Your Name: Key

Calculus I, Math 151-06, Quiz #2

1. **[10 points total]**

   (a) **[4 points]** State the precise definition of a limit as it applies to the statement $\lim_{x\to 3}(5-4x) = -7$.

   For each $\varepsilon > 0$ there is a corresponding $\delta > 0$ for which, if $0 < |x-3| < \delta$, then $|(5-4x)-(-7)| < \varepsilon$.

   (b) **[6 points]** Determine the δ that corresponds to a given generic ε in the precise definition for the statement $\lim_{x\to 3}(5-4x) = -7$. You do not need to write a formal proof that the limit holds.

   Want $|(5-4x)-(-7)| < \varepsilon$

   $|-4x+12| < \varepsilon$

   $|-4||x-3| < \varepsilon$

   $4|x-3| < \varepsilon$

   $|x-3| < \frac{\varepsilon}{4}$

   Have $0 < |x-3| < \delta$

   Choose $\delta = \frac{\varepsilon}{4}$.

2. **[10 points total]**

(a) **[4 points]** State the Intermediate Value Theorem.

Suppose f is a continuous function on $[a,b]$,
and suppose N is any number between $f(a)$ and $f(b)$.
Then there is a number c between a and b where $f(c)=N$.

(b) **[6 points]** Use the Intermediate Value Theorem to prove that the function $f(x) = x + \cos x$ has at least one real root.

f is the sum of a continuous polynomial and a continuous trig function, so
f is continuous on $\mathbb{R}$. In particular, f is continuous on $[\frac{-\pi}{2}, 0]$.

$f(0) = 0 + \cos 0 = 1 > 0$
$f(\frac{-\pi}{2}) = \frac{-\pi}{2} + \cos(\frac{-\pi}{2}) = \frac{-\pi}{2} < 0$.

So 0 is between $f(\frac{-\pi}{2})$ and $f(0)$.

By IVT, there exists a c between $\frac{-\pi}{2}$ and 0 so that $f(c)=0$.
This c is a real root of f.

3. **[5 points]** Evaluate the limit $\lim_{t\to 2^-} e^{1/(x^2-4)}$. Show your work.

$= e^{\frac{1}{(2^-)^2-4}} = e^{\frac{1}{4^- -4}} = e^{\frac{1}{0^-}} = e^{-\infty} = \boxed{0}$