UMBC

Name: Key

Math 151 Exam #3

Spring 2023

Wednesday, May 3, 2023

[2 points] Put your name on the test above, put a check mark next to your discussion section in the following chart, and read and sign the honor pledge below:

Section 07, Sai Naga, 2 - 2:50 (MP010)		Section 09, Grace, 3 - 3:50 (MP008)	
Section 08, Grace, 2 - 2:50 (MP008)		Section 10, Sai Naga, 3 - 3:50 (MP010)	

Signature:________________________________

Question	Possible points	Score
0	2	
1	16	
2	20	
3	16	
4	24	
5	32	
TOTAL	100	

Instructions:

- Read each problem carefully.
- Write legibly.
- Show all your work on these sheets.
- We will **not accept** answers without justification, unless otherwise noted.
- This exam has 6 pages, and 5 questions. Please make sure that all pages are included.
- There are 110 possible points available on this exam. It will be graded out of 100 points.
- You may not use books, notes or calculators.
- You have 50 minutes to complete this exam.

Good luck!

Question 1. [16 points total] Evaluate the following limits, using whatever techniques are most appropriate. Show all of your work.

(a) **[8 points]** $\lim_{x\to-\infty} 5x^2 + 2x^5$ $= \infty - \infty$

$= \lim_{x\to-\infty} x^2(5+2x^3) = \infty(5-\infty) = \infty(-\infty) = \boxed{-\infty}$

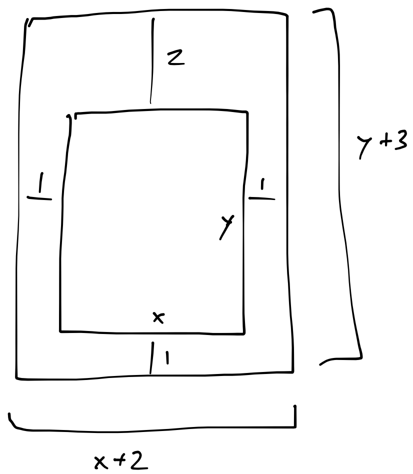
(b) **[8 points]** $\lim_{x\to 0^+} x^{1/x}$ $= 0^{\infty}$

$L = \lim_{x\to 0^+} x^{1/x}$

$\ln L = \lim_{x\to 0^+} \ln x^{1/x} = \lim_{x\to 0^+} \frac{1}{x}\ln x = \lim_{x\to 0^+} \frac{\ln x}{x} = \frac{-\infty}{0^+} = -\infty$

$L = e^{-\infty} = \boxed{0}$

Question 2. [20 points] A rectangular poster is going to be designed to have 1 inch margins on the bottom and sides, and a 2 inch margin on top. The printable area of the poster (inside the margins) is to be 54 square inches. Find the dimensions of the poster that minimize the total amount of paper used. Include an argument that the critical point you find is actually a minimum.



$xy = 54 \qquad y = \frac{54}{x}$

$P = (x+2)(y+3) = (x+2)\left(\frac{54}{x}+3\right) = 54 + \frac{108}{x} + 3x + 6$

$P = 3x + \frac{108}{x} + 60$

$P' = 3 - \frac{108}{x^2} = 0 \qquad \frac{108}{x^2} = 3 \qquad 108 = 3x^2$

$x^2 = 36$

$x = 6$

$P'' = \frac{216}{x^3} > 0$ at $x = 6$ — minimum

$y = \frac{54}{6} = 9$

Overall dimensions should be 6+2, 9+3 inches, i.e. 8″ × 12″.

Question 3. [16 points] An object travels back and forth on a straight line in such a way that its acceleration at time t seconds is given by $a(t) = -2 + 6t\ m/s^2$. Its velocity at time $t = 4$ seconds is $3\ m/s$, and its position at that same time is 0 meters. At what position did the object start at time $t = 0$?

$a(t) = -2 + 6t$

$v(t) = -2t + 3t^2 + C \qquad v(4) = 3 \qquad -2(4) + 3(4)^2 + C = 3$

$-8 + 48 + C = 3$

$C = -37$

$v(t) = -2t + 3t^2 - 37$

$p(t) = -t^2 + t^3 - 37t + d \qquad p(4) = 0 \qquad -(4)^2 + 4^3 - 37(4) + d = 0$

$-16 + 64 - 148 + d = 0$

$d = 100$

$p(t) = -t^2 + t^3 - 37t + 100 \qquad p(0) = 100$

initial position is $\boxed{100 \text{ meters}}$

Question 4. [24 points total] Let $f(x) = \dfrac{\sqrt{\frac{3}{2} - x^2}}{x}$.

Note that $f'(x) = \dfrac{-3}{x^2\sqrt{\frac{3}{2} - x^2}}$, and $f''(x) = \dfrac{-(9x^2 - 9)}{2x^3\sqrt{\frac{3}{2} - x^2}^{3}}$.

f has domain $(-\sqrt{3/2}, 0) \cup (0, \sqrt{3/2})$, x-intercepts at $(-\sqrt{3/2}, 0)$ and $(\sqrt{3/2}, 0)$, and no y-intercepts. f is an odd function and has a vertical asymptote at $y = 0$.

