Your Name: Key

Calculus I, Math 151-06, Quiz #2

1. **[10 points total]**

   (a) **[4 points]** Determine whether the following statement of the precise definition of a limit is correct. If it is incorrect, provide a corrected version of the statement.

   $\lim_{x \to a} f(x) = L$ means that for every $\varepsilon > 0$ there is a $\delta > 0$ so that $0 < |x - a| < \delta$. Then $|f(x) - L| < \varepsilon$.

   hypothesis not conclusion.

   ...for every $\varepsilon > 0$ there is a $\delta > 0$ so that if $0 < |x-a| < \delta$, then $|f(x) - L| < \varepsilon$.

   (b) **[6 points]** Determine the δ that corresponds to a given generic ε in the precise definition for the statement $\lim_{x \to -2} (-4x - 5) = 3$. You do not need to write a formal proof that the limit holds.

   Want $|(-4x-5) - 3| < \varepsilon$

   $|-4x - 8| < \varepsilon$

   $|-4||x+2| < \varepsilon$

   $4|x+2| < \varepsilon$

   $|x+2| < \frac{\varepsilon}{4}$

   Have $0 < |x - (-2)| < \delta$

   $0 < |x+2| < \delta$

   $\boxed{\text{Choose } \delta = \frac{\varepsilon}{4}}$

2. [**4 points**] State the Intermediate Value Theorem.

Suppose f is continuous on the closed interval $[a,b]$, and suppose N is any number between $f(a)$ and $f(b)$. Then there is a point c in (a,b) such that $f(c)=N$.

3. [**6 points**] Let $f(x) = \begin{cases} 3x - q, & x \leq 4 \\ qx^2 + 5x, & x > 4 \end{cases}$. Determine the value(s) of q for which $f(x)$ is continuous at $x = 4$. Justify your answer using the definition of continuity.

Need $\lim_{x \to 4} f(x) = f(4)$. For $\lim_{x\to 4} f(x)$ to exist, need $\lim_{x\to 4^-} f(x) = \lim_{x \to 4^+} f(x)$.

$\lim_{x\to 4^-} f(x) = \lim_{x\to 4^-} (3x - q) = 12 - q$

$\lim_{x\to 4^+} f(x) = \lim_{x\to 4^+} (qx^2 + 5x) = 16q + 20$

Need $12 - q = 16q + 20$

$-17q = 8$ $\quad q = \frac{-8}{17}$

$f(4) = 3(4) - q = 12 - q = \lim_{x\to 4^-} f(x) = \lim_{x \to 4} f(x)$ if it exists, so f is continuous if $\boxed{q = \frac{-8}{17}}$

4. [**5 points**] Evaluate the limit $\lim_{t\to 0^+} \ln\left(3 + e^{1/x}\right)$. Show your work.

$= \ln(3 + e^{1/0^+}) = \ln(3 + e^{\infty}) = \ln(3 + \infty) = \ln(\infty) = \boxed{\infty}$