                path[nb.F] = u;
```

```
            }
        }
    }
}

int main()
{
    ios_base::sync_with_stdio(false);
    cin.tie(nullptr);

    int n, s, t;
    cin >> n;
    s = 0, t = 4;
    graph = vector<vector<pi>>(MAX + 5, vector<pi>());
    int d = 0;

    // adjacency matrix
    for (int i = 0; i < n; i++){
        for (int j = 0; j < n; j++){
            cin >> d;
            if (d > 0){
                graph[i].push_back(pi(j, d));
            }
        }
    }

    Dijkstra(s);
    int ans = dist[t];
    cout << ans << endl;

    return 0;
}
```

11.8.2 Bellman-Ford

bellmanford.h

Description: Bellman-Frod

Time: $\mathcal{O}(MN)$

d41d8c, 23 lines

```
const long long INF = 2000000000000000000LL;
struct Edge {
    int u, v;
    long long w;
};

void bellmanFord(int n, int S, vector<Edge> &e, vector<long
     long> &D, vector<int> &trace) {
    D.resize(n, INF);
    trace.resize(n, -1);

    D[S] = 0;
    for(int T = 1; T < n; T++) {
        for (auto E : e) {
            int u = E.u;
            int v = E.v;
            long long w = E.w;
            if (D[u] != INF && D[v] > D[u] + w) {
                D[v] = D[u] + w;
                trace[v] = u;
            }
        }
    }
}
```

11.8.3 Floyd Warshall

floyd-warshall.h

Description: Floyd warshall

Time: $\mathcal{O}(N^3)$

d41d8c, 24 lines

```
void init_trace(vector<vector<int>> &trace) {
    int n = trace.size();
    for (int u = 0; u < n; u++) {
        for (int v = 0; v < n; v++) {
            trace[u][v] = u;
        }
    }
}

void floydWarshall(int n, vector<vector<long long>> &w, vector<
     vector<long long>> &D, vector<vector<int>> &trace) {
    D = w;
    init_trace(trace); // only if trace is needed

    for (int k = 0; k < n; k++) {
        for (int u = 0; u < n; u++) {
            for (int v = 0; v < n; v++) {
                if (D[u][v] > D[u][k] + D[k][v]) {
                    D[u][v] = D[u][k] + D[k][v];
                    trace[u][v] = trace[k][v];
                }
            }
        }
    }
}
```

11.8.4 Trace path

tracepath.h

Description: Trace back the shortest path

d41d8c, 12 lines

```
vector<int> trace_path(vector<int> &trace, int S, int u) {
    if (u != S && trace[u] == -1) return vector<int>(0);

    vector<int> path;
    while (u != -1) {
        path.push_back(u);
        u = trace[u];
    }
    reverse(path.begin(), path.end());

    return path;
}
```

11.8.5 0-1 BFS

0-1BFS.h

Description: 0-1 BFS, find shortest path in 0-1 weighted graph. App: find the minimum of edges that is needed to be reversed in direction to make the path 1->N possible

Time: better than Dijkstra

d41d8c, 28 lines

```
int n, m;
int d[maxN];
vector < pair <int, int> > g[maxN];

void bfs(int s) {
    fill_n(d, n + 1, inf);
    deque <int> q;
    q.push_back(s);
    d[s] = 0;
    while (!q.empty()) {
        int u = q.front();
        q.pop_front();

        if (u == n) return;

        for (auto edge : g[u]) {
            int v = edge.second;
            int w = edge.first;
            if (d[v] > d[u] + w) {
                d[v] = d[u] + w;
                if (w) q.push_back(v);
                else q.push_front(v);
            }
        }
    }
    d[n] = -1;
}
```

11.9 Min Spanning Tree

11.9.1 Kruskal

kruskal.h

Description: Kruskal Algorithm

Time: if the graph is densed, use Prim for better performance

d41d8c, 57 lines

```
struct DSU {
    vector<int> lab;

    DSU(int n) : lab(n+1, -1) {}

    int getRoot(int u) {
        if (lab[u] < 0) return u;
        return lab[u] = getRoot(lab[u]);
    }

    bool merge(int u, int v) {
        u = getRoot(u); v = getRoot(v);
        if (u == v) return false;
        if (lab[u] > lab[v]) swap(u, v);
        lab[u] += lab[v];
        lab[v] = u;
        return true;
    }

    bool same_component(int u, int v) {
        return getRoot(u) == getRoot(v);
    }

    int component_size(int u) {
        return -lab[getRoot(u)];
    }
};

int main()
{
    ios_base::sync_with_stdio(false);
    cin.tie(nullptr);

    int V, E, mst_cost = 0, num_taken = 0;
    cin >> V >> E;
    vector<iii> EL(E);
    DSU g(V + 5);

    for (int i = 0; i < E; i++){
        int u, v, w;
        cin >> u >> v >> w;
        EL[i] = {w, u, v};
    }

    sort(EL.begin(), EL.end()); // sort by w

    for (auto &[w, u, v] : EL){
        if (g.same_component(u, v)) continue;
        mst_cost += w;
        g.merge(u, v);
        ++num_taken;
```

