Monte-Carlo methods
TP2

The simulations should be written as a **Jupyter Notebook**. At the end of the computer sessions,

please send an email with your jupyter notebook to `randal.douc@telecom-sudparis.eu` with the subject: TP2JVN.

You can add any additional graphs and comments on your results!!!!

Exercice 1. Recall that if $(S_t)_{t\geq 0}$ satisfies the Stochastic Differential Equation (SDE)

$$dS_t = rS_t dt + \sigma S_t dW_t \tag{1}$$

where $r \in (0,1)$ and $\sigma > 0$, then we can show that for all $t \geq 0$,

$$S_t = S_0 e^{(r-\sigma^2/2)t+\sigma W_t} \tag{2}$$

1. Using (1), write the Euler discretization scheme for (S_t) on a grid of $[0,T]$ with equal segments of size T/N: this should be a function named **euler**(T, N, S_0). The output is the vector S such that $S[n]$ is the value of the discretized process $S^N_{t_n}$ at $t_n = nh$ where $h = \frac{T}{N}$.

2. We now choose for example $\boxed{T = 1,\ r = 0.05,\ \sigma = 1}$ and try some values of N (for example $N = 1000$ but this is just a suggestion). Compare the distribution of S^N_1 using a sample of size $M = 1000$ with the distribution of S_1, using another sample of size $M = 1000$. You can use the function *qqplotSample* (see the padlet site) to compare the two samples. Any comments?

3. Write a function which outputs a sample of the maximum conditionally to an Euler Scheme (see Algorithm 8 in the lecture notes). Can we just consider $\max(S^N_{t_n})_{0\leq n\leq N}$ to approximate the maximum $\max(S_t)_{0\leq t\leq 1}$.