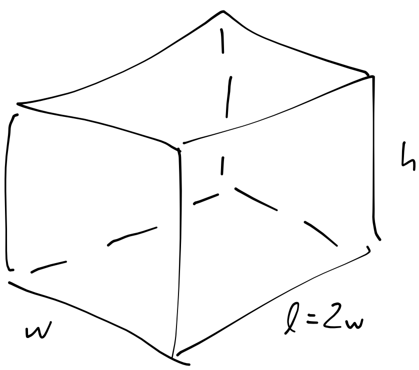
Your Name: Key

Calculus I, Math 151-06, Quiz #9

1. **[15 points]** A box has measurements width w, length ℓ, and height h. Given that a certain box is to have length exactly twice its width, and is to hold a volume of 72 cubic inches, what dimensions for this box will minimize its surface area? Provide an argument that your answer really does give a minimum.



$$V = lwh = (2w)wh = 2w^2h = 72$$

$$h = \frac{72}{2w^2} = \frac{36}{w^2}$$

$$A = 2lw + 2lh + 2wh$$
$$= 2(2w)w + 2(2w)h + 2wh$$
$$= 4w^2 + 6wh = 4w^2 + 6w\left(\frac{36}{w^2}\right)$$

$$A = 4w^2 + \frac{216}{w}$$

$$A' = 8w - \frac{216}{w^2} = 0$$

$$8w = \frac{216}{w^2}$$

$$w^3 = \frac{216}{8} = 27$$

$$w = 3$$

$$A'' = 8 + \frac{432}{w^3} > 0 \qquad w=3 \text{ is a } \underline{\text{minimum}}.$$

$$l = 2w = 6 \qquad h = \frac{36}{w^2} = \frac{36}{9} = 4$$

The box should be 6 inches long, 3 inches wide, and 4 inches tall.

2. **[10 points]** Let $h(x) = 3x^2 - 8x + 1$. Using Newton's Method to approximate a root of h with an initial guess of $x_1 = 2$, find x_2 and x_3. You do not need to simplify x_3.

$$h'(x) = 6x - 8$$

$$x_2 = x_1 - \frac{h(x_1)}{h'(x_1)} = 2 - \frac{h(2)}{h'(2)} = 2 - \frac{3(2)^2 - 8(2) + 1}{6(2) - 8} = 2 - \frac{-3}{4} = \boxed{\frac{11}{4}}$$

$$x_3 = x_2 - \frac{h(x_2)}{h'(x_2)} = \frac{11}{4} - \frac{h(\frac{11}{4})}{h'(\frac{11}{4})} = \boxed{\frac{11}{4} - \frac{3(\frac{11}{4})^2 - 8(\frac{11}{4}) + 1}{6(\frac{11}{4}) - 8}}$$