Your Name: Key

Calculus I, Math 151-06, Quiz #5

1. **[7 points]** Use implicit differentiation to solve for $\frac{dy}{dx}$, given the curve $4x - \tan^{-1} y = y^2$.

$$4 - \frac{1}{1+y^2}\frac{dy}{dx} = 2y\frac{dy}{dx}$$

$$4 = 2y\frac{dy}{dx} + \frac{1}{1+y^2}\frac{dy}{dx}$$

$$4 = \frac{dy}{dx}\left(2y + \frac{1}{1+y^2}\right)$$

$$\boxed{\frac{dy}{dx} = \frac{4}{2y + \frac{1}{1+y^2}}}$$

2. **[8 points]** Find the derivative of $f(x) = \dfrac{e^{\tan x}}{x^{12}(x^4+5x)^7}$, using the most appropriate technique(s) available.

$$\ln f(x) = \ln \frac{e^{\tan x}}{x^{12}(x^4+5x)^7}$$

$$\ln f(x) = \ln e^{\tan x} - \ln x^{12} - \ln (x^4+5x)^7$$

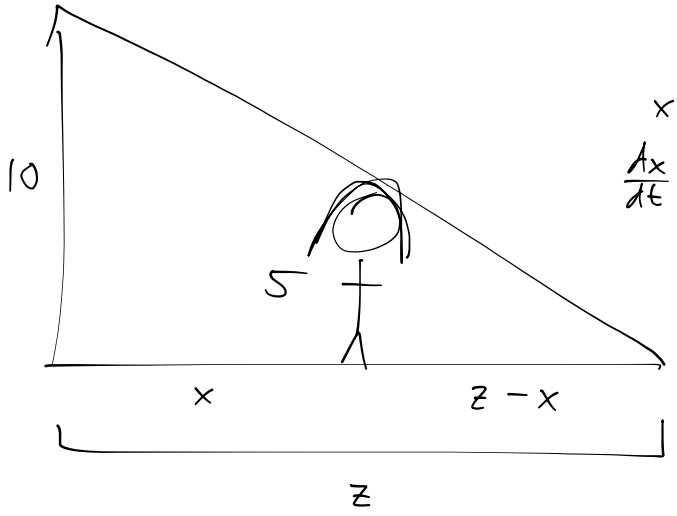
$$\ln f(x) = \tan x - 12\ln x - 7\ln(x^4+5x)$$

$$\frac{f'(x)}{f(x)} = \sec^2 x - 12\frac{1}{x} - 7\frac{4x^3+5}{x^4+5x}$$

$$f'(x) = f(x)\left(\sec^2 x - 12\frac{1}{x} - 7\frac{4x^3+5}{x^4+5x}\right)$$

$$f'(x) = \left(\frac{e^{\tan x}}{x^{12}(x^4+5x)^7}\right)\left(\sec^2 x - 12\frac{1}{x} - 7\frac{4x^3+5}{x^4+5x}\right)$$

3. **[10 points]** Jennifer, who is 5 feet tall, walks at a constant speed of 2 ft/s past a lamppost that is 10 feet tall. When Jennifer is 6 feet past the lamppost, how fast is the tip of her shadow moving?



$x = 6$ want $\frac{dz}{dt}$

$\frac{dx}{dt} = 2$

$\frac{10}{z} = \frac{5}{z-x}$

$10z - 10x = 5z$

$5z = 10x$

$5\frac{dz}{dt} = 10\frac{dx}{dt}$

$\frac{dz}{dt} = 2\frac{dx}{dt} = 2(2) = \boxed{4 \text{ ft/s}}$