

Your Name:

Key

Calculus I, Math 151-06, Quiz #7

Evaluate the following limits. Show all of your work.

1. [8 points] $\lim_{x \rightarrow 0} \frac{3 - 3 \cos x}{x^2} = \frac{3 - 3 \cos 0}{0^2} = \frac{3 - 3(1)}{0} = \frac{0}{0}$

$$\stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0} \frac{3 \sin x}{2x} = \frac{3 \sin 0}{2(0)} = \frac{0}{0}$$

$$\stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0} \frac{3 \cos x}{2} = \boxed{\frac{3}{2}}$$

2. [8 points] $\lim_{x \rightarrow \infty} 4x^2 e^{-x^2} = 4(\infty)^2 e^{-\infty^2} = \infty(0)$

$$= \lim_{x \rightarrow \infty} \frac{4x^2}{e^{x^2}} = \frac{\infty}{\infty} = \frac{\infty}{\infty}$$

$$\stackrel{\text{L'H}}{=} \lim_{x \rightarrow \infty} \frac{8x}{2x e^{x^2}} = \lim_{x \rightarrow \infty} \frac{4}{e^{x^2}} = \frac{4}{\infty} = \boxed{0}$$

3. [9 points] $\lim_{x \rightarrow 0^+} (1+3x)^{1/x} = (1+0)^{1/0^+} = 1^\infty$

$$L = \lim_{x \rightarrow 0^+} (1+3x)^{1/x}$$

$$\ln L = \lim_{x \rightarrow 0^+} \ln (1+3x)^{1/x} = \lim_{x \rightarrow 0^+} \frac{1}{x} \ln(1+3x) = \lim_{x \rightarrow 0^+} \frac{\ln(1+3x)}{x} = \frac{\ln(1+0)}{0} = \frac{\ln 1}{0} = \frac{0}{0}$$

$$\stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0^+} \frac{\frac{3}{1+3x}}{1} = \lim_{x \rightarrow 0^+} \frac{3}{1+3x} = \frac{3}{1+3(0)} = \frac{3}{1} = 3$$

$$\ln L = 3$$

$$e^{\ln L} = e^3$$

$$L = \boxed{e^3}$$