

Your Name: Key

Calculus I, Math 151-06, Quiz #4

1. [7 points] Use implicit differentiation and a reference triangle to prove that $\frac{d}{dx} \cot^{-1} x = \frac{-1}{1+x^2}$.

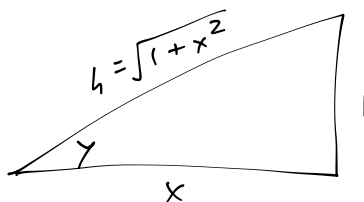
$$y = \cot^{-1} x$$

$$\cot y = x$$

$$-\csc^2 y \cdot \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{-1}{\csc^2 y}$$

$$\frac{dy}{dx} = \frac{-1}{(\sqrt{1+x^2})^2} = \frac{-1}{1+x^2}$$



$$\cot y = \frac{x}{1} = \frac{A}{O}$$

$$1^2 + x^2 = h^2$$

$$h = \sqrt{1+x^2}$$

$$\csc y = \frac{H}{O} = \frac{\sqrt{1+x^2}}{1} = \sqrt{1+x^2}$$

2. [8 points] Solve for $\frac{dy}{dx}$ for the curve $y^3 - 4x^2y + x^3 = 1$.

$$3y^2 \frac{dy}{dx} - [4x^2 \frac{dy}{dx} + 8xy] + 3x^2 = 0$$

$$3y^2 \frac{dy}{dx} - 4x^2 \frac{dy}{dx} - 8xy + 3x^2 = 0$$

$$3y^2 \frac{dy}{dx} - 4x^2 \frac{dy}{dx} = 8xy - 3x^2$$

$$\frac{dy}{dx} (3y^2 - 4x^2) = 8xy - 3x^2$$

$$\boxed{\frac{dy}{dx} = \frac{8xy - 3x^2}{3y^2 - 4x^2}}$$

3. [10 points] Find an equation of the tangent line to the curve $2e^{(3x-y)} = \frac{6x}{y}$ at the point $(1, 3)$.

$$2e^{3x-y} \left(3 - \frac{dy}{dx} \right) = \frac{y(6) - 6x \left(\frac{dy}{dx} \right)}{y^2}$$

$$2e^{3(1)-3} \left(3 - \frac{dy}{dx} \right) = \frac{(3)(6) - 6(1) \frac{dy}{dx}}{(3)^2}$$

$$2e^0 \left(3 - \frac{dy}{dx} \right) = \frac{18 - 6 \frac{dy}{dx}}{9}$$

$$2(1) \left(3 - \frac{dy}{dx} \right) = 2 - \frac{2}{3} \frac{dy}{dx}$$

$$6 - 2 \frac{dy}{dx} = 2 - \frac{2}{3} \frac{dy}{dx}$$

$$4 = \frac{4}{3} \frac{dy}{dx}$$

$$3 = \frac{dy}{dx}$$

tangent line:

$$\boxed{y - 3 = 3(x - 1)}$$