(a) **[8 points]** Find all intervals of increase and decrease of f. Find all critical points of f (x- and y-values) and classify each as a local maximum, local minimum, or neither.

$f'(x) = 0$

$\dfrac{-3}{x^2\sqrt{\frac{3}{2}-x^2}} = 0$

$-3 = 0$

no solutions

$f'(x)$: $-\ -\ -$ DNE $-\ -\ -$

x: $-\sqrt{\frac{3}{2}}$ $\quad 0 \quad$ $\sqrt{\frac{3}{2}}$

f is decreasing on its entire domain.

No local maxes/mins

(b) **[8 points]** Find all intervals of concavity for f and identify any inflection points (again, x- and y-values).

$f''(x) = 0$

$\dfrac{9x^2-9}{2x^3\sqrt{\frac{3}{2}-x^2}^{3}} = 0$

$9x^2 - 9 = 0$

$9x^2 = 9$

$x^2 = 1$

$x = \pm 1$

$f''(x)$: $+\ +\ 0\ -\ -\ -$ DNE $+\ +\ 0\ -\ -$

x: $-\sqrt{\frac{3}{2}}$ $\quad -1 \quad$ 0 $\quad 1 \quad$ $\sqrt{\frac{3}{2}}$

$-(9x^2-9)$	$-$	$+$	$+$	$-$
$2x^3$	$-$	$-$	$+$	$+$
$\sqrt{\frac{3}{2}-x^2}^{3}$	$+$	$+$	$+$	$+$
$f''(x)$	$+$	$-$	$+$	$-$

$f(1) = \sqrt{\frac{1}{2}}$

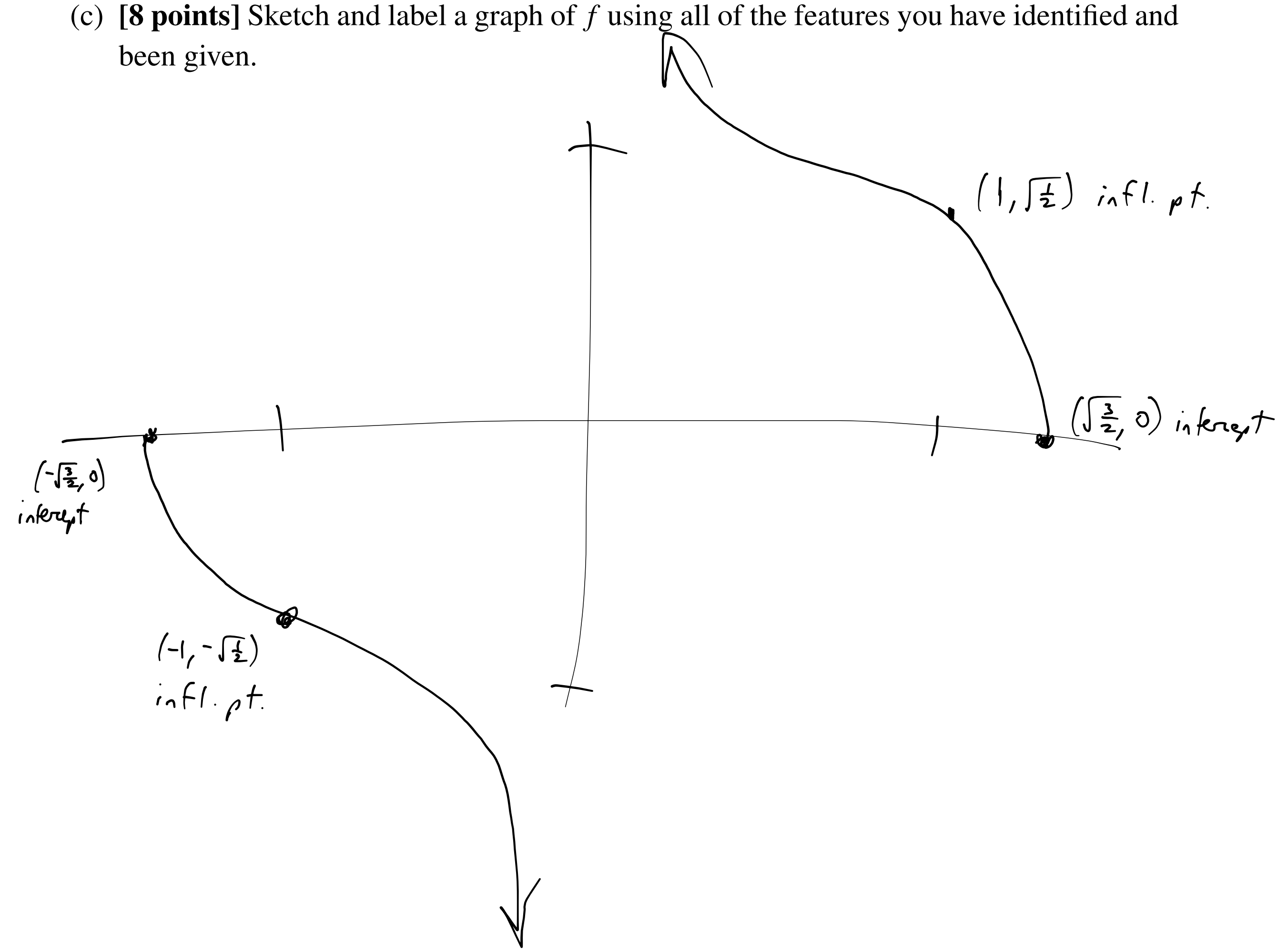
$f(-1) = -\sqrt{\frac{1}{2}}$

concave down on $(-1, 0)$ and $(1, \sqrt{\frac{3}{2}})$

concave up on $(-\sqrt{\frac{3}{2}}, -1)$ and $(0, 1)$

inflection points at $(-1, -\sqrt{\frac{1}{2}})$ and $(1, \sqrt{\frac{1}{2}})$

(c) **[8 points]** Sketch and label a graph of f using all of the features you have identified and been given.



Question 5. [32 points total] For each of the following, **circle the letter of the correct answer**. Remember to show your work for possible partial credit.

(a) **[8 points]** Approximate a solution of the equation $x^2 - 4x + 1 = 0$ by using Newton's Method. With an initial guess of $x_1 = 0$, find the next two guesses, x_2 and x_3.

(A) $x_2 = 1/4$, $x_3 = 15/56$ (B) $x_2 = -1/4$, $x_3 = -51/72$

(C) $x_2 = 4$, $x_3 = 8$ (D) $x_2 = -4$, $x_3 = -48/11$

$f(x) = x^2 - 4x + 1$

$f'(x) = 2x - 4$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0 - \frac{f(0)}{f'(0)} = 0 - \frac{1}{-4} = \frac{1}{4}$$

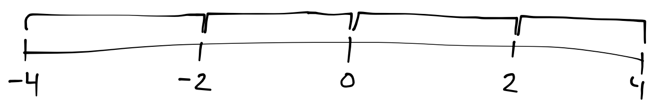
$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = \frac{1}{4} - \frac{f(\frac{1}{4})}{f'(\frac{1}{4})} = \frac{1}{4} - \frac{\frac{1}{16} - 1 + 1}{\frac{1}{2} - 4} = \frac{1}{4} - \frac{1/16}{-7/2} = \frac{1}{4} + \frac{1}{56} = \frac{14}{56} + \frac{1}{56} = \frac{15}{56}$$

(b) **[8 points]** Find the approximation R_4 for $\int_{-4}^{4} (x^2 - x + 2)\, dx$.

$f(x) = x^2 - x + 2$

(A) 28 (B) 56 (C) 72 (D) 100

$\Delta x = \frac{4-(-4)}{4} = \frac{8}{4} = 2$



$R_4 = 2f(-2) + 2f(0) + 2f(2) + 2f(4) = 2(8) + 2(2) + 2(4) + 2(14) = 56$

(c) **[8 points]** Determine which of the following choices correctly expresses $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\left(1 + \frac{3i}{n}\right)^2 \ln\left(1 + \frac{3i}{n}\right)\right) \frac{3}{n}$ as a definite integral.

$x_i^2 \quad \ln \quad x_i \quad \Delta x$

$\Delta x = \frac{3}{n}$ $\quad x_i = a + i\Delta x$

$\frac{b-a}{n} = \frac{3}{n}$ $\quad = 1 + \frac{3i}{n}$ $\quad a = 1$

$\frac{b-1}{n} = \frac{3}{n}$ $\quad b = 4$

(A) $\int_0^1 \left((1 + x^2)\ln(1 + x)\right) dx$ (B) $\int_0^3 \left(x^2 \ln(x)\right) dx$

(C) $\int_1^4 \left((1 + x)^2 \ln(1 + x)\right) dx$ (D) None of the other choices.

$\int_1^4 x^2 \ln x \, dx$

(d) **[8 points]** Suppose that $\int_{-2}^{6} f(x)\, dx = 3$, $\int_{-1}^{2} f(x)\, dx = 6$, and $\int_{2}^{6} f(x)\, dx = 4$. Find $\int_{-2}^{-1} 3f(x)\, dx$.

$\int_{-2}^{-1} f(x)dx + \int_{-1}^{2} f(x)dx + \int_{2}^{6} f(x)dx = \int_{-2}^{6} f(x)dx$

(A) −21 (B) −7 (C) 0 (D) 7 (E) 21

$\int_{-2}^{-1} f(x)dx + 6 + 4 = 3$

$\int_{-2}^{-1} f(x)dx = -7$

$\int_{-2}^{-1} 3f(x)dx = 3(-7) = -21$