Your Name: Key

Calculus I, Math 151-11, Quiz #7

Evaluate the following limits.

1. **[8 points]** $\lim_{x \to 0} \dfrac{\sin x - x}{x^3} = \dfrac{0-0}{0} = \dfrac{0}{0}$

$$\overset{L'H}{=} \lim_{x \to 0} \frac{\cos x - 1}{3x^2} = \frac{1-1}{0} = \frac{0}{0}$$

$$\overset{L'H}{=} \lim_{x \to 0} \frac{-\sin x}{6x} = \frac{0}{0}$$

$$\overset{L'H}{=} \lim_{x \to 0} \frac{-\cos x}{6} = \boxed{\frac{-1}{6}}$$

2. **[8 points]** $\lim\limits_{x\to 0^+} \frac{1}{x} - \frac{1}{e^x - 1}$ $= \frac{1}{0^+} - \frac{1}{e^{0^+}-1} = \infty - \frac{1}{1^+-1} = \infty - \frac{1}{0^+} = \infty - \infty$

$$= \lim_{x\to 0^+} \frac{e^x-1}{x(e^x-1)} - \frac{x}{x(e^x-1)} = \lim_{x\to 0^+} \frac{e^x - x - 1}{x(e^x-1)} = \frac{1-0-1}{0(1-1)} = \frac{0}{0}$$

$$\overset{L'H}{=} \lim_{x\to 0^+} \frac{e^x - 1}{xe^x + e^x - 1} = \frac{1-1}{0+1-1} = \frac{0}{0}$$

$$\overset{L'H}{=} \lim_{x\to 0^+} \frac{e^x}{xe^x + 2e^x} = \lim_{x\to 0^+} \frac{1}{x+2} = \frac{1}{0+2} = \boxed{\frac{1}{2}}$$

3. **[9 points]** $\lim\limits_{x\to\infty} \left(1 + \frac{3}{x}\right)^{5x}$ $= \left(1+\frac{3}{\infty}\right)^{\infty} = (1+0)^{\infty} = 1^{\infty}$

$$L = \lim_{x\to\infty} \left(1+\frac{3}{x}\right)^{5x}$$

$$\ln L = \lim_{x\to\infty} \ln\left(1+\frac{3}{x}\right)^{5x} = \lim_{x\to\infty} 5x\ln\left(1+\frac{3}{x}\right) = \infty\cdot\ln 1 = \infty\cdot 0$$

$$\ln L = \lim_{x\to\infty} \frac{5\ln\left(1+\frac{3}{x}\right)}{\frac{1}{x}} = \frac{0}{1/\infty} = \frac{0}{0}$$

$$\overset{L'H}{=} \lim_{x\to\infty} \frac{5\,\frac{\frac{-3}{x^2}}{1+\frac{3}{x}}}{-1/x^2} = \lim_{x\to\infty} \frac{15}{1+\frac{3}{x}} = \frac{15}{1+0} = 15.$$

$$\ln L = 15 \qquad e^{\ln L} = e^{15} \qquad \boxed{L = e^{15}}$$