UMBC

**Name:** Key

# Math 151 Exam #1

## Spring 2023

## Wednesday, March 1st, 2023

**[2 points] Put your name on the test above, put a check mark next to your discussion section in the following chart, and read and sign the honor pledge below:**

| | | | |
|---|---|---|---|
| Section 07, Saeed, 2 - 2:50 (MP010) | | Section 09, Grace, 3 - 3:50 (MP008) | |
| Section 08, Grace, 2 - 2:50 (MP008) | | Section 10, Saeed, 3 - 3:50 (MP010) | |

Student Academic Conduct Policy: As per university policy, by enrolling in this course, each student assumes the responsibilities of an active participant in UMBC's scholarly community in which everyone's academic work and behavior are held to the highest standards of honesty. Cheating, fabrication, plagiarism, and helping others to commit these acts are all forms of academic dishonesty, and they are wrong. Academic misconduct could result in disciplinary action that may include, but is not limited to, suspension or dismissal. To read the full student academic conduct policy, consult the UMBC student handbook, the faculty handbook, or the UMBC policies section of the UMBC directory.

I agree not to engage in academic misconduct. I further agree not to tolerate misconduct among other students.

Signature:________________________________________

| Question | Possible points | Score |
|---|---|---|
| 0 | 2 | |
| 1 | 16 | |
| 2 | 16 | |
| 3 | 16 | |
| 4 | 16 | |
| 5 | 16 | |
| 6 | 28 | |
| TOTAL | 100 | |

**Instructions:**

- Read each problem carefully.
- Write legibly.
- Show all your work on these sheets.
- We will **not accept** answers without justification, unless otherwise noted.
- This exam has 7 pages, and 6 questions. Please make sure that all pages are included.
- There are 110 possible points available on this exam. It will be graded out of 100 points.
- You may not use books, notes or calculators.
- You have 50 minutes to complete this exam.

**Good luck!**

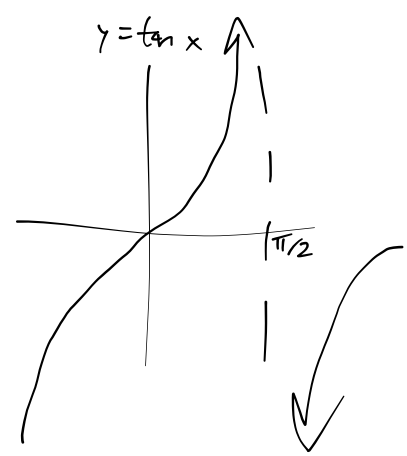
**Question 1. [16 points total]** Evaluate the following limits.

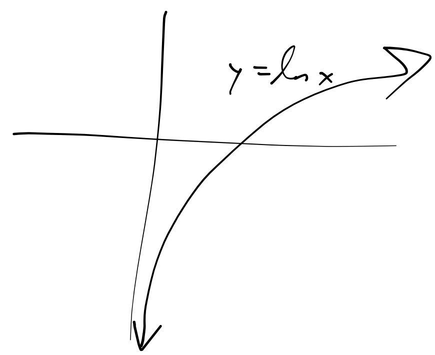
(a) **[8 points]** $\lim_{t\to 2} \dfrac{1/t - 1/2}{2-t}$ $= \dfrac{\frac{1}{2} - \frac{1}{2}}{2-2} = \dfrac{0}{0}$

$$= \lim_{t\to 2} \frac{\frac{2}{2t} - \frac{t}{2t}}{2-t} = \lim_{t\to 2} \frac{\frac{2-t}{2t}}{(2-t)/1} = \lim_{t\to 2} \frac{2-t}{2t} \cdot \frac{1}{2-t} = \lim_{t\to 2} \frac{1}{2t} = \boxed{\frac{1}{4}}$$

(b) **[8 points]** $\lim_{\theta\to\pi/2^-} \dfrac{\theta}{\ln(\tan(\theta))}$ $= \dfrac{\pi/2}{\ln(\tan(\frac{\pi}{2}^-))} = \dfrac{\pi/2}{\ln(\infty)}$

$$= \frac{\pi/2}{\infty} = \boxed{0}$$



**Question 2. [16 points total]**

(a) **[6 points]** Write the definition of continuity for a function $f(x)$ at a point $x = a$.

$$\lim_{x \to a} f(x) = f(a)$$

(b) **[10 points]** Let $f(x) = \begin{cases} 3x + a, & x < -1 \\ b, & x = 1 \\ ax + 3, & x > -1 \end{cases}$. Use the definition you gave in part (a) to find the value(s) of $a$ and $b$ for which $f(x)$ is continuous at $x = -1$.

Need $\lim_{x \to -1^-} f(x) = \lim_{x \to -1^+} f(x)$

$\lim_{x \to -1^-} (3x + a) = \lim_{x \to -1^+} ax + 3$

$3(-1) + a = a(-1) + 3$

$-3 + a = -a + 3$

$2a = 6$

$a = 3$

So if $a = 3$, $\lim_{x \to -1} f(x)$ exists and equals $3(-1) + (3)$ and $(3)(-1) + 3$, i.e. $0$.

Need $\lim_{x \to -1} f(x) = f(-1)$

$0 = b$

$$\boxed{a = 3, \quad b = 0}$$

**Question 3. [16 points total]**

(a) **[6 points]** Write the precise definition of a limit as it applies to the statement $\lim_{x\to -5}(x^2+3x) = 10$.

