

Homework 1 Answers

1.

2. Beta-reduce lambda expression:

$$\begin{aligned}
 & (((\lambda x. \lambda y. \lambda z. ((x \ y) \ z) \ \lambda f. \lambda a. (f \ a)) \ \lambda i. i) \ \lambda j. j) \\
 & \rightarrow ((\lambda y. \lambda z. ((\lambda f. \lambda a. (f \ a) \ y) \ z) \ \lambda i. i) \ \lambda j. j) \\
 & \rightarrow (\lambda z. ((\lambda f. \lambda a. (f \ a) \ \lambda i. i) \ z) \ \lambda j. j) \\
 & \rightarrow ((\lambda f. \lambda a. (f \ a) \ \lambda i. i) \ \lambda j. j) \\
 & \rightarrow (\lambda a. (\lambda i. i \ a) \ \lambda j. j) \\
 & \rightarrow (\lambda i. i \ \lambda j. j) \\
 & \rightarrow \lambda j. j
 \end{aligned}$$

3. Beta-reduce lambda expression:

$$\begin{aligned}
 & (\lambda h. ((\lambda a. \lambda f. (f \ a) \ h) \ h) \ \lambda f. (f \ f)) \\
 & \rightarrow (\lambda h. (\lambda f. (f \ h) \ h) \ \lambda f. (f \ f)) \\
 & \rightarrow (\lambda h. (h \ h) \ \lambda f. (f \ f)) \\
 & \rightarrow (\lambda f. (f \ f) \ \lambda f. (f \ f)) \\
 & \rightarrow (\lambda f. (f \ f) \ \lambda f. (f \ f)) \dots \text{and repeat to infinity and beyond.}
 \end{aligned}$$

4. Use α conversion to ensure unique names in the following expression:

$$\begin{aligned}
 & \lambda x. \lambda y. (\lambda x. y \ \lambda y. x) \\
 & \rightarrow \lambda x. \lambda y. (\lambda m. y \ \lambda n. x)
 \end{aligned}$$

5. Define a calculus representation for implies. You should be able to reduce your answer down so that it's in terms of x, and y and maybe true, and/or false. Notice, when X is true, Implies is the same as Y and when X is False, Implies is True. Assuming:

- $def\ cond = e1.e2.c.((c \ e1) \ e2)$
- $def\ not = x.(((cond \ false) \ true) \ x)$
- $def\ or = x.y.((x \ true) \ y)$

We have:

$$\begin{aligned}
 & (or \ (not \ x) \ y) \\
 & \rightarrow ((\lambda x. \lambda y. ((x \ true) \ y)) (\lambda x. (((cond \ false) \ true) \ x) \ x)) \ y)
 \end{aligned}$$

$$\begin{aligned}
&\rightarrow (\lambda y.(((\lambda x.(((cond\ false)\ true)\ x)\ x)\ true)\ y)\ y) \\
&\rightarrow (((\lambda x.(((cond\ false)\ true)\ x)\ x)\ true)\ y) \\
&\rightarrow (((((cond\ false)\ true)\ x)\ true)\ y) \\
&\rightarrow ((((\lambda e2.\lambda c.((c\ false)\ e2)\ true)\ x)\ true)\ y) \\
&\rightarrow (((\lambda c.((c\ false)\ true)\ x)\ true)\ y) \\
&\rightarrow (((x\ false)\ true)\ true)\ y)
\end{aligned}$$