UMBC

Name: Key

Math 151 Exam #1

Fall 2023

Tuesday, October 3rd, 2023

[2 points] Put your name on the test above, put a check mark next to your discussion section in the following chart, and read and sign the honor pledge below:

Section 07, Satvik, 8 - 8:50 (MP008)		Section 09, Sandun, 9 - 9:50 (MP010)	
Section 08, Sandun, 8 - 8:50 (MP010)		Section 10, Kara, 9 - 9:50 (MP008)	

Student Academic Conduct Policy: As per university policy, by enrolling in this course, each student assumes the responsibilities of an active participant in UMBC's scholarly community in which everyone's academic work and behavior are held to the highest standards of honesty. Cheating, fabrication, plagiarism, and helping others to commit these acts are all forms of academic dishonesty, and they are wrong. Academic misconduct could result in disciplinary action that may include, but is not limited to, suspension or dismissal. To read the full student academic conduct policy, consult the UMBC student handbook, the faculty handbook, or the UMBC policies section of the UMBC directory.

I agree not to engage in academic misconduct. I further agree not to tolerate misconduct among other students.

Signature:________________________________

Question	Possible points	Score
0	2	
1	16	
2	16	
3	16	
4	16	
5	16	
6	28	
TOTAL	100	

Instructions:

- Read each problem carefully.
- Write legibly.
- Show all your work on these sheets.
- We will **not accept** answers without justification, unless otherwise noted.
- This exam has 6 pages, and 6 questions. Please make sure that all pages are included.
- There are 110 possible points available on this exam. It will be graded out of 100 points.
- You may not use books, notes or calculators.
- You have 50 minutes to complete this exam.

Good luck!

Question 1. [16 points total] Evaluate the following limits.

(a) **[8 points]** $\lim_{x \to 4} \frac{\sqrt{8-x}-2}{x^2}$ $= \frac{\sqrt{4}-2}{4^2} = \frac{0}{16} = \boxed{0}$

(b) **[8 points]** $\lim_{x \to \pi/2^+} e^{\left(\frac{-3}{\tan(x)}\right)}$ $= e^{\left(-3/\tan \frac{\pi}{2}^+\right)} = e^{\left(-3/-\infty\right)} = e^0 = \boxed{1}$

Question 2. [16 points total]

(a) **[6 points]** Finish the limit definition of continuity: A function f is **continuous** at the point a if...

$$\lim_{x \to a} f(x) = f(a)$$

(b) **[10 points]** Use the definition you just gave to determine the value(s) of a and b for which $f(x)$ is continuous at the point where $x = -1$, given that

$$f(x) = \begin{cases} 4ax + 5, & \text{if } x < -1, \\ x + 2a, & \text{if } x > -1, \\ b, & \text{if } x = -1. \end{cases}$$

Need $\lim_{x \to -1} f(x)$ exists: $\lim_{x \to -1^-} f(x) = \lim_{x \to -1^+} f(x)$

$$\lim_{x \to -1^-} (4ax + 5) = \lim_{x \to -1^+} (x + 2a)$$

$$4a(-1) + 5 = (-1) + 2a$$

$$-4a + 5 = 2a - 1$$

$$-6a = -6$$

$$\underline{a = 1}$$

Given $a = 1$, need $\lim_{x \to -1} f(x) = f(-1) = b$.

$$\lim_{x \to -1} f(x) = \lim_{x \to -1^-} f(x) = 4(1)(-1) + 5 = 1.$$

Need $b = 1$.

$$= \lim_{x \to -1^+} f(x) = -1 + 2(1) = 1$$

$$\boxed{a = 1,\ b = 1}$$

Question 3. [16 points]

Use the limit definition of the derivative to compute $f'(4)$, where $f(x) = \dfrac{6}{2x+1}$.

$$f'(4) = \lim_{h\to 0} \frac{f(4+h)-f(4)}{h} = \lim_{h\to 0} \frac{\frac{6}{2(4+h)+1} - \frac{6}{2(4)+1}}{h} = \lim_{h\to 0} \frac{\frac{6}{2h+9} - \frac{2}{3}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{6(3)}{(2h+9)(3)} - \frac{2(2h+9)}{(2h+9)(3)}}{h/1} = \lim_{h\to 0} \frac{18-4h-18}{6h+27} \cdot \frac{1}{h} = \lim_{h\to 0} \frac{-4h}{h(6h+27)}$$

$$= \lim_{h\to 0} \frac{-4}{6h+27} = \boxed{\frac{-4}{27}}$$

Question 4. [16 points] State the precise definition of a limit as it applies to the statement $\lim_{x\to 4}(9-5x) = -11$. Then, find the δ that corresponds with a generic given ε in this definition. You do not need to write a formal proof that the limit holds.

$\lim_{x\to 4}(9-5x) = -11$ if for each $\varepsilon > 0$ there exists $\delta > 0$ so that if $0 < |x-4| < \delta$, then $|(9-5x) - (-11)| < \varepsilon$.

Want $|(9-5x) - (-11)| < \varepsilon$ Have $0 < |x-4| < \delta$.

$|-5x+20| < \varepsilon$

$|-5||x-4| < \varepsilon$

$5|x-4| < \varepsilon$

$|x-4| < \frac{\varepsilon}{5}$

Choose $\boxed{\delta = \frac{\varepsilon}{5}}$

Question 5. [16 points total] Find the derivatives of the following functions.

(a) **[8 points]** $k(p) = 4^{(\sin p)(\cos p)}$ $= 4^w$

$w = \sin p \cos p$

$k'(p) = 4^w \ln 4 \cdot w'$

$w' = \sin p(-\sin p) + (\cos p)\cos p$ (product rule)

$= \cos^2 p - \sin^2 p$

$$\boxed{k'(p) = 4^{\sin p \cos p} \ln 4 \cdot (\cos^2 p - \sin^2 p)}$$

(b) **[8 points]** $q(t) = \dfrac{t}{(t^2+1)^5}$

$$q'(t) = \frac{(t^2+1)^5(1) - t\left[5(t^2+1)^4 \cdot 2t\right]}{(t^2+1)^{10}}$$

Question 6. [28 points total] Choose the best answer for each question. Show your work to be eligible for partial credit.

(a) **[7 points]** Circle **True** or **False**:

True **False** If a function f is positive at a point a and negative at a point b, then f must have a root at a point between a and b. f needs to be continuous to make this conclusion using IVT.

(b) **[7 points]** Circle **True** or **False**:

True **False** The derivative $\frac{d}{dx}$ of the function e^4 is $4e^3$.

$\frac{d}{dx}(e^4) = 0$ since e^4 is a constant.

For each of the following, **circle the letter of the correct answer**.

(c) **[7 points]** Determine $\lim_{x\to\infty} \frac{-12x^3 + 2x + 7}{4x^2 - 3x} \cdot \frac{1/x^2}{1/x^2} = \lim_{x\to\infty} \frac{-12x + \frac{2}{x} + 7/x^2}{4 - 3/x} = \frac{-12\infty + 0 + 0}{4 - 0}$

$= -\infty$

(A) 0.

(B) 3.

(C) ∞.

(D) $-\infty$.

(E) Some other answer.

(F) There is not enough information to find a concrete answer.

(d) **[7 points]** Suppose that $f(3) = 2$, $f'(3) = -4$, $g(3) = -3$, and $g'(3) = 5$. Let $h(x) = \frac{f(x)}{g(x)}$. Find $h'(3)$.

$h'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$

$h'(3) = \frac{g(3)f'(3) - f(3)g'(3)}{(g(3))^2} = \frac{(-3)(-4) - (2)(5)}{(-3)^2} = \frac{12-10}{9} = \frac{2}{9}$

(A) $-\frac{2}{9}$.

(B) $-\frac{4}{5}$.

(C) 22.

(D) $-\frac{2}{3}$.

(E) Some other answer.

(F) There is not enough information to find a concrete answer.