

## Homework for Day 7

Name: \_\_\_\_\_

1. Use the properties of expected value and show that for a random variable  $X$  we have

$$\text{Var}(X) = E(X^2) - [E(X)]^2$$

where  $\text{Var}(X) = E((X - \mu)^2)$  and  $\mu = E(X)$ .

$$\text{Solution: } \text{Var}(X) = E((X - \mu)^2) = E(X^2 - 2\mu X + \mu^2) =$$

2. Use the properties of expected value and the formula  $\text{Var}(X) = E(X^2) - [E(X)]^2$  and show that for any real number  $c$ , we have

a.  $\text{Var}(X + c) = \text{Var}(X)$

$$\text{Solution: } \text{Var}(X + c) = E((X + c)^2) - [E(X + c)]^2 =$$

b.  $\text{Var}(cX) = c^2 \text{Var}(X)$

- c. Let  $X$  and  $Y$  be independent random variables.

$$\text{Show } \text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y).$$

Hint: You will need to use  $E(XY) = E(X)E(Y)$ , which is true for independent random variables.

3. Suppose the random variable  $X$  has the distribution function given in the table to the right.

$x$	$f_X(x)$
-1	0.2
0	0.1
2	0.3
3	0.1
4	0.3

a. Find  $E(X)$

b. Find  $E(X^2)$

c. Find  $\text{Var}(X)$

d. Find  $\sigma_X$

4. Tiffany is tutoring in the tutoring lab. The number of students she sees in any fifteen minute period follows a Poisson distribution. She sees an average of 0.5 students in any fifteen minute period.

a. What is the probability that Tiffany will see exactly two students in the next fifteen minute period she tutors?

b. What is the probability that Tiffany will see at least two students in the next fifteen minute period?

c. What is the probability that Tiffany will see more than two students in the next fifteen minute period?

d. Suppose during finals week the average number of students Tiffany sees in a fifteen minute period jumps to 4.5 students. Now what is the probability that Tiffany will see exactly two students in the next fifteen minute period?

5. Suppose Bryson plays football and is a quarterback. Suppose his chance of completing any particular pass is 60%.
- a. What is the chance he will complete his next three passes?
  - b. What is the chance he will complete at least six of his next ten passes?
  - c. What is the chance he will complete exactly six of his next ten passes?
  - d. What is the chance he will complete less than 14 of his next 25 passes?
  - e. What is the chance that the first pass he completes will be on his second attempt?
  - f. What is the chance that the fourth pass he completes will be on his sixth attempt?
  - g. What is the chance that fourth pass he completes will be on his third attempt?
6. Suppose Matt is playing piano pieces in a recital. The chance that Matt is satisfied with his performance of any particular piece is 80%.
- a. What is the chance that Matt is satisfied with his performance of the next seven pieces he plays?
  - b. What is the chance that Matt is not satisfied with at least one of the next seven pieces he plays?

- c. What is the chance that Matt is not satisfied with exactly one of the next seven pieces he plays?
  - d. What is the chance that he is not satisfied with exactly two of the next seven pieces he plays?
  - e. What is the chance that the first piece he is not satisfied with is the fifth piece he plays?
  - f. What is the chance that the fifth piece he plays is the third piece he is not satisfied with?
  - g. What is the chance that the third piece he is not satisfied with is the fifth piece he plays?
7. Samih is at a party. He sees ten people in the room he would like to talk to. He has a 70% chance of striking up an interesting conversation with anybody he talks to.
- a. In how many ways can he plan to talk the ten people he would like to have a conversation with?
  - b. What is the chance that he will have an interesting conversation with the first three people he talks to?

Suppose Samih talks to all ten people he wants to talk with.

- c. What is the chance that he has his sixth interesting conversation with the eighth person he talks to?
- d. What is the chance that he has his first interesting conversation with the second person he talks to?

- e. What is the chance that he has an interesting conversation with at least eight of them?
- f. What is the chance that he has an interesting conversation with exactly eight of them?
- g. What is the chance that he has an interesting conversation with no more than eight of them?
8. Minh has an urn. Some are blue balls and the rest are red balls. Each ball is either stone, glass, or plastic. 20% of the balls are stone, 30% are glass, and 50% are plastic. The probability of picking a red ball, given that the ball is stone, is 90%. The probability of picking a red ball, given that the ball is glass, is 20%. The probability of picking a red ball, given that the ball is plastic, is 10%. Jacob picks a ball and it is red. What is the probability that it is also stone?
9. Emma has an urn filled with balls. The balls are either white, green, or red. The balls are either marble, crystal, or ceramic. There are 81 balls in the urn.

- a. It is equally likely to pick a white, red, or green ball. How many white balls are in the urn?
- b. The probability of picking a marble ball is the same as the probability of picking a red ball. How many marble balls are in the urn?
- c. Picking a white ball is independent of picking a marble ball. How many white marble balls are in the urn?
- d. The probability of picking a crystal white ball is 1.25 times the probability of picking a ceramic white ball. How many ceramic white balls are in the urn?
- e. There are only 13 ceramic balls in the urn. Ceramic and green are mutually exclusive. How many ceramic red balls are in the urn?
- f. The probability of picking a marble ball, given that the ball is red, is the same as the probability of picking a crystal ball, given that the ball is red. How many green crystal balls are in the urn?
- g. What is the probability of picking a crystal ball?
- j. What is the probability of picking a red ball, given that the ball is not marble?

10. Carissa is conducting an experiment to see if a drug helps to cure the common cold faster. She finds 100 people who have just caught a cold and randomly assigns half to get the drug and the other half she puts in a control group. The average length of the colds for the volunteers in the control is  $l_c$  (for length-control). The average length of the colds for the volunteers who got the drug is  $l_d$  (for length-drug). Write out the null hypothesis and the alternative hypothesis for this test. Is this a one-tailed or a two-tailed test?
11. Riley is conducting a test to see if the temperature of a room has any effect on test scores of his students. He randomly assigns half of his students to take the test in a cold room with a temperature of  $18^{\circ}\text{C}$  and the other half in a hot room with a temperature of  $31^{\circ}\text{C}$ . The average score in the cold room was  $s_c$  (for score-cold) and the average score in the hot room was  $s_h$  (for score-hot). Write out the null hypothesis and the alternative hypothesis for this test. Is this a one-tailed or a two-tailed test?

Practice quiz (not part of the homework to be graded)

1. Ariana has created a list of holiday music she likes to listen to. Forty percent of the songs are vocals without instruments (a cappella). Seventy percent of the songs include at least some vocals. A song is picked at random from her list and it turns out to include at least some vocals. What is the probability that it is also a cappella?

The distribution for a discrete random variable  $Y$  is given below.

$y$	$f(y)$
0	0.60
6	0.10
7	0.30

Find

2.  $E(Y)$

3.  $E(Y^2)$

4.  $E(10Y+3)$

Answers to practice quiz: #1.  $4/7$ ; #2.  $2.7$ ; #3.  $18.3$ ; #4.  $30$