Student: Minh Pham
UTEID: mlp2279
CSID: minhpham

Homework 1 Answers

1.
2. Beta-reduce lambda expression:

$$(((\lambda x.\lambda y.\lambda z.((x\ y)\ z)\ \lambda f.\lambda a.(f\ a))\ \lambda i.i)\ \lambda j.j)$$
$$\rightarrow ((\lambda y.\lambda z.((\lambda f.\lambda a.(f\ a)\ y)\ z)\ \lambda i.i)\ \lambda j.j)$$
$$\rightarrow (\lambda z.((\lambda f.\lambda a.(f\ a)\ \lambda i.i)\ z)\ \lambda j.j)$$
$$\rightarrow ((\lambda f.\lambda a.(f\ a)\ \lambda i.i)\ \lambda j.j)$$
$$\rightarrow (\lambda a.(\lambda i.i\ a)\ \lambda j.j)$$
$$\rightarrow (\lambda i.i\ \lambda j.j)$$
$$\rightarrow \lambda j.j$$

3. Beta-reduce lambda expression:

$$(\lambda h.((\lambda a.\lambda f.(f\ a)\ h)\ h)\ \lambda f.(f\ f))$$
$$\rightarrow (\lambda h.(\lambda f.(f\ h)\ h)\ \lambda f.(f\ f))$$
$$\rightarrow (\lambda h.(h\ h)\ \lambda f.(f\ f))$$
$$\rightarrow (\lambda f.(f\ f)\ \lambda f.(f\ f))$$
$$\rightarrow (\lambda f.(f\ f)\ \lambda f.(f\ f))$$ ... and repeat to infinity and beyond.

4. Use α conversion to ensure unique names in the following expression:

$$\lambda x.\lambda y.(\lambda x.y\ \lambda y.x)$$
$$\rightarrow \lambda x.\lambda y.(\lambda m.y\ \lambda n.x)$$

5. Define a calculus representation for implies. You should be able to reduce your answer down so that it's in terms of x, and y and maybe true, and/or false. Notice, when X is true, Implies is the same as Y and when X is False, Implies is True. Assuming:
   - $def cond = e1.e2.c.((c\ e1)\ e2)$
   - $def not = x.(((cond\ false)\ true)\ x)$
   - $def or = x.y.((x\ true)\ y)$

We have:

$$
\begin{aligned}
& (or\ (not\ x)\ y) \\
\rightarrow\ & ((\lambda x.\lambda y.((x\ true)\ y)(\lambda x.(((cond\ false)\ true)\ x)\ x))\ y) \\
\rightarrow\ & (\lambda y.(((\lambda x.(((cond\ false)\ true)\ x)\ x)\ true)\ y)\ y) \\
\rightarrow\ & (((\lambda x.(((cond\ false)\ true)\ x)\ x)\ true)\ y) \\
\rightarrow\ & (((((cond\ false)\ true)\ x)\ true)\ y) \\
\rightarrow\ & ((((\lambda e2.\lambda c.((c\ false)\ e2)\ true)\ x)\ true)\ y) \\
\rightarrow\ & (((\lambda c.((c\ false)\ true)\ x)\ true)\ y) \\
\rightarrow\ & ((((x\ false)\ true)\ true)\ y)
\end{aligned}
$$

Interpretation: If x is false then the expression is $true$, otherwise the expression is equal to y.