Your Name: Key

Calculus I, Math 151-06, Quiz #3

1. [**10 points**] Use the limit definition of a derivative to compute $f'(6)$ for $f(x) = \sqrt{3x-2}$. (Hint: Compute $f'(6)$ directly, rather than computing $f'(x)$ and then plugging in $x = 6$ at the end.)

$$f'(6) = \lim_{h\to 0} \frac{f(6+h) - f(6)}{h} = \lim_{h\to 0} \frac{\sqrt{3(6+h)-2} - \sqrt{3(6)-2}}{h}$$

$$= \lim_{h\to 0} \frac{(\sqrt{3h+16} - 4)}{h}\frac{(\sqrt{3h+16} + 4)}{(\sqrt{3h+16} + 4)} = \lim_{h\to 0} \frac{3h+16-16}{h(\sqrt{3h+16}+4)} = \lim_{h\to 0} \frac{3h}{h(\sqrt{3h+16}+4)}$$

$$= \lim_{h\to 0} \frac{3}{\sqrt{3h+16}+4} = \frac{3}{\sqrt{16}+4} = \frac{3}{4+4} = \boxed{\frac{3}{8}}$$

2. **[8 points]** Find an equation for the tangent line to the curve $g(x) = x^3 - 4\sqrt{x} + 3$ at the point where $x = 4$.

$g(4) = 64 - 4\sqrt{4} + 3 = 64 - 8 + 3 = 59$

$g'(x) = 3x^2 - \frac{4}{2\sqrt{x}} = 3x^2 - \frac{2}{\sqrt{x}}$

$g'(4) = 3(16) - \frac{2}{\sqrt{4}} = 48 - 1 = 47$

$$\boxed{y - 59 = 47(x - 4)}$$

3. **[7 points]** Find $\frac{dy}{dx}$ for the function $y = 5\sqrt[5]{x^4} - \frac{4}{x^7} + 8e^x - e^8$.

$y = 5x^{4/5} - 4x^{-7} + 8e^x - \text{const}$

$y' = 5 \cdot \frac{4}{5} x^{-1/5} - 4(-7)x^{-8} + 8e^x + 0$

$y' = \frac{4}{\sqrt[5]{x}} + \frac{28}{x^8} + 8e^x$