Student: Minh Pham

EID: mlp2279

Homework 1

1. (5 points) Vector $v = [x\ y\ z\ 0]^T$ is rotated about the x-axis until it lies in the xz plane. The new vector is v'. What is $|v'|$ (the length of v')?

   Answer:

   Rotation does not affect a vector's magnitude (or length, the term used in the question). Therefore, the magnitude of the new vector is equal to that of the original, i.e. $|v'| = |v|$.
   We have: $|v'| = |v| = x^2 + y^2 + z^2$.

2. (5 points) Using v' from problem 1, v'' is the projection of v' onto the z axis. What is $|v''|$?

   Answer:

   Let $u = \frac{v}{|v|}$ be the unit vector parallel to v. Then $u = [\cos\theta_x\ \ \cos\theta_y\ \ \cos\theta_z\ \ 0]^T$, with $\cos\theta_x$, $\cos\theta_y$, $\cos\theta_z$ being the angle v form with the positive x, y, and z axes, respectively. We also have

   $$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1 \quad (1)$$

   since u is a unit vector.

   Let u' the vector that results from applying the same rotation that we applied to v to u. Then u' lies on the xz plane, and form the angles $\cos\theta'_x$, $\cos\theta'_y$, $\cos\theta'_z$ with the x, y, and z axes, respectively. We have:

   $\cos\theta'_x = \cos\theta_x$ since the rotation is done around the x axis, and

   $\cos\theta'_y = 0$ since the u' is in the xz plane perpendicular to the y axis.

   We can rewrite u' as $u' = [\cos\theta_x\ 0\ \cos\theta'_z\ 0]^T$. Since rotation does not affect the magnitude of a vector, u' is also a unit vector, so:

   $$\cos^2\theta_x + \cos^2\theta_z^{'} = 1 \quad (2)$$

   From equations (1) and (2), we have:

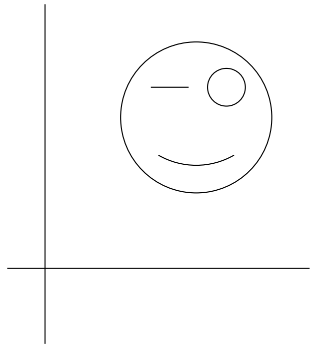
   $$\cos^2\theta_z^{'} = \cos^2\theta_y + \cos^2\theta_z \leftrightarrow \cos\theta_z^{'} = \sqrt{\cos^2\theta_y + \cos^2\theta_z}$$

   Let u'' be the projection of u' onto the z axis. We have $|u''| = |u'|\cos\theta_z^{'} = |u'|\sqrt{\cos^2\theta_y + \cos^2\theta_z}$.
   Applying the same transformation to v instead of u, we have:

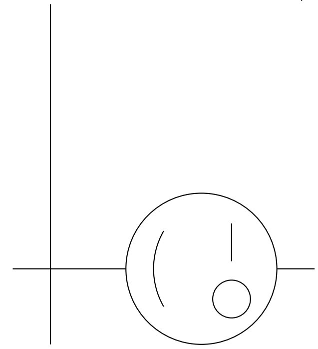
   $$|v''| = |v'|\sqrt{\cos^2\theta_y + \cos^2\theta_z} = (x^2 + y^2 + z^2)\sqrt{\cos^2\theta_y + \cos^2\theta_z}$$

3. (10 points) Happy Harry is happy even when he's sleeping. Give a series of 3x3 2D transformation matrices (using homogeneous coordinates) in the proper order to transform Happy Harry from his

awake position centered at $(x,\ y)$ (figure a) to his sleeping position centered at $(x,\ 0)$ (figure b). Leave any trigonometric functions unevaluated (leave rotation matrices in terms of sine and cosine).



(a)



(b)

Answer:

To get figure (b) from figure (a), we do as follow, in order:

(a) Translate the figure from its original position $(x\ y)$ to the origin with matrix T_1

(b) Rotate the figure 90 degrees (or $pi/2$ radian) clock-wise with matrix R

(c) Translate the figure to its final position at $(x, 0)$ with matrix T_2

We have the values for the matrices as follow:

$$T_2 = \begin{bmatrix} 1 & 0 & x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad R = \begin{bmatrix} \cos 90 & \sin 90 & 0 \\ -\sin 90 & \cos 90 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad T_1 = \begin{bmatrix} 1 & 0 & -x \\ 0 & 1 & -y \\ 0 & 0 & 1 \end{bmatrix}$$

The concatenated matrix that do the above transformations in the order specified is:

$$M = T_2 R T_1$$

$$M = \begin{bmatrix} 1 & 0 & x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 90 & \sin 90 & 0 \\ -\sin 90 & \cos 90 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x \\ 0 & 1 & -y \\ 0 & 0 & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} \cos 90 & \sin 90 & x \\ -\sin 90 & \cos 90 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x \\ 0 & 1 & -y \\ 0 & 0 & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} \cos 90 & \sin 90 & -x\cos 90 - y\sin 90 + x \\ -\sin 90 & \cos 90 & x\sin 90 - y\cos 90 \\ 0 & 0 & 1 \end{bmatrix}$$

4. (5 points) A wireframe cube (see wikipedia: "wire-frame model") is placed at the origin. The camera is placed using `gluLookAt(0, 0, 5, 0, 0, 0, 0, 1, 0)`. Using pespective projection, sketch what will be rendered on the screen.

Answer:

5. (10 points) The camera is placed using `gluLookAt(0, 10, 5, 0, 5, 0, 0, 1, 0)`. What are the coordinate axes u, v, n? Show your work.

Answer:

We have:

$eye = [0\ 10\ 5]^T$

$at = [0\ 5\ 0]^T$

$up = [0\ 1\ 0]^T$

The coordinate axes are:

$n = at - eye = [0\ \ -5\ \ -5]^T$

$u = up \times n = [0\ 1\ 0]^T \times [0\ \ -5\ \ -5]^T = [-5\ 0\ 0]^T$

$v = n \times u = [0\ \ -5\ \ -5]^T \times [-5\ 0\ 0]^T = [0\ 25\ \ -25]^T$

6. Consider the following code:

```
glutInitWindowSize(500, 500);
glMatrixMode(GL_PROJECTION);
glLoadIdentity();
glFrustum(-1, 1, -1/3.0, 1/3.0, 1, 3);
glMatrixMode(GL_MODELVIEW);
glLoadIdentity();
gluLookAt(0, 0, 2, 0, 0, 0, 0, 1, 0);
glColor3f(0, 0, 0);
glutWireCube(2);
```

(a) (5 points) Sketch what will be rendered.

(b) (10 points) What percentage of the cube's volume lies inside the view frustum?

Answer: