Your Name: Key

Calculus I, Math 151-06, Quiz #8

1. **[25 points total]** Let $f(x) = x\sqrt{x+4}$. Note that $f'(x) = \dfrac{3x+8}{2\sqrt{x+4}}$, and $f''(x) = \dfrac{3x+16}{4\sqrt{x+4}^3}$.

   (a) **[7 points]** Find the domain of f, the intercepts of f, and any symmetries and asymptotes of f.

Domain: Need $x+4 \geq 0$ $\quad x \geq -4$ domain $[-4, \infty)$

Intercepts: $f(x)=0$ $\quad x\sqrt{x+4}=0$ $\quad x=0$ $(0,0)$ or $\sqrt{x+4}=0$, $x+4=0$, $x=-4$ $(-4,0)$

$f(0) = 0\sqrt{4} = 0$ $(0,0)$

Intercepts $(0,0)$ and $(-4,0)$

Symmetries: $f(-x) = (-x)\sqrt{(-x)+4} = -x\sqrt{-x+4} \neq f(x)$

$\neq -f(x) = -x\sqrt{x+4}$

No symmetries

Asymptotes $\quad \lim_{x\to\infty} x\sqrt{x+4} = \infty\sqrt{\infty+4} = \infty\sqrt{\infty} = \infty\cdot\infty = \infty$ no horiz. asymptote

$\lim_{x\to -4} x\sqrt{x+4} = -4\sqrt{-4+4} = -4\sqrt{0} = 0$ no vert. asymptote

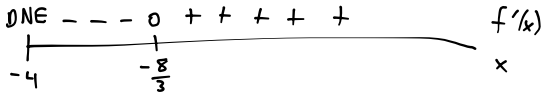
   (b) **[6 points]** Find all intervals of increase or decrease for f. Find all critical points, and label each as a local minimum of f, local maximum of f, or neither. Find all local minimum and maximum values of f.

$f'(x) = \dfrac{3x+8}{2\sqrt{x+4}} = 0$

$3x+8=0$

$3x=-8$

$x=\frac{-8}{3}$



$f(\frac{-8}{3}) = \frac{-8}{3}\sqrt{\frac{-8}{3}+4} = \frac{-8}{3}\sqrt{\frac{4}{3}} = \frac{-8}{3}\cdot\frac{2}{\sqrt{3}} = \frac{-16}{3\sqrt{3}}$

f is increasing on $(\frac{-8}{3}, \infty)$ and decreasing on $(-4, \frac{-8}{3})$.

f has a local min at $(\frac{-8}{3}, \frac{-16}{3\sqrt{3}})$.

Recall that $f(x) = x\sqrt{x+4}$, $f'(x) = \dfrac{3x+8}{2\sqrt{x+4}}$, and $f''(x) = \dfrac{3x+16}{4\sqrt{x+4}^3}$.

(c) **[6 points]** Analyze the concavity of f, and find all points of inflection for f.

$f''(x) = \dfrac{3x+16}{4\sqrt{x+4}^3} = 0$

$3x + 16 = 0$

$3x = -16$

$x = \dfrac{-16}{3}$ out of domain

DNE $+ + + + \vdash + + +$ $f''(x)$

-4 x

f is concave up on $[-4, \infty)$

f has no inflection points

(d) **[6 points]** Use the results of parts (a) – (c) to sketch a graph of f as accurately as possible. Label the important points and features on your graph.