UMBC **Name:** Key

Math 151 Exam #2

Fall 2023

Tuesday, October 31, 2023

[2 points] Put your name on the test above, put a check mark next to your discussion section in the following chart, and read and sign the honor pledge below:

Section 07, Satvik, 8 - 8:50 (MP008)		Section 09, Sandun, 9 - 9:50 (MP010)	
Section 08, Sandun, 8 - 8:50 (MP010)		Section 10, Kara, 9 - 9:50 (MP008)	

Student Academic Conduct Policy: As per university policy, by enrolling in this course, each student assumes the responsibilities of an active participant in UMBC's scholarly community in which everyone's academic work and behavior are held to the highest standards of honesty. Cheating, fabrication, plagiarism, and helping others to commit these acts are all forms of academic dishonesty, and they are wrong. Academic misconduct could result in disciplinary action that may include, but is not limited to, suspension or dismissal. To read the full student academic conduct policy, consult the UMBC student handbook, the faculty handbook, or the UMBC policies section of the UMBC directory.

I agree not to engage in academic misconduct. I further agree not to tolerate misconduct among other students.

Signature:____

Question	Possible points	Score
0	2	
1	24	
2	12	
3	15	
4	20	
5	16	
6	21	
TOTAL	100	

Instructions:

- Read each problem carefully.
- Write legibly.
- Show all your work on these sheets.
- We will **not accept** answers without justification, unless otherwise noted.
- This exam has 7 pages, and 7 questions. Please make sure that all pages are included.
- There are 110 possible points available on this exam. It will be graded out of 100 points.
- You may not use books, notes or calculators.
- You have 50 minutes to complete this exam.

Good luck!

Question 1. [24 points total]

(a) **[8 points]** Find the derivative of $h(z) = \tan^{-1}(\cosh z)$.

$$h'(z) = \frac{\sinh z}{1 + \cosh^2 z}$$

(b) **[8 points]** Solve for $\dfrac{dy}{dx}$ if $y^3 + ye^x = 4x^2$.

$$3y^2 \frac{dy}{dx} + \left[\frac{dy}{dx} e^x + ye^x\right] = 8x$$

$$3y^2 \frac{dy}{dx} + \frac{dy}{dx} e^x = 8x - ye^x$$

$$\frac{dy}{dx}(3y^2 + e^x) = 8x - ye^x$$

$$\boxed{\frac{dy}{dx} = \frac{8x - ye^x}{3y^2 + e^x}}$$

(c) **[8 points]** Find the derivative of $s(t) = t^{\ln t}$.

$$\ln s(t) = \ln t^{\ln t}$$

$$\ln s(t) = (\ln t)(\ln t)$$

$$\frac{s'(t)}{s(t)} = \frac{1}{t}\ln t + (\ln t)\left(\frac{1}{t}\right)$$

$$s'(t) = s(t)\left[\frac{2}{t}\ln t\right]$$

$$\boxed{s'(t) = t^{\ln t}\left(\frac{2}{t}\ln t\right)}$$

Question 2. [12 points] Use an appropriate linear approximation to estimate the number $(3.993)^{3/2}$. Simplify your answer.

$f(x) = x^{3/2}$ $\qquad f'(x) = \frac{3}{2}x^{1/2}$ $\qquad a = 4$

$f(a) = 4^{3/2} = 8$ $\qquad f'(a) = \frac{3}{2}(4)^{1/2} = 3$

$$L(x) = f(a) + f'(a)(x-a)$$

$$L(x) = 8 + 3(x-4)$$

$$\sqrt{3.993} = f(3.993) \approx L(3.993) = 8 + 3(3.993 - 4)$$

$$= 8 + 3(-.007) = 8 - .021 = \boxed{7.979}$$

Question 3. [15 points total]

(a) **[7 points]** State Rolle's Theorem.

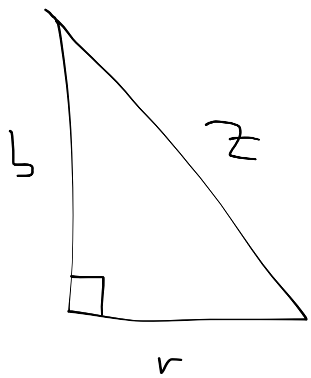
Suppose f is continuous on $[a,b]$ and differentiable on (a,b), and suppose $f(a)=f(b)$. Then there is a point c in (a,b) so that $f'(c)=0$.