For every $\varepsilon > 0$ there exists a corresponding $\delta > 0$ so that if $0 < |x - -5| < \delta$, then $|(x^2+3x) - 10| < \varepsilon$.

(b) **[10 points]** Determine for a given $\varepsilon$, which $\delta$ should be used in the precise definition of the limit $\lim_{x\to 0}\sqrt[3]{x} = 0$. You do not need write a formal proof that $\lim_{x\to 0}\sqrt[3]{x} = 0$.

Want $|\sqrt[3]{x} - 0| < \varepsilon$

$|\sqrt[3]{x}| < \varepsilon$

$|\sqrt[3]{x}|^3 < \varepsilon^3$

$|x| < \varepsilon^3$

Have $0 < |x - 0| < \delta$

$|x| < \delta$

$\boxed{\text{Choose } \delta = \varepsilon^3}$

**Question 4. [16 points]** Use the limit definition of the derivative to compute $f'(-3)$ if $f(x) = \sqrt{28+4x}$. Find $f'(-3)$ directly; don't find $f'(x)$ in general and then plug in $-3$ for $x$. Use your answer to find an equation of the tangent line to $f(x)$ at $x = -3$.

$$f'(-3) = \lim_{h\to 0} \frac{f(-3+h) - f(-3)}{h} = \lim_{h\to 0} \frac{\sqrt{28+4(-3+h)} - \sqrt{28+4(-3)}}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{28-12+4h} - \sqrt{16}}{h} = \lim_{h\to 0} \frac{\sqrt{16+4h} - 4}{h} \qquad \left(= \frac{0}{0}\right)$$

$$= \lim_{h\to 0} \frac{(\sqrt{16+4h} - 4)}{h} \frac{(\sqrt{16+4h} + 4)}{(\sqrt{16+4h} + 4)} = \lim_{h\to 0} \frac{16+4h-16}{h(\sqrt{16+4h}+4)} = \lim_{h\to 0} \frac{4h}{h(\sqrt{16+4h}+4)}$$

$$= \lim_{h\to 0} \frac{4}{\sqrt{16+4h}+4} = \frac{4}{\sqrt{16}+4} = \frac{4}{4+4} = \frac{4}{8} = \boxed{\frac{1}{2}}$$

point $(-3, f(-3)) = (-3, 4)$ slope $\frac{1}{2}$

Equation of tangent line

$$\boxed{y - 4 = \frac{1}{2}(x+3)}$$

**Question 5. [16 points total]** Find the derivatives of the following functions.

(a) **[8 points]** $g(t) = 3t^6 - \dfrac{5}{t^2} + 2te^t - 3\csc t$

$g(t) = 3t^6 - 5t^{-2} + 2te^t - 3\csc t$

$g'(t) = 18t^5 + 10t^{-3} + \underbrace{2t(e^t) + (2)e^t}_{\text{product rule}} + 3\csc t \cot t$

(b) **[8 points]** $k(p) = \dfrac{e^p \tan p}{p^3}$

$k'(p) = \dfrac{p^3[e^p(\sec^2 p) + (e^p)\tan p] - e^p \tan p\,(3p^2)}{p^6}$

**Question 6. [28 points total]**

(a) **[7 points]** Circle **True** or **False**:

**True** **False** A function $f(x)$ is always differentiable at every point where it is continuous.

e.g. $|x|$ at $0$ is continuous but not differentiable

(b) **[7 points]** Circle **True** or **False**:

**True** **False** A function that is continuous everywhere must always hit every $y$-value that lies between any two of its outputs.

by IVT

For each of the following, **circle the letter of the correct answer**.

(c) **[7 points]** Determine $\lim_{x\to\infty} \frac{12x^3+2x+7}{3x-4x^2}$ $\cdot \frac{1/x^2}{1/x^2} = \lim_{x\to\infty} \frac{12x + \frac{2}{x} + \frac{7}{x^2}}{\frac{3}{x} - 4} = \frac{12\infty + \frac{2}{\infty} + \frac{7}{\infty^2}}{\frac{3}{\infty} - 4}$

$= \frac{\infty + 0 + 0}{0 - 4} = \frac{\infty}{-4} = -\infty$

(A) 0.

(B) 3.

(C) $\infty$.

(D) $-\infty$.

(E) Some other answer.

(F) There is not enough information to find a concrete answer.

(d) **[7 points]** Suppose that $f(-1) = 2$, $f'(-1) = 3$, $g(-1) = 4$, and $g'(-1) = -2$. Let $h(x) = \frac{f(x) + 2x}{g(x)}$. Find $h'(-1)$.

$h'(x) = \frac{g(x)[f'(x)+2] - [f(x)+2x](g'(x))}{(g(x))^2}$

$h'(-1) = \frac{g(-1)[f'(-1)+2] - [f(-1)+2(-1)](g'(-1))}{(g(-1))^2}$

(A) 0.

(B) $-\frac{5}{2}$.

(C) 1.

(D) $\frac{5}{4}$.

(E) Some other answer.

(F) There is not enough information to find a concrete answer.

$h'(-1) = \frac{4[3+2] - [2-2](-2)}{4^2} = \frac{20}{16} = \frac{5}{4}$