Your Name: Key

Calculus I, Math 151-11, Quiz #3

1. **[5 points]** Evaluate $\displaystyle\lim_{x\to\infty} \frac{3x - x^3}{x^2+14}$. Show all of your work. You may not use techniques that have not yet been introduced in this course.

$$\lim_{x\to\infty} \frac{3x-x^3}{x^2+14}\cdot\frac{1/x^2}{1/x^2} = \lim_{x\to\infty}\frac{\frac{3}{x}-x}{1+14/x^2} = \frac{\frac{3}{\infty}-\infty}{1+14/\infty^2} = \frac{0-\infty}{1+14/\infty} = \frac{-\infty}{1+0} = \boxed{-\infty}$$

2. **[10 points]** Use the limit definition of a derivative to compute $f'(2)$ for $f(x) = \dfrac{x^2}{1+2x}$. (Hint: Compute $f'(2)$ directly, rather than computing $f'(x)$ and then plugging in $x=2$ at the end.)

$$f'(2) = \lim_{h\to 0}\frac{f(2+h)-f(2)}{h} = \lim_{h\to 0}\frac{\frac{(2+h)^2}{1+2(2+h)} - \frac{4}{5}}{h} = \lim_{h\to 0}\frac{\frac{h^2+4h+4}{5+2h} - \frac{4}{5}}{h}$$

$$= \lim_{h\to 0}\frac{\frac{5h^2+20h+20}{5(5+2h)} - \frac{4(5+2h)}{5(5+2h)}}{h} = \lim_{h\to 0}\frac{\frac{5h^2+20h+20}{25+10h} - \frac{20+8h}{25+10h}}{h}$$

$$= \lim_{h\to 0}\frac{\frac{5h^2+12h}{25+10h}}{h/1} = \lim_{h\to 0}\frac{h(5h+12)}{25+10h}\cdot\frac{1}{h} = \lim_{h\to 0}\frac{5h+12}{25+10h} = \boxed{\frac{12}{25}}$$

3. **[10 points]** Use the limit definition of the derivative to compute $g'(t)$, where $g(t) = \sqrt{3+4t}$.

$$g'(t) = \lim_{h \to 0} \frac{g(t+h) - g(t)}{h} = \lim_{h \to 0} \frac{\sqrt{3+4(t+h)} - \sqrt{3+4t}}{h} = \lim_{t \to 0} \frac{\sqrt{3+4t+4h} - \sqrt{3+4t}}{h}$$

$$= \lim_{t \to 0} \frac{\left(\sqrt{3+4t+4h} - \sqrt{3+4t}\right)}{h} \frac{\left(\sqrt{3+4t+4h} + \sqrt{3+4t}\right)}{\left(\sqrt{3+4t+4h} + \sqrt{3+4t}\right)}$$

$$= \lim_{t \to 0} \frac{3+4t+4h-(3+4t)}{h\left(\sqrt{3+4t+4h} + \sqrt{3+4t}\right)} = \lim_{h \to 0} \frac{4h}{h\left(\sqrt{3+4t+4h} + \sqrt{3+4h}\right)} = \lim_{h \to 0} \frac{4}{\sqrt{3+4t+4h} + \sqrt{3+4t}}$$

$$= \frac{4}{\sqrt{3+4t} + \sqrt{3+4t}} = \frac{4}{2\sqrt{3+4t}} = \boxed{\frac{2}{\sqrt{3+4t}}}$$