

Your Name: Key

Calculus I, Math 151-06, Quiz #5

1. [10 points] Find the derivative of $f(x) = \frac{(x^3 - 4x)^6 x^{17}}{e^{\cos x} \cos^5 x}$, using logarithmic differentiation.

$$\ln f(x) = \ln \left(\frac{(x^3 - 4x)^6 x^{17}}{e^{\cos x} \cos^5 x} \right)$$

$$\ln f(x) = \ln((x^3 - 4x)^6 x^{17}) - \ln(e^{\cos x} \cos^5 x)$$

$$\ln f(x) = \ln(x^3 - 4x)^6 + \ln x^{17} - \ln e^{\cos x} - \ln \cos^5 x$$

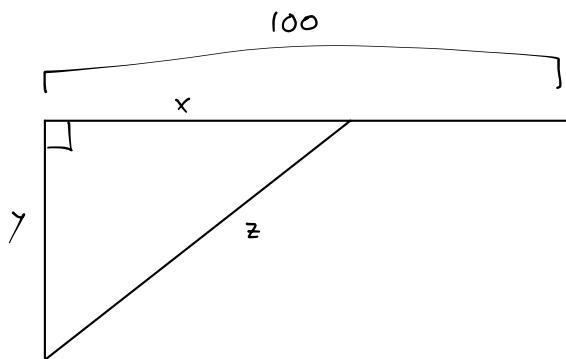
$$\ln f(x) = 6\ln(x^3 - 4x) + 17\ln x - \cos x - 5\ln \cos x$$

$$\frac{f'(x)}{f(x)} = 6 \left(\frac{3x^2 - 4}{x^3 - 4x} \right) + \frac{17}{x} + \sin x - 5 \frac{-\sin x}{\cos x}$$

$$f'(x) = f(x) \left[6 \left(\frac{3x^2 - 4}{x^3 - 4x} \right) + \frac{17}{x} + \sin x + 5 \tan x \right]$$

$$f'(x) = \frac{(x^3 - 4x)^6 x^{17}}{e^{\cos x} \cos^5 x} \left[6 \left(\frac{3x^2 - 4}{x^3 - 4x} \right) + \frac{17}{x} + \sin x + 5 \tan x \right]$$

2. [15 points] At 9pm, Jonathan sets out due south from Cleveland at a constant speed of 40 mph. At the same time, Simone leaves a point exactly 100 miles east of Cleveland and travels west towards Cleveland at a constant speed of 20 mph. At 11pm, how is the distance between Jonathan and Simone changing?



$$x = 100 - 2(20) = 60$$

$$\frac{dx}{dt} = -20$$

x is getting smaller

$$y = 2(40) = 80$$

$$\frac{dy}{dt} = 40$$

y is getting bigger

want $\frac{dz}{dt}$

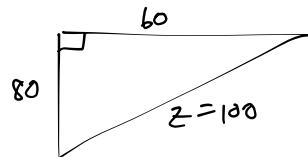
$$x^2 + y^2 = z^2$$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}$$

$$2(60)(-20) + 2(80)(40) = 2(100) \frac{dz}{dt}$$

$$-2400 + 6400 = 200 \frac{dz}{dt}$$

$$4000 = 200 \frac{dz}{dt}$$



$$\frac{dz}{dt} = 20 \text{ mph.}$$

Jonathan and Simone are getting further away from each other at 20 mph.