Monte-Carlo methods
Examination TP1 Numeric simulations

The simulations should be written as a **Jupyter Notebook**. At the end of the computer sessions,

please send an email with your jupyter notebook to `randal.douc@telecom-sudparis.eu` with the subject: TP1JVN.

You can add any additional graphs and comments on your results!!!!

Exercice 1. Recall that $\max_{0\leq s\leq T} B_s \stackrel{\mathcal{L}}{=} |B_T|$ for any standard Brownian motion $(B_s)_{s\geq 0}$.

1. Write a function which gives the trajectory of a Brownian motion $(B_s)_{0\leq s\leq T}$ between 0 and T on some grid containing N equal segments. The input variables of the function should be T and N. The output should be the values of the Brownian motion on the grid. There should be no loop in this function.
2. From now on, we set $\boxed{T=1}$. Check that the maximum of the Brownian motion a grid on $[0,T]$ has approximately the same distribution as $|B_T|$ (we can use qqplotSample which is given in the padlet).
3. Write in the jupyter notebook, the calculations that gives the theoretical value of $\mathbb{E}[\max_{0\leq s\leq T} B_s]$. (In latex, equations are between dollars, integrals are written with *int* and fraction using *frac*).
4. Illustrate that the Law of Large Numbers $\lim_{m\to\infty} \frac{1}{m}\sum_{i=1}^m |B_T^i| = \mathbb{E}[|B_T|]$, $\mathbb{P}$-a.s. where $(B_T^i)_{i\geq 1}$ are iid Brownian motions.
5. Plot an histogram of an approximation of $\max_{0\leq s\leq T} B_s$ on the grid (see question 1) and compare with the theoretical density of $|B_T|$.