Introduction

Linear Algebra
(spring,2016)

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Syllabus

- course webpage:
  http://www.minho-kim.com/courses/16sp71007
- Textbook:
  “Coding the Matrix: Linear Algebra through Computer Science Applications” by Phillip Klein
  http://codingthematrix.com

Introduction to Linear Algebra

What is *algebra*(대수학 : 代數學)?

- "Algebra is the branch of mathematics concerning the study of the rules of **operations** and **relations**, and the constructions and concepts arising from them, including **terms**, **polynomials**, **equations** and **algebraic structures**..." (from Wikipedia)
- Elementary algebra
  - operations(연산자): $+$, $-$, $\times$,...
  - relations(관계): $>$, $<$, $=$, $\leqslant$,...
  - variables/unknowns(미지수): x, y,...
  - terms(항): '$3x$' and '$4y$' in "$3x + 4y$"
  - polynomials(다항식): $ax^2 + by + cz$
  - functions
  - algebraic structures(대수적 구조): $\mathbb{Z}$(정수) under addition & multiplication

Introduction to Linear Algebra (cont'd)

What is *linear algebra*?

- "Linear algebra is a branch of mathematics that studies **vector spaces**, also called **linear spaces**, along with **linear functions** that input one vector and output another... (from Wikipedia)
- Vector space is an algebraic structure. → operations?

What You've Already Learned in High School...

- 수학관련 교과목 : 수학 (고1과정), 수학의활용, 수학I, 미적분과통계기본, 수학II, 적분과통계, 기하와벡터
- 7차교육과정
- "대한민국의 고등학교 수학 교과목" (Wikipedia korea)

Related topics:

- 수학 (고1과정)
  - 수와 연산 (elementary algebra: 기초대수학)
- 수학I
  - 행렬 (matrices) 과 그래프
- 기하와 벡터
  - 일차변환 (linear transformations) 과 행렬 (matrices)
  - 공간도형 (three-dimensional geometries) 과 공간좌표 (three-dimensional coordinates)
  - 벡터 (vectors)

Review on Vectors

Let’s refresh your memory...

- Definition
- Vector *representation* using coordinates
- Algebra of vectors
- Dot product (점곱) or scalar product (스칼라곱)
  NOTE: Dot(scalar) product *is an* inner product (내적), but *not vice versa*.
- Length of a vector

Python

- This lecture is based on Python 3.x.
- Try Python Anywhere for least hassles.
- You can try installing stand-alone packages yourselves. (No instruction will be given though...)
- You can install a Python 3.x interpreter on your smartphones/tablets – for example, QPython3 for Android, Python 3.4 for iOS, etc.

Table of Contents of the Textbook

Applications

- Security (Cryptography, Authentication)
- Graph Theory
- Image Processing (Filtering, Compression)
- Coding Theory
- Computer Graphics
- Machine Learning

Fundamental Questions

- How can we tell whether a solution to a **linear system** is unique?
- How can we find the number of solutions to a linear system over $GF(2)$?
- How can we tell if a set $\mathcal{V}$ of vectors is equal to the **span** of vectors $\mathbf{v}_1, \ldots, \mathbf{v}_n$?
- For a system of linear equations, what other linear equations are implied?
- How can we tell if a matrix is **invertible**?
- Can every **vector space** be represented as the solution set of a **homogeneous** linear system?

Fundamental Computational Problems

- Find the solution to a matrix equation $M\mathbf{x} = \mathbf{b}$.
- Find the vector $\mathbf{x}$ minimizing the **distance** between $M\mathbf{x}$ and $\mathbf{b}$.
- Given vector $\mathbf{b}$, find the **closest vector** to $\mathbf{b}$ whose representation in a given basis is k**-sparse**.
- Find the solution to a matrix inequality $M\mathbf{x} \leqslant \mathbf{b}$.
- Given a matrix M, find the **closest matrix** to M whose **rank** is at most k.