UMBC

Name: Key

Math 151 Exam #3

Fall 2023

Tuesday, November 28, 2023

[2 points] Put your name on the test above, put a check mark next to your discussion section in the following chart, and read and sign the honor pledge below:

Section 07, Satvik, 8 - 8:50 (MP008)		Section 09, Sandun, 9 - 9:50 (MP010)	
Section 08, Sandun, 8 - 8:50 (MP010)		Section 10, Kara, 9 - 9:50 (MP008)	

Student Academic Conduct Policy: As per university policy, by enrolling in this course, each student assumes the responsibilities of an active participant in UMBC's scholarly community in which everyone's academic work and behavior are held to the highest standards of honesty. Cheating, fabrication, plagiarism, and helping others to commit these acts are all forms of academic dishonesty, and they are wrong. Academic misconduct could result in disciplinary action that may include, but is not limited to, suspension or dismissal. To read the full student academic conduct policy, consult the UMBC student handbook, the faculty handbook, or the UMBC policies section of the UMBC directory.

I agree not to engage in academic misconduct. I further agree not to tolerate misconduct among other students.

Signature:____________________________________

Question	Possible points	Score
0	2	
1	16	
2	16	
3	16	
4	24	
5	12	
6	24	
TOTAL	100	

Instructions:

- Read each problem carefully.
- Write legibly.
- Show all your work on these sheets.
- We will **not accept** answers without justification, unless otherwise noted.
- This exam has 6 pages, and 6 questions. Please make sure that all pages are included.
- There are 110 possible points available on this exam. It will be graded out of 100 points.
- You may not use books, notes or calculators.
- You have 50 minutes to complete this exam.

Good luck!

Question 1. [16 points total] Evaluate the following limits, using whatever techniques are most appropriate. Show all of your work.

(a) **[8 points]** $\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{2}{\tan x}\right)$ $= \infty - \infty$

$= \lim_{x\to 0^+}\left(\frac{\tan x}{x\tan x} - \frac{2x}{x\tan x}\right) = \lim_{x\to 0^+} \frac{\tan x - 2x}{x\tan x} = \frac{0-0}{0} = \frac{0}{0}$

$\overset{L'H}{=} \lim_{x\to 0^+} \frac{\sec^2 x - 2}{x\sec^2 x + \tan x} = \frac{1-2}{0+0} = \frac{-\infty}{0^+} = \boxed{-\infty}$

(b) **[8 points]** $\lim_{t\to\infty}(2t)^{1/t}$ $= \infty^{1/\infty} = \infty^0$

$L = \lim_{t\to\infty}(2t)^{1/t}$

$\ln L = \lim_{t\to\infty} \ln(2t)^{1/t} = \lim_{t\to\infty}\frac{1}{t}\ln(2t) = \lim_{t\to\infty}\frac{\ln(2t)}{t} = \frac{\infty}{\infty}$

$\overset{L'H}{=} \lim_{t\to\infty} \frac{\frac{2}{2t}}{1} = \frac{2}{\infty} = 0$

$\ln L = 0$

$e^{\ln L} = e^0 = 1$

$\boxed{L = 1}$

Question 2. [16 points] Donkey Kong throws a flaming barrel off of the roof of a construction site, with an initial velocity of 6 m/s upward. 3 seconds later, the barrel hits the ground and causes a fiery explosion. Assuming that acceleration due to gravity is $-10\ m/s^2$, what was the initial height from which Donkey Kong threw the barrel?

$a(t) = -10$

$v(t) = -10t + c$ $\quad v(0) = 6 = -10(0) + c \quad c = 6$

$v(t) = -10t + 6$

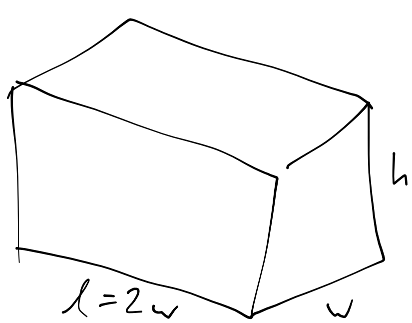
$h(t) = -5t^2 + 6t + d$ $\quad h(3) = 0 = -5(3)^2 + 6(3) + d$

$0 = -45 + 18 + d$

$27 = d$

$h(t) = -5t^2 + 6t + 27$ $\quad$ initial height $= h(0) = \boxed{27 \text{ meters}}$

Question 3. [16 points] Princess Peach wants to assemble a closed rectangular box with length exactly twice its width, and whose surface area is 600 in^2. What should the dimensions of the box be to maximize its volume? Include an argument that the critical point you find gives a maximum.



