

Student: Minh Pham

EID: mlp2279

Homework 1

1. (5 points) Vector $v = [x \ y \ z \ 0]^T$ is rotated about the x-axis until it lies in the xz plane. The new vector is v' . What is $|v'|$ (the length of v')?

Answer:

Rotation does not affect a vector's magnitude (or length, the term used in the question). Therefore, the magnitude of the new vector is equal to that of the original, i.e. $|v'| = |v|$.

We have: $|v'| = |v| = \sqrt{x^2 + y^2 + z^2}$.

2. (5 points) Using v' from problem 1, v'' is the projection of v' onto the z axis. What is $|v''|$?

Answer:

Let $u = \frac{v}{|v|}$ be the unit vector parallel to v . Then $u = [\cos \theta_x \ \cos \theta_y \ \cos \theta_z \ 0]^T$, with $\cos \theta_x$, $\cos \theta_y$, $\cos \theta_z$ being the angle v form with the positive x , y , and z axes, respectively. We also have

$$\cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z = 1 \quad (1)$$

since u is a unit vector.

Let u' the vector that results from applying the same rotation that we applied to v to u . Then u' lies on the xz plane, and form the angles $\cos \theta'_x$, $\cos \theta'_y$, $\cos \theta'_z$ with the x , y , and z axes, respectively. We have:

$\cos \theta'_x = \cos \theta_x$ since the rotation is done around the x axis, and

$\cos \theta'_y = 0$ since the u' is in the xz plane perpendicular to the y axis.

We can rewrite u' as $u' = [\cos \theta_x \ 0 \ \cos \theta'_z \ 0]^T$. Since rotation does not affect the magnitude of a vector, u' is also a unit vector, so:

$$\cos^2 \theta_x + \cos^2 \theta'_z = 1 \quad (2)$$

From equations (1) and (2), we have:

$$\cos^2 \theta'_z = \cos^2 \theta_y + \cos^2 \theta_z \leftrightarrow \cos \theta'_z = \sqrt{\cos^2 \theta_y + \cos^2 \theta_z}$$

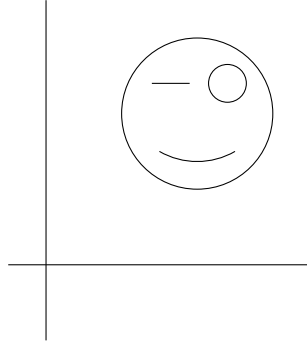
Let u'' be the projection of u' onto the z axis. We have $|u''| = |u'| \cos \theta'_z = |u'| \sqrt{\cos^2 \theta_y + \cos^2 \theta_z}$.

Applying the same transformation to v instead of u , we have:

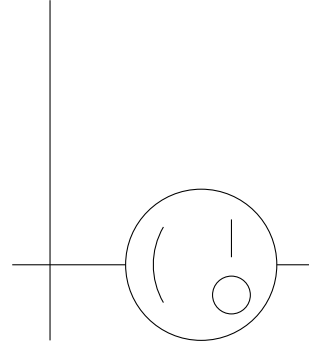
$$|v''| = |v'| \sqrt{\cos^2 \theta_y + \cos^2 \theta_z} = (x^2 + y^2 + z^2) \sqrt{\cos^2 \theta_y + \cos^2 \theta_z}$$

3. (10 points) Happy Harry is happy even when he's sleeping. Give a series of 3x3 2D transformation matrices (using homogeneous coordinates) in the proper order to transform Happy Harry from his

awake position centered at (x, y) (figure a) to his sleeping position centered at $(x, 0)$ (figure b). Leave any trigonometric functions unevaluated (leave rotation matrices in terms of sine and cosine).



(a)



(b)

Answer:

To get figure (b) from figure (a), we do as follow, in order:

- (a) Translate the figure from its original position (x, y) to the origin with matrix T_1
- (b) Rotate the figure 90 degrees (or $\pi/2$ radian) clock-wise with matrix R
- (c) Translate the figure to its final position at $(x, 0)$ with matrix T_2

We have the values for the matrices as follow:

$$T_2 = \begin{bmatrix} 1 & 0 & x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad R = \begin{bmatrix} \cos 90 & \sin 90 & 0 \\ -\sin 90 & \cos 90 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad T_1 = \begin{bmatrix} 1 & 0 & -x \\ 0 & 1 & -y \\ 0 & 0 & 1 \end{bmatrix}$$

The concatenated matrix that do the above transformations in the order specified is:

$$\begin{aligned} M &= T_2 R T_1 \\ M &= \begin{bmatrix} 1 & 0 & x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 90 & \sin 90 & 0 \\ -\sin 90 & \cos 90 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x \\ 0 & 1 & -y \\ 0 & 0 & 1 \end{bmatrix} \\ M &= \begin{bmatrix} \cos 90 & \sin 90 & x \\ -\sin 90 & \cos 90 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x \\ 0 & 1 & -y \\ 0 & 0 & 1 \end{bmatrix} \\ M &= \begin{bmatrix} \cos 90 & \sin 90 & -x \cos 90 - y \sin 90 + x \\ -\sin 90 & \cos 90 & x \sin 90 - y \cos 90 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

4. (5 points) A wireframe cube (see wikipedia: "wire-frame model") is placed at the origin. The camera is placed using `gluLookAt(0, 0, 5, 0, 0, 0, 0, 1, 0)`. Using perspective projection, sketch what will be rendered on the screen.

Answer:

5. (10 points) The camera is placed using `gluLookAt(0, 10, 5, 0, 5, 0, 0, 1, 0)`. What are the coordinate axes u, v, n ? Show your work.

Answer:

Since the syntax of `gluLookAt()` is `gluLookAt(eyex,)`

6. Consider the following code:

```
glutInitWindowSize(500, 500);  
glMatrixMode(GL_PROJECTION);  
glLoadIdentity();  
glFrustum(-1, 1, -1/3.0, 1/3.0, 1, 3);  
glMatrixMode(GL_MODELVIEW);  
glLoadIdentity();  
gluLookAt(0, 0, 2, 0, 0, 0, 0, 1, 0);  
glColor3f(0, 0, 0);  
glutWireCube(2);
```

- (a) (5 points) Sketch what will be rendered.
- (b) (10 points) What percentage of the cube's volume lies inside the view frustum?

Answer: