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Homework 1

1. (10 points) Given:

```
glutInitWindowSize(800, 600);
gluOrtho2D(-100.0, 100.0, -100.0, 100.0);
```

Convert the following object coordinates to window coordinates.

Solution:

	Object Coordinates	Window Coordinates
(a)	(0, 0)	(400, 300)
(b)	(-50, 50)	(200, 150)
(c)	(-75, -100)	(100, 600)
(d)	(90, 10)	(760, 270)
(e)	(0, -40)	(400, 420)

2. (5 points) Let:

- α, β, γ be scalars
- A, B, C be points
- u, v, w be vectors

Are the following operations defined?
Answer T/F/? if operation is defined/undefined/don't know.

	Operation	Defined?
(a)	$v - u$	T
(b)	$v - A$	F
(c)	$A - v$	T
(d)	$A + \alpha(B - A)$	T
(e)	$\alpha A + v$	F

3. (5 points) Find a homogeneous-coordinate representation of a plane.

The following solution assumes we are working in 3 dimensions.
Suppose we had a point $P(x_0, y_0, z_0, 1)$ and two non-parallel vectors a and

b. These completely define a plane in 3 dimensions. Let $n(x_n, y_n, z_n, 0)$ be the cross product of a and b. Then n is a vector perpendicular to the plane defined by P, a and b.
Let $Q(x, y, z, 1)$ be an arbitrary point in 3 dimensions.
Let $v(x-x_0, y-y_0, z-z_0, 0)$ be the vector from P to Q. Q lies on the plane defined by P, a, and b if and only if v is a linear combination of a and b. This implies that v is perpendicular to n, since n is the cross-product of the two.
Because v and n are perpendicular, their dot product must be 0.
A homogeneous-coordinate representation of a plane is therefore:

$$n \cdot v = 0$$

$$\Leftrightarrow x_n * (x - x_0) + y_n * (y - y_0) + z_n * (z - z_0) = 0$$

4. (15 points) If we are interested in only two-dimensional graphics, we can use three-dimensional homogeneous coordinates by representing a point as $p = [x\ y\ 1]^T$ and a vector as $v = [a\ b\ 0]^T$.
   Find the 3x3 rotation, translation, scaling, and shear matrices R, T, S, and H, respectively. How many degrees of freedom are there in an affine transformation for transforming two-dimensional points?
   In two dimensions we have:

   - Rotation matrix for rotating an angle θ counter-clockwise:
     $$R = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
   - Translation matrix for translating by a vector $[\alpha_x\ \alpha_y\ 0]^T$:
     $$T = \begin{bmatrix} 1 & 0 & \alpha_x \\ 0 & 1 & \alpha_y \\ 0 & 0 & 1 \end{bmatrix}$$
   - Scaling matrix for scaling by β_x in the x direction, and β_y in the y direction:
     $$S = \begin{bmatrix} \beta_x & 0 & 0 \\ 0 & \beta_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
   - Shear matrix for shearing along the x axis by the angle θ:
     $$H_x = \begin{bmatrix} 1 & \cot\theta & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
   - Shear matrix for shearing along the y axis by the angle θ:

$$H_y = \begin{bmatrix} 1 & 0 & 0 \\ \cot\theta & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

5. (15 points) Derive a rotation matrix where we rotate first about the x-axis $R_x(\theta_x)$, then about the y-axis $R_y(\theta_y)$, and then about the z-axis $R_z(\theta_z)$. Assume that the fixed point is the origin.

   Assuming we are working in 3 dimension, in a right-handed coordinate system.

   We have the following rotation matrices around each of the axes:

$$R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_y = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_z = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

   The transformation that rotates a vector v first about the x-axis by θ_x, then about the y-axis by θ_y, and then about the z-axis by θ_z is $R_z(\theta_z)R_y(\theta_y)R_x(\theta_x)v$. We get the combined transformation matrix M by multiplying the three individual rotation matrices together. We have:

$$M = R_z(\theta_z)R_y(\theta_y)R_x(\theta_x)$$

$$M = \begin{bmatrix} \cos\theta_z & -\sin\theta_z & 0 & 0 \\ \sin\theta_z & \cos\theta_z & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_y & 0 & \sin\theta_y & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta_y & 0 & \cos\theta_y & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta_x & -\sin\theta_x & 0 \\ 0 & \sin\theta_x & \cos\theta_x & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} \cos\theta_z\cos\theta_y & -\sin\theta_z & \cos\theta_z\sin\theta_y & 0 \\ \sin\theta_z\cos\theta_y & \cos\theta_z & \sin\theta_z\sin\theta_y & 0 \\ -\sin\theta_y & 0 & \cos\theta_y & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta_x & -\sin\theta_x & 0 \\ 0 & \sin\theta_x & \cos\theta_x & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} \cos\theta_z \cos\theta_y & -\sin\theta_z \cos\theta_x + \cos\theta_z \sin\theta_y \sin\theta_x & \sin\theta_z \sin\theta_x + \cos\theta_z \sin\theta_y \cos\theta_x & 0 \\ \sin\theta_z \cos\theta_y & \cos\theta_z \cos\theta_x + \sin\theta_z \sin\theta_y \sin\theta_x & -\cos\theta_z \sin\theta_x + \sin\theta_z \sin\theta_y \cos\theta_x & 0 \\ -\sin\theta_y & \cos\theta_y \sin\theta_x & \cos\theta_y \cos\theta_x & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$