```
        if (num_taken == V - 1) break;
    }

    cout << mst_cost << endl;
    return 0;
}
```

11.9.2 Min Spanning Subgraph

mss.h

Description: Minimum Spanning Subgraph of MST problem. Some edges in the given graph have already been fixed and must be taken as part of the solution. For Kruskal's algorithm, we first take into account all the fixed edges and their costs. Then, we continue running Kruskal's algorithm on the remaining free edges until we have a spanning subgraph (or spanning tree). For Prim's algorithm, we give higher priorities to these fixed edges so that we will always take them and their costs.

11.9.3 Second-Best Spanning Tree

sbst.h

Description: Second-Best Spanning Tree is a variant of MST problem, We can see that the second best ST is actually the MST with just two edges difference. One edge is taken out from the MST and another chord edge is added into the MST. Next, for each edge in the MST (there are at most V-1 edges in the MST), temporarily flag it so that it cannot be chosen, then try to find the MST again in O(E) but now excluding that flagged edge. Note that we do not have to re-sort the edges at this point. The best spanning tree found after this process is the second best ST.

11.10 Math Related

11.10.1 Prime Check

isPrime.h

Description: check if a number is Prime

Time: $\mathcal{O}\left(\sqrt{(N)}\right)$

d41d8c, 5 lines

```
bool isPrime(int n) {
    for (int i = 2; i*i <= n; i++)
        if (n % i == 0) return false;
    return n > 1;
}
```

11.10.2 Sieve of Eratosthenes

sieve.h

Description: Sieve of Eratosthenes

Time: $\mathcal{O}(N \log N)$

d41d8c, 34 lines

```
// find all prime number in range [1, N]
void sieve(int N) {
    bool isPrime[N+1];
    for(int i = 0; i <= N;++i) {
        isPrime[i] = true;
    }
    isPrime[0] = false;
    isPrime[1] = false;
    for(int i = 2; i * i <= N; ++i) {
         if(isPrime[i] == true) {
            for(int j = i * i; j <= N; j += i)
                isPrime[j] = false;
        }
    }
}

// find all prime number in range [L, R]
vector<bool> isPrime(R - L + 1, true);  // x is prime <=>
    isPrime[x - l] == true
for (long long i = 2; i * i <= R; ++i) {
    for (long long j = max(i * i, (L + i - 1) / i * i); j <= R;
         j += i) {
        isPrime[j - L] = false;
    }
}

if (1 >= L) {  // case number 1
    isPrime[1 - L] = false;
}

for (long long x = L; x <= R; ++x) {
    if (isPrime[x - L]) {
        // i is prime
    }
}
```

11.10.3 Factorize a number

factorize.h

Description: Factorize a number

Time: $\mathcal{O}\left(\sqrt{N}\right)$

d41d8c, 42 lines

```
// Sol1: O(\sqrt{N})
vector<int> factorize(int n) {
    vector<int> res;
    for (int i = 2; i * i <= n; ++i) {
        while (n % i == 0) {
            res.push_back(i);
            n /= i;
        }
    }
    if (n != 1) {
        res.push_back(n);
    }
    return res;
}

//  Sol2: O(\logN)
int minPrime[n + 1];
for (int i = 2; i * i <= n; ++i) {
    if (minPrime[i] == 0) { //if i is prime
        for (int j = i * i; j <= n; j += i) {
            if (minPrime[j] == 0) {
                minPrime[j] = i;
            }
        }
    }
}
for (int i = 2; i <= n; ++i) {
    if (minPrime[i] == 0) {
        minPrime[i] = i;
    }
}

vector<int> factorize(int n) {
    vector<int> res;
    while (n != 1) {
        res.push_back(minPrime[n]);
        n /= minPrime[n];
    }
    return res;
}

// If n = (p1^q1)(p2^q2)...(pk^qk) then n have (q1 + 1)(q2 + 2)
    ...(qk + k) divisors
```

11.10.4 GCD and LCM

gcdlcm.h

Description: Find GCD and LCM

d41d8c, 2 lines

```
template<class T> T gcd(T a, T b){ T r; while (b != 0) { r = a
    % b; a = b; b = r; } return a;}
template<class T> T lcm(T a, T b) { return a / gcd(a, b) * b; }
```

11.11 Sorting

sorting.h

Description: Sorting Using Library

d41d8c, 23 lines

```
int arr2[] = {5, 1, 3, 2, 4};
sort(arr2 + 1, arr2 + 4); // 5 1 2 3 4

// By default, C++ pairs are sorted by first element and then
    second element in case of a tie. Tuples are sorted
    similarly.
vector<pair<int, int>> v{{1, 5}, {2, 3}, {1, 2}};
sort(v.begin(), v.end());

// technique 1, create a custom comparison function
bool cmp(const int a, const int b) {
    return a > b; // non-decreasing order
}

sort(A.begin(), A.end(), cmp);

// technique 2, use an anonymous function (lambda expression)
sort(A.begin(), A.end(), [](const int a, const int b) {
    return a > b;
});

// technique 3, use reverse iterator
sort(A.rbegin(), A.rend());

// technique 4, add minus sign
```

11.12 Others

11.12.1 RMQ - ST

RMQ-ST.h

Description: Range min querry problem using Sparse Table, DP

Time: Preprocess: $\mathcal{O}(N \log N)$, Querry: $\mathcal{O}(1)$

d41d8c, 18 lines

```
// M[i][j] is the index of the minimum value in the range
    starting at i and has a length of 2^j
void process2(int M[MAXN][LOGMAXN], int A[MAXN], int N)
{
  int i, j;

  for (i = 0; i < N; i++)
    M[i][0] = i;

  for (j = 1; 1 << j <= N; j++)
    for (i = 0; i + (1 << j) - 1 < N; i++)
      if (A[M[i][j - 1]] < A[M[i + (1 << (j - 1))][j - 1]])
        M[i][j] = M[i][j - 1];
      else
        M[i][j] = M[i + (1 << (j - 1))][j - 1];
}

// Find RMQ(i, j) by comparing two ranges of length 2^k that
    cover [i,j].
// One starts at i and the other ends at j
```

11.12.2 Lowest Common Ancestor

LCA.h

Description: Lowest Common Ancestor, Euler Tour + RMQ
Time: $\mathcal{O}(M \log N)$

d41d8c, 20 lines

```
int L[2*MAX_N], E[2*MAX_N], H[MAX_N], idx;

// init L, E, H
void dfs(int cur, int depth) {
    H[cur] = idx;
    E[idx] = cur;
    L[idx++] = depth;
    for (auto &nxt : children[cur]) {
        dfs(nxt, depth+1);
        E[idx] = cur; // backtrack to cur
        L[idx++] = depth;
    }
}

void buildRMQ() {
    idx = 0; memset(H, -1, sizeof H);
    dfs(0, 0); // root is at index 0
}

// the solution is given by LCA(u, v) = E[RMQ(H[u], H[v])]
    where RMQ(i, j) is executed on the L array.
```

11.12.3 Calculating Tree Diameter

treediameter.h

Description: The diameter of a tree is the maximum length of a path between two nodes.
Time: $\mathcal{O}(N)$ for both algorithm

d41d8c, 96 lines

```
// First Algorithm
A general way to approach tree problems is to first root the
    tree
arbitrarily and then solve the problem separately for each
    subtree
An important observation is that every path in a rooted tree
    has a highest point:
the highest node that belongs to the path. Thus, we can
    calculate for each node x the
length of the longest path whose highest point is x. One of
    those paths corresponds
to the diameter of the tree.
toLeaf(x): the maximum length of a path from x to any leaf
maxLength(x): the maximum length of a path whose highest point
    is x
First, to calculate toLeaf(x), we go through the children of x,
choose a child c with the maximum toLeaf(c), and add one to
    this value. Then,
to calculate maxLength(x), we choose two distinct children a
    and b such that the
sum toLeaf(a) + toLeaf(b) is maximum and add two to this sum. (
    The cases
where x has less than two children are easy special cases.)

// Second Algorithm
Another efficient way to calculate the diameter of a tree is
    based
on two depth-first searches. First, we choose an arbitrary node
     a in the tree and find
the farthest node b from a. Then, we find the farthest node c
    from b. The diameter
of the tree is the distance between b and c.

// Apply second Algo, Use LCA to find dist between 2 nodes
const int N = 2e5 + 8;
int n, k, root;
vector<vi> g(N), group(N >> 1);

int h[N], up[N][18];

void dfs(int u) {
    for (int v : g[u]) {
        h[v] = h[u] + 1;

        for (int j = 1; j < 18; ++j)
            up[v][j] = up[up[v][j - 1]][j - 1];

        dfs(v);
    }
}

int lca(int u, int v) {
    if (h[u] != h[v]) {
        if (h[u] < h[v]) swap(u, v);

        int k = h[u] - h[v];
        for (int j = 0; (1 << j) <= k; ++j)
            if (k >> j & 1)
                u = up[u][j];
    }
    if (u == v) return u;

    int k = __lg(h[u]);
    for (int j = k; j >= 0; --j)
        if (up[u][j] != up[v][j])
            u = up[u][j], v = up[v][j];
    return up[u][0];
}

int dist(int u, int v) {
    int p = lca(u, v);
    return h[u] + h[v] - 2 * h[p];
}

int diameter(vector<int> &meeting) {
    int A = meeting[0], max_dist = 0, B = A, d;

    for (int x : meeting) {
        d = dist(A, x);
        if (max_dist < d) {
            max_dist = d;
            B = x;
        }
    }

    max_dist = 0;
    for (int x : meeting) {
        d = dist(B, x);
        max_dist = max(max_dist, d);
    }
    return max_dist;
}

int main() {
    cin.tie(NULL)->sync_with_stdio(false);
    cin >> n >> k;
    for (int i = 1, x; i <= n; ++i) {
        cin >> x >> up[i][0];
        group[x].emplace_back(i);
        g[up[i][0]].push_back(i);
        if (up[i][0] == 0) root = i;
    }

    dfs(root);

    for (int i = 1; i <= k; ++i)
        cout << diameter(group[i]) << '\n';
}
```

11.13 Dynamic Programming

11.13.1 Max 1-D range sum

mrs1D.h

Description: Max 1-D range sum, DP
Time: $\mathcal{O}(N^2)$

d41d8c, 7 lines

```
int n = 9, a[] = {4, -5, 4, -3, 4, 4, -4, 4, -5};
int sum = 0, ans = 0;
for (int i = 0; i < n; i++){
    sum += a[i];
    ans = max(ans, sum);
    if (sum < 0) sum = 0;
}
```

11.13.2 Longest range sum divisible by k

qbseq.h

Description: Longest range that has sum divisible by k, DP

d41d8c, 29 lines

```
int sub(int a, int b){
    int res = (a - b) % k;
    if (res >= 0) return res;
    return res + k;
}

int main()
{
    ios_base::sync_with_stdio(false);
    cin.tie(nullptr);

    cin >> n >> k;
    vi a(n);
    for (int i = 0; i < n; i++){
        cin >> a[i];
        sum += a[i];
    }
    memset(f, INF, sizeof(f));
    f[0][0] = 0;
    for (int i = 1; i < n; i++){
        for (int t = 0; t < k; t++){
            f[i][t] = min(f[i - 1][t], 1 + f[i - 1][sub(t, a[i
                ])]);
        }
    }

    cout << max(n - f[n - 1][sum % k], 0) << endl;

    return 0;
}
```

11.13.3 Longest common substring

lcs.h

Description: Longest common substring, DP
Time: $\mathcal{O}(N^2)$

d41d8c, 43 lines

```
int main()
{
    ios_base::sync_with_stdio(false);
    cin.tie(nullptr);

    string s, t, ans = "";
    cin >> s >> t;
    int m = s.length(), n = t.length(), init = max(m, n);
```

```
    for (int i = 0; i <= init; i++){
        dp[i][0] = 0; dp[0][i] = 0;
    }

    for (int i = 1; i <= m; i++){
        for (int j = 1; j <= n; j++){
            if (s[i - 1] == t[j - 1]){
                dp[i][j] = dp[i - 1][j - 1] + 1;
                ans += s[i - 1];
            }
            else{
                dp[i][j] = max(dp[i - 1][j], dp[i][j - 1]);
            }
        }
    }

    string res = "";
    while (m != 0 && n != 0){
        if (s[m - 1] == t[n - 1]){
            res += s[m - 1]; m--; n--;
        }
        else if (dp[m][n] == dp[m - 1][n]){
            m--;
        }
        else{
            n--;
        }
    }

    reverse(res.begin(), res.end());
    cout << res << endl;

    return 0;
}
```

11.13.4 Coin Exchange 1

coinexchange.h

Description: Coin exchange, DP, Returns number of ways we can exchange k using set of coins

d41d8c, 10 lines

```
count[0] = 1;
const int MOD = 1e9;
for (int x = 1; x <= n; x++){
    for (auto c : coins){
        if (x - c >= 0){
            count[x] += count[x - c];
            count[x] %= MOD;
        }
    }
}
```

11.13.5 Coin Exchange 2 - Counting Solutions

coinexchange2.h

Description: Coin exchange, DP, Returns minimum number of coins we can exchange k using set of coins

d41d8c, 17 lines

```
// value[x] is the ans for exchanging x
value[0] = 0;
for (int x = 1; x <= n; x++){
    value[x] = INF;
    for (auto c : coins){
        if (x - c >= 0 && value[x - c] + 1 < value[x]){
            value[x] = value[x - c] + 1;
            first[x] = c; // used to trace back answer
        }
    }
}

// trace back
while(n > 0){
    cout << first[n] << endl;
    n -= first[n];
}
```