

TIPS

Tip 01 Linear Function

The functions are called “linear” because they are precisely the functions whose graph in the xy -plane is a straight line.

Such a function can be written as

- 1) Slope-intercept form

$f(x) = mx + b$, where m is the slope and b is the y -intercept.

- 2) Point-slope form

$y - y_1 = m(x - x_1)$, where (x_1, y_1) is the known point on the line.

- 3) General form

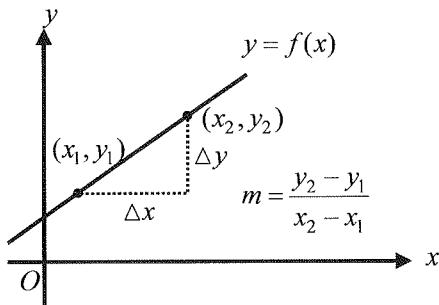
$$ax + by + c = 0$$

- 4) Standard form

$$Ax + By = C$$

The slope between any two points on the line is constant.

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$



SAT Practice

1. For a linear function f , $f(0) = 2$ and $f(3) = 5$. If $k = f(5)$, what is the value of k ?

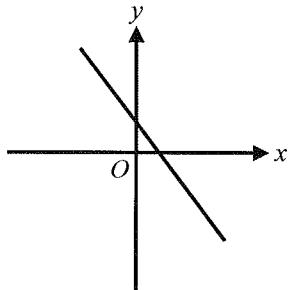
- A) 5
- B) 6
- C) 7
- D) 8

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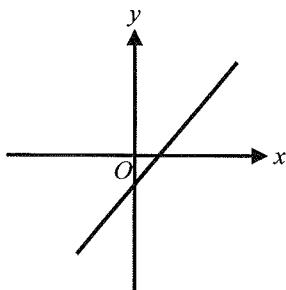
x	$f(x)$
0	a
1	12
2	b

2. The table above shows some values for the function f . If f is a linear function, what is the value of $a+b$?
- A) 24
B) 36
C) 48
D) 60
-
3. A linear function is given by $ax+by+c=0$ and $a > 0$, $b < 0$, and $c > 0$. Which of the following graphs best represents the graph of the function?

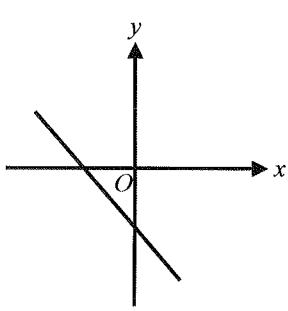
A)



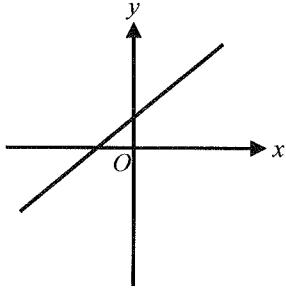
B)



C)



D)



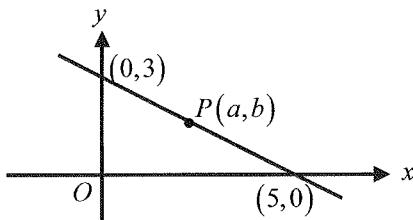
TIPS

4. If f is a linear function and $f(3) = 2$ and $f(5) = 6$, what is the y -intercept of the graph of f ?

- A) 4
- B) 2
- C) -2
- D) -4

-
5. If f is a linear function and $f(3) = -2$ and $f(4) = -4$, what is the x -intercept of the graph of f ?

- A) 3
 - B) 2.5
 - C) 2
 - D) 0
-



6. The graph of a function f is shown in the xy -plane above. If $b = 2a$, what is the value of a ?

- A) $\frac{5}{2}$
- B) $\frac{5}{4}$
- C) $\frac{15}{13}$
- D) $\frac{16}{15}$

TIPS

x	$f(x)$
-1	6
0	4
1	2
2	0

7. The table above shows some values of the linear function f for selected values of x . Which of the following represents the function f ?

- A) $f(x) = 4 - x$
- B) $f(x) = 4 - 2x$
- C) $f(x) = 4 + 2x$
- D) $f(x) = 4 + x$

$$F = \frac{9}{5}C + 32$$

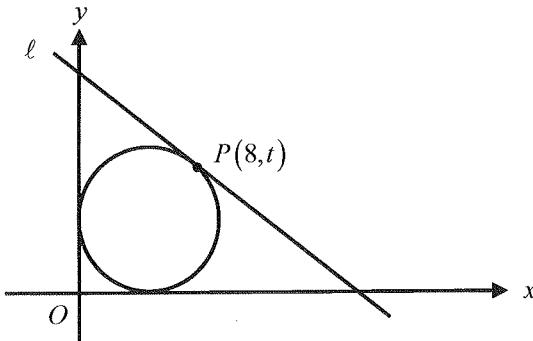
8. Fahrenheit (F) and Celsius (C) are related by the equation above. If Fahrenheit temperature increased by 27 degrees, what is the degree increase in Celsius?

- A) 15
- B) 20
- C) 32
- D) 81

9. In the formula $P = \frac{7}{12}K + 60$, if P is increased by 35, what is the increase in K ?

- A) 35
- B) 60
- C) 80
- D) 140

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10. In the xy -plane above, a circle is tangent to line ℓ , the x -axis, and the y -axis. If the radius of the circle is 5, what is the value of t ?
- A) 7
B) 8
C) 9
D) 10

Tip 02 Slope of a Line

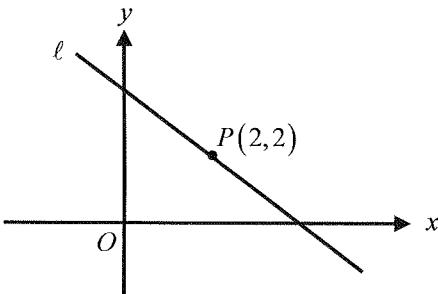
One of the most important properties of a straight line is its angle from the horizontal. This concept is called “slope”. To find the slope, we need two points from the line.

- 1) From two points (x_1, y_1) and (x_2, y_2) \rightarrow Slope $m = \frac{y_2 - y_1}{x_2 - x_1}$
- 2) From slope-intercept form of a line $y = mx + b \rightarrow m = \text{slope}$ and $b = y\text{-intercept}$
- 3) The slope between any two points on the line is constant.

SAT Practice

1. If f is a linear function and $f(3) = 6$ and $f(5) = 12$, what is the slope of the graph of f ?
- A) 2
B) 3
C) 4
D) 5

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2. In the xy -plane above, line ℓ passes through point P and has a slope of $-\frac{1}{2}$. What is the x -intercept of line ℓ ?
- A) $(4, 0)$
 B) $(5, 0)$
 C) $(6, 0)$
 D) $(7, 0)$
-

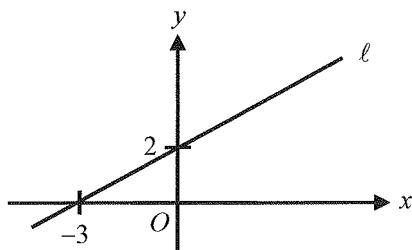
x	$f(x)$
2	5
4	a
8	23
a	b

3. The table above shows values of the linear function f for selected values of x . What is the value of b ?
- A) 11
 B) 22
 C) 32
 D) 42
-

x	$f(x)$
2	a
5	6
8	b

4. The table above gives values of the linear function f for selected values of x . What is the value of $a + b$?
- A) 8
 B) 10
 C) 12
 D) 18

TIPS



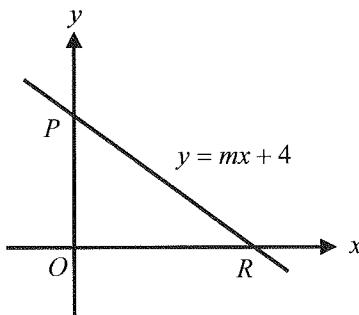
5. In the xy -plane above, point $P(42, m)$ lies on line ℓ . What is the value of m ?

- A) 24
- B) 30
- C) 36
- D) 42

Tip 03 Area enclosed by Lines

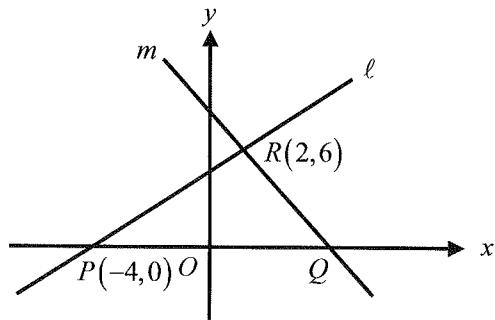
In order to find the area enclosed by lines, mostly we need to find x -intercept, y -intercept, and points of intersection of lines

SAT Practice

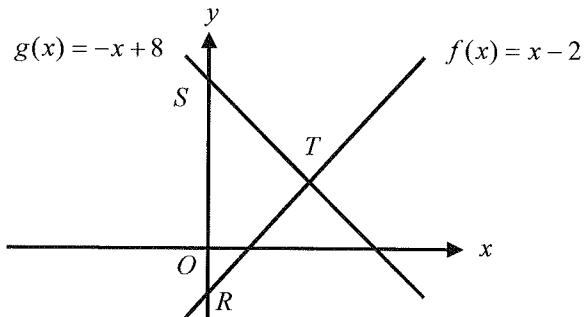


1. The graph of $y = mx + 4$ is shown in the xy -plane above. If the area of triangle POR is 6, what is the value of m ?
- A) -2
 - B) $-\frac{4}{3}$
 - C) $-\frac{3}{4}$
 - D) $-\frac{1}{4}$

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2. In the xy -plane above, line m and line ℓ are perpendicular and intersect at point $R(2, 6)$. What is the area of triangle PQR ?
- A) 18
B) 24
C) 32
D) 36
-



3. The graphs of the functions f and g are shown in the xy -plane above. What is the area of $\triangle RST$?
- A) 25
B) 50
C) 75
D) 100

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Tip 04 Midpoint and Distance between Two Points

The midpoint of a line segment: Each coordinate of the midpoint of a line segment is equal to the average of the corresponding coordinates of the endpoints of the line segment. Given the two end points (x_1, y_1) and (x_2, y_2) , the coordinates of the midpoint of the line segment are

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).$$

The distance between two points: The distance d between two points (x_1, y_1) and (x_2, y_2) is given by the formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

SAT Practice

1. In the xy -plane, the midpoint of \overline{AB} is $(10, 4)$. If the coordinates of point A are $(5, 1)$, what are the coordinates of point B ?
A) $(5, 3)$
B) $(6, 4)$
C) $(15, 7)$
D) $(20, 10)$

2. If point $M(5, -3)$ is the midpoint of the line segment connecting point $A(2a, b)$ and point $B(b, a)$, what is the value of a ?
A) 8
B) 12
C) 16
D) 20

3. In triangle ABC in the xy -plane, the coordinates of point A are $(-4, 4)$ and the coordinates of point B are $(4, 4)$. If the area of $\triangle ABC$ is 24, which of the following could be the coordinates of point C ?
A) $(3, 8)$
B) $(2, 10)$
C) $(2, -5)$
D) $(-6, -4)$

TIPS

4. If the distance between $(a, 3)$ and $(b, 8)$ is 13, what is the value of $|a - b|$?

A) 4
B) 8
C) 12
D) 16

Tip 05 Line Reflection

Reflecting across the x -axis : When we reflect a point (x, y) across the x -axis, the x -coordinate remains the same, but the y -coordinate is transformed into its opposite as follows.

Reflecting across the x -axis: $P(x, y) \rightarrow P'(x, -y)$

Reflecting across the y -axis: $P(x, y) \rightarrow P'(-x, y)$

Reflecting across the $y = x$: $P(x, y) \rightarrow P'(y, x)$

Reflecting across the $y = -x$: $P(x, y) \rightarrow P'(-y, -x)$

Reflecting across the origin: $P(x, y) \rightarrow P'(-x, -y)$

SAT Practice

1. In the xy -plane, line ℓ is the reflection of line m across the x -axis. If the equation of line m is $y = \frac{1}{5}x - 6$, what is the slope of line ℓ ?
- A) -5 B) $-\frac{1}{5}$ C) $\frac{1}{5}$ D) 5
-
2. In the xy -plane, line ℓ is the reflection of line m across the y -axis. If these two lines intersect at point (a, b) , which of the following must be true?
- A) $a = -2$ B) $a = 0$ C) $a = 2$ D) $a > 0$
-
3. If the graph of $2x - 3y = 6$ is reflected across the x -axis, which of the following represents the equation of the reflected graph?
- A) $2x + 3y = -6$ B) $2x + 3y = 6$ C) $2x - 3y = -6$ D) $-2x - 3y = 6$

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Tip 06 Parallel and Perpendicular Lines

1. Two non-vertical lines are parallel if and only if their slopes are equal.
2. Two non-vertical lines are perpendicular if and only if the product of their slopes is -1 . (Negative reciprocal each other)

SAT Practice

1. Which of the following is an equation for the line passing through the point $(-4, 1)$ that is parallel to $4x - 2y = 3$?
A) $y = 2x - 9$
B) $y = 2x + 9$
C) $y = -2x - 9$
D) $y = -2x + 9$
2. Which of the following is an equation for the line passing through the point $(-4, 1)$ that is perpendicular to $4x - 2y = 3$?
A) $y = -\frac{1}{2}x - 1$
B) $y = -\frac{1}{2}x + 1$
C) $y = \frac{1}{2}x - 1$
D) $y = \frac{1}{2}x + 1$

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Tip 07 System of Linear Equations

A system of linear equations means two or more linear equations. If two linear equations intersect, that point of intersection is called the solution to the system of equations.

1) The system has exactly one solution.

When two lines have different slopes, the system has only one and only one solution.

2) The system has no solution.

When two lines are parallel and have different y -intercept, the system has no solution.

3) The system has infinitely many solutions.

When two lines are parallel and the lines have the same y -intercept.

From the standard form for the system of equations

$$a_1x + b_1y = c_1 \quad \text{and} \quad a_2x + b_2y = c_2$$

1) If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ One solution

2) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ No solution

3) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ Infinitely many solutions

From the slope-intercept form for the system of equations

$$y = m_1x + b_1 \quad \text{and} \quad y = m_2x + b_2$$

1) If $m_1 \neq m_2$ one solution

2) If $m_1 = m_2$ and $b_1 \neq b_2$ no solution

3) If $m_1 = m_2$ and $b_1 = b_2$ infinitely many solution

SAT Practice

$$2x - 5y = 8$$

$$4x + ky = 17$$

1. For which of the following values of k , will the system of equations above has no solution?

- A) 10 B) 5 C) -5 D) -10

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$$5x - 2y = 3$$

$$ax + by = 6$$

2. In the system of equations above, a and b are constants. If the system has infinitely many solutions, what is the value of $a + b$?
- A) 6 B) 4 C) 0 D) -4
-

$$3x + by = 3$$

$$ax - 4y = 6$$

3. In the system of equations above, a and b are constants. For which of the following values of $\{a, b\}$ will the system have no solution?
- A) $\{-1, 2\}$ B) $\{1, 1\}$ C) $\{2, 1\}$ D) $\{3, -4\}$
-

$$ax + 3y = 6$$

$$(a-1)x + (a-1)y = 2$$

4. In the system of equations above, a is a constant. If the system has no solution, what is the value of a ?
- A) -3 B) 1 C) 3 D) 5
-

5. The cost of long distance telephone call is determined by a basic fixed charge for the first 5 minutes and a fixed charge for each additional minute. If a 15-minute call costs \$3.50 and a 20-minute call costs \$4.75, what is the total cost, in dollars, of a 40-minute call?
- A) 8.25 B) 9.50 C) 9.75 D) 10.25
-

6. The tickets for a movie cost \$8.00 for adults and \$5.00 for children. If the total of 200 tickets were sold and the total amount of \$1360 was collected, how many adult tickets were sold?