(b) **[8 points]** Ryan throws a ball straight up in the air and catches it at the height of release 2 seconds later. Explain, using Rolle's Theorem, why the ball must be motionless at some point during its flight.

Let $h(t)$ = the height of the ball at time t.
a = the time when Ryan throws the ball, and let b = the time when Ryan catches the ball. Since he releases and catches the ball at the same time, $h(a)=h(b)$. Since the ball follows the usual laws of physics, $h(t)$ is everywhere continuous and differentiable.

Therefore there is a point c between a and b where $h'(c)=0$.
The ball is motionless at time c.

Question 4. [20 points] Ruth walks directly north from the True Grit statue at a rate of 2 ft/s. Bridget walks directly west towards the True Grit statue at a rate of 3 ft/s. When Ruth is 30 ft from the statue and Bridget is 40 feet from the statue, how is the distance between the two girls changing?



$r = 30 \qquad \frac{dr}{dt} = 2$

$b = 40 \qquad \frac{db}{dt} = -3$

$r^2 + b^2 = z^2$

(at the time in question: $30^2 + 40^2 = z^2$
$z = 50$)

$$2r\frac{dr}{dt} + 2b\frac{db}{dt} = 2z\frac{dz}{dt}$$

$$2(30)(2) + 2(40)(-3) = 2(50)\frac{dz}{dt}$$

$$120 - 240 = 100\frac{dz}{dt}$$

$$\frac{-120}{100} = \frac{dz}{dt}$$

$$-\frac{6}{5} = \frac{dz}{dt}$$

Ruth and Bridget are getting closer together at $\frac{6}{5}$ ft/sec.

Question 5. [16 points total] Let $f(x) = 2x^3 - 3x^2 - 36x + 10$.

(a) **[8 points]** Find the critical points (x- and y-values) of $f(x)$ and classify each one as a local maximum, a local minimum, or neither.

$f'(x) = 6x^2 - 6x - 36 = 0$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 3,\ x = -2$

$f''(x) = 12x - 6$

$f''(3) = 30 > 0$ $\quad f(3) = 2(3)^3 - 3(3)^2 - 36(3) + 10 = -71$

f has a local min at $(3, -71)$

$f''(-2) = -30 < 0$ $\quad f(-2) = 2(-2)^3 - 3(-2)^2 - 36(-2) + 10 = 54$

f has a local max at $(-2, 54)$

(b) **[8 points]** Find the absolute maximum and absolute minimum values of $f(x)$ on the interval $[-1, 4]$.

$x = -2$ is not in the domain.

$f(-1) = 2(-1)^3 - 3(-1)^2 - 36(-1) + 10 = 41$

$f(3) = -71$ (as above)

$f(4) = 2(4)^3 - 3(4)^2 - 36(4) + 10 = -54$

f has an absolute maximum at $(-1, 41)$ and an absolute minimum at $(3, -71)$.

Question 6. [21 points total] True/False. Mark each statement true or false. **(7 points each)**

(a) If the side length of a cube is measured to be 4 inches, with a possible error in measurement of ±0.2 inches, then the estimated error in the volume of the cube is ±9.6 cubic inches.

$V = s^3$ $\qquad dV = 3s^2\,ds$

$= 3(4)^2(\pm 0.2)$

$= \pm 9.6$

True **False**

(b) $\coth x = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$.

$$\coth x = \frac{\cosh x}{\sinh x} = \frac{\frac{1}{2}(e^x + e^{-x})}{\frac{1}{2}(e^x - e^{-x})} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

True **False**

(c) Mark the correct choice below. If $g'(x) > 2$ for all x, and if $g(4) = 3$, what is the possible range of $g(1)$?

g differentiable for all x, therefore cts for all x. MVT valid

(i) $g(1) > -3$. (ii) $g(1) < -3$. (iii) $g(1) > 9$. (iv) $g(1) < 9$. (v) Some other answer.

Applying MVT to g on $[1,4]$, there is a c in $(1,4)$ where $g'(c) = \dfrac{g(4) - g(1)}{4-1}$.

Since $g'(c) > 2$, $\quad \dfrac{g(4) - g(1)}{4-1} > 2 \qquad \dfrac{3 - g(1)}{3} > 2$

$3 - g(1) > 6 \qquad -g(1) > 3 \qquad g(1) < -3$