$A = 2lw + 2wh + 2lh$

$= 4w^2 + 2wh + 4wh$

$= 4w^2 + 6wh = 600$

$6wh = 600 - 4w^2$

$h = \frac{600}{6w} - \frac{4w^2}{6w} = \frac{100}{w} - \frac{2}{3}w$

$V = lwh = (2w)w\left(\frac{100}{w} - \frac{2}{3}w\right) = 200w - \frac{4}{3}w^3$

$V' = 200 - 4w^2 = 0$ $\quad V'' = -8w < 0$ so $w = \sqrt{50}$ is a max

$4w^2 = 200$

$w^2 = 50$

$w = \sqrt{50}$

$\boxed{\text{Optimal dimensions are } w = \sqrt{50} \text{ in}, \ l = 2\sqrt{50} \text{ in}, \ h = \frac{100}{\sqrt{50}} - \frac{2}{3}\sqrt{50} \text{ in}}$

Question 4. [24 points total] Let $f(x) = \dfrac{x^3+1}{x}$. Note that $f'(x) = \dfrac{2x^3-1}{x^2}$, and $f''(x) = \dfrac{2x^3+2}{x^3}$. f has domain $(-\infty, 0) \cup (0, \infty)$, an x-intercept at $(-1, 0)$, no y-intercept, is neither an even nor an odd function, and has a vertical asymptote at $x = 0$.

(a) **[8 points]** Find all intervals of increase and decrease of f. Find all critical points of f (x- and y-values) and classify each as a local maximum, local minimum, or neither.

$f'(x) = 0$

$\dfrac{2x^3-1}{x^2} = 0$

$2x^3 - 1 = 0$

$x^3 = \frac{1}{2}$

$x = \sqrt[3]{\frac{1}{2}}$

$f\left(\sqrt[3]{\frac{1}{2}}\right) = \dfrac{\frac{1}{2}+1}{\sqrt[3]{\frac{1}{2}}} = \dfrac{3}{\sqrt[3]{4}}$

$- \; - \; -$ DNE $-$ 0 $+ \; + \; +$ $\quad f'(x)$

$0 \quad \sqrt[3]{\frac{1}{2}} \quad x$

f is decreasing on $(-\infty, 0)$ and $(0, \sqrt[3]{\frac{1}{2}})$ and increasing on $(\sqrt[3]{\frac{1}{2}}, \infty)$ and has a local minimum at $\left(\sqrt[3]{\frac{1}{2}}, \frac{3}{\sqrt[3]{4}}\right)$.

(b) **[8 points]** Find all intervals of concavity for f and identify any inflection points (again, x- and y-values).

$f''(x) = \dfrac{2x^3+2}{x^3} = 0$

$2x^3 + 2 = 0$

$2x^3 = -2$

$x^3 = -1$

$x = -1$

$f(-1) = \dfrac{-1+1}{-1} = 0$

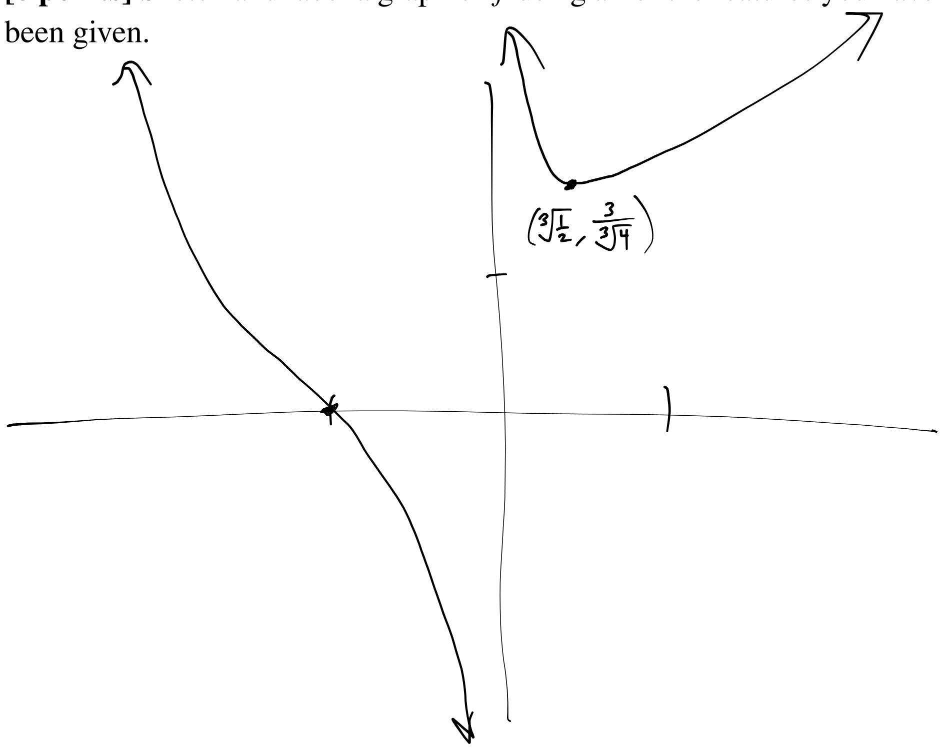
$+ \; + \; +$ 0 $- \; -$ DNE $+ \; +$ $\quad f''(x)$

$-1 \quad 0 \quad x$

f is concave up on $(-\infty, -1)$ and $(0, \infty)$ and is concave down on $(-1, 0)$.

f has an inflection point at $(-1, 0)$.

(c) **[8 points]** Sketch and label a graph of f using all of the features you have identified and been given.



Question 5. [12 points] Write $\lim_{n\to\infty}\sum_{i=1}^{n}\left(\left(-3+\frac{7i}{n}\right)^2-5\left(-3+\frac{7i}{n}\right)+6\right)\frac{7}{n}$ as a definite integral.

$(x_i^2 - 5x_i + 6)\Delta x$

$\Delta x = \frac{7}{n}$

$x_i = a + i\Delta x = -3 + \frac{7i}{n}$ $\quad a = -3$

$\Delta x = \frac{b-a}{n} = \frac{7}{n}$

$\frac{b - -3}{n} = \frac{7}{n}$

$b+3 = 7 \quad b = 4$

$\boxed{\int_{-3}^{4} (x^2 - 5x + 6)\,dx}$

Question 6. [24 points total] For each of the following, **circle the letter of the correct answer**.

(a) **[8 points]** Approximate a solution of the equation $x^3 - 2x - 2 = 0$ by using Newton's Method. With an initial guess of $x_1 = 1$, find the next two guesses, x_2 and x_3.

$f'(x) = 3x^2 - 2$

(a) $x_2 = -2$, $x_3 = -13/5$ (b) $x_2 = 4/3$, $x_3 = 259/93$ (c) $x_2 = 3$, $x_3 = -21/25$

(d) $x_2 = 4$, $x_3 = 9/5$ (e) $x_2 = 4$, $x_3 = 65/23$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1 - \frac{f(1)}{f'(1)} = 1 - \frac{-3}{1} = 4$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 4 - \frac{f(4)}{f'(4)} = 4 - \frac{54}{46} = \frac{92}{23} - \frac{27}{23} = \frac{65}{23}$

(b) **[8 points]** Find the approximation R_4 for $\int_{-3}^{5} (x^2 - 2)\, dx$.

$\Delta x = \frac{5-(-3)}{4} = 2$

(a) 12 (b) 24 (c) 56 (d) 70

$-3 \quad -1 \quad 1 \quad 3 \quad 5$

$R_4 = \Delta x \left(f(-1) + f(1) + f(3) + f(5)\right)$

$= 2(-1 + -1 + 7 + 23) = 2(28) = 56$

(c) **[8 points]** Suppose that $\int_{-1}^{6} g(x)\, dx = 4$, $\int_{-1}^{2} g(x)\, dx = 6$, and $\int_{5}^{6} g(x)\, dx = -5$. Find $\int_{5}^{2} g(x)\, dx$. $= -\int_2^5 g(x)dx$

$\int_{-1}^{2} g(x)dx + \int_2^5 g(x)dx + \int_5^6 g(x)dx = \int_{-1}^{6} g(x)dx$

$6 + \int_2^5 g(x)dx + -5 = 4$

(a) −11 (b) −3 (c) 0 (d) 3 (e) 11

$\int_2^5 g(x)dx = 3$

$\int_5^2 g(x)dx = -3$