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Tip 08 Quadratic Function

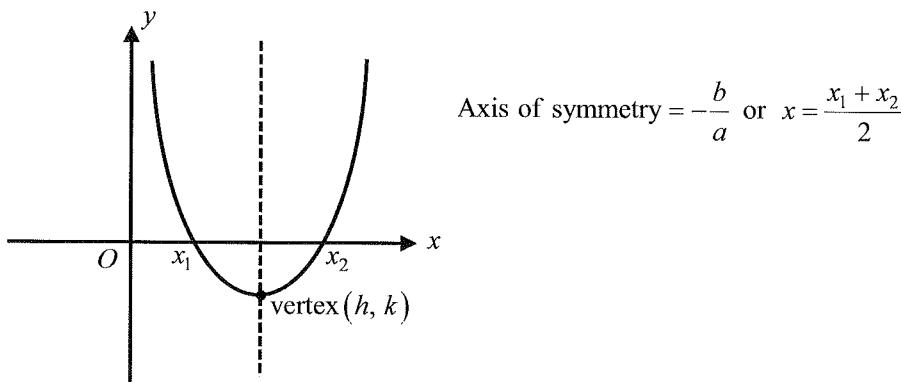
In algebra, a quadratic function is a polynomial function in which the highest degree term is of the second degree.

- 1) Standard form: $f(x) = ax^2 + bx + c \quad (a \neq 0)$
- 2) Vertex form: $f(x) = a(x - h)^2 + k \quad (a \neq 0)$ and vertex (h, k)
- 3) Factored from: $f(x) = a(x - x_1)(x - x_2)$, where x_1 and x_2 are the roots of the quadratic function.

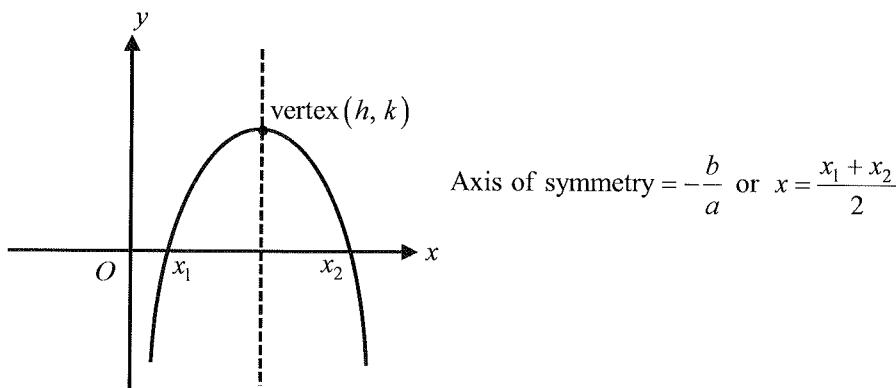
Axis of symmetry

- 1) Standard form: $x = -\frac{b}{2a} \rightarrow \text{Vertex}\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$
- 2) Vertex form: $x = h \rightarrow \text{Vertex}(h, k)$
- 3) Factored form: $x = \frac{x_1 + x_2}{2} \rightarrow \text{Vertex}\left(\frac{x_1 + x_2}{2}, f\left(\frac{x_1 + x_2}{2}\right)\right)$

- 1) $a > 0$ and $y = ax^2 + bx + c$



- 2) $a < 0$ and $y = ax^2 + bx + c$



TIPS

SAT Practice

Questions 1 and 2 refer to the following information.

$$h = 256t - 16t^2$$

A ball is thrown straight up from the ground with an initial velocity of 256 feet per second. The equation above describes the height the ball can reach in t seconds.

1. If the ball reaches its maximum height in k seconds, what is the value of k ?

- A) 8
- B) 12
- C) 16
- D) 24

2. What is the maximum height, in feet, that the ball will reach?

- A) 370
- B) 384
- C) 1024
- D) 1200

$$f(x) = \frac{1}{2}(x + 2)(x - 10)$$

3. If the function f above has a vertex at point (h, k) in the xy -plane, what is the value of k ?

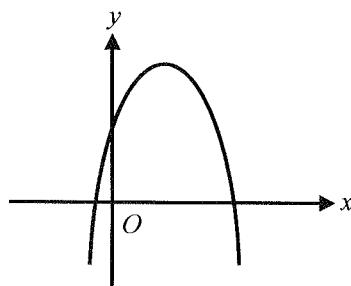
- A) -36
 - B) -18
 - C) 9
 - D) 18
-

$$y = 3x^2 - 6x + 10$$

4. Which of the following is equivalent to the equation above?

- A) $y = 3x^2 + 10$
- B) $y = 3(x - 1)^2 + 7$
- C) $y = 3(x - 1)^2 + 10$
- D) $y = 3(x + 2)^2 - 2$

TIPS



5. The graph of $y = ax^2 + bx + c$ is shown in the xy -plane above, where a , b , and c are constants. Which of the following must be true?
- I. $a < 0$ II. $b > 0$ III. $c > 0$
- A) I only B) I and II only C) I and III only D) I, II, and III

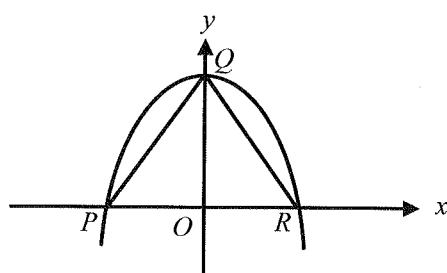
$$f(x) = a(x - h)^2 + k$$

6. The function f is defined by the equation above, where a , h , and k are constants. If a and k are negative, which of the following CANNOT be true?
- A) $f(5) < 0$ B) $f(-5) < 0$ C) $f(1) = k$ D) $f(0) = -k$

Tip 09 Area enclosed by Curves

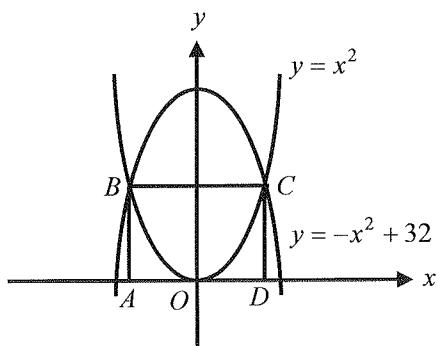
In order to find the area enclosed by curves, mostly we need to find x -intercept, y -intercept, and points of intersection of curves.

SAT Practice

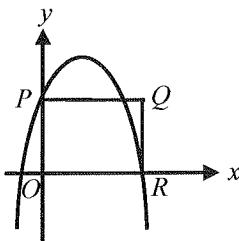


1. The graph of $y = -x^2 + k$ is shown in the xy -plane above, where k is a constant. If the area of $\triangle PQR$ is 64, what is the value of k ?

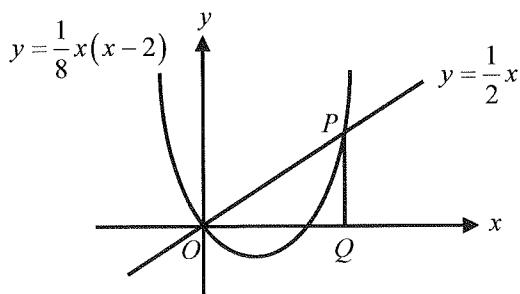
TIPS



2. The graphs of $y = x^2$ and $y = -x^2 + 32$ are shown in the xy -plane above. What is the area of rectangle $ABCD$?



3. The graph of $y = -x^2 + 4x + 5$ is shown in the xy -plane above. What is the area of rectangle $OPQR$?



4. The graphs of $y = \frac{1}{8}x(x-2)$ and $y = \frac{1}{2}x$ are shown in the xy -plane above. What is the area of right triangle OPQ ?

- A) 18
- B) 15
- C) 9
- D) 6

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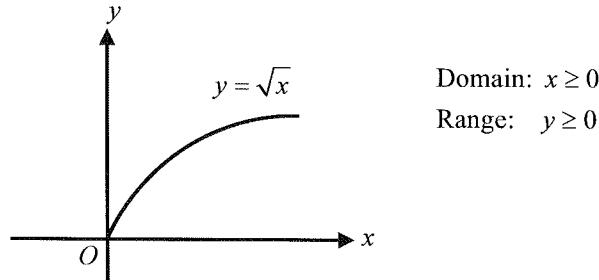
Tip 10 | Domain and Range

The **domain** of a given function is the complete set of “input” values for which the function is defined. In the xy -plane, the domain is represented on the x -axis (or abscissa).

- Note: 1) The denominator of a fraction CANNOT be zero.
2) The number inside a square root sign must be positive.

The **range** of a function is the set of all “output” values produced by that function. In the xy -plane the range is represented on the y -axis (or ordinate).

Example:



SAT Practice

$$f(x) = \frac{\sqrt{x}}{x - 3}$$

1. In the function f above, which of the following represents its domain?

- A) $x \geq 0$
- B) $x \neq 3$
- C) $x \geq 3$
- D) $x \geq 0$ and $x \neq 3$

$$g(x) = \sqrt{x - 2} - 5$$

2. Which of the following represents the range of the function f above?

- A) $y \geq 0$
- B) $y \geq 2$
- C) $y \geq -2$
- D) $y \geq -5$

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Tip 11 Composition of Functions

Composition of functions is applying one function to the result of the first function.(Combining functions)

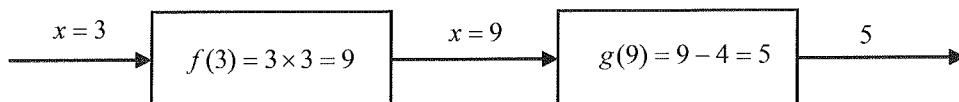


The result of $f(x)$ is sent through $g()$. The composition is written in the form

$$g(f(x)) \text{ or } (g \circ f)(x)$$

Example 1:

Given the functions $f(x) = 3x$ and $g(x) = x - 4$, then $g(f(3)) =$



$$(g \circ f)(x) = 3x - 4 \rightarrow (g \circ f)(3) = 3 \times 3 - 4 = 5$$

Example 2:

For the given functions $f(x) = 3x - 4$ and $g(x) = 2f(x) + 3$, if $g(k) = 0$, what is the value of k ?

Since $g(x) = 2(3x - 4) + 3 = 6x - 5$, $g(k) = 6k - 5 = 0 \rightarrow k = \frac{5}{6}$.

SAT Practice

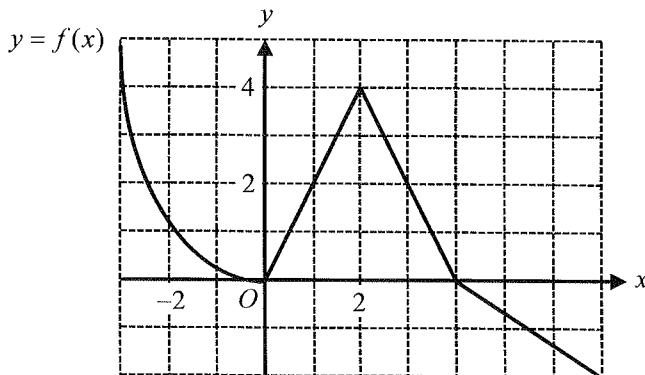
- The functions f and g are defined by $f(x) = 5x + 3$ and $g(x) = 3f(x) - k$. If $g(2) = 25$, what is the value of k ?
A) 18
B) 14
C) 12
D) 10

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$$f(x) = ax + b$$
$$g(x) = 2f(x) - 3$$

2. In the functions f and g above, if $g(1) = 3$ and $g(3) = 5$, what is the value of b ?

A) $\frac{1}{2}$ B) $\frac{3}{2}$ C) $\frac{5}{2}$ D) $\frac{7}{2}$



3. The graph of $y = f(x)$ is shown in the xy -plane above for $-3 \leq x \leq 7$. The function g is defined by $g(x) = 3f(x) - 1$. For how many values of k are there such that $g(k) = 8$?
- A) One B) Two C) Three D) Four

Tip 12 Identical Equation

The two expressions, left hand side and right hand side, are always equal for all values we give to the variable. The equations that are true for all values of the variable are called identical equations.

- 1) $5x - 10 = 0$ is an algebraic equation because the equation is true only for $x = 2$.
- 2) $5x - 10 = 5x - 10$ is an identical equation because the equation is true for all values of x .
 - The equation has infinitely many solutions.
 - The expressions of both sides must be equal.

Example 1:

If $2x + 5 = ax + b$ for all values of x , what are values of a and b ?

Solution) The expressions are equal. $a = 2$ and $b = 5$

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Example 2:

If $ax^2 + bx + c = 0$ is true for all values of x , what are the values of a , b , and c ?

Solution) Since $ax^2 + bx + c = 0x^2 + cx + 0$, $a = 0$, $b = 0$, and $c = 0$.

SAT Practice

1. If $x(k - 2) = 0$ for all values of x , what is the value of k ?

- A) 0
- B) 2
- C) 4
- D) 6

2. If $ax^2 + bx + c = 0$ for all values of x , what is the value of $a + b + c$?

- A) 0
- B) 1
- C) 2
- D) It cannot be determined from the information given.

$$(k+1)x + 5 = ax + k$$

3. In the equation above, k and a are constants. If the equation above is true for all values of x , what is the value of a ?

- A) 6
- B) 5
- C) 2
- D) 0

$$a(x+1) + b(x-1) = 2x + 4$$

4. In the equation above, a and b are constants. If the equation above is true for all values of x , what is the value of a ?

- A) 1
- B) 2
- C) 3
- D) 4

TIPS

Tip 13 Factoring

Factoring is to write an expression as a product of factors.

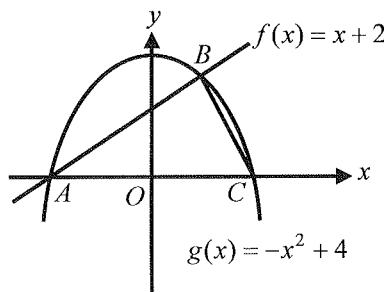
- 1) $a^2 + 2ab + b^2 = (a + b)^2$
- 2) $a^2 - 2ab + b^2 = (a - b)^2$
- 3) $a^2 - b^2 = (a + b)(a - b)$
- 4) $a^2 - 2a - 3 = (a - 3)(a + 1)$
- 5) $6a^2 + a + 1 = (3a + 1)(2a + 1)$
- 6) $12a^2 + 2a - 2 = 2(6a^2 + a - 1) = 2(3a + 1)(2a - 1)$: Complete factoring

SAT Practice

1. If $(x - 3)(x + 3) = a$, then $(2x - 6)(x + 3) =$
 - A) $2a$
 - B) $3a$
 - C) $4a$
 - D) $6a$

2. Which of the following is equivalent to $\left(n - \frac{1}{n}\right)^2 + 4$?
 - A) 4
 - B) $\left(n + \frac{1}{n}\right)^2$
 - C) $n^2 + \frac{1}{n^2}$
 - D) $n^2 - \frac{1}{n^2}$

TIPS



3. The graphs of the functions f and g are shown in the xy -plane above. What is the area of $\triangle ABC$?
- A) 18
B) 12
C) 9
D) 6
-
4. If $x^2 - y^2 = 24$, where x and y are positive integers ($x > y$), what is one possible value of x ?

Tip 14 Direct Variation (Direct Proportion)

When two variables are related in such a way that $y = kx$, the two variables are said to be in direct variation.

Expression of direct variation:

- 1) $y = kx$
- 2) $\frac{y}{x} = k$ or $\frac{y_1}{x_1} = \frac{y_2}{x_2} = \frac{y_3}{x_3} = \dots = k$ (constant)

Geometric interpretation:

$y = kx$ is a special linear equation, where the y -intercept is $(0, 0)$. In the xy -plane, k is the slope of the graph.

SAT Practice

1. The value of y varies directly proportional to the value of x . If $y = 15$ when $x = 5$, what is the value of y when $x = 12.5$?

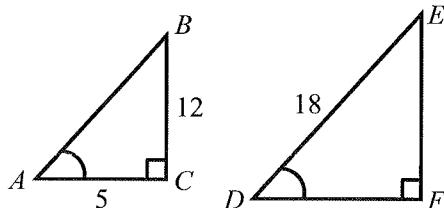
TIPS

2. A group of workers can harvest all the grapes from 10 square meters of a vineyard in 20 minutes. At this rate, how many minutes will the group need to harvest all the grapes from 300 square meters of this vineyard?

- A) 60
- B) 200
- C) 400
- D) 600

-
3. To make an orange dye, 5 parts of red dye are mixed with 3 parts of yellow dye. To make a green dye, 4 parts of blue dye are mixed with 2 parts of yellow dye. If equal amount of green and orange dye are mixed, what fraction of the new mixture is yellow dye?

- A) $\frac{1}{3}$
- B) $\frac{17}{48}$
- C) $\frac{9}{24}$
- D) $\frac{1}{2}$



4. In the figure above, $\triangle ABC$ is similar to $\triangle DEF$. What is the length of \overline{DF} ?

- A) 6
- B) $\frac{82}{13}$
- C) $\frac{90}{13}$
- D) $\frac{25}{3}$

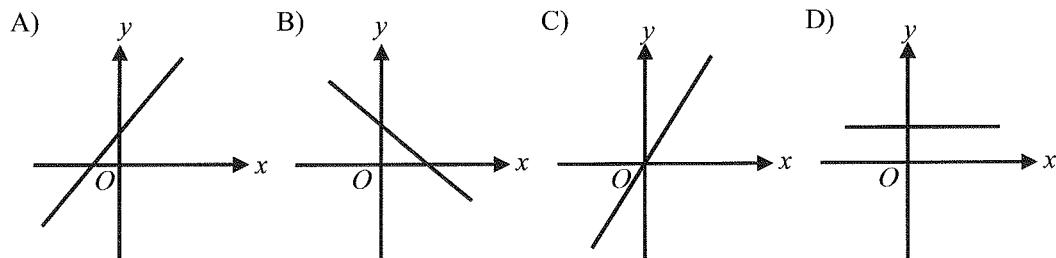
TIPS

x	y
1	a
a	$5a$

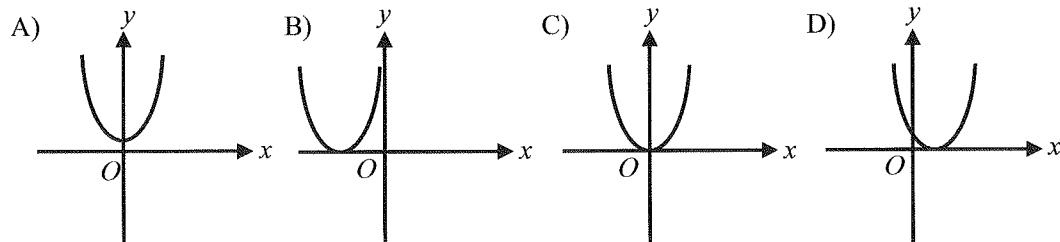
5. In the table above, y is directly proportional to x and $a \neq 0$. Which of the following is the value of a ?

- A) 0
- B) 1
- C) 5
- D) 10

-
6. If y is directly proportional to x , which of the following could represent the graph of $y = f(x)$?



-
7. If y is directly proportional to x^2 , which of the following could be the graph of $y = f(x)$?



TIPS

Tip 15 Inverse Variation

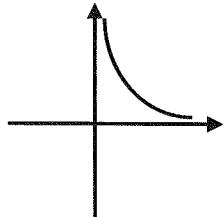
When two variables are related in such a way that $xy = k$, the two variables are said to be in inverse variation.

Properties:

- 1) The values of two variables change in an opposite way, that is, as one variable increases, the other decreases.
- 2) The product k is unchanged.

In the xy -plane, the graph of $y = f(x)$ is as follows.

- 1) $k > 0$



SAT Practice

1. The cost of hiring a bus for a trip to Niagara Falls is \$400. If 25 people go on the trip, what is the cost per person in dollars?
2. If four typists can complete the typing of a manuscript in 9 days, how long would it take 12 typists to complete the manuscript?
A) 3 days B) 4 days C) 5 days D) 10 days
3. If a man can drive from his home to Albany in 5 hours at 45 miles per hour, how long would it take him if he drove at 50 miles per hour?
A) 4 hours
B) 4 hours 30 minutes
C) 5 hours
D) 5 hours 30 minutes

TIPS

x	y
2	25
4	a
5	10
8	b

4. In the table above, y varies inversely as x . What is the value of $a+b$?

A) 16 B) 18 C) 18.75 D) 20.25

5. If a job can be completed by 2 people in 10 days, then how many people, working at the same rate, are needed to complete the same job in 5 days?

A) 1 B) 2 C) 3 D) 4

6. The length of a rectangle varies inversely with the width. If the length is 10 when the width is 20, what is the length when the width is 40?

A) 2 B) 5 C) 10 D) 20

7. A certain job can be completed by p persons in h hours. How long would it take n persons, working at the same rate, to complete the same job?

A) $\frac{hn}{p}$ B) $\frac{n}{hp}$ C) $\frac{hp}{n}$ D) $\frac{np}{h}$

8. If 5 people take d days to install the plumbing for a house, then how many days would it take 2 people to complete one third of the same job? (Assume the people work at the same rate.)

A) $\frac{3d}{2}$
B) $\frac{5d}{2}$
C) $\frac{5d}{6}$
D) $\frac{d}{2}$

TIPS

Tip 16

Sum and Product of Quadratic Equations

For a quadratic equation $ax^2 + bx + c = 0$,

$$\text{Sum of the roots} = -\frac{b}{a} \quad \text{and} \quad \text{Product of the roots} = \frac{c}{a}$$

SAT Practice

1. If one of roots of a quadratic equation $2x^2 + x - k = 0$ is 1, what is the other root of the equation?

A) $-\frac{5}{2}$ B) $-\frac{3}{2}$ C) $-\frac{1}{2}$ D) $\frac{3}{2}$

2. If the roots of the equation $x^2 + 4x - 12 = 0$ are α and β , what is the value of $\frac{1}{\alpha} + \frac{1}{\beta}$?

A) $\frac{1}{4}$ B) $\frac{1}{3}$ C) $\frac{1}{2}$ D) $\frac{3}{2}$

Tip 17

Remainder Theorem

When polynomial $f(x)$ is divided by $(x - a)$, the remainder R is equal to $f(a)$.

Polynomial $f(x)$ can be expressed as follows.

$$f(x) = (x - a)Q(x) + R, \text{ where } Q(x) \text{ is the quotient and } R \text{ is the remainder.}$$

The identical equation above is true for all values of x , especially $x = a$.

$$\text{Therefore, } f(a) = (a - a)Q(a) + R \rightarrow f(a) = R.$$

Examples:

- 1) Interpretation of $f(2) = 5 \rightarrow$ The remainder is 5 when $f(x)$ is divided by $(x - 2)$.
- 2) Interpretation of $f(-5) = -3 \rightarrow$ The remainder is -3 when $f(x)$ is divided by $(x + 5)$.

TIPS

SAT Practice

1. When $f(x) = x^2 + 3x + k$ is divided by $x - 3$, the remainder is 25. What is the value of k ?

-
2. What is the remainder when $x^3 - x^2 - 3x - 1$ is divided by $(x + 3)$?

A) -36 B) -28 C) 14 D) 36

3. Find the value of k for which the remainder is zero when $x^3 - 5x^2 + x + k$ is divided by $(x - 1)$.

Tip 18 Factor Theorem

If $f(a) = 0$, then $f(x)$ has a factor of $(x - a)$.

$f(x)$ can be expressed with a factor of $(x - a)$ as follows.

$$f(x) = (x - a)Q(x)$$

Therefore, $f(a) = 0$ means that the remainder is 0.

SAT Practice

1. If $(x - 3)$ is a factor of $x^3 - 4x + k$, what is the value of the constant k ?

A) -15 B) -10 C) 10 D) 15

2. If a polynomial $P(x) = x^2 + kx - 8$ has a factor of $(x - 2)$, what is the value of k ?

TIPS

Tip 19 Circle

A circle is the locus of points equidistant from a given point known as the center.

The standard equation of a circle whose center is at the point (h, k) is

$$(x - h)^2 + (y - k)^2 = r^2, \text{ where } r = \text{radius}.$$

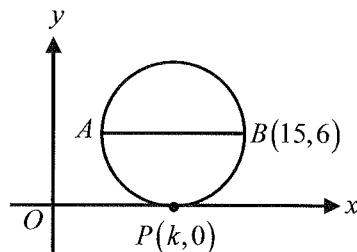
SAT Practice

1. What is the area of the circle whose equation is $x^2 - 4x + y^2 + 2y = 11$?

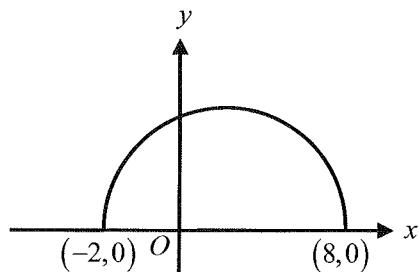
A) 11π B) 12π C) 16π D) 25π

2. What is the circumference of a circle whose equation is $x^2 + y^2 - 6y = 16$?

A) 10π B) 20π C) 30π D) 40π



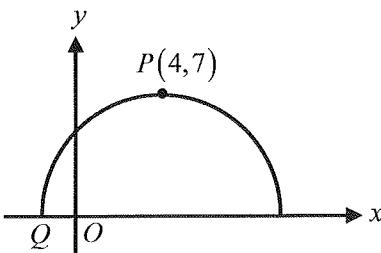
3. In the xy -plane above, \overline{AB} is the diameter of the circle and parallel to the x -axis. What is the value of k ?
-



4. The graph of a semicircle is shown in the xy -plane above. Which of the following are the x -coordinates of two points on this semicircle whose y -coordinates are equal?

A) 0 and 7 B) 1 and 6 C) 1 and 5 D) 2 and 3

TIPS



5. In the xy -plane above, the semicircle has a maximum at point P . What are the coordinates of point Q ?

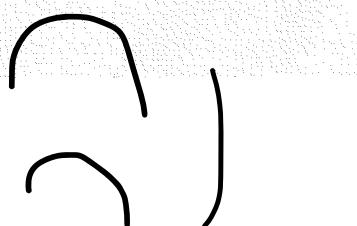
- A) $(-4, 0)$
- B) $(-3, 0)$
- C) $(-2, 0)$
- D) $(-1, 0)$

Tip 20

Average Speed

Average speed is the total distance divided by the total time taken.

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$



Example:

Peter travelled from city A to city B at 60 miles per hour, and then he travelled back along the same route at 40 miles per hour. What is his average speed for the entire trip?

Solution)

D = distance between city A and city B

$$\text{Total distance} = 2D \text{ and Total time taken} = \frac{D}{60} + \frac{D}{40} = \frac{5D}{120} = \frac{D}{24}$$

$$\text{Therefore, average speed} = \frac{2D}{\frac{D}{24}} = 48 \text{ mph}$$



Or

We can use any convenient number for D .

$$\text{If } D = 120, \text{ then } t_1 = \frac{120}{60} = 2 \text{ and } t_2 = \frac{120}{40} = 3.$$

$$\text{Therefore, average speed} = \frac{\text{Total distance}}{\text{Total time}} = \frac{120 + 120}{2 + 3} = 48 \text{ mph}$$

TIPS

SAT Practice

1. Jason travelled from city A to city B in 4 hours. For the first hour, he drove at a constant speed of 50 miles per hour. Then he increased his speed and kept it at 60 miles per hour for the next 3 hours. What is his average speed, miles per hour, for the trip?

- A) 55.5
- B) 56
- C) 57.5
- D) 58

2. Claire travelled from city C to city D . The first half of the way, she drove at the constant speed of 40 miles per hour. Then she increased her speed and travelled the remaining distance at 50 miles per hour. What is her average speed, miles per hour, for the trip?

- A) $44\frac{4}{9}$
- B) $44\frac{2}{3}$
- C) 45
- D) $45\frac{1}{3}$



$t_1 = \frac{x}{40}$



$t_2 = \frac{x}{50}$

$$\frac{1}{2} - 40$$

3. Jackson drove a car from Amherst to Boston at the constant speed of 60 miles per hour. On the way back along the same route, he drove at a constant speed of 40 miles per hour. If he took 6 hours for the entire trip, what is the distance, in miles, from Amherst to Boston?

- A) 120
- B) 144
- C) 160
- D) 240

$$\frac{x}{60} + \frac{x}{40} = 6$$

TIPS

Tip 21 Percentage

- % of increase = $\frac{\text{Amount of increase}}{\text{Original amount}} \times 100$
- % of decrease = $\frac{\text{Amount of decrease}}{\text{Original amount}} \times 100$

SAT Practice

- If 20% of 30% of a positive number is equal to 10% of $k\%$ of the same number, what is the number?
A) 80 B) 60 C) 40 D) 20

- The price of a music CD was first increased by 15 % and then the new price was decreased by 30 %. Which of the following is true about the price after these two changes?
A) The price decreases by 15%.
B) The price decreases by 19.5%.
C) The price decreases by 35%
D) The price decreases by 45%.

- If $2a + 3b$ is equal to 250 percent of $6b$, what is the value of $\frac{a}{b}$?
A) $\frac{1}{3}$
B) 3
C) 6
D) 9

- If 25 percent of m is 50, what is 15 percent of $2m$?
A) 80
B) 60
C) 50
D) 40

TIPS

5. The cost of an automobile increases each by 2.5%, and the cost this year is \$20,000. If the cost of the automobile is given by $C(n) = 20,000x^n$, what is the value of x ?
- A) 1.25
B) 1.025
C) 0.25
D) 0.025
-
6. Tom's weekly salary was increased from \$500 to \$1,000 this week. By what percent was his salary increased?
- A) 50%
B) 100%
C) 200%
D) 400%
-
7. If the price of a stock rises by 6 percent one day falls 5 percent next day, what was the change in the price of the stock after these two days?
- A) The price rose by 0.5%
B) The price rose by 0.7%
C) The price rose by 1%
D) The price rose by 1.5%
-
8. If a is 25 percent of $2b$, then b is what percent of a ?
- A) 50%
B) 75%
C) 100%
D) 200%

TIPS

Tip 22 Ratios and Proportion

Ratio

A ratio is a comparison of two numbers. We can write this as 8:12 or as a fraction $\frac{8}{12}$, and we say the ratio is *eight to twelve*.

Proportion

A proportion is an equation with a ratio on each side. It is a statement that two ratios are equal.

$\frac{3}{4} = \frac{6}{8}$ is an example of a proportion.

Rate

A rate is a ratio that expresses how long it takes to do something, such as traveling a certain distance. To walk 3 kilometers in one hour is to walk at the rate of 3 km/h. The fraction expressing a rate has units of distance in the numerator and units of time in the denominator.

Example 1:

Juan runs 4 km in 30 minutes. At that rate, how far could he run in 45 minutes?

Solution)

Give the unknown quantity the name n . In this case, n is the number of km Juan could run in 45 minutes at the given rate. We know that running 4 km in 30 minutes is the same as running n km in 45 minutes; that is, the rates are the same. So we have the proportion $\frac{4}{30} = \frac{n}{45}$.

Finding the cross products and setting them equal, we get $30 \times n = 4 \times 45$, or $30n = 180$. Dividing both sides by 30, we find that $n = 180 \div 30 = 6$ and the answer is 6 km.

SAT Practice

- If the cost of a 6-minute telephone call is \$1.20, then at this rate, what is the cost of a 15-minute call?
 - \$2.00
 - \$3.00
 - \$3.25
 - \$3.75

TIPS

2. In 5 years the ratio of Julie's age to Song's age will be 3:5. In 10 years the ratio of Julie's age to Song's age will be 2:3. What is the sum of their current ages?
- A) 15
B) 20
C) 30
D) 35
-
3. If $\frac{x}{y} = \frac{2}{3}$ and $2x + 5y = 76$, what is the value of x ?
- A) 2
B) 4
C) 8
D) 16
-
4. If a is divisible by 2, b is divisible by 5, and $\frac{a}{b} = \frac{7}{9}$, where a and b are positive numbers, and $a+b < 400$, what is one possible value of $a+b$?

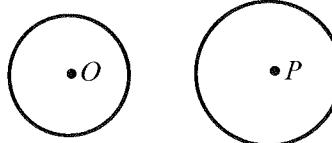
Tip 23 Ratios in Similar Figures

Two polygons are similar if and only if their corresponding angles are congruent and their corresponding sides are in proportion.

Remember:

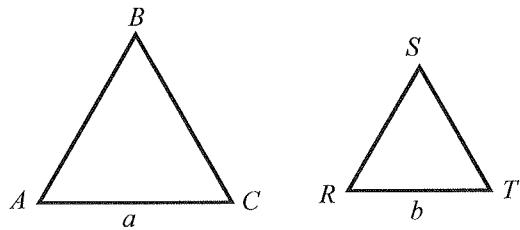
If the ratio of the corresponding lengths is $a:b$, then the ratio of the areas is $a^2:b^2$ and the ratio of the volumes is $a^3:b^3$.

SAT Practice

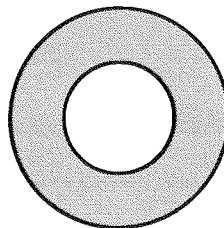


1. In the figure above, the ratio of the circumference of circle O to the circumference of circle P is 2:3. If the area of circle O is 20, what is the area of circle P ?

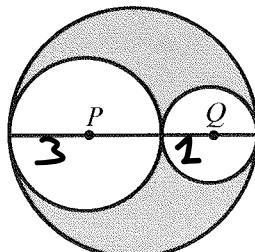
TIPS



2. The figure above shows two equilateral triangles with a side a and a side b . If $\frac{a}{b} = \frac{5}{2}$ and the area of $\triangle ABC$ is 30, what is the area of $\triangle RST$?



3. In the figure above, the radius of the larger circle is $\frac{5}{2}$ times the radius of the smaller circle. If the area of the smaller circle is 28, what is the area of the shaded region?
- A) 70
B) 112
C) 147
D) 175



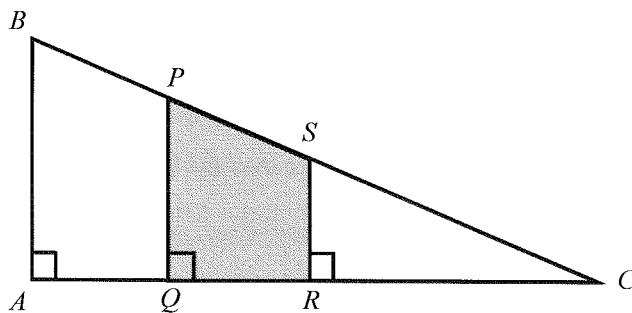
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$$16K - 10K = 96\pi$$

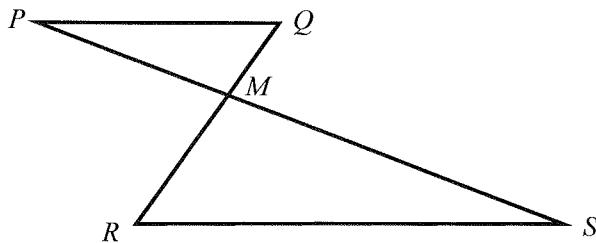
4. In the figure above, circles P and Q are tangent each other and internally tangent to the largest circle. The ratio of the radius of circle P to the radius of circle Q is 3:1. If the area of the shaded region is 96π , what is the radius of circle Q ?
- A) 2
B) 4
C) 6
D) 8

$$K = 16\pi$$

TIPS



5. In the figure above, the ratio of the lengths $AQ:QR:RC = 2:2:3$. If the area of quadrilateral $PQRS$ is 48, what is the area of $\triangle ABC$?
- A) 96
B) 124
C) 147
D) 192



6. In the figure above, \overline{PQ} is parallel to \overline{RS} . The ratio of the area of $\triangle PQM$ to the area of $\triangle SRM$ is 4:9. If the perimeter of $\triangle PQM$ is 15, what is the perimeter of $\triangle SRM$?
- A) 22.5
B) 33.75
C) 35.5
D) 37.5

TIPS

Tip 24 Percent of a Solution (Mixture)

The percent of a solution is expressed as the percentage of solute over the total amount of solution.

$p\%$ of a solution is

$$\frac{\text{Solute}}{\text{Total amount of solution}} = \frac{p}{100}$$

Or

$$\frac{\text{Solute}}{\text{Total amount of solution}} \times 100 = p\%$$

SAT Practice

1. How many gallons of water must be added to 40 gallons of a 10% alcohol solution to produce an 8% alcohol solution?
 - A) 5
 - B) 8
 - C) 10
 - D) 12

2. How many gallons of a 20% salt solution must be added to 10 gallons of a 50% salt solution to produce 30% salt solution?
 - A) 5 gallons
 - B) 10 gallons
 - C) 15 gallons
 - D) 20 gallons

3. How many quarts of alcohol must be added to 10 quarts of a 25% alcohol solution to produce a 40% alcohol solution?
 - A) 2.5 quarts
 - B) 8 quarts
 - C) 10 quarts
 - D) 15 quarts

TIPS

4. How many gallons of acid must be added to G gallons of a $k\%$ acid solution to bring it up to an $m\%$ solution?

A) $\frac{G}{100 - m}$

B) $\frac{Gm}{100 - m}$

C) $\frac{G(m - k)}{100 - m}$

D) $\frac{100 - m}{G(m - k)}$

Tip 25 Exponents

The exponent is the number of times the base is used as a factor.

$$5^2 = 25 \begin{cases} 5 = \text{base} \\ 2 = \text{exponent} \\ 25 = \text{power} \end{cases}$$

The mathematical operations of exponents are as follows.

1) $a^m a^n = a^{m+n}$

2) $(a^m)^n = a^{mn}$

3) $(ab)^m = a^m b^m$

4) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

5) $a^{-n} = \frac{1}{a^n}$

6) $a^{\frac{m}{n}} = \sqrt[n]{a^m}$

TIPS

SAT Practice

1. If $\{(-2)^3(8)^2\}^4 = (2^4)^n$, what is the positive value of n ?

- A) 6
- B) 7
- C) 8
- D) 9

2. If $4^3 + 4^3 + 4^3 + 4^3 = 2^n$, what is the value of n ?

- A) 2
- B) 4
- C) 6
- D) 8

3. If m and n are positive and $5m^5n^{-3} = 20m^3n$, what is the value of m in terms of n ?

- A) $\frac{1}{4n}$
- B) $\frac{4}{n^2}$
- C) $\frac{4}{n^3}$
- D) $2n^2$

4. If a and b are positive integers, $(a^{-4}b)^{-1} = 16$, and $b = a^2$, which of the following could be the value of a ?

- A) 0
- B) 2
- C) 4
- D) 8

5. If $k^{-2} \times 2^3 = 2^7$, what is the value of k ?

- A) 2
- B) 4
- C) 8
- D) $\frac{1}{4}$

TIPS

6. If p and q are positive numbers, $p^{-3} = 2^{-6}$, and $q^{-2} = 4^2$, what is the value of pq ?

- A) 1
- B) 2
- C) 3
- D) 4

-
7. If a and b are positive integers and $(a^6b^4)^{\frac{1}{2}} = 675$, what is the value of $a+b$?

- A) 2
- B) 4
- C) 6
- D) 8

Tip 26 Defined Operations

The **defined operations** are mathematical models (symbolic representations/notational systems/sign systems) of certain situations.

Example:

If the operation \blacktriangle is defined by $\blacktriangle a = a^a$, what is the value of $\frac{\blacktriangle 8}{\blacktriangle 4}$?

Solution)

$$\blacktriangle 8 = 8^8 = (2^3)^8 = 2^{24} \quad \text{and} \quad \blacktriangle 4 = 4^4 = (2^2)^4 = 2^8$$

Therefore, the answer is $\frac{\blacktriangle 8}{\blacktriangle 4} = \frac{2^{24}}{2^8} = 2^{24-8} = 2^{16}$.

SAT Practice

1. Let the operation \odot be defined for all numbers by $a \odot b = \frac{a+b}{a-b}$. If $p \odot q = 3$, what is the value of $\frac{p}{q}$?

- A) $\frac{1}{2}$
- B) 1
- C) $\frac{3}{2}$
- D) 2

TIPS

2. Let the operation Δ be defined by $a\Delta b = \frac{a}{b}$ for all positive numbers. If $4\Delta(k\Delta 6) = 3$, what is the value of k ?
- A) 4
B) 8
C) 12
D) 20
-
3. Let the operation be defined by $n^\Delta = n(n-1)(n-2)(n-3)\dots(2)(1)$, where n is a positive integer. Which of the following is equivalent to $(n+1)^\Delta$?
- A) $n(n^\Delta)$
B) $(n+1)(n+1)^\Delta$
C) $n(n-1)^\Delta$
D) $(n+1)n^\Delta$

Tip 27 Functions as Models

Functions as Models

A function can serve as a simple kind of mathematical model, or a simple piece of a larger model. Remember that a function is just a rule. We can think of the rule (given in our model as a graph, a formula, or a table of values) as a representation of some natural cause and effect relationship.

SAT Practice

1. The total cost c , in dollars, of repairing shoes is given by the function $c(x) = \frac{200x - 400}{x} + k$, where x is the number of repairing shoes and k is a constant. If 50 shoes were repaired at a cost of \$300, what is the value of k ?
- A) 100
B) 108
C) 126
D) 150

TIPS

2. The value of a computer decreases each year by 1.2 percent. This year the price of the computer was \$1,200. If the price p of the computer n years from now is given by the function $p(n) = 1,200c^n$, what is the value of c ?
- A) 0.012
B) 0.88
C) 0.988
D) 1.012
-
3. Let the function m , average rate of change between a and b in the domain of the function, be defined by $m(x) = \frac{f(b) - f(a)}{b - a}$. If $f(x) = x^2$, what is the value of m between -2 and 3 ?
- A) -2
B) -1
C) 0
D) 1
-
4. The present value p of a certain car that depreciates for a number of years is defined by $p(t) = k\left(1 - \frac{r}{100}\right)^t$, where k is the initial value of the car, r is the percent of depreciation per year, and t is the number of years. If a person purchases the car for \$20,000 and the value of the car depreciates by 10% per year, how much will the value of the car be after three years from the date of purchase?
- A) \$18,000
B) \$16,200
C) \$14,580
D) \$14,000

TIPS

Tip 28 Combined Rate of Work

These problems involve two people (or any machines) working at different rates. The general formula for solving combined work rate problems are as follows.

$$1) \text{Work rate} = \frac{1}{\text{Time taken}}$$

$$2) \text{Time taken together} = \frac{1}{\text{Combined work rate}}$$

Let's assume we have two workers, John and Chris.

1) John can finish a job in a hours when walking alone.

2) Chris can finish a job in b hours when working alone.

If two workers are working together, the number of hours they need to complete the job is given by

Worker	Rate	Combined work rate	Time
John	$1/a$	$\frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab}$	$\frac{ab}{a+b}$
Chris	$1/b$		

$$\text{John's work rate} = \frac{1}{a} \quad \text{Chris' work rate} = \frac{1}{b}$$

$$\text{Combined work rate} = \frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab}$$

$$\text{Time taken together} = \frac{1}{\text{Combined work rate}} = \frac{ab}{a+b}$$

SAT Practice

1. Worker A can finish a job in 5 hours. When worker A works together with worker B , they can finish the job in 4 hours. How long does it take for worker B to finish the job if he works alone?
- A) 8 hours
B) 12 hours
C) 16 hours
D) 20 hours

TIPS

2. Raymond and Peter can paint a house in 20 hours when working together at the same time. If Raymond works twice as fast as Peter, how long would it take Peter to paint the house if he works alone?
- A) 20 hours
B) 30 hours
C) 40 hours
D) 60 hours
-
3. The swimming pool can be filled by pipe A in 5 hours and by pipe B in 8 hours. How long would it take to fill the pool if both pipes were used?
- A) $3\frac{1}{13}$ hours
B) $5\frac{2}{3}$ hours
C) 7 hours
D) $8\frac{1}{3}$ hours
-
4. If it takes 5 people 12 hours to paint 3 identical houses, then how many hours will it take 4 people working at the same rate to paint 5 identical houses? (Assume they work at the same rate.)
- A) 18 hours
B) 19 hours
C) 20 hours
D) 25 hours

Tip 29

Combined Range of Two Intervals

For the interval of x , $a \leq x \leq b$

a = minimum value and b = maximum value

Minimum \leq Combined Range \leq Maximum

Example:

$$5 \leq A \leq 10 \text{ and } 2 \leq B \leq 5$$

- 1) $7 \leq A + B \leq 15$ 2) $10 \leq A \cdot B \leq 50$
3) $0 \leq A - B \leq 8$ 4) $1 \leq \frac{A}{B} \leq 5$

TIPS

SAT Practice

1. Given $2 \leq P \leq 8$ and $1 \leq Q \leq 4$. By how much is the maximum value of $\frac{P}{Q}$ greater than the minimum value of $\frac{P}{Q}$?

2. If $-2 \leq A \leq 2$, and $-6 \leq B \leq -2$, and $C = (A - B)^2$, what is the smallest value of C ?

-8

3. If $1 \leq P \leq 6$, and $3 \leq Q \leq 10$, what is the smallest value of $P \times Q$?

4. If $-2 < x < 4$ and $-3 < y < 2$, what are all possible values of $x - y$?

- A) $-4 < x - y < 2$
- B) $1 < x - y < 7$
- C) $1 < x - y < 4$
- D) $-4 < x - y < 7$

TIPS

Tip 30 Absolute Value

The absolute value of x , denoted " $|x|$ " (and which is read as "the absolute value of x "), is regarded as the distance of x from zero.

Properties of absolute value:

- 1) If $|x|=a$ and $a>0$, then $x=a$ or $-a$
- 2) If $|x|<a$ and $a>0$, then $-a < x < a$
- 3) If $|x|>a$ and $a>0$, then $x>a$ or $x<-a$
- 4) $|x|<5 \leftrightarrow x^2 < 25 \leftrightarrow -5 < x < 5$
- 5) $|x|>5 \leftrightarrow x^2 > 25 \leftrightarrow x < -5$ or $x > 5$
- 6) $|x-10|=|10-x|$

How do we convert the general interval into an expression using the absolute value?

Example 1:

For $10 \leq x \leq 30$,

Step 1: Find the midpoint of 10 and 30. $\rightarrow \frac{10+30}{2} = 20$

Step 2: Find the distance from the midpoint to the end point. $\rightarrow 30-20=10$ or $20-10=10$

Step 3: Substitute in the form.

$$|x - \text{midpoint}| \leq \text{distance}$$

Therefore, $|x-20| \leq 10$.

Example2:

For $x \leq 10$ or $x \geq 30$,

$$|x-20| \geq 10.$$

Example 3:

If $-8 < x < 2$, then express the interval using absolute value.

Solution)

Step 1: Find the midpoint between -8 and 2 $\rightarrow \frac{-8+2}{2} = -3$

Step 2: Find the distance from the midpoint to the endpoint. $\rightarrow 2 - (-3) = 5$

Step 3: From the figure above, the interval can be expressed with absolute value.

$$\begin{array}{c} \text{midpoint} = -3 \\ \swarrow \quad \searrow \\ -8 < x < 2 \quad \longleftrightarrow \quad |x - (-3)| < 5 \quad = \quad |x + 3| < 5 \end{array}$$

TIPS

SAT Practice

1. An art class of 20 students took a final exam and ten of the students scored between 78 and 86 in the exam. If s is defined as the scores of the ten students, which of the following describes all possible values of s ?

- A) $|s - 82| = 4$
- B) $|s + 82| = 4$
- C) $|s - 82| < 4$
- D) $|s + 82| < 4$

2. At a bottling company, a computerized machine accepts a bottle only if the number of fluid ounces is greater than or equal to $5\frac{3}{7}$, and less than or equal to $6\frac{4}{7}$. If the machine accepts a bottle containing f fluid ounces, which of the following describes all possible values of f ?

- A) $|f - 6| < \frac{4}{7}$
- B) $|f - 6| \leq \frac{3}{7}$
- C) $|f + 6| > \frac{4}{7}$
- D) $|6 - f| \leq \frac{4}{7}$

3. At the O.K Daily Milk Company, machine X fills a box with milk, and machine Y eliminates milk-box if the weight is less than 450 grams, or greater than 500 grams. If the weight of the box that will be eliminated by machine Y is E , in grams, which of the following describes all possible values of E ?

- A) $|E - 475| < 25$
- B) $|E - 500| > 450$
- C) $|475 - E| = 25$
- D) $|E - 475| > 25$

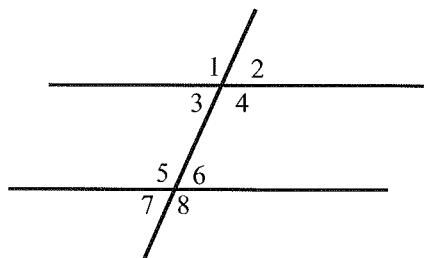
< 450
 > 500

TIPS

Tip 31 Parallel Lines with Transversal

A transversal is a line that intersects parallel lines. When it intersects parallel lines, many angles are congruent.

If a set of parallel lines are cut by a transversal, each of the parallel lines has 4 angles surrounding the intersection as follows.



$\angle 1 \cong \angle 4$ and $\angle 2 \cong \angle 3$: Vertical angles

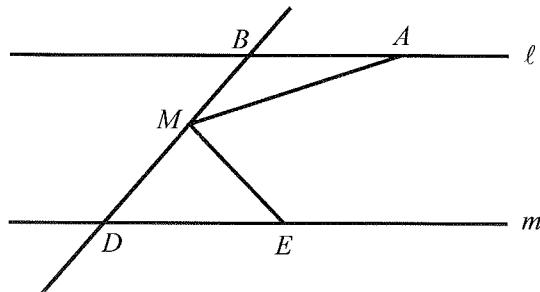
$\angle 2 \cong \angle 6$ and $\angle 4 \cong \angle 8$: Corresponding angles

$\angle 3 \cong \angle 6$ and $\angle 4 \cong \angle 5$: Alternate angles

$\angle 3 + \angle 5 = 180^\circ$ and $\angle 4 + \angle 6 = 180^\circ$:

Sum of the interior angles in the same side is 180° .

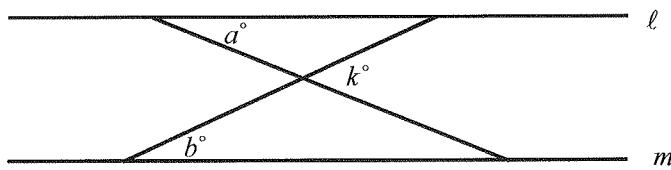
SAT Practice



Note: Figure not drawn to scale.

1. In the figure above, If $AB = BM = DM = DE$ and $\ell \parallel m$, what is the measure of $\angle AME$?
A) 50°
B) 60°
C) 75°
D) 90°

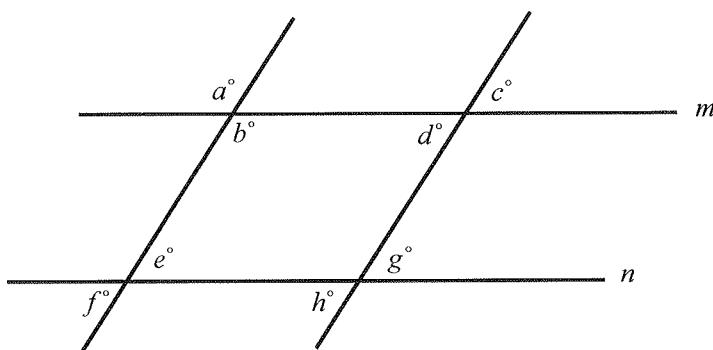
TIPS



Note: Figure not drawn to scale.

2. In the figure above, $\ell \parallel m$, $a = 65$, and $b = 45$. What is the value of k ?

- A) 80
 - B) 90
 - C) 100
 - D) 110
-



3. In the figure above, line m is parallel to line n . Which of the following must be true?

- I. $a = c$
- II. $d = g$
- III. $b + e = 180$

- (A) I only
- (B) II only
- (C) III only
- (D) II and III only

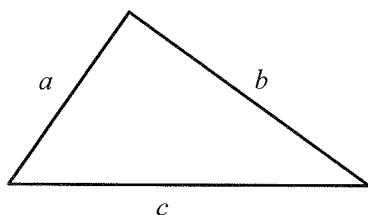
TIPS

Tip 32 Triangle Inequality

Triangle Inequality Theorem

Theorem 1:

The length of one side of a triangle is less than the sum of the other two sides and is greater than the difference of the other two sides.



We can say that

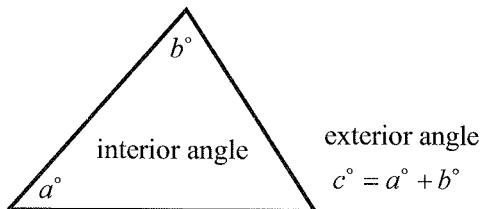
$$a + b < c < a + b$$

Theorem 2:

In a triangle, the longest side has the opposite largest angle.

Theorem 3:

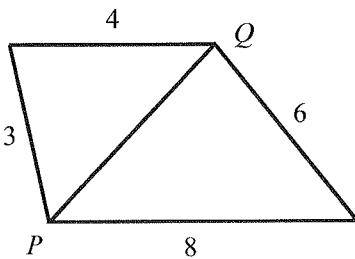
The measure of an exterior angle of a triangle is equal to the sum of the measures of its two nonadjacent interior angles.



SAT Practice

- If the lengths of the sides of $\triangle ABC$ is 3, $x + 3$, and 9, which of the following could be the value of x ?
 - 1
 - 2
 - 3
 - 4

TIPS



2. In the figure above, which of the following could be the length of \overline{PQ} ?

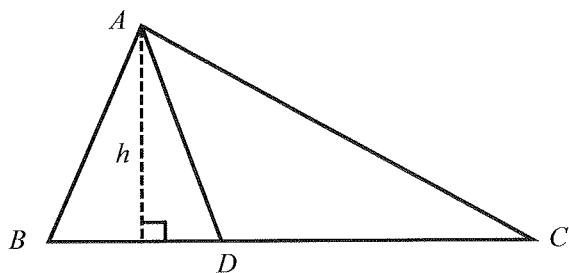
- A) 9
- B) 8
- C) 7
- D) 6

3. Which of the following CANNOT be possible to construct a triangle with the given side lengths?

- A) 6, 7, 11
- B) 3, 6, 9
- C) 28, 34, 39
- D) 35, 120, 125

Tip 33

Ratio of Areas of Triangles with the same height

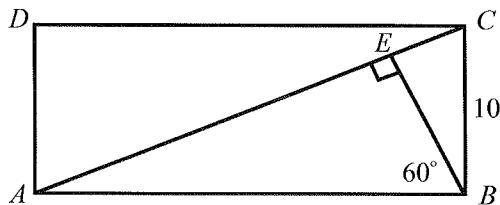


Two triangles with the same height h are shown above. If the ratio of BD to DC is $a:b$, then the ratio of the areas of $\triangle ABD$ to $\triangle ADC$ is also $a:b$.

$$\text{Ratio of areas} \rightarrow \frac{BD \times h}{2} : \frac{DC \times h}{2} = BD : DC = a : b$$

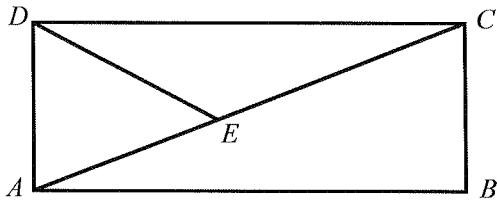
TIPS

SAT Practice



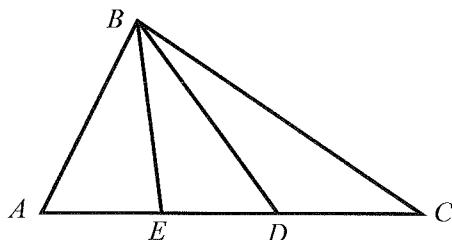
1. In rectangle $ABCD$ above, $\angle ABE = 60^\circ$ and $BC = 10$. what is the area of triangle ABC ?

- A) 50
 - B) $50\sqrt{3}$
 - C) 55
 - D) $60\sqrt{3}$
-



2. In the figure above, the ratio of AE to EC is 3:5. If the area of $\triangle ADE$ is 24, what is the area of rectangle $ABCD$?

- A) 64
 - B) 80
 - C) 100
 - D) 128
-

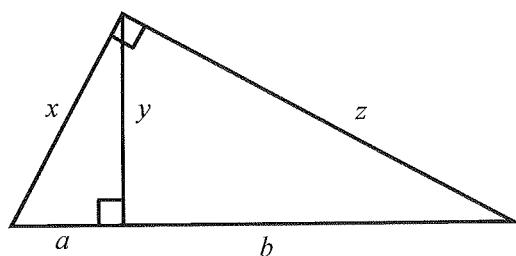


Note: Figure not drawn to scale.

3. In the figure above, the ratio of the lengths AE to ED to DC is 2:3:4. If the area of $\triangle ABD$ is 40, what is the area of $\triangle BDC$?

TIPS

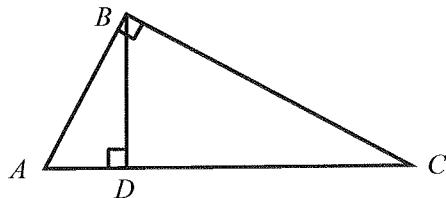
Tip 35 Proportions in a Right Triangle



Memorize the formulas.

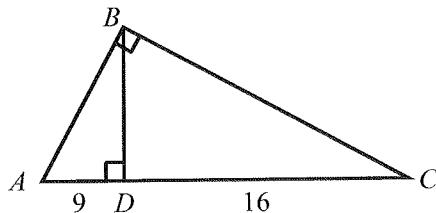
$$1) x^2 = a(a+b) \quad 2) y^2 = a \cdot b \quad 3) z^2 = b(b+a) \quad 4) \frac{xz}{2} = \frac{(a+b)y}{2} \text{ (area of the triangle)}$$

SAT Practice



1. In $\triangle ABC$ above, $AB = 6$ and $AD = 3$. What is the length of \overline{CD} ?

Questions 2-4 refer to the following information.



In the figure above, $AD = 9$ and $CD = 16$.

2. What is the length of \overline{AB} ?

- A) 12
- B) 14
- C) 15
- D) 18

TIPS

3. What is the length of \overline{BC} ?

- A) 18
- B) 20
- C) 25
- D) 28

4. What is the length of \overline{BD} ?

- A) 10
- B) 12
- C) 15
- D) 18

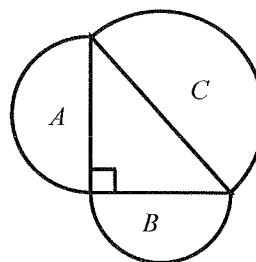
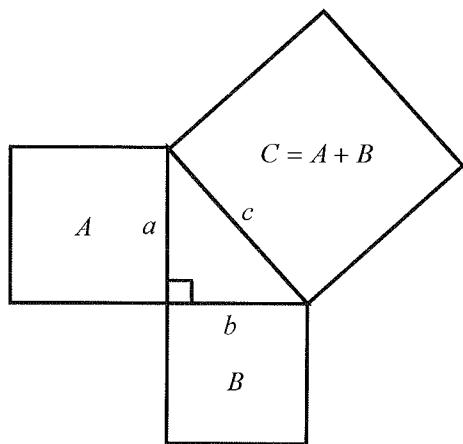
Tip 36 Pythagorean Theorem

In mathematics, the **Pythagorean Theorem** is a relation in Euclidean geometry among the three sides of a right triangle. The theorem is named after the Greek mathematician Pythagoras, who by tradition is credited with its discovery and proof, although knowledge of the theorem almost certainly predates him. The theorem is as follows:

In any right triangle, the area of the square whose side is the hypotenuse (the side opposite the right angle) is equal to the sum of the areas of the squares whose sides are the two legs (the two sides that meet at a right angle).

$$a^2 + b^2 = c^2 \text{ (side)}$$

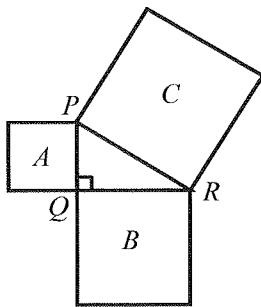
$$A + B = C \text{ (area)}$$



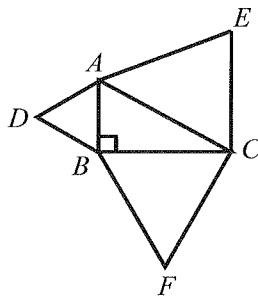
Remember: If the figures attached to the right triangle are similar, $C = A + B$ is always true.

TIPS

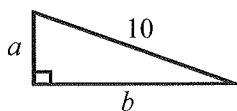
SAT Practice



1. In the figure above, the area of square A is 16 and the area of square B is 20. What is the length of \overline{PR} ?
- A) 4 B) 6 C) 25 D) 36
-

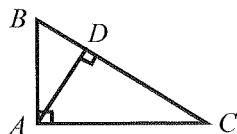


2. In the figure above, $\triangle ABD$, $\triangle ACE$, and $\triangle BCF$ are equilateral triangles, and the ratio of AB to BC is 1:2. If the area of $\triangle ACE$ is 20, what is the area of $\triangle ABD$?
-



3. In the figure above, which of the following is true about the lengths a and b ?

- A) $10 < (a+b)^2 < 40$ B) $40 < (a+b)^2 < 80$ C) $80 < (a+b)^2 < 100$ D) $100 < (a+b)^2$
-



4. In $\triangle ABC$ above, if $AB = 10$ and $AC = 20$, what is the length of \overline{AD} ?
- A) 5 B) $4\sqrt{5}$ C) $6\sqrt{3}$ D) $8\sqrt{3}$

TIPS

Tip 37 Transformation

Types of transformation in math are:

- Translation: involves “sliding” the object from one position to another.
- Reflection: involves “flipping” the object over a line called the line of reflection.
- Rotation: involves “turning” the object about a point called the center of rotation.
- Dilation: involves a resizing of the object. It could result in an increase in size (enlargement) or a decrease in size (reduction).

Translation of a graph

If the graph of

$$y = f(x)$$

Is translated a units horizontally and b units vertically, then the equation of the translated graph is

$$y - b = f(x - a) \quad \text{or} \quad y = f(x - a) + b$$

SAT Practice

1. The graph of the line represented by the equation $y = -2x + 5$ is moved to the left by 3 units and up 1 unit, what is the equation of the graph of the new line?

- A) $y = -2x$
- B) $y = -2x + 4$
- C) $y = -2x - 5$
- D) $y = -2x + 5$

$$y - 1 = -2(x + 3) + 5$$

2. How does the graph of $g(x) = x^2 - 1$ compare to the graph of $f(x) = (x - 2)^2 + 1$?

- A) The vertex of the graph of $f(x)$ moved to the right by 2 units and down by 2 units.
- B) The vertex of the graph of $f(x)$ moved to the left by 2 units and down by 2 units.
- ~~C) The vertex of the graph of $f(x)$ moved to the right by 2 units and up by 2 units.~~
- ~~D) The vertex of the graph of $f(x)$ moved to the left by 2 units and up by 2 units.~~

$$g - 2$$

TIPS

Tip 38 Classifying a group in two different ways

Organize the information in a table and use a convenient number.

Example:

In a certain group of only senior and junior students, $\frac{3}{5}$ of the students are boys, and the ratio of seniors to juniors is 4:5. If $\frac{2}{3}$ of girls are seniors, what fraction of the boys are juniors?

	Boys	Girls	Total
Seniors		$\frac{4}{15}$	$\frac{4}{9}$
Juniors	A	B	$\frac{5}{9}$
Total	$\frac{3}{5}$	$\frac{2}{5}$	1

$$\frac{2}{3} \text{ of girls are seniors} \rightarrow \frac{2}{3} \times \frac{2}{5} = \frac{4}{15} \quad B = \frac{2}{5} - \frac{4}{15} = \frac{2}{15}$$

$$\text{Therefore, } A = \frac{5}{9} - \frac{2}{15} = \frac{25}{45} - \frac{6}{45} = \frac{19}{45}.$$

Question: What fraction of boys are juniors?

$$\text{Final answer is } \frac{19/45}{3/5} = \frac{19/45}{27/45} = \frac{19}{27}.$$

Or

Use a convenient number 45 for the number of students in the group.

	Boys	Girls	Total
Seniors	8	12	20
Juniors	19	6	25
Total	27	18	45

TIPS

SAT Practice

1. Of the 24 company presidents attending a corporate meeting, $\frac{3}{4}$ of the presidents are males and $\frac{2}{3}$ of the presidents have children. If 2 female presidents do not have children, how many male presidents have children?
- A) 8
B) 10
C) 12
D) 14

Tip 39

Permutation and Combination

Permutation is an arrangement of objects in some specific order. A selection in which order is not important is called a combination.

- 1) If the order does matter, it is a permutation. The number of permutations of n things taken n at a time is

$${}_n P_n = n(n-1)(n-2)(n-3)\cdots 2 \cdot 1$$

- Example 1:

How many different orders can the program for a music recital be arranged if 5 students are to perform?

$${}_5 P_5 = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$$

- Example 2: Permutation with repetition

How many 8-letter arrangements can be made of the letters of the word “abscissa”?

This is a permutation where two a and three s are identical. Therefore, the solution is

$$\frac{{}_8 P_8}{2!3!} = \frac{8!}{2!3!} = 3360.$$

- 2) If the order doesn't matter, it is a combination.

$${}_n C_r = \frac{n!}{(n-r)!r!}$$

- Example:

How many ways can two boys be selected from a group of 8 boys?

$${}_8 C_2 = \frac{8!}{6!2!} = \frac{8 \cdot 7}{2} = 28.$$

TIPS

SAT Practice

1. In how many different ways can five students be arranged in a row?

- A) 60
- B) 80
- C) 120
- D) 160

$$5!$$

2. How many distinct arrangements of the letters of the word LETTER are possible that begins and ends with a T?

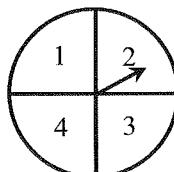
- A) 6
- B) 12
- C) 24
- D) 120

T _ _ _ T

$$\begin{array}{r} 4! \\ \hline 2! \end{array}$$

3. If a fair die is thrown three times, what is the probability that a 5 comes up exactly two times?

- A) $\frac{5}{216}$
- B) $\frac{5}{72}$
- C) $\frac{1}{5}$
- D) $\frac{5}{24}$



4. In the figure above, the arrow is spun twice on a wheel containing four equally likely regions, numbered 1 through 4. What is the probability that the first digit spun is larger than the second?

- A) $\frac{1}{8}$
- B) $\frac{1}{4}$
- C) $\frac{3}{8}$
- D) $\frac{1}{2}$

2 1

3 2

5 1

4 3

4 2

4 1

6

TIPS

Tip 40 Handshakes

If there are five people in a room, and they shake each other's hands once and only once, how many handshakes are there altogether?

(A, B) (A, C) (A, D) (A, E) ----- 4 handshakes

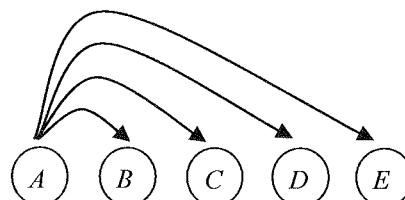
(B, C) (B, D) (B, E) ----- 3 handshakes

(C, D) (C, E) ----- 2 handshakes

(D, E) ----- 1 handshake

Therefore there will be

$$4+3+2+1=10 \text{ handshakes}$$



Or, we can use a combination.

The number of handshakes is equal to ways to select two people from 8, because two people make one handshake. Therefore,

$${}_5C_2 = \frac{5 \times 4}{2!} = 10$$

SAT Practice

1. If you have 12 people in a group and each person shakes everyone else's hand only once, how many handshakes take place?

- A) 132
- B) 112
- C) 88
- D) 66

$$\begin{array}{r} 1211 \\ \hline 2 \end{array}$$

TIPS

2. At a party, everybody shakes hands with each other once. If there are 45 handshakes, how many people are there at the party?

- A) 9
- B) 10
- C) 11
- D) 12

$$\frac{n(n - 1)}{2} = 45$$

Tip 41 Consecutive Integers I

Integers which follow each other in order, without gaps, from smallest to largest are consecutive integers.

12, 13, 14 and 15 are consecutive integers.

12, 14, 16, 18 are consecutive even integers.

11, 13, 15, 17 are consecutive odd integers.

Properties of consecutive integers:

For consecutive numbers (integers), if the first term is a_1 and the last term is a_n , the average is equal to median.

$$\text{Average (Arithmetic mean)} = \frac{a_1 + a_n}{2}$$

Median = Average

Sum of consecutive = median \times number of integers

Or, Sum of consecutive = average \times number of integers

Example:

For the sequence of consecutive numbers

2, 3, 4, 5, 6, 7, 8, 9, 10

$$\text{Average} = \frac{2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10}{9} = 6$$

But simply we use the formula average = $\frac{a_1 + a_n}{2}$.

$$\text{Average} = \frac{2 + 10}{2} = 6$$

That is

$$\text{Median} = \text{Average} = 6$$

Therefore, in the list of consecutive numbers, the average value is equal to the median.

TIPS

SAT Practice

1. What is the sum of 11 consecutive integers if the middle one is 30?
 - A) 60
 - B) 120
 - C) 330
 - D) 660

2. If the median of a list of 99 consecutive integers is 80, what is the greatest integer in the list?
 - A) 99
 - B) 128
 - C) 129
 - D) 157

3. The median of a list of 10 consecutive even integers is 77. What is the sum of the integers?
 - A) 700
 - B) 770
 - C) 780
 - D) 800

4. If the median of a list of 30 consecutive odd integers is 120, what is the greatest integer in the list?
 - A) 145
 - B) 147
 - C) 149
 - D) 151

TIPS

Tip 42 Consecutive Integers II

Example:

The smallest integer of a set of consecutive integers is -10 . If the sum of these integers is 23 , how many integers are in this set?

Solution)

$$\underbrace{-10, -9, -8, -7, \dots, 0, \dots, 7, 8, 9, 10, 11, 12}_{\text{sum} = 0} \quad \underbrace{\qquad\qquad\qquad}_{\text{sum} = 23}$$

We know that the sum of the consecutive integers from -10 to $+10$ is zero.

Therefore, the number of integers is

$$1, 2, 3, 4, \dots, 10 = 10 \text{ integers}$$

$$0 = 1 \text{ integer}$$

$$-1, -2, -3, \dots, -10 = 10 \text{ integers}$$

We need two more integers 11 and 12 to have a sum of 23 .

Hence, there are $10 + 1 + 10 + 2 = 23$ integers.

SAT Practice

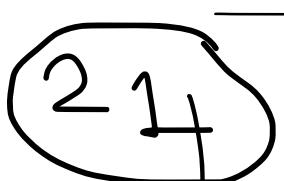
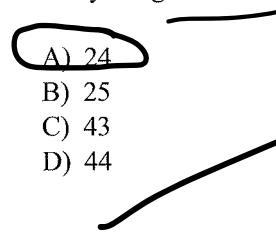
1. If the sum of the consecutive integers from -30 to x , inclusive, is 96 , what is the value of x ?

- A) 30
- B) 31
- C) 32
- D) 33



2. The smallest integer of a set of even consecutive integers is -20 . If the sum of these integers is 72 , how many integers are in the set?

- A) 24
- B) 25
- C) 43
- D) 44



22 24 26

TIPS

Tip 43 Number of Factors (Positive Divisors)

Let n be a natural number with prime factorization

$n = a^{k_1} b^{k_2} c^{k_3}$. The number of factors of the number is $(k_1 + 1)(k_2 + 1)(k_3 + 1)$.

The number of factors (positive divisors) can be found by adding one to all exponents of prime factors and multiplying those results together.

Example: For a natural number 12,

$$12 = 2^2 \times 3^1$$

From the prime factorization, the number of factors is $(2 + 1)(1 + 1) = 6$.

12 have factors of 1, 2, 3, 4, 6, 12 as follows.

2^2	3^1			
2^0	3^0	$2^0 \times 3^0 = 1$	$2^1 \times 3^0 = 2$	$2^2 \times 3^0 = 4$
2^1	3^1	$2^0 \times 3^1 = 3$	$2^1 \times 3^1 = 6$	$2^2 \times 3^1 = 12$
2^2				
3	x 2	= 6		

Each product generate all of the factors.

SAT Practice

- Let a positive integer n be defined by $n = p^2 \times q^4$, where p and q are distinct prime numbers. How many factors does the number n have?
 - 6
 - 8
 - 12
 - 15
- Let a positive integer k be defined by $k = 24p^2$, where p is a prime number greater than 5. How many factors does the number k have?
 - 8
 - 16
 - 24
 - 32

TIPS

Tip 44 Must be true or Could be true

$$(a+b)^2 = (a-b)^2$$

The questions can be as follows

(1) If the statement above is true, which of the following must also be true (always true)?

or

(2) If the statement above is true, which of the following could be true (possibly true)?

Solution)

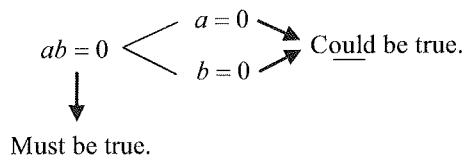
Simplify the equation until we can make a conclusion as follows.

$$(a+b)^2 = a^2 + 2ab + b^2 \quad (a-b)^2 = a^2 - 2ab + b^2$$

Then

$$a^2 + 2ab + b^2 = a^2 - 2ab + b^2$$

$$4ab = 0, \text{ or } ab = 0$$



For the question (1), the answer is

$$\underline{ab = 0}$$

The answer $ab = 0$ implies that

- (a) When $a = 0$, b is all real numbers and
- (b) When $b = 0$, a is all real numbers.

For the question (2), there are two answers.

$$\underline{a = 0}, \text{ or } \underline{b = 0}$$

- (a) When $a = 0$, b can be any real number.
Or
- (b) When $b = 0$, a can be any real number.

SAT Practice

1. If $k(a-b) = a-b$, which of the following could be true?

- I. $k = 1$
- II. $a = 2$ and $b = 2$
- III. $a = b$

- A) II only
- B) III only
- C) I and III only
- D) I, II, and III

TIPS

2. If $a^2 + b^2 = 2ab$, which of the following must be true?

- I. $a = 1$
- II. $a = b$
- III. $a = 0$ and $b = 0$

- A) I only
B) II only
C) III only
~~D) I and II only~~

$$(a - b)^2 = 0 > \boxed{a - b}$$

~~$$(0 - 0)^2$$~~

Tip 45 Complex Numbers

Imaginary number: $i = \sqrt{-1}$ and $i^2 = -1$

Example:

$$\sqrt{-4} = i\sqrt{4} = 2i \quad \sqrt{-12} = i\sqrt{12} = 2i\sqrt{3}$$

Complex number: $a + bi$, where a and b are real numbers and $i = \sqrt{-1}$.

a = the real part of $a + bi$ and b = imaginary part of $a + bi$

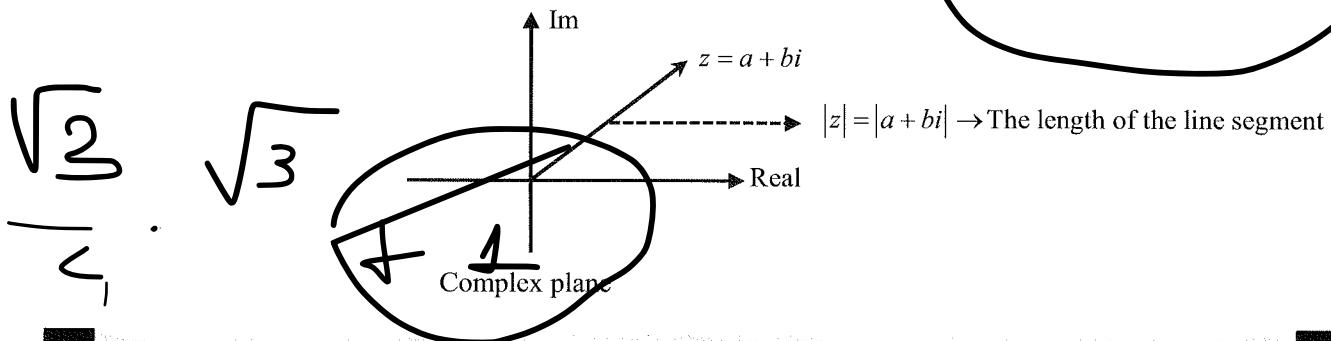
Property:

- 1) Two complex numbers are equal if and only if their real parts are equal and their imaginary parts are equal.

$$a + bi = c + di \rightarrow a = b \text{ and } c = d$$

2) $|a + bi| = \sqrt{a^2 + b^2}$ (Distance from the origin)

$$|z| = 1$$



SAT Practice

1. If $a + b + (a - b)i = 6 - 4i$, what is the value of a ?

$$a + b = 6$$

$$a - b =$$

TIPS

$$\frac{2+3i}{1-2i} = a+bi$$

2. In the equation above, $i = \sqrt{-1}$. What is the value of b ?

- A) $-\frac{4}{5}$ B) $\frac{4}{5}$ C) $\frac{7}{5}$ D) $\frac{9}{5}$
-

$$\sqrt{36} \quad i^2$$

3. What is the value of $\sqrt{-12} \times \sqrt{-3}$?

- A) -36
B) -6
C) 6
D) 36
-

$$\begin{aligned}\sqrt{12} &= \sqrt{12} \cdot \sqrt{1} \\ &= \sqrt{12} \cdot \sqrt{3} \end{aligned}$$

4. In the equation above, a and b are real numbers and $i = \sqrt{-1}$. What is the value of b ?
-

5. If $i = \sqrt{-1}$, which of the following is equal to i^{126} ?

- A) i B) -1 C) $-i$ D) 1
-

6. Which of the following could be the solution of $x^3 - 2x^2 + 3x = 0$?

- A) 1
B) 3
C) $1+i\sqrt{2}$
D) $\frac{1}{2}+\frac{\sqrt{2}}{2}i$

TIPS

Tip 46 Circle (Geometry)

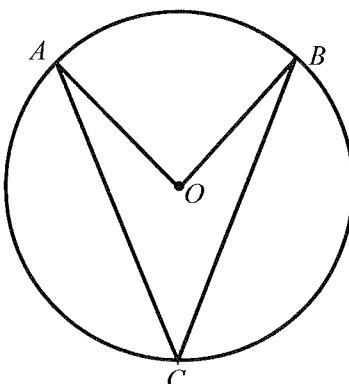
1. Central angle: $\angle AOB$

2. Inscribed angle: $\angle ACB$

3. Major arc: \widehat{ACB}

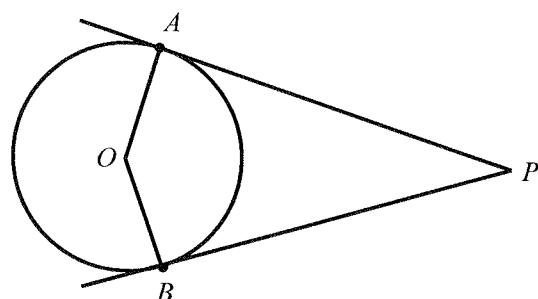
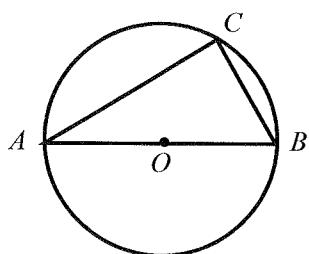
4. Minor arc: \widehat{AB}

5. Central angle = 2 × Inscribed angle



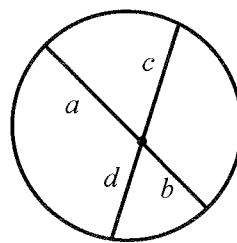
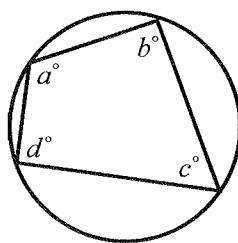
6. $\angle ACB = 90^\circ$

7. $AP = BP$ and $\angle AOB + \angle APB = 180^\circ$, $\overline{AO} \perp \overline{AP}$, $\overline{BO} \perp \overline{BP}$

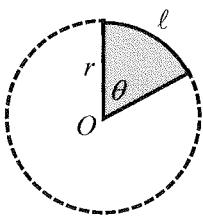


8. $a + c = 180^\circ$ and $b + d = 180^\circ$

9. $a \times b = c \times d$



10.

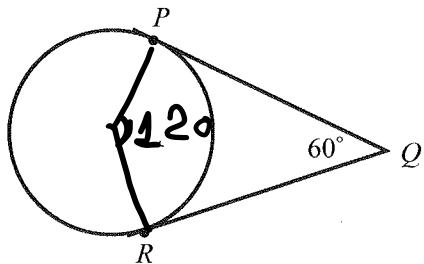


The length of the minor arc: $\ell = r\theta$
The area of the sector: $A = \frac{1}{2}(r^2\theta)$, where θ is in radian measure.

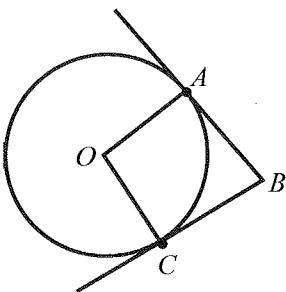
11. $\pi(\text{radian}) = 180^\circ$

TIPS

SAT Practice

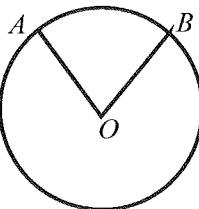


1. In the figure above, \overline{PQ} and \overline{RQ} are tangent to the circle at points P and R . If the radius of the circle is 12, what is the length of minor arc \widehat{PR} ?
- A) 4π B) 6π C) 8π D) 10π
-



2. In the figure above, point O is the center of the circle, \overline{BA} and \overline{BC} are tangent to the circle at points A and C . If the measure of $\angle AOC$ is 105° , what is the measure of $\angle ABC$ in degrees?
-

Questions 3 and 4 refer to the following figure.



In the figure above, the measure of $\angle AOB$ is $\frac{5\pi}{12}$ radians and the radius of the circle is 36.

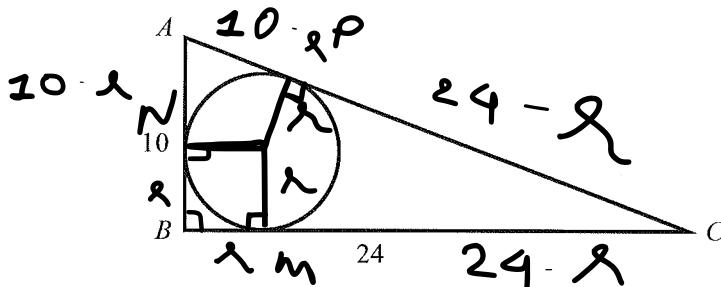
3. What is the length of the minor arc \widehat{AB} ?
- A) 10π B) 15π C) 20π D) 36π

TIPS

4. What is the area of sector AOB ?

A) 135π B) 180π C) 270π D) 360π

$\Delta C = 26$

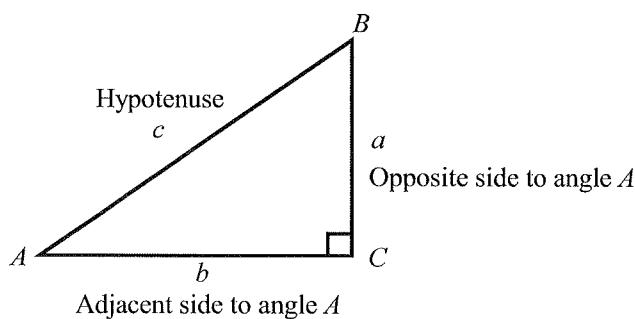


5. In the figure above, each side of right triangle ABC with $\angle B = 90^\circ$ are tangent to the circle. The length of \overline{AB} is 10 and the length of \overline{BC} is 24. What is the radius of the circle?

Tip 47

Trigonometric Function

Trigonometric Function:



$$\sin \theta = \frac{a}{c} \quad \cos \theta = \frac{b}{c} \quad \tan \theta = \frac{a}{b}$$

Cofunction:

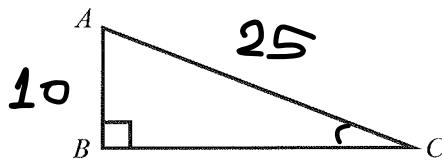
$\sin A = \frac{a}{c}$ and $\cos B = \frac{a}{c} \rightarrow \sin A = \cos B$, because the triangle is a right triangle ($\angle C = 90^\circ$ or $\angle A + \angle B = 90^\circ$).

Definition: In right triangle above,

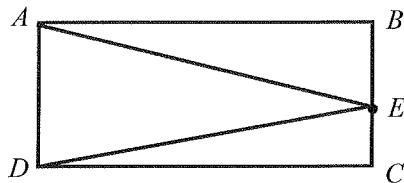
- 1) If $\angle A + \angle B = 90^\circ$, then $\sin \angle A = \cos \angle B$. 2) If $\sin \angle A = \cos \angle B$, then $\angle A + \angle B = 90^\circ$

TIPS

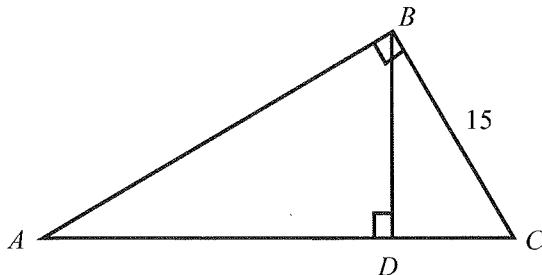
SAT Practice



1. In the figure above, the length of \overline{AB} is 10. If the value of $\sin \angle ACB$ is 0.4, what is the length of \overline{AC} ?



2. In rectangle $ABCD$ above, the value of $\tan \angle BAE = 0.6$ and the value of $\tan \angle EDC = 0.25$. What is the value of $\frac{BE}{EC}$?



3. In the figure above, the length of \overline{BC} is 15 and the value of $\cos \angle BAC$ is 0.8. What is the length of \overline{BD} ?

- A) 8 B) 10 C) 12 D) 13

4. If $\sin(3x + 5)^\circ = \cos(2x - 15)^\circ$ in a right triangle, what is the value of x ?

TIPS

Tip 48 Probability

The probability of an event is the number of ways that the event can occur, divided by the total number of possible outcomes. The symbolic form is

$$P(E) = \frac{n(E)}{n(S)}.$$

- 1) $P(E)$ represents the probability of event E ;
- 2) $n(E)$ represents the number of ways that event E can occur;
- 3) $n(S)$ represents the number of possible outcomes in sample space S .

Example 1:

A bag contains 3 red marbles and 4 blue marbles. What is the probability that you select one red marble and one blue, at random, from the bag?

Solution)

There are two different selections.

RB or BR

Therefore,

$$P(R \text{ and } B) = \frac{3}{7} \times \frac{4}{6} = \frac{12}{42}, \quad P(B \text{ and } R) = \frac{4}{7} \times \frac{3}{6} = \frac{12}{42}$$

$$\text{The answer is } \frac{12}{42} + \frac{12}{42} = \frac{24}{42} = \frac{4}{7}.$$

SAT Practice

1. A bag contains 3 red marbles and 3 blue marbles. What is the probability that you draw two red marbles without replacement?

- A) $\frac{1}{9}$
- B) $\frac{1}{6}$
- C) $\frac{1}{5}$
- D) $\frac{1}{3}$

TIPS

3 5 9

2. The three cards shown above were taken from a box of ten cards, each with a different integer on it from 1 to 10. What is the probability that the next two cards selected from the box will have both even integer on it?
- A) $\frac{10}{21}$
B) $\frac{12}{23}$
C) $\frac{4}{7}$
D) $\frac{5}{7}$
-
3. In a box, there are b blue marbles and g green marbles. If a person selects two marbles, what is the probability that both marbles are blue?
- A) $\frac{b}{b+g}$
B) $\frac{b}{b+g+1}$
C) $\frac{b \times b}{(b+g)(b+g-1)}$
D) $\frac{b(b-1)}{(b+g)(b+g-1)}$
-
4. If a number is chosen at random from the set $\{-15, -10, -5, 0, 5, 10, 15, 20\}$, what is the probability that it is a member of the solution set of $|x+2| < 8$?
- (A) 0
(B) $\frac{1}{4}$
(C) $\frac{3}{8}$
(D) $\frac{1}{2}$

TIPS

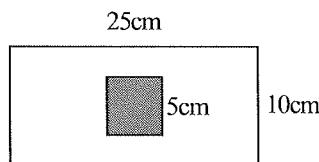
Tip 49

Geometric Probability

"**Geometric probability**" is the probability dealing with the areas of regions instead of the "number" of outcomes.
The equation becomes

$$\text{Probability} = \frac{\text{Favorable region}}{\text{Area of total region}}$$

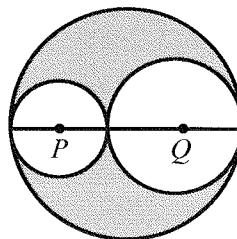
A typical problem might be this: If you are throwing a dart at the rectangular target below and are equally likely to hit any point on the target, what is the probability that you will hit the small square?



$$\text{Probability} = \frac{\text{favorable}}{\text{total}} = \frac{25\text{cm}^2}{250\text{cm}^2} = \frac{1}{10}$$

This means that there is a 1 in 10 chance that a dart thrown at the rectangle will hit the small square.

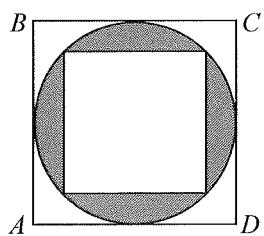
SAT Practice



1. In the figure above, the radius of the circle P is 2, the radius of the circle Q is 4, and \overline{AB} is the diameter of the largest circle. If a dart is thrown at the circular target and is equally likely to hit any point on the target, what is the probability that the dart will hit the shaded region?

A) $\frac{2}{9}$
B) $\frac{1}{3}$
C) $\frac{4}{9}$
D) $\frac{5}{9}$

TIPS



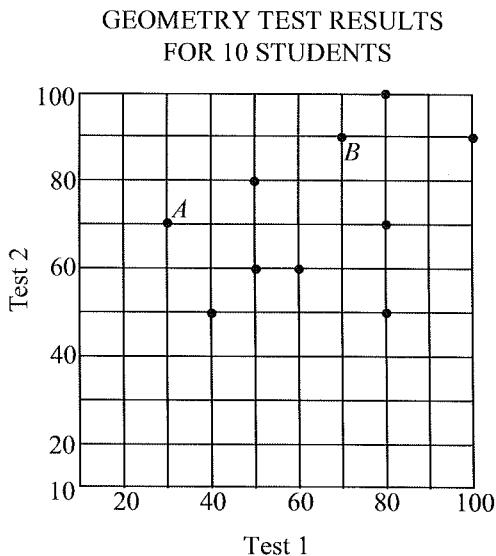
2. In the figure above, a circle is inside of and outside of a square. If a point is chosen at random from square $ABCD$, what is the probability that the point is chosen from the shaded region?

- A) $\frac{1}{4}$
- B) $\frac{2\pi - 50}{100}$
- C) $\frac{\pi - 2}{8}$
- D) $\frac{\pi - 2}{4}$

TIPS

Tip 50 | Data Interpretation

Data interpretation problems usually require two basic steps. First, you have to read a chart or graph in order to obtain certain information. Then, second, you have to apply or manipulate the information in order to obtain an answer. Be sure to read all notes related to the data.



In the scatter plot above, student *A* got 30 on test 1 and 70 on test 2. Student *B* got 70 on test 1 and 90 on test 2.

- 1) What is the median score on test 1 and 2?

(Sol) On test 1

30, 40, 50, 50, 60, 70, 80, 80, 80, 100

The median is $\frac{60+70}{2} = 65$.

(Sol) On test 2

50, 50, 60, 60, 70, 70, 80, 90, 90, 100

The median is 70.

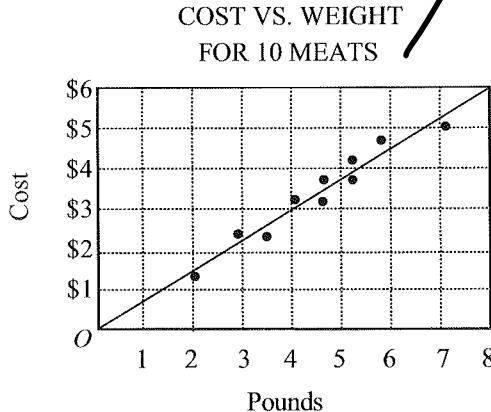
- 2) What is the average (arithmetic mean) on test 1?

(Sol) The average is

$$\frac{30 + 40 + 50 + 50 + 60 + 70 + 80 + 80 + 80 + 100}{10} = 64.$$

TIPS

SAT Practice



1. For 10 meats of different weights, the cost and weight of each are displayed in the scatter plot above, and the line of best fit for the data is shown. Which of the following is closest to the average (arithmetic mean) cost per pound for the 10 meats?
- A) \$0.18 B) \$0.56 C) \$0.62 D) \$0.73

ITEMS PURCHASED
BY EACH CUSTOMER

Numbers of Customers	Number of Items
10	10
25	8
45	5
50	Fewer than 5

2. The table above shows the number of items 130 customers purchased from a stationery store during on Sunday. Which of the following can be obtained from the information in the table?
- I. The average (arithmetic mean) number of items
II. The median number of items.
III. The mode of the number of items.
- A) I only
B) II only
C) III only
D) I and II only

TIPS

Tip 51 | Expected Value

Expected value $E(X)$ is a weighted average that involves multiplying each possible outcome with its probability to get the expected value.

The expected value or mean is computed as follows.

$$E(X) = n \cdot P(n)$$

Where,

$E(X)$ = expected value

n = an outcome

$P(n)$ = probability of that outcome

Expected value is merely an average. If we were to flip a coin 20 times, the frequency weighted average of Head will be 10 times. Because the probability of getting a head is $\frac{1}{2}$. Therefore

$$E(\text{head}) = n \times P = 20 \times \frac{1}{2} = 10$$

Example:

When you roll a die, you will be \$5 for odd number and \$2 for even number. What is the expected value of money you get paid for one roll of the die?

X	1	2	3	4	5	6
P	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$
Amount	5	2	5	2	5	2

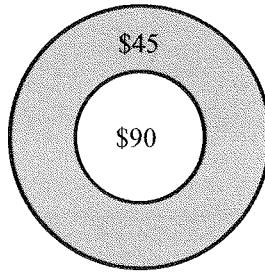
Therefore

$$E(X) = 5\left(\frac{1}{6}\right) + 2\left(\frac{1}{6}\right) + 5\left(\frac{1}{6}\right) + 2\left(\frac{1}{6}\right) + 5\left(\frac{1}{6}\right) + 2\left(\frac{1}{6}\right) = \$3.50$$

SAT Practice

- If the probability of a boy's being born is $\frac{1}{3}$, and if a family plans to have 6 children, what is the expected number of boys?
A) 1 B) 2 C) 3 D) 4

TIPS



2. A carnival game consists of tossing a dart, which lands at a random spot within the larger circle. The shaded region loses \$45 and the unshaded region wins \$90. If the ratio of the radius of the smaller circle to the radius of the larger circle is 1:3, which of the following can be expected in this game?
- A) lose \$30
B) lose \$10
C) earn \$10
D) earn \$20

Tip 52

Linear Correlation Coefficient

The **linear correlation coefficient**, r , measures the **strength** and the **direction** of a linear relationship between two variables.

$$-1 \leq r \leq 1$$

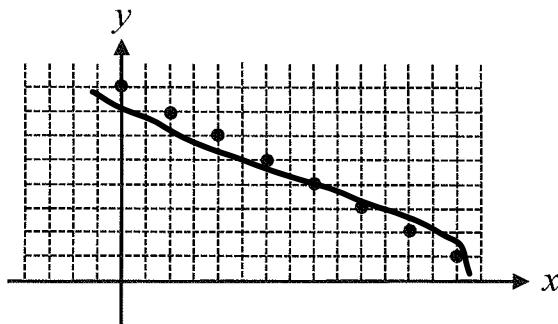
- 1) Positive correlation: If variables x and y have a strong positive linear correlation, $0.8 \leq r \leq 1$.
For x increases, y also increases.
- 2) Negative correlation: If variables x and y have a strong negative correlation, $-1 \leq r \leq -0.8$.
For x increases, y decreases.
- 3) No correlation: If there is no linear correlation or a weak linear correlation, r is close to 0.
Nonlinear relationship between two variables.
- 4) Perfect correlation: The data points all lie exactly on a straight line. $r = 1$ or $r = -1$.

The strength of the correlation:

- 1) $-1 \leq r \leq -0.8 \rightarrow$ strong negative correlation
- 2) $0.8 \leq r \leq 1 \rightarrow$ strong positive correlation
- 3) $-0.8 < r < 0.8 \rightarrow$ weak correlation

TIPS

SAT Practice

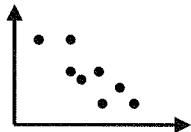


1. In the scatterplot above, which of the following best represents the correlation coefficient between two variables?

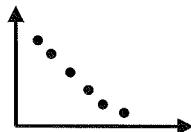
A) -1 B) -0.9 C) -0.5 D) 1

2. Which of the following graphs best represents a strong positive association between x and y?

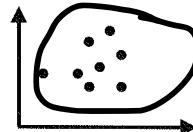
A)



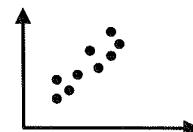
B)



C)



D)



Tip 53 Standard Deviation

Standard deviation is a measure of dispersion of a set of data values from the mean.

“The greater standard deviation, the greater the spread of data values from the mean”

- 1) A standard deviation close to 0: The data points tend to be very close to the mean.
- 2) A high standard deviation: The data points are spread out over a wider range of values.

Example:

For each data set: $\{10, 10, 10, 10, 10, 10, 10\}$ → Standard deviation is 0.

✓ $\{8, 9, 10, 10, 10, 9, 8\}$ → Standard deviation is 0.83.

$\{7, 8, 9, 10, 9, 8, 7\}$ → Standard deviation is 1.03.

TIPS

SAT Practice

Questions 1 and 2 refer to the following information.

Data Set 1: $\{1, 1, 1, 3, 3, 3, 3, 5, 6, 8\}$

Data Set 2: $\{2, 2, 3, 3, 3, 4, 4, 4, 5, 5\}$

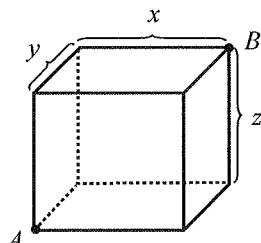
Data Set 3: $\{3, 3, 3, 3, 3, 4, 4, 4, 4, 4\}$

1. From the data sets above, which data set appears to have the largest standard deviation?
A) Data set 1 B) Data set 2 C) Data set 3 D) It cannot be determined from the information given.

2. Which data set appears to have the smallest standard deviation?
A) Data set 1 B) Data set 2 C) Data set 3 D) It cannot be determined from the information given.

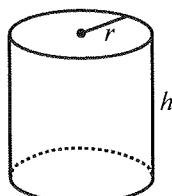
Tip 54 Solid

Rectangular solid:



▲ Surface area = $2(xy + yz + zx)$ ▲ Volume = xyz ▲ Length of diagonal $AB = \sqrt{x^2 + y^2 + z^2}$

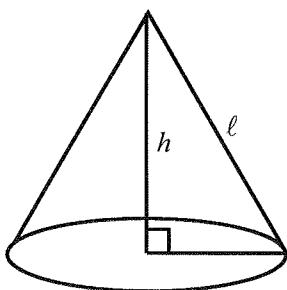
Cylinder:



▲ Surface area = $2\pi r^2 + 2\pi rh = 2\pi r(r + h)$ ▲ Volume = $\pi r^2 h$ ▲ Length of $\overline{AB} = \sqrt{(2r)^2 + h^2}$

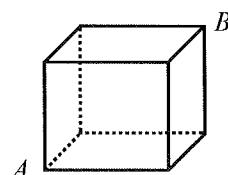
TIPS

Cone:



$$\blacktriangle \text{ Surface area} = \pi r^2 + \pi r \ell \quad \blacktriangle \text{ Lateral area} = \pi r \ell \quad \blacktriangle \text{ volume} = \frac{1}{3}(\pi r^2 h)$$

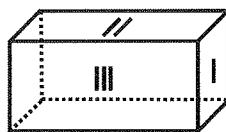
SAT Practice



1. In the figure above, if the volume of the cube is 64, what is the length of \overline{AB} (not shown)?
A) 4 B) $4\sqrt{2}$ C) $4\sqrt{3}$ D) 8

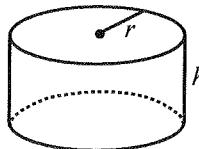
2. If the surface area of a cube is 96, what is the volume of the cube?
A) 8
B) 27
C) 64
D) 81

TIPS



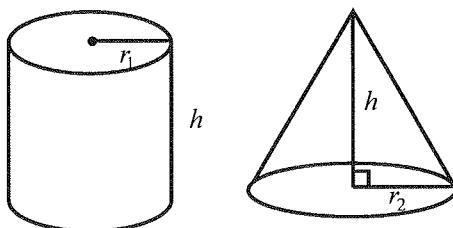
3. In the rectangular solid above, the area of region I (side) is 8, the area of region II (top) is 10, and the area of region III (front) is 20. What is the volume of the solid?

- A) 40
 - B) 60
 - C) 80
 - D) 100
-



4. The cylinder shown above has a radius of r and a height of h . If $r = h$, what is the surface area of the cylinder?

- A) $2\pi r^2$
 - B) $2\pi r^3$
 - C) $4\pi r^2$
 - D) $4\pi r^3$
-



5. In the figure above, the volume of the cylinder is equal to the volume of the cone. What is the value of $\frac{r_2}{r_1}$?

- A) $\sqrt{2}$
- B) $\sqrt{3}$
- C) 3
- D) 9