Your Name: Key

Calculus I, Math 151-06, Quiz #8

1. **[16 points total]** Let $f(x) = \dfrac{\sqrt{9-x^2}}{x}$. Note that $f'(x) = \dfrac{-9}{x^2\sqrt{9-x^2}}$, and $f''(x) = \dfrac{-27(x^2-6)}{x^3(9-x^2)^{3/2}}$.

   (a) **[4 points]** The domain of f is $[-3,0)\cup(0,3]$. Find the intercepts of f, and any symmetries and asymptotes of f.

   $f(0)$ DNE – no y-intercept

   $\dfrac{\sqrt{9-x^2}}{x} = 0 \longrightarrow \sqrt{9-x^2} = 0 \longrightarrow 9 - x^2 = 0 \longrightarrow x^2 = 9 \longrightarrow x = \pm 3$

   x-intercepts $(-3,0)$, $(3,0)$

   $f(-x) = \dfrac{\sqrt{9-x^2}}{-x} = -f(x)$ f is odd

   Vertical asymptote at $x = 0$.

   (b) **[4 points]** Find all intervals of increase or decrease for f. Find all critical points, and label each as a local minimum of f, local maximum of f, or neither. Find all local minimum and maximum values of f.

   $f'(x) = 0$ has no solutions

   $f'(x)$: – – – – – DNE – – – –
   x: -3 0 3

   f is decreasing on $[-3,0)\cup(0,3]$. No critical points.

   (c) **[4 points]** Analyze the concavity of f, and find all points of inflection for f.

   $f''(x) = 0 \longrightarrow -27(x^2-6) = 0 \longrightarrow x^2 = 6 \longrightarrow x = \pm\sqrt{6}$

   +++0 – – – DNE +++ 0 – – –
   -3 $-\sqrt{6}$ 0 $\sqrt{6}$ 3

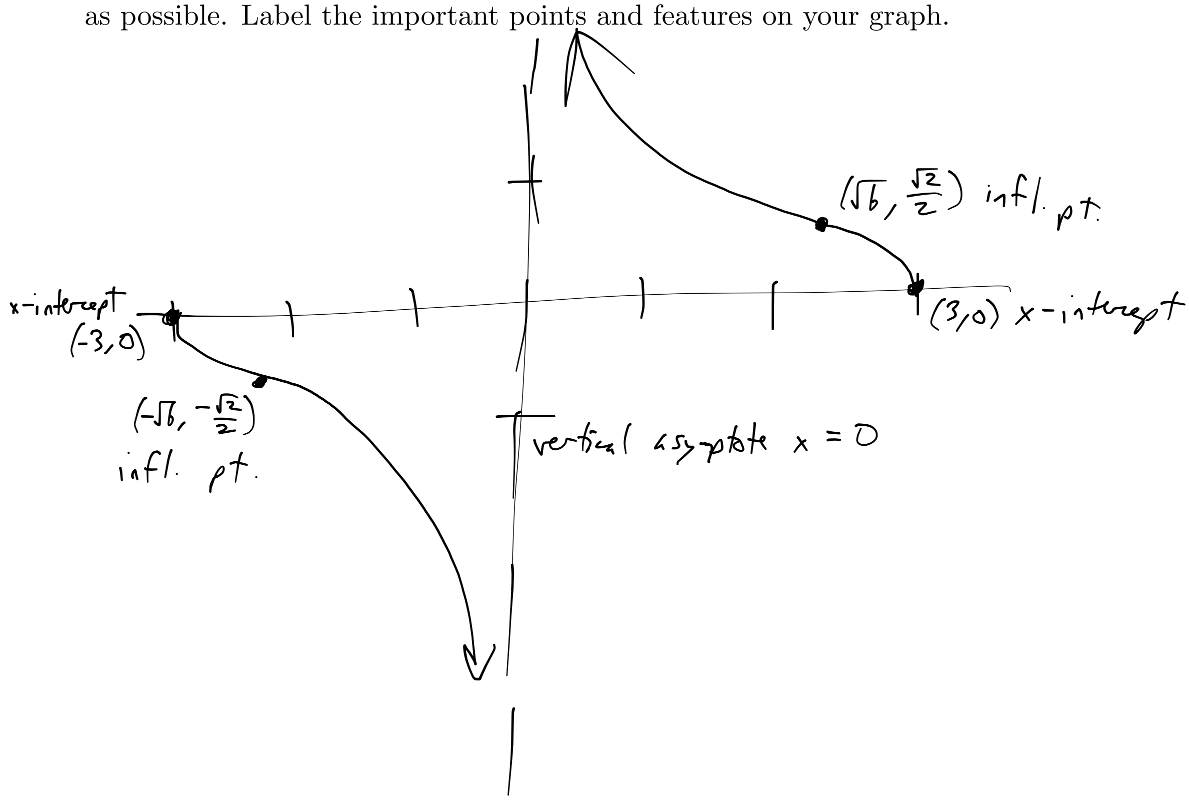
   $f(-\sqrt{6}) = \dfrac{\sqrt{9-6}}{-\sqrt{6}} = -\sqrt{\dfrac{3}{6}} = \dfrac{-1}{\sqrt{2}}$ or $\dfrac{-\sqrt{2}}{2}$

   $f(\sqrt{6}) = \dfrac{\sqrt{9-6}}{\sqrt{6}} = \sqrt{\dfrac{1}{2}} = \dfrac{1}{\sqrt{2}}$ or $\dfrac{\sqrt{2}}{2}$

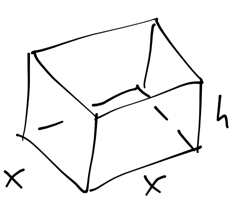
   f is concave up on $[-3,-\sqrt{6})\cup(0,\sqrt{6})$, and concave down on $(-\sqrt{6},0)\cup(\sqrt{6},3]$.

   f has inflection pts at $\left(-\sqrt{6}, \frac{-\sqrt{2}}{2}\right)$ and $\left(\sqrt{6}, \frac{\sqrt{2}}{2}\right)$.

(d) [**4 points**] Use the results of parts (a) – (c) to sketch a graph of f as accurately as possible. Label the important points and features on your graph.



2. [**9 points**] Dameon has 300 in^2 of cardboard which he wants to use to build an open-top box with a square base. What are the dimensions of such a box that will maximize its volume?



square base - length and width are the same

$$\text{surface area} = x^2 + 4xh = 300$$
$$4xh = 300 - x^2$$
$$h = \frac{300}{4x} - \frac{x^2}{4x}$$
$$h = \frac{75}{x} - \frac{x}{4}$$

$$Vol = x^2 h = x^2\left(\frac{75}{x} - \frac{x}{4}\right) = 75x - \frac{1}{4}x^3$$

$$Vol' = 75 - \frac{3}{4}x^2 = 0$$

$$\frac{3}{4}x^2 = 75$$
$$x^2 = 100$$
$$x = \pm 10$$

side length can't be negative, so $x = 10$

$$Vol'' = -\frac{3}{2}x \qquad Vol''(10) = -15 < 0$$

Vol has a max at $x = 10$

Corresponding h is $\frac{75}{10} - \frac{10}{4} = 7.5 - 2.5 = 5$

Maximal volume is with a 10in × 10in × 